$f(x) = \ln(x)$ $\qquad\qquad$ $\ln(1) = 0$

$$\text{Taylor} \Rightarrow \sum_{m=0}^{\infty} f^{(m)}(a)\cdot\frac{(x-a)^m}{m!}$$

$m = 3 \quad a = 1$

$$= \overset{m=0}{\cancel{\ln(1)}}^{\,0} + \overset{m=1}{\frac{1}{a}\cdot\cancel{\frac{(x-a)}{1!}}}^{\,x-1} + \overset{m=2}{\left(-\frac{1}{\cancel{a^2}_{\,1}}\right)\cdot\cancel{\frac{(x-a)^2}{2!}}}^{\,-\frac{(x-1)^2}{2}} + \overset{m=3}{\frac{\cancel{2}^{\,2}}{\cancel{a^3}_{\,1}}\cdot\cancel{\frac{(x-a)^3}{3!}}}^{\,\frac{(x-1)^3}{3}}$$

$$= 0 + (x-1) + \left(\frac{-(x-1)^2}{2}\right) + \left(\frac{2(x-1)^3}{6}\right)\dots$$

vamos analisar para $m=4$ e $m=5$ para ter certeza das razões e padrões:

$\Rightarrow m=4$

$$\boxed{\frac{2}{a^3} = 2\cdot\left(\frac{1}{a^3}\right)' \Rightarrow \left(\frac{1}{a^3}\right)' = \frac{0\cdot a^3 - 1\cdot 3a^2}{a^6} = 2\cdot\left(\frac{-3a^2}{a^6}\right) = \cancel{\frac{-6}{a^4}\cdot\frac{(x-1)^4}{4!}}^{\,-\frac{(x-1)^4}{4}}}$$

$$\boxed{\left(\frac{-6}{a^4}\right)' = -6\left(\frac{1}{a^4}\right)' \Rightarrow \frac{-1\cdot 4a^3}{a^8} = -6\cdot\left(\frac{-4a^3}{a^8}\right) = \cancel{\frac{-24}{a^5}\cdot\frac{(x-1)^5}{5!}}^{\,\frac{(x-1)^5}{5}}} \Rightarrow m=5$$

em forma reduzida temos:

$$= \overset{m=0}{0} + \overset{m=1}{(x-1)} - \overset{m=2}{\frac{1}{2}(x-1)^2} + \overset{m=3}{\frac{1}{3}(x-1)^3} - \overset{m=4}{\frac{1}{4}(x-1)^4} + \overset{m=5}{\frac{1}{5}(x-1)^5}\dots$$

a formula de Taylor reduzida pode ser então:

$$\sum^{\infty} \frac{(-1)^{m+1}}{m}\cdot(x-1)^m$$

$m=1$