Orbit Reference

March 26, 2021

This is a quick reference of various useful equations relating to astrodynamics.

1 Ellipse

An ellipse is defined as a circular shape with two axes of freedom, the long axis is called the major axis, the shorter axis is the minor axes, the difference in length of the major and minor axes determines the eccentricity.

Eccentricity ε ranges from greater than zero to less than 1 for an ellipse, zero is a circle, one is a parabola, greater than one is a hyperbola.

The equation for an ellipse in polar coordinates is:

$r = \frac{a(1-\varepsilon^2)}{1+\varepsilon\cos\theta}$

An alternative is:

$r = \frac{\ell^2}{m^2\mu}\frac{1}{1+\varepsilon\cos\theta}$ where $\ell = mvr$ and $\mu = GM$

Note: distance and velocity should be from the periapsis.

1.1 Conversion to rectangular coordinates

The equation to convert from polar to rectangular is:

$x = r\cos\theta$
$y = r\sin\theta$

1.2 Distance from centre

The minimum and maximum distance from the centre of orbit to the centre of the body (pericentre and apocentre) can be calculated with:

$r_{min} = \frac{a(1-\varepsilon^2)}{1+\varepsilon}$

$r_{max} = \frac{a(1-\varepsilon^2)}{1-\varepsilon}$

The same applies to the alternative equation, just drop the cosine θ.

1.3 Apoapsis and Periapsis

The apocentre and pericentre distance can be converted to apoapsis and periapsis (from centre of orbit to surface of body) by adding the radius of the body.

1.4 Semi-major axis

Semi-major axis (half the major axis length) can be calculated from the apocentre and pericentre distance:

$a = \frac{r_{min}r_{max}}{2}$

1.5 Semi-minor axis

The semi-minor axis (half the minor axis length) can be calculated with:

$b = \sqrt{r_{min}r_{max}}$

1.6 Linear eccentricity

Linear eccentricity is the distance between centre of the ellipse and the focal point:

$c = \sqrt{a^2 - b^2}$

1.7 Eccentricity

The eccentricity of an ellipse is:

$\frac{c}{a}$ or $\sqrt{1 - \frac{b^2}{a^2}}$ or $\frac{ApC - PeC}{ApC + PeC}$

1.8 Semi-latus rectum

The semi-latus rectum which is the line at a right angle to the focus is:

$\ell = \frac{b^2}{a}$ or $a(1 - \varepsilon^2)$

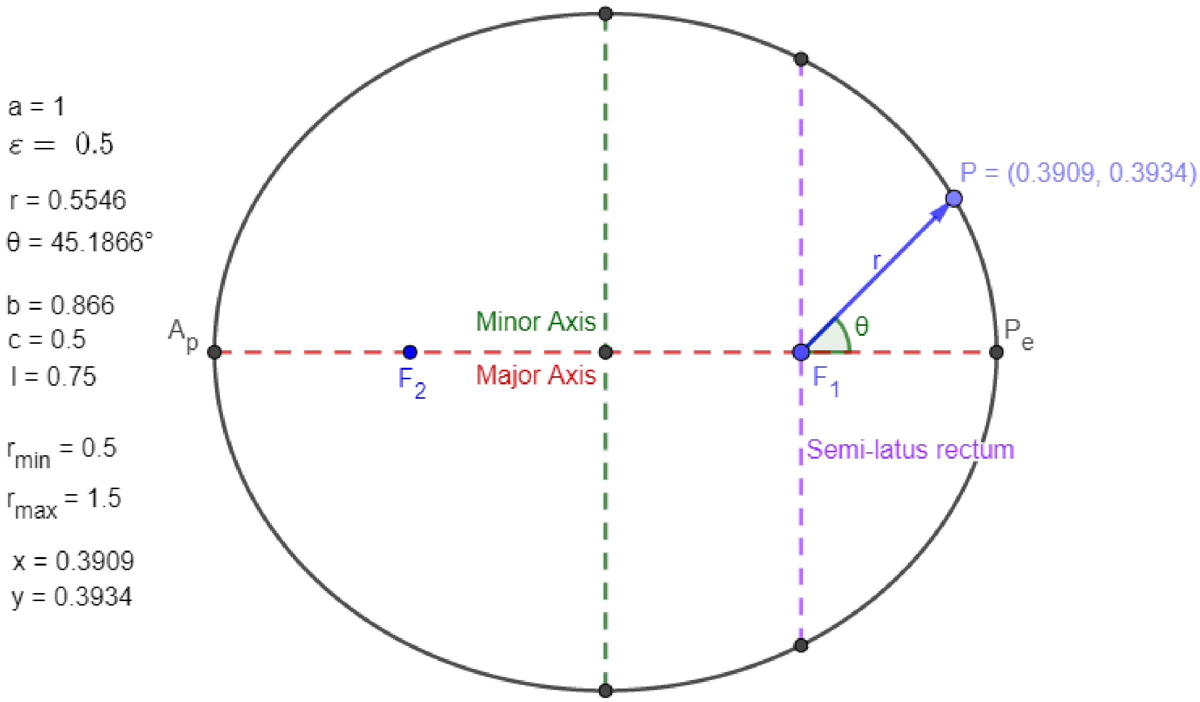

Figure 1: Ellipse

2 Time

2.1 Gregorian Calendar to Julian Day Number

$a = \frac{14-M}{12}$

$y = Y + 4800 - a$

$m = M + 12a - 3$

$J_{DN} = D + \frac{153m+2}{5} + 365y + \frac{y}{4} - \frac{y}{100} + \frac{y}{400} - 32045$

Where:
M - Integer month (1-12)
Y = Year
D = Integer day (1-31)

Note: All division is integer division which in python uses the // operator.

2.2 Julian Calendar to Julian Day Number

$a = \frac{14-M}{12}$

$y = Y + 4800 - a$

$m = M + 12a - 3$

$J_{DN} = D + \frac{153m+2}{5} + y365 + \frac{y}{4} - 32083$

Note: All division is integer division which in python uses the // operator.

2.3 Julian Date

$J_D = J_{DN} + \frac{H-12}{24} + \frac{M}{1440} + \frac{S}{86400}$

Where:
H = Hour (0-23)
M = Minute (0-59)
S = Second (0-59)

2.4 Difference in seconds between Julian dates

$\Delta t = (JD_1 - JD_2)86400$

2.5 Mean Motion

Mean motion is how many radians of the total orbit are covered per second, naturally one full orbital period should equal 2π radians.

$n = \sqrt{\frac{\mu}{a^3}}$

2.6 Mean Anomaly

$M = M_0 n \Delta t$

Where:
M_0 = Mean anomaly at epoch in degrees
n = Mean motion in degrees/s
Δt = Time in seconds relative to the epoch

2.7 Eccentric Anomaly

Eccentric anomaly is calculated from the mean anomaly, since it's a transcendental equation it requires a root finding algorithm to solve, in this case I am using Newton's method:

$E_{n+1} = \frac{E_n - \varepsilon \sin E_n - M}{1 - \varepsilon \cos E_n} - E_n$

2.8 True Anomaly

$v = 2 \cdot \text{atan2}(\sqrt{1+\varepsilon} \sin \frac{E}{2}, \sqrt{1-\varepsilon} \cos \frac{E}{2})$

3 Keplerian elements to state vectors

Transforming the Keplerian elements to state vectors is relatively easy, it requires three rotations in the z, x and z axes, the first rotation is the argument of periapsis ω, the second is inclination i and the final rotation is longitude of the ascending node Ω, the same rotations are also applied to the velocity vector.

$r = a \frac{1-\varepsilon^2}{1+\varepsilon \cos v}$

$r_p = \frac{\sqrt{\mu a}}{r}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r \cos v \\ r \sin v \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} r_p - (\sin E) \\ r_p \sqrt{1-\varepsilon^2} \cos E \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix} \begin{bmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}' \\ \dot{y}' \\ \dot{z}' \end{bmatrix} = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & \sin i & \cos i \end{bmatrix} \begin{bmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

$r = R_z(-\Omega) R_x(-i) R_Z(-\omega) o$

$\dot{r} = R_z(-\Omega) R_x(-i) R_Z(-\omega) \dot{o}$