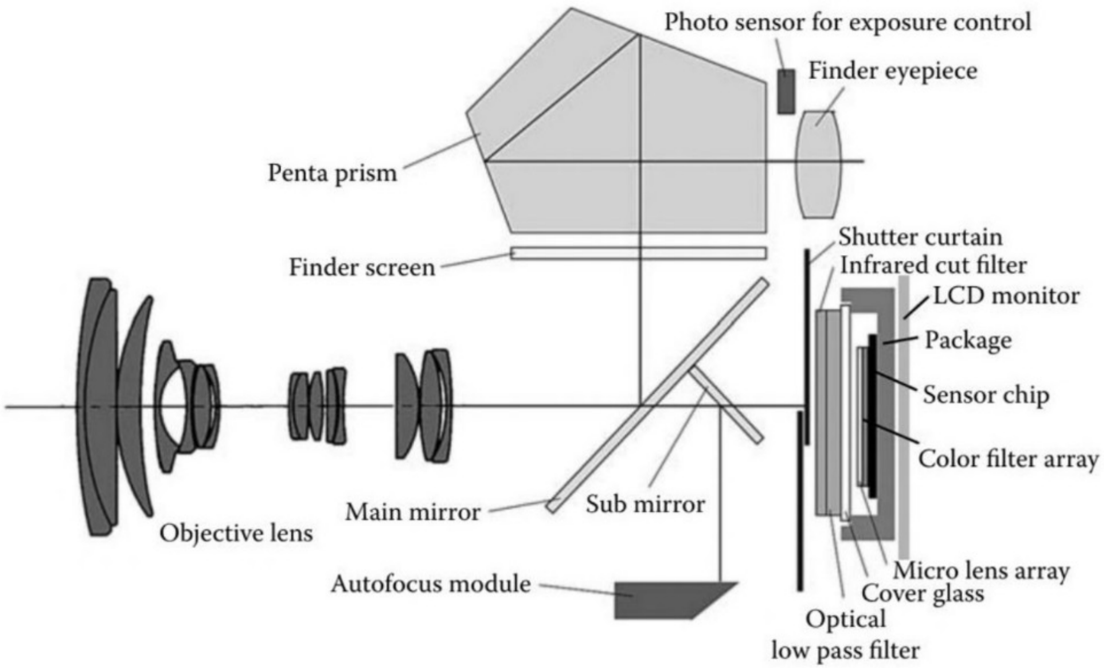
Inside a digital single lens reflex (DSLR)

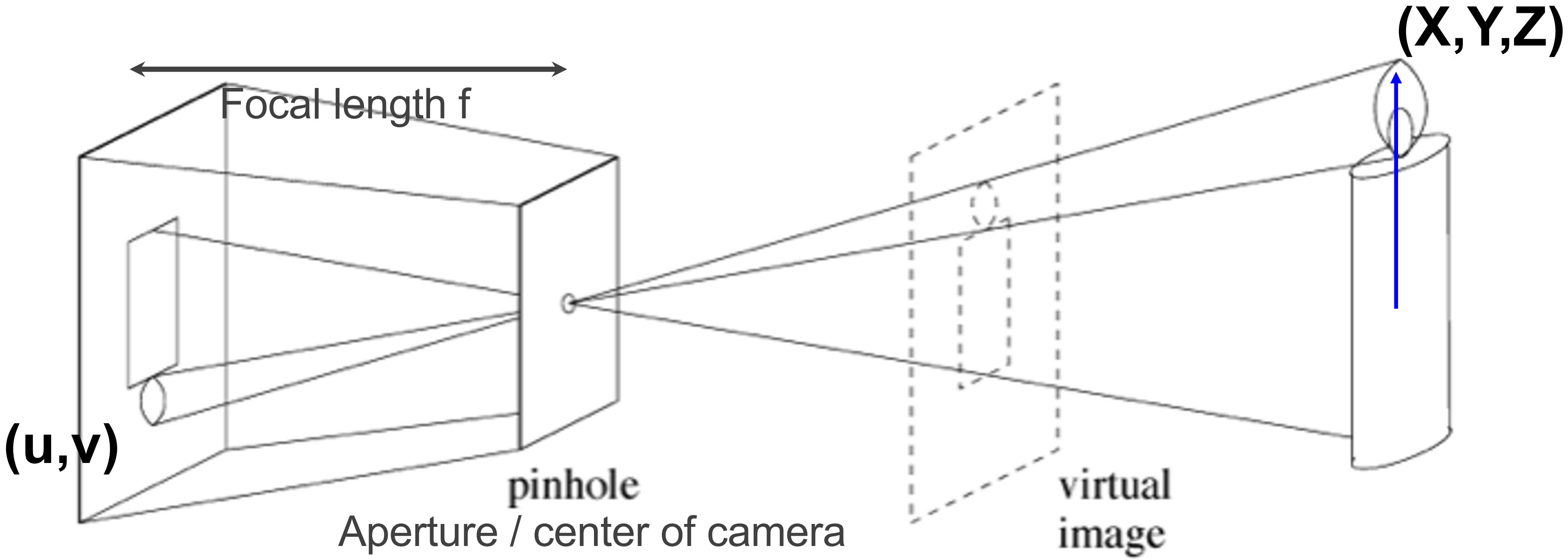


Source: Toyoda, “Digital still cameras at a glance”

(a) changing the f-number (b) changing the focal length

https://www.theverge.com/2012/1/2/2663464/camera-buyers-guide

Pinhole Camera Model



How to compute a 2D pixel location (u,v) image from a 3D location (X,Y,Z)?

Image Sensing Pipeline

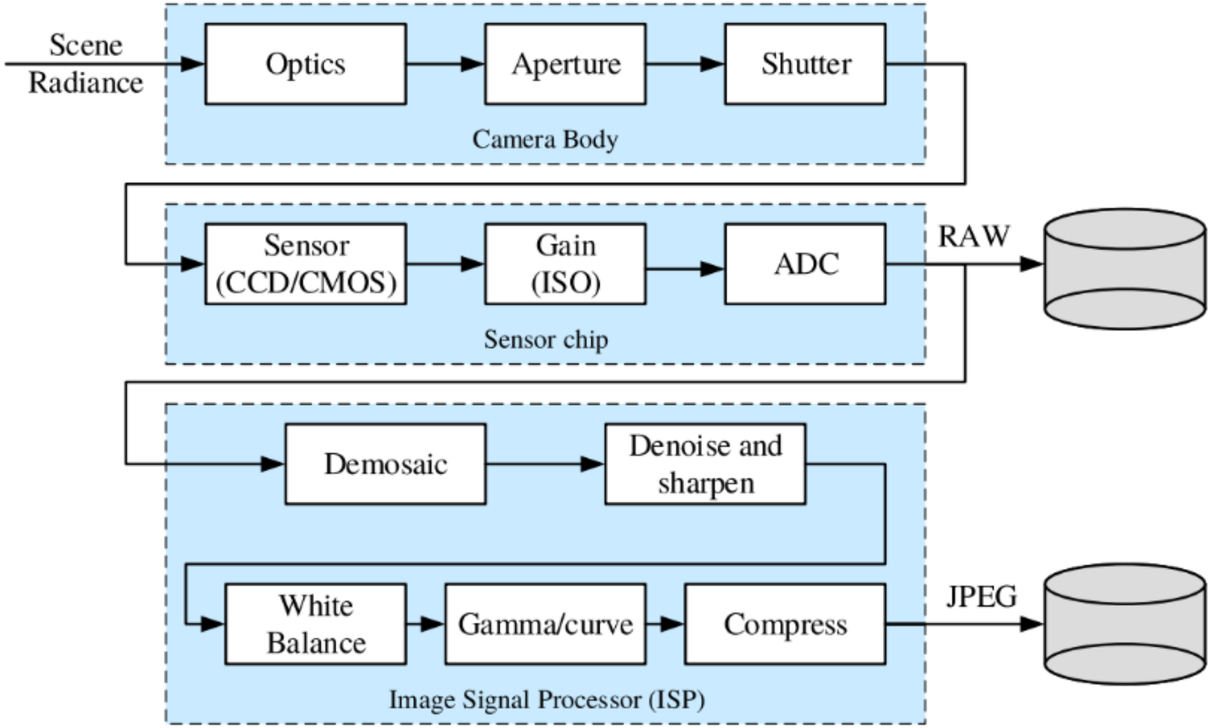
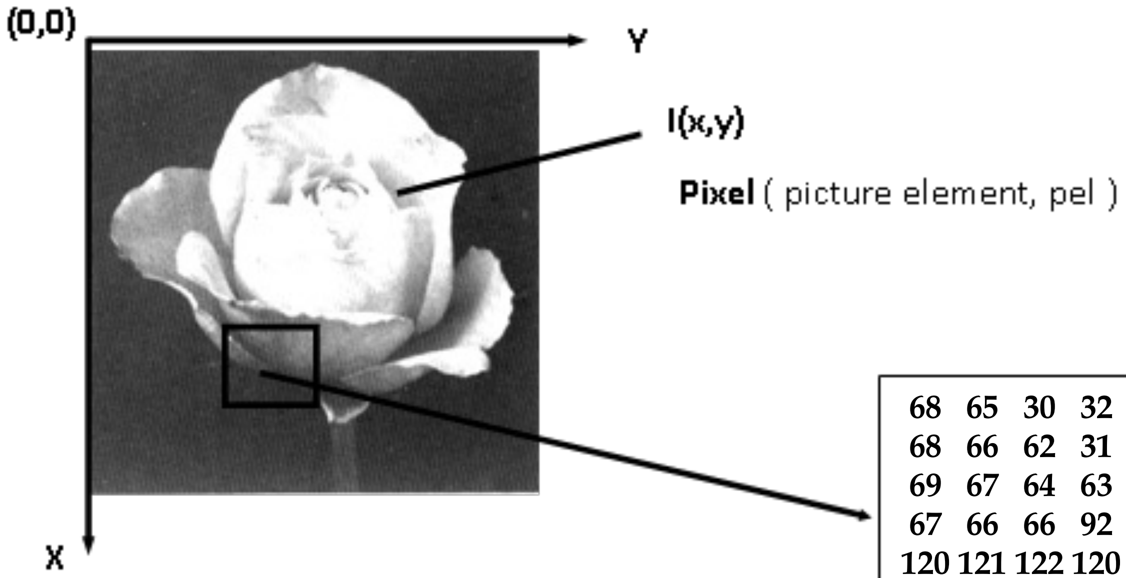


Image Representation



68	65	30	32	31	32
68	66	62	31	30	32
69	67	64	63	29	89
67	66	66	92	89	91
120	121	122	120	121	124
123	120	123	122	125	122
120	119	120	123	122	124

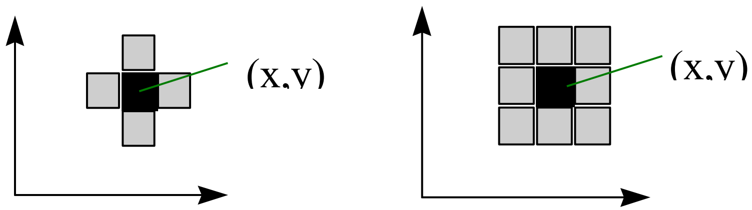
Represented as a 2-D function I(x,y)

I: brightness
x,y: spatial coordinates

Discretization of I: **quantization**
Discretization of x and y: **sampling**

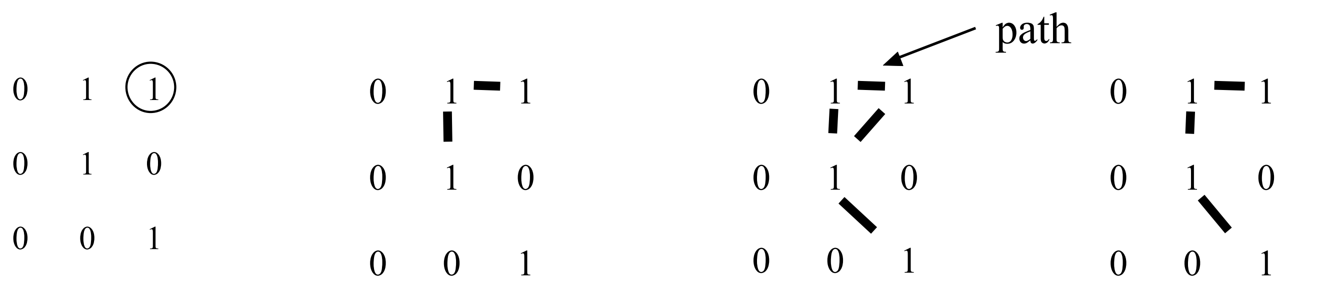
Relationship between Pixels

Neighbor



4-neighbors 8-neighbors

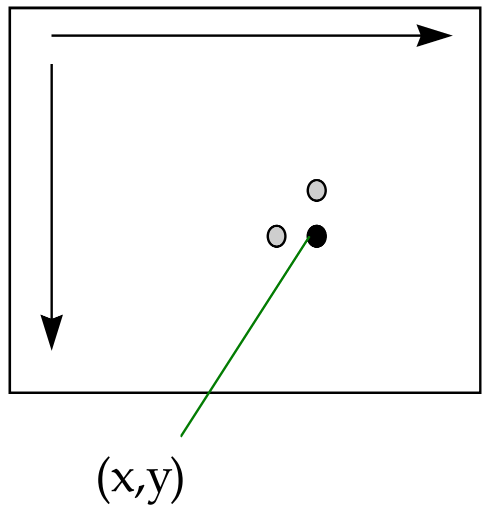
Connectivity



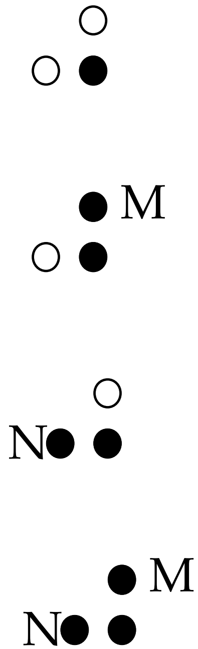
4-connectivity 8-connectivity m-connectivity

Connected Component Analysis

Scan: Left to Right, Top to Bottom



4-connectivity



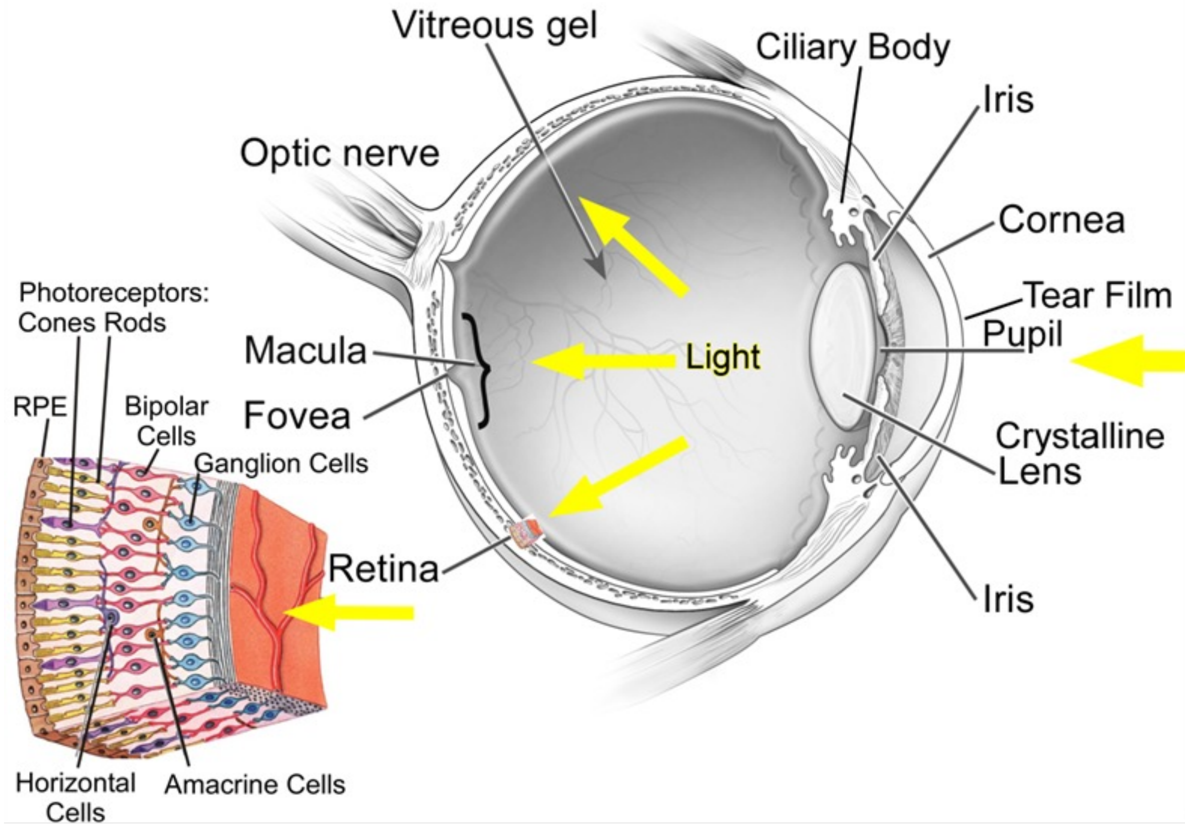
assign a new label

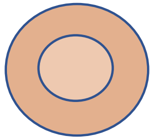
labeled M

labeled N

labeled M, make a note that
M and N are equivalent

Credit: Prof. Sheng-Jyh Wang NYCU EE


Vitreous gel
Ciliary Body
Iris
Optic nerve
Cornea
Tear Film
Pupil
Photoreceptors:
Cones Rods
Macula
Light
RPE
Bipolar
Cells
Fovea
Ganglion Cells
Crystalline
Lens
Retina
Iris
Horizontal
Cells
Amacrine Cells



Point Spread Function

$h(x, y)$

Image projected
on receptor layer

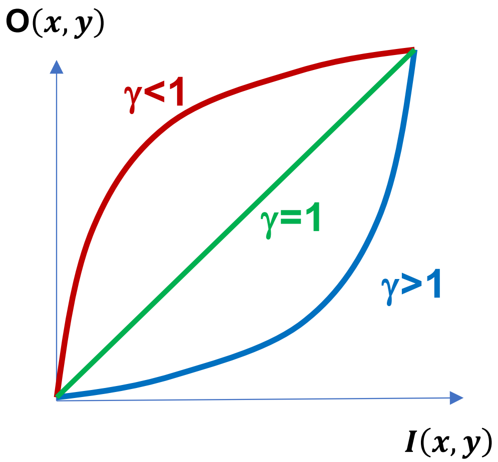
$f(x, y)$

Linear Shift-Invariant
System

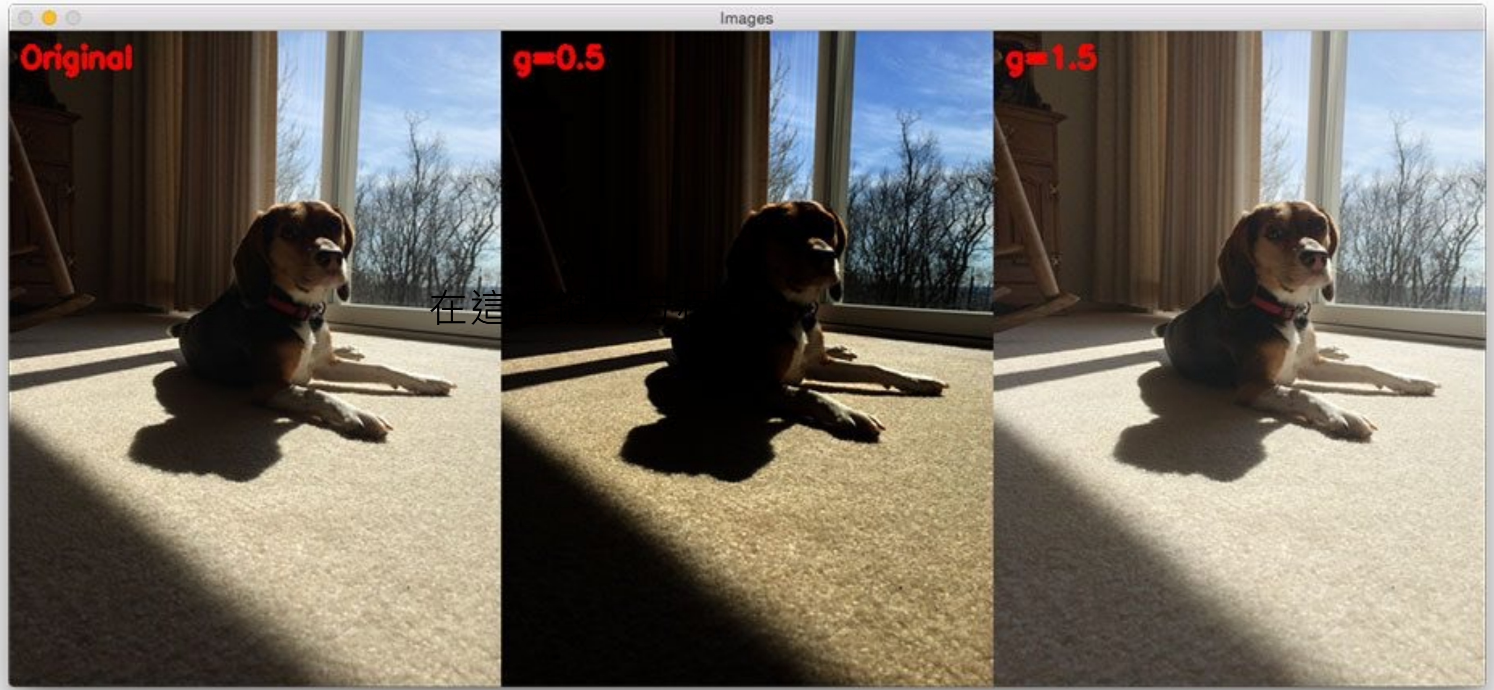
Image on ganglion layer

$f(x, y) * h(x, y)$

$*$: convolution operation

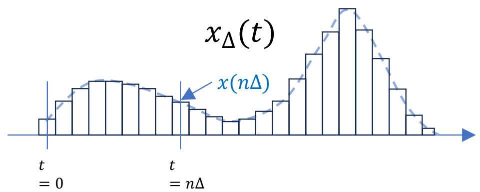


$$g = \frac{1}{\gamma}$$



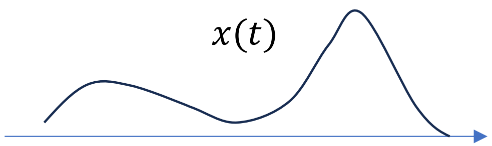
https://pyimagesearch.com/2015/10/05/opencv-gamma-correction/

Superposition

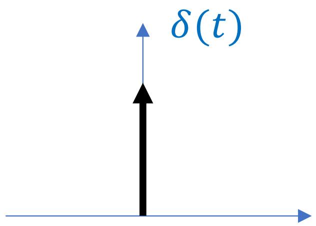


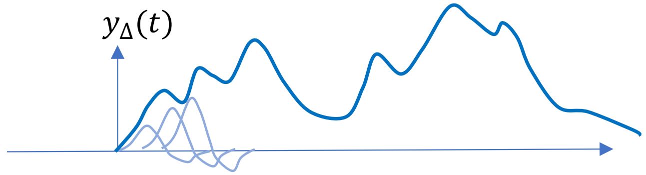
$$x_\Delta(t) = \sum_{n=-\infty}^{\infty} x(n\Delta)\delta_\Delta(t-n\Delta)\Delta$$

$\Delta \rightarrow 0$



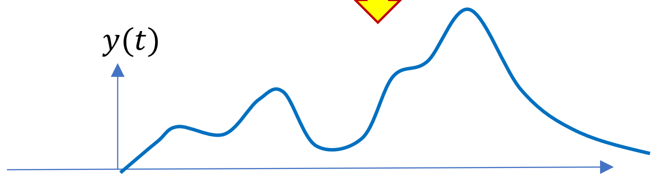
$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$





$$y_\Delta(t) = \sum_{n=-\infty}^{\infty} x(n\Delta)h_\Delta(t-n\Delta)\Delta$$

$\Delta \rightarrow 0$

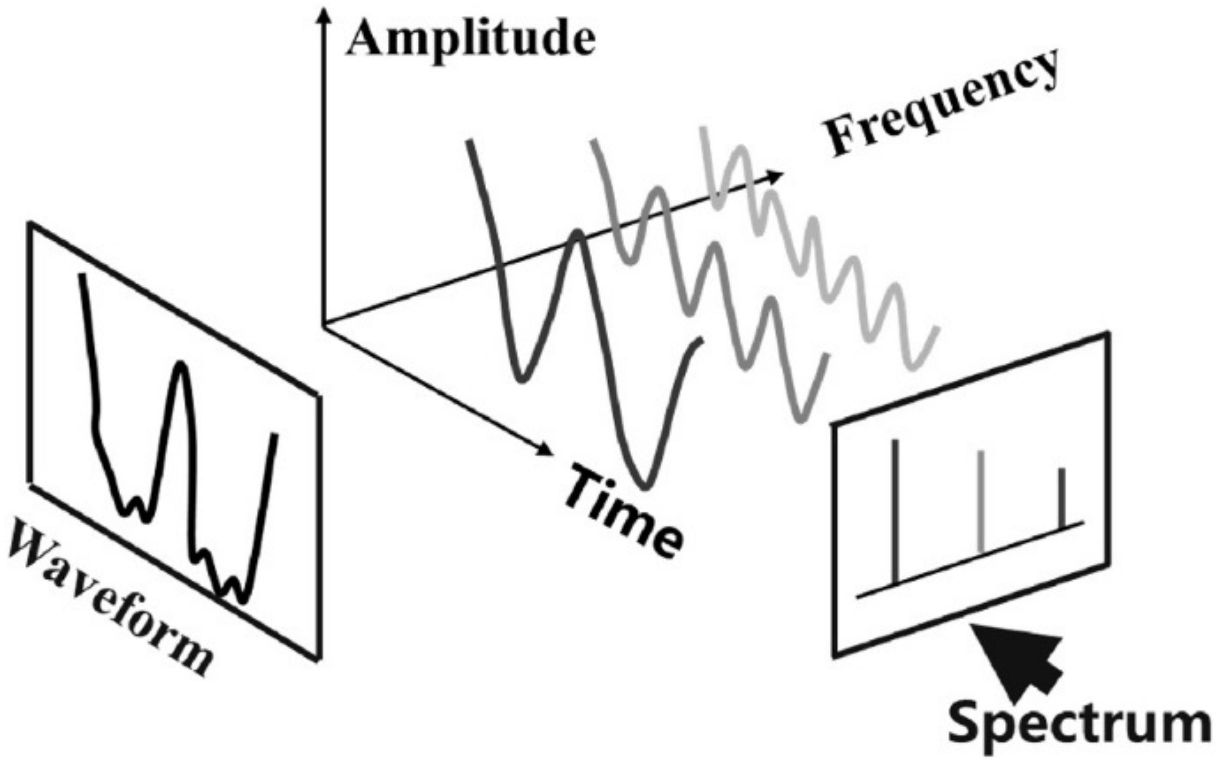


$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$ Convolution

where $h(t) = T\{\delta(t)\}$

Impulse response

Credit: Prof. Sheng-Jyh Wang NYCU EE


Amplitude
Frequency
Time
Waveform
Spectrum

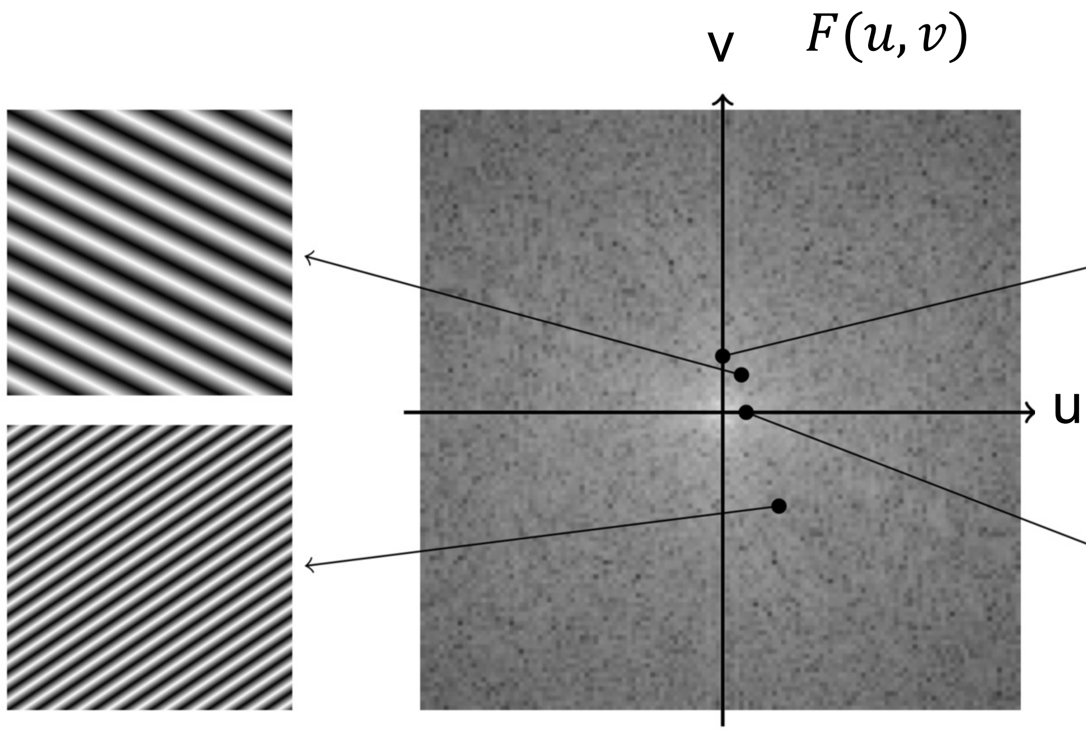
Frequency-Domain Analysis (1/3)

$$e^{j2\pi ft} = \cos(2\pi ft) + j\sin(2\pi ft)$$

$$x(t) = e^{j2\pi ft} \longrightarrow \boxed{T} \longrightarrow y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

$$\begin{aligned}
&= \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau \\
&= \int_{-\infty}^{\infty} e^{j2\pi f(t-\tau)}h(\tau)d\tau \\
&= e^{j2\pi ft}\int_{-\infty}^{\infty} e^{-j2\pi f\tau}h(\tau)d\tau \\
&\equiv H(f)e^{j2\pi ft}
\end{aligned}$$

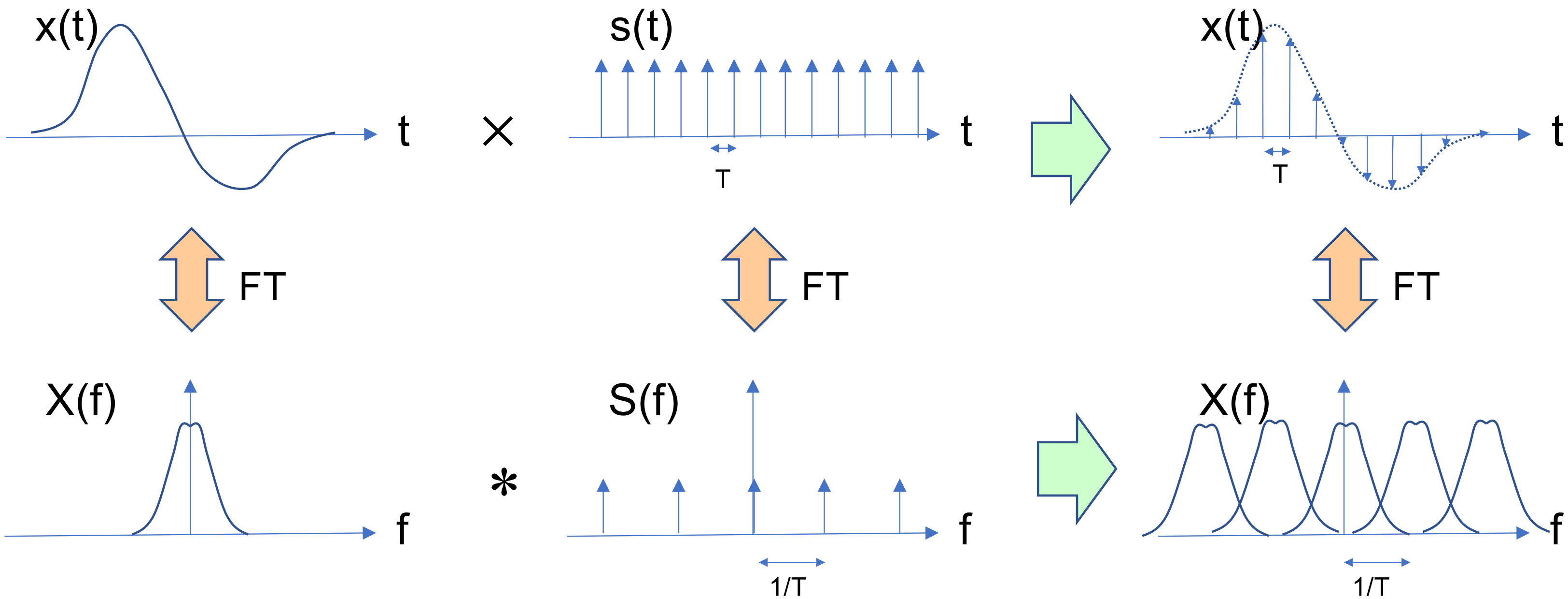
$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)\, e^{j2\pi(ux+vy)} du dv$$



https://commons.wikimedia.org/wiki/File:2D_Fourier_Transform_and_Base_Images.png

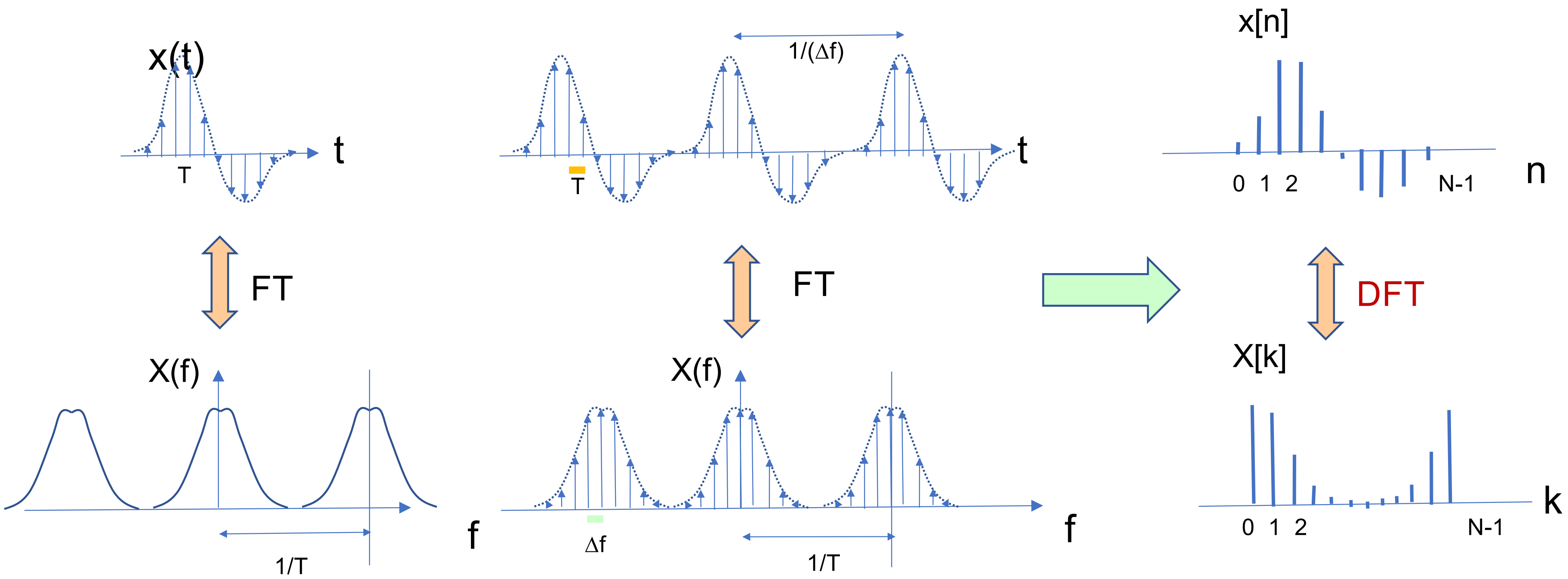
Sampling

Whittaker-Shannon Sampling Theorem

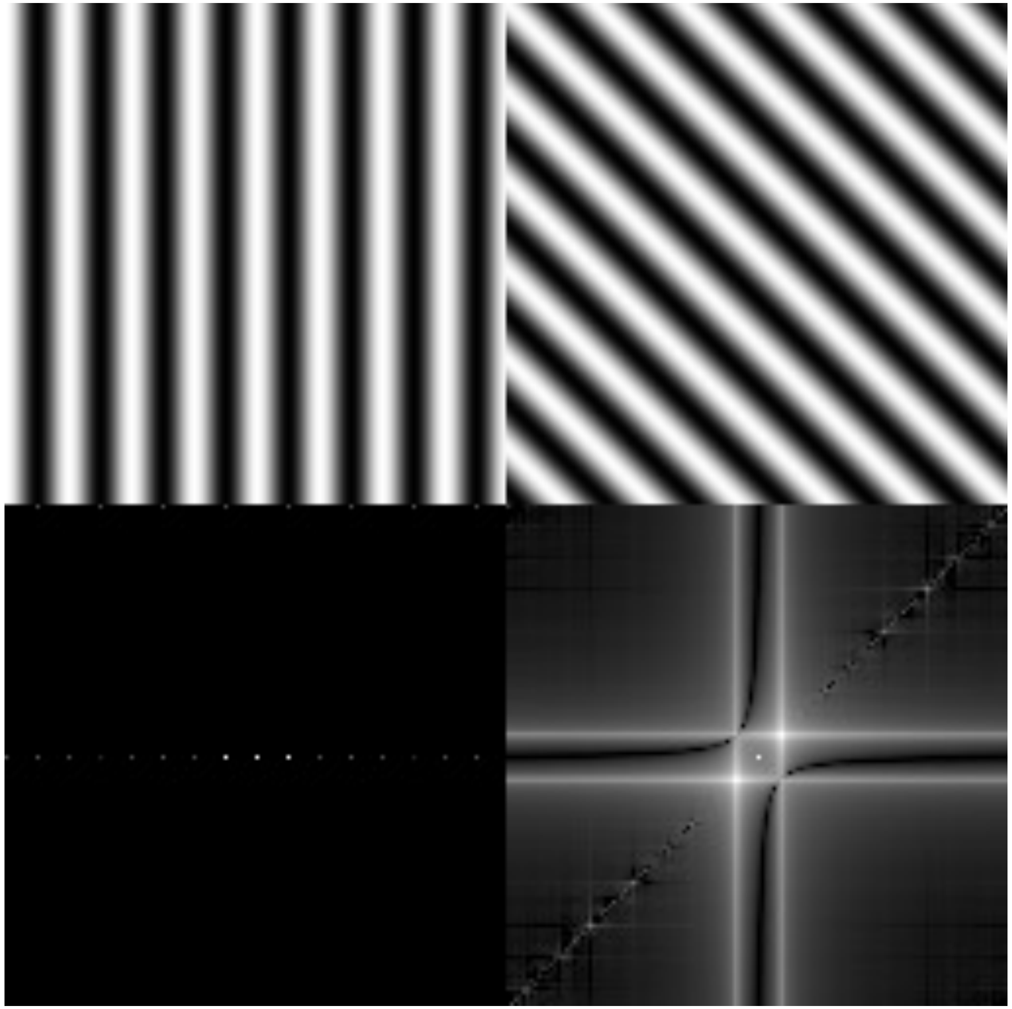


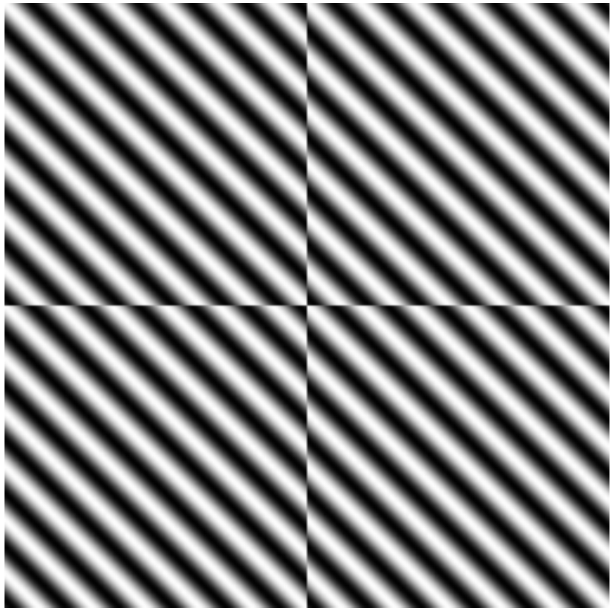
Credit: Prof. Sheng-Jyh Wang NYCU EE

Discrete Fourier Transform (1/2)

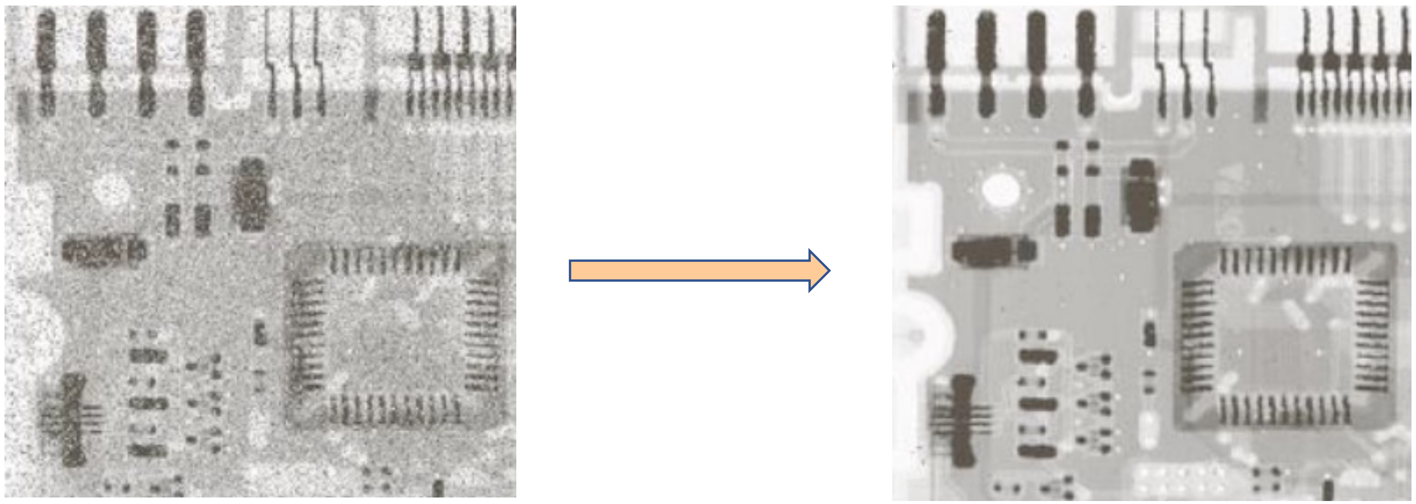


Credit: Prof. Sheng-Jyh Wang NYCU EE





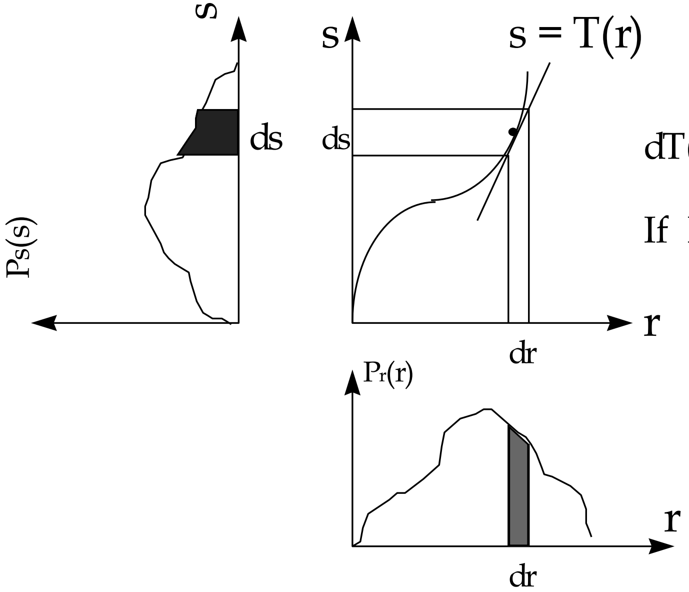
Ref: http://www.cs.unm.edu/~brayer/vision/fourier.html



Histogram Equalization (2/3)

(a) $T(r)$ is a monotonic[†] increasing function in the interval $0 \le r \le L-1$; and

(b) $0 \le T(r) \le L-1$ for $0 \le r \le L-1$.



$$P_r(r)\,dr = P_s(s)\,ds$$

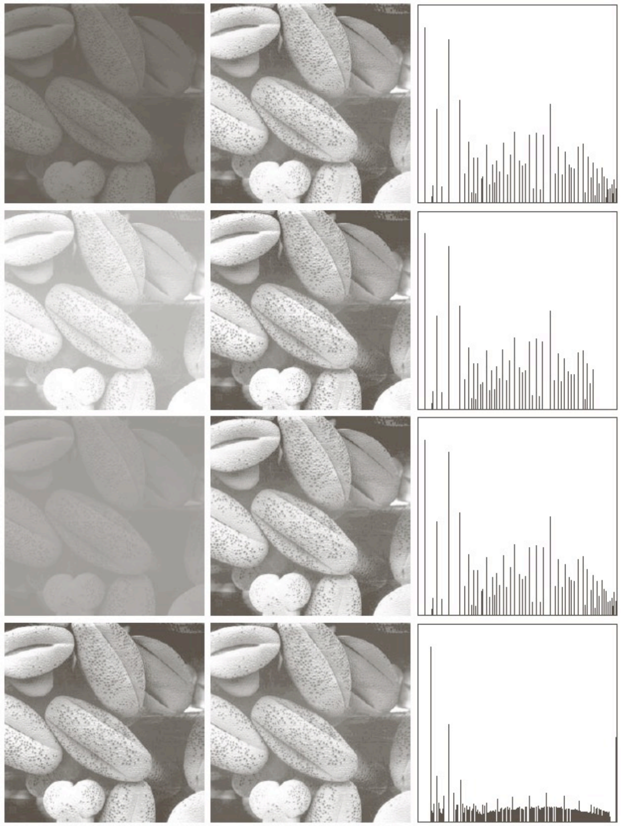
$$dT(r)/dr = ds/dr = P_r(r)/P_s(s)$$

If $P_s(s)$ = constant = K
$= 1/(\mathbf{r}_{max} - \mathbf{r}_{min})$

$$\mathbf{T(r) = \frac{1}{K}\int_{r_{min}}^{r} P_r(r)\,dr + r_{min}}$$

for $\mathbf{r}_{min} \le \mathbf{r} \le \mathbf{r}_{max}$.

Histogram Equalization (3/3)



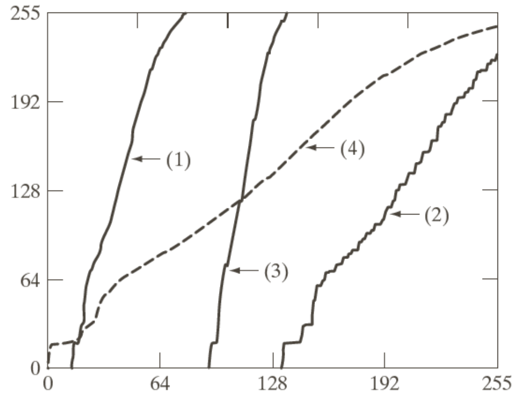
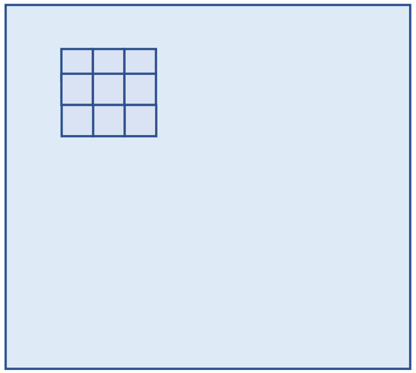


FIGURE 3.21 Transformation functions for histogram equalization. Transformations (1) through (4) were obtained from the histograms of the images (from top to bottom) in the left column of Fig. 3.20 using Eq. (3.3-8).

Local Averaging



$$\hat{I}(x,y) = \frac{1}{9}\sum_{i=-1}^{1}\sum_{j=-1}^{1} I(x+i,y+j)$$

where $I(x,y) = S(x,y) + N(x,y)$

signal noise

$$E[\hat{I}(x,y)] = \frac{1}{9}\sum_{i=-1}^{1}\sum_{j=-1}^{1} S(x+i,y+j) + E\left[\frac{1}{9}\sum_{i=-1}^{1}\sum_{j=-1}^{1} N(x+i,y+j)\right]$$
$$= \frac{1}{9}\sum_{i=-1}^{1}\sum_{j=-1}^{1} S(x+i,y+j)$$

$$\begin{aligned}\mathrm{Var}(X) &= \mathrm{E}\left[(X-\mathrm{E}[X])^2\right] \\ &= \mathrm{E}\left[X^2 - 2X\,\mathrm{E}[X] + (\mathrm{E}[X])^2\right] \\ &= \mathrm{E}\left[X^2\right] - 2\,\mathrm{E}[X]\,\mathrm{E}[X] + (\mathrm{E}[X])^2 \\ &= \mathrm{E}\left[X^2\right] - (\mathrm{E}[X])^2\end{aligned}$$

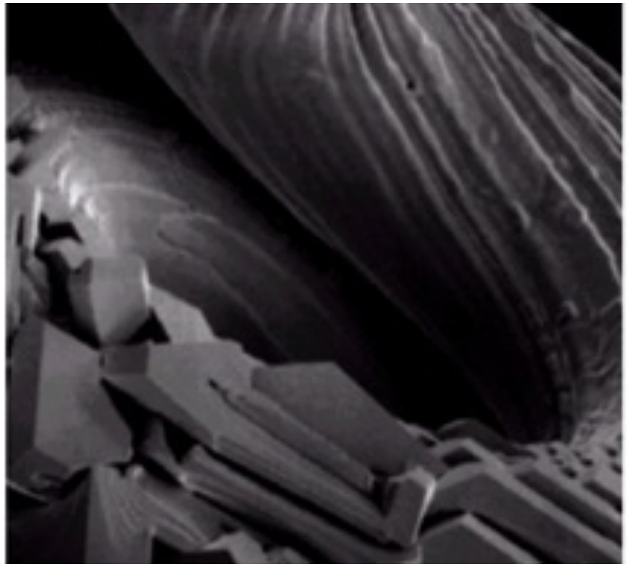
$$Var[\hat{I}(x,y)] = E[(\hat{I}(x,y) - E(\hat{I}(x,y))^2] = E\left[\{\frac{1}{9}\sum_{i=-1}^{1}\sum_{j=-1}^{1} N(x+i,y+j)\}^2\right]$$

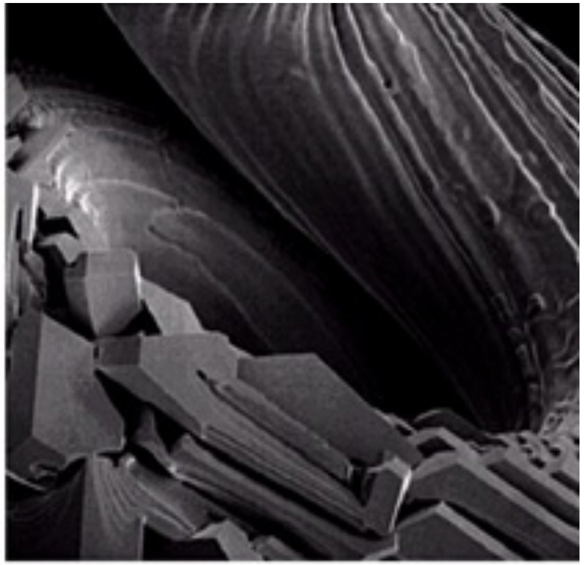
$$= \frac{1}{81}\{\sum_{i=-1}^{1}\sum_{j=-1}^{1} E[(N(x+i,y+j))^2] + cross\ terms\} = \frac{1}{81}\{\sum_{i=-1}^{1}\sum_{j=-1}^{1} Var[N(x+i,y+j)]\} = \frac{1}{9}Var[N(x,y)]$$

Credit: Prof. Sheng-Jyh Wang NYCU EE

Sharpness Enhancement

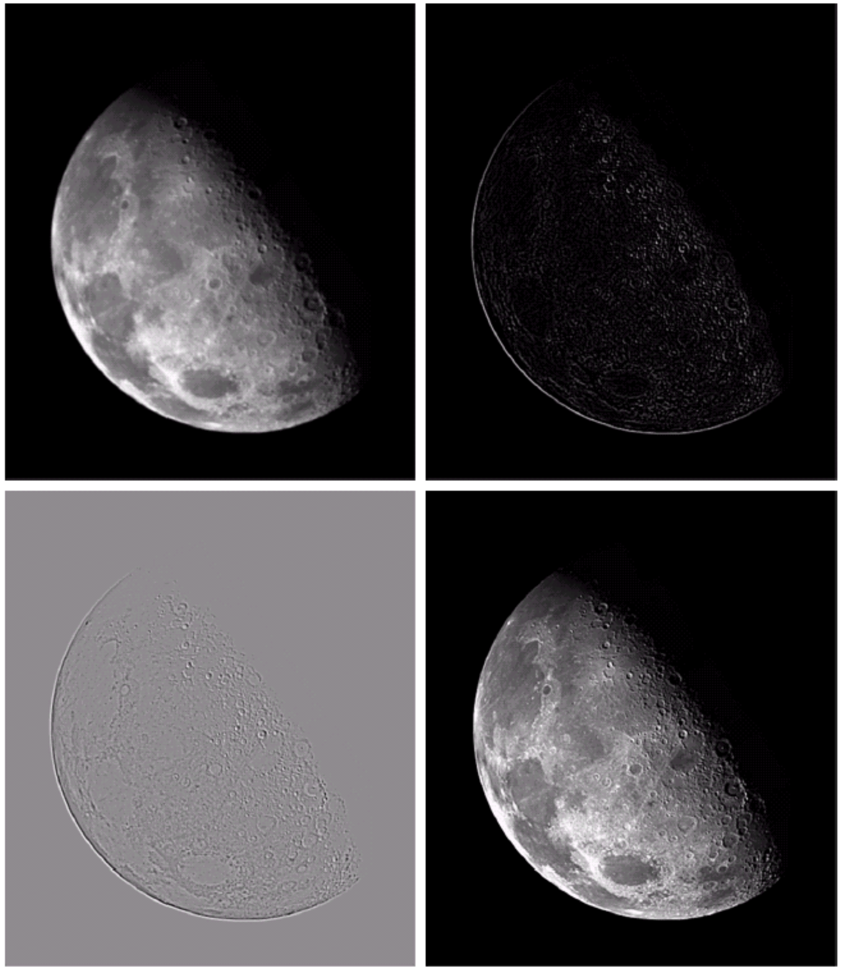
- Purpose: highlight or enhance fine detail.





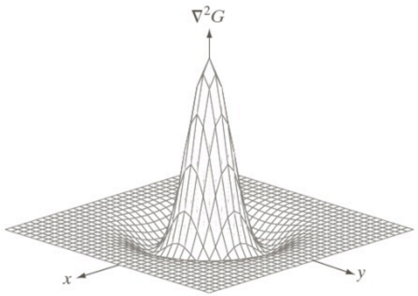
a b
c d

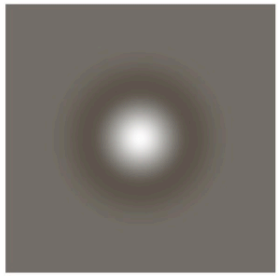
FIGURE 3.40
(a) Image of the North Pole of the moon.
(b) Laplacian-filtered image.
(c) Laplacian image scaled for display purposes.
(d) Image enhanced by using Eq. (3.7-5).
(Original image courtesy of NASA.)

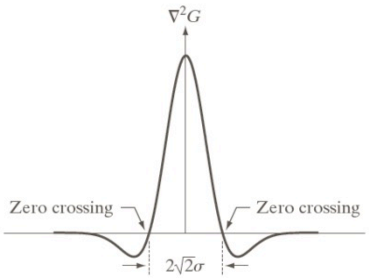


LOG (Laplacian of Gaussian) Operator

$$\nabla^2 G(x,y) = \frac{-1}{\pi\sigma^4}\left(1 - \frac{x^2+y^2}{2\sigma^2}\right)e^{-\frac{x^2+y^2}{2\sigma^2}}$$







0	0	−1	0	0
0	−1	−2	−1	0
−1	−2	16	−2	−1
0	−1	−2	−1	0
0	0	−1	0	0

- **DOG (Difference of Gaussian) Operator**

Approximate $\nabla^2 G(x,y) = \dfrac{-1}{\pi\sigma^4}(1-\dfrac{x^2+y^2}{2\sigma^2})e^{-\frac{x^2+y^2}{2\sigma^2}}$

with $h(x,y) = \dfrac{1}{2\pi\sigma_1^2}e^{-\frac{x^2+y^2}{2\sigma_1^2}} - \dfrac{1}{2\pi\sigma_2^2}e^{-\frac{x^2+y^2}{2\sigma_2^2}}$

where $\dfrac{\sigma_2}{\sigma_1} \approx 1.6$ and $\sigma^2 = \dfrac{\sigma_1^2\sigma_2^2}{\sigma_1^2-\sigma_2^2}\ln[\dfrac{\sigma_1^2}{\sigma_2^2}]$

Ideal Low-pass Filters

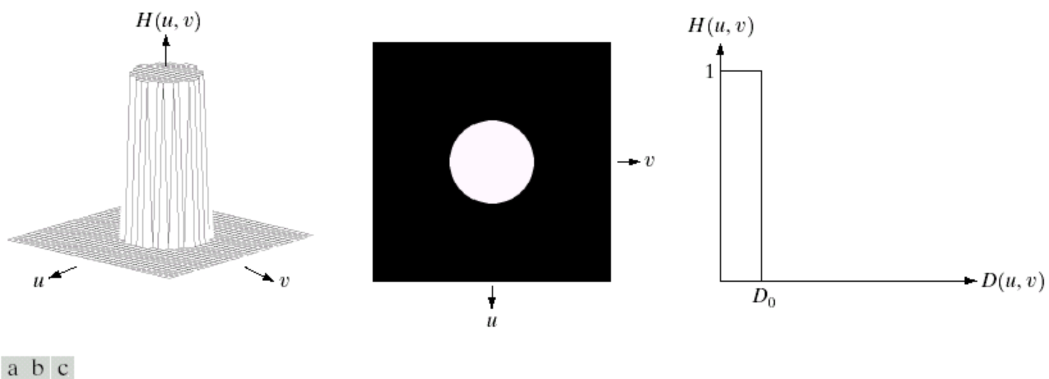
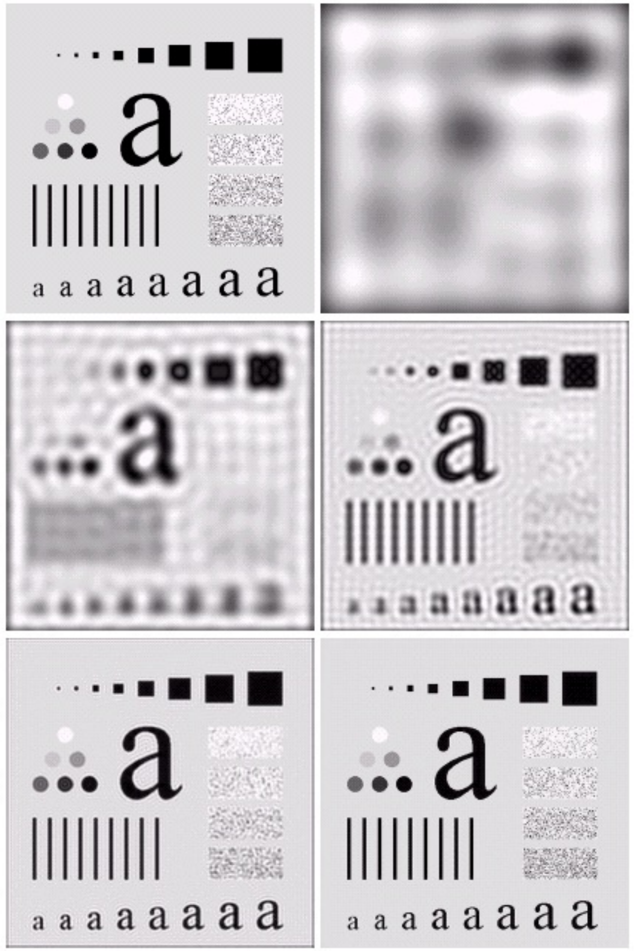
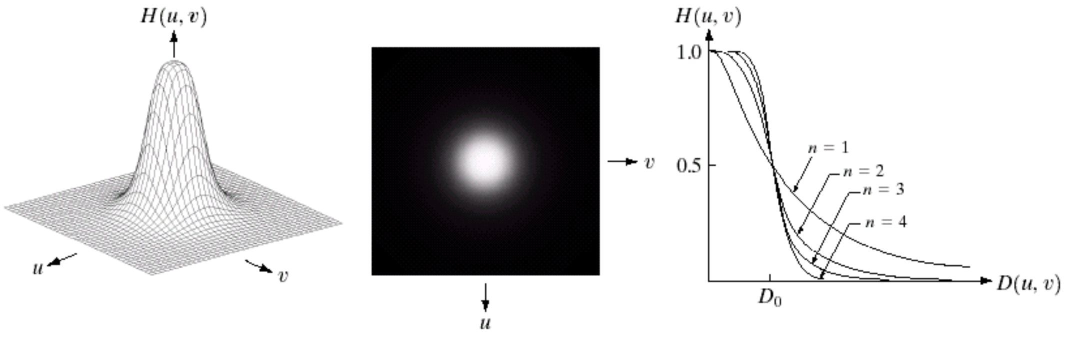


FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.



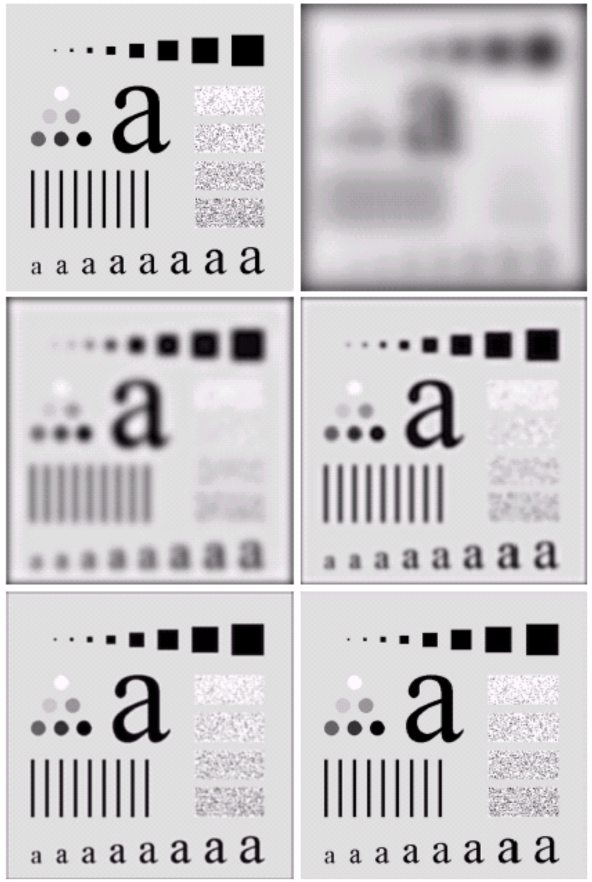
Butterworth Low-pass Filters

$$H(u,v) = \frac{1}{1+[D(u,v)/D_0]^{2n}}$$



a b c

FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.



a b c d e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Imaging Degradation

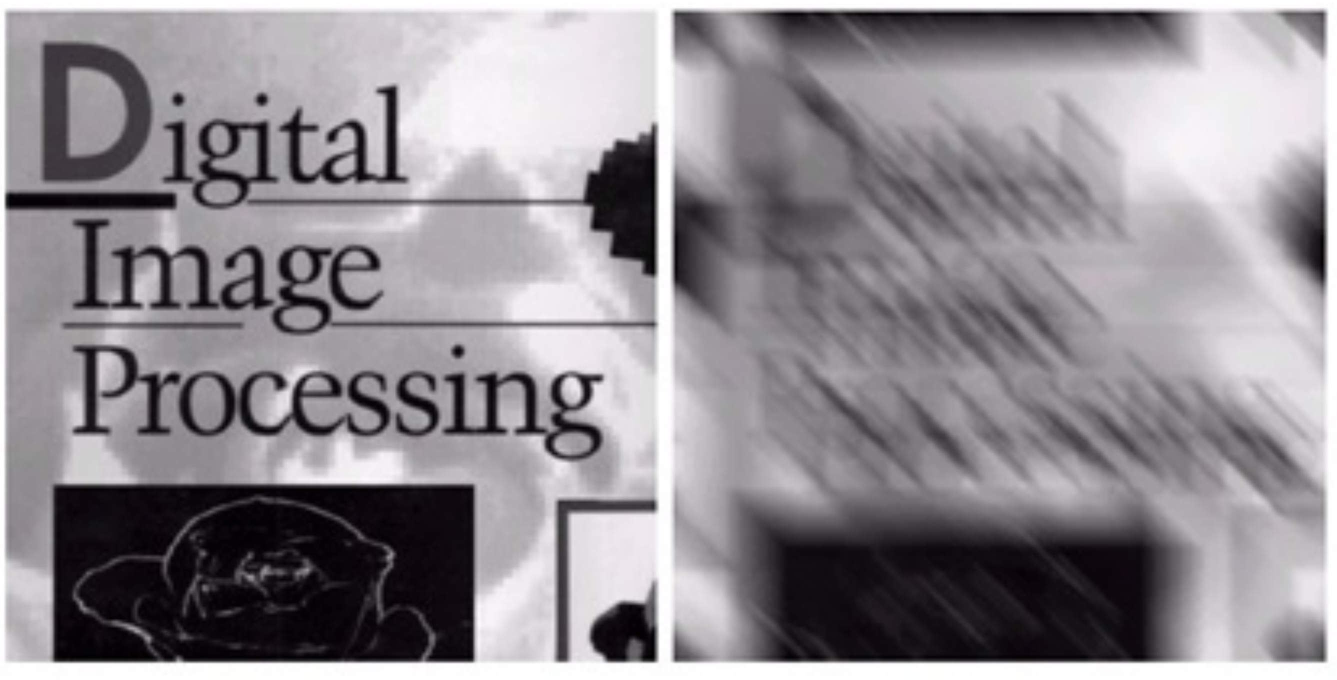
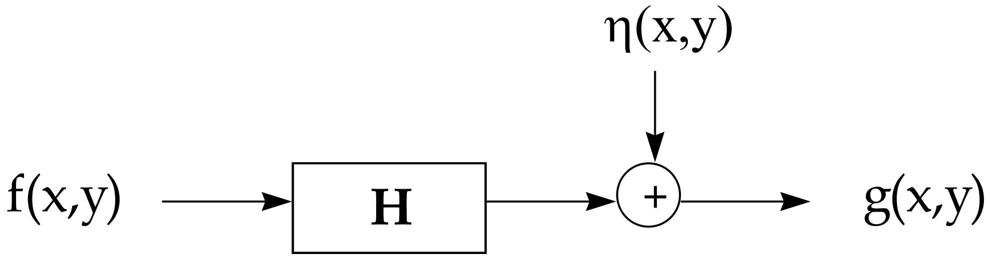


Image Restoration

Spatial Domain: $g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$

Frequency Domain: $G(u,v) = H(u,v)F(u,v) + N(u,v)$



Restoration: given g(x,y) and H, try to recover f(x,y).

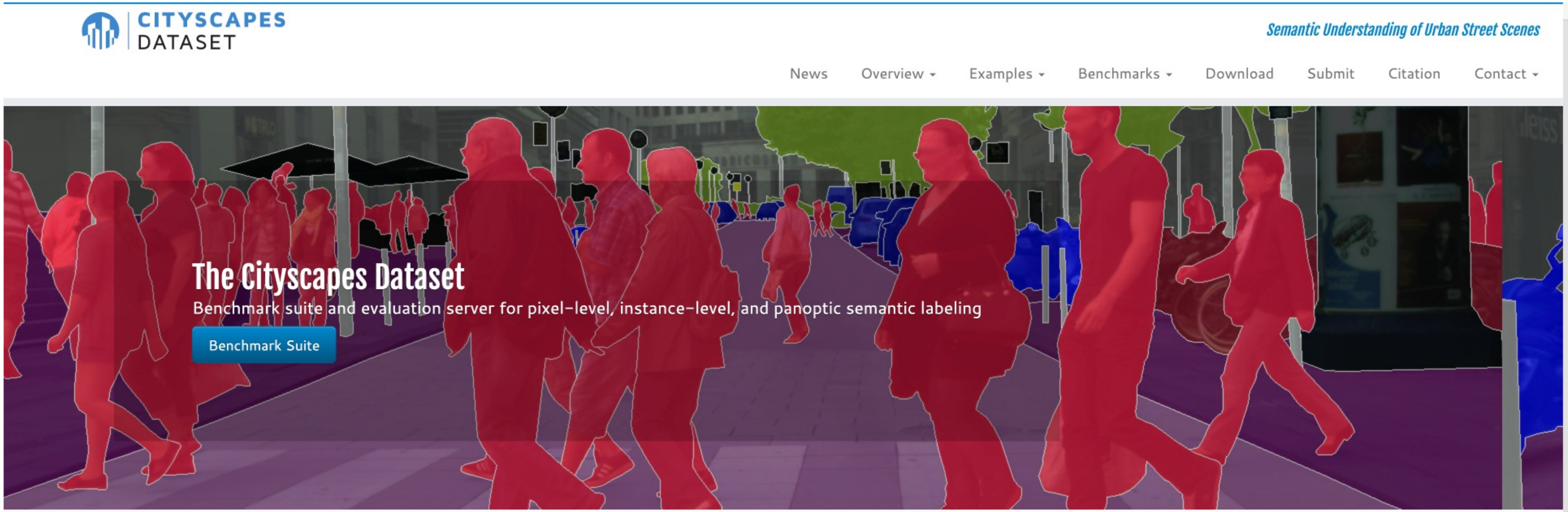
Remark: **Geometric Mean Filter**

$$\hat{F}(u,v)=\left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha}\left[\frac{H^*(u,v)}{|H(u,v)|^2+\beta\left[\frac{S_\eta(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha}G(u,v)$$

$\alpha = 1 \Rightarrow$ inverse filter
$\alpha = 0 \Rightarrow$ parametric Wiener filter
$\alpha = 0 \;\; \beta = 1 \Rightarrow$ standard Wiener filter
$\alpha = 1/2 \;\; \beta = 1 \Rightarrow$ spectrum equalization filter

a b c

FIGURE 5.28 Comparison of inverse- and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.


CITYSCAPES
DATASET
Semantic Understanding of Urban Street Scenes
News
Overview
Examples
Benchmarks
Download
Submit
Citation
Contact
The Cityscapes Dataset
Benchmark suite and evaluation server for pixel-level, instance-level, and panoptic semantic labeling
Benchmark Suite

Edge-based Segmentation

Figure 3.18
Hockenbury/Nolan, *Discovering Psychology*,
7e, © 2016 Worth Publishers

image

↓

Edge Detection — **edge detector**

- Template matching
- Statistical methods
- Edge fitting
- Derivative-based methods

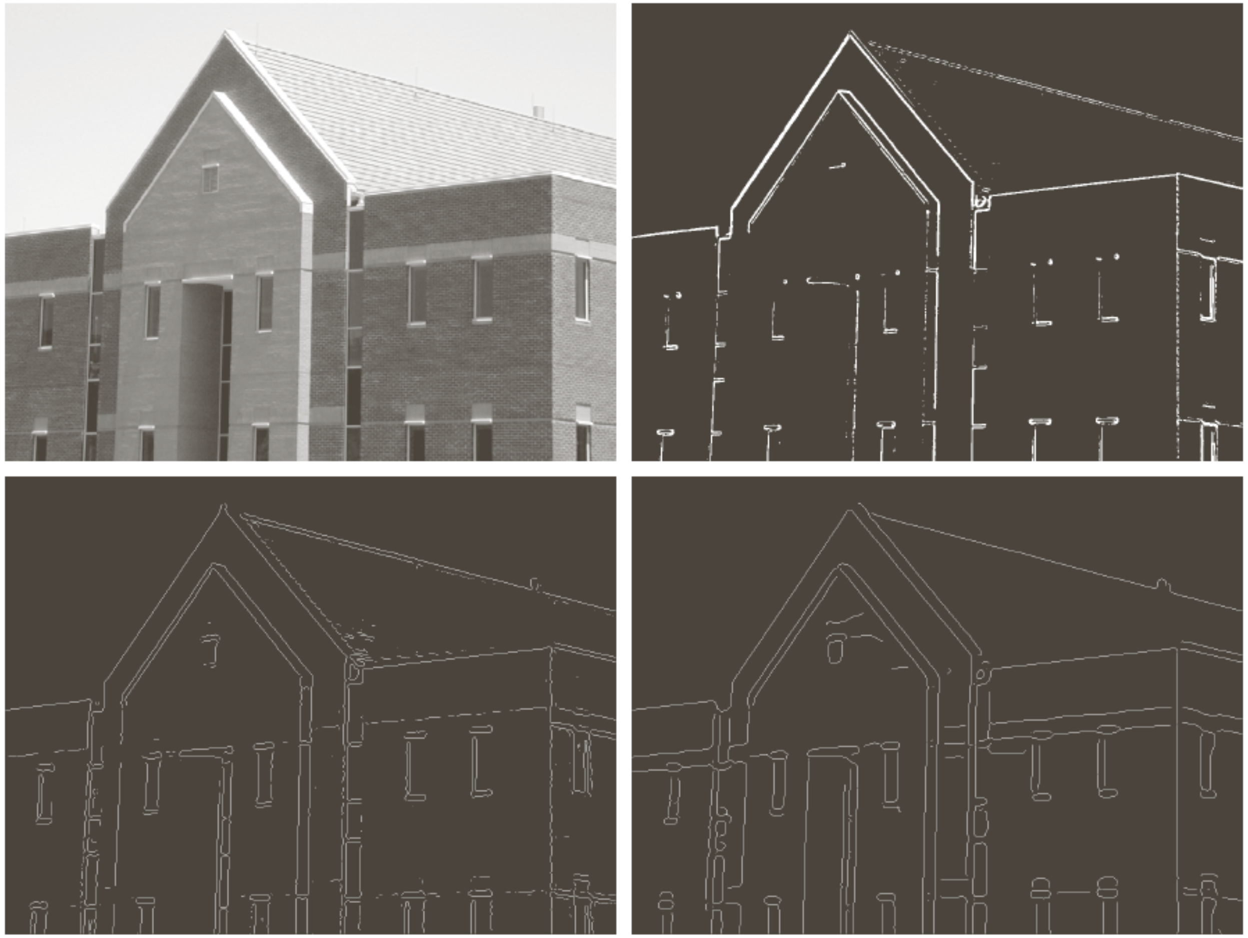
↓ *edge elements (edgels)*

Edge Linking — **edge linker**

- Local approach
- Global approach

↓

boundaries

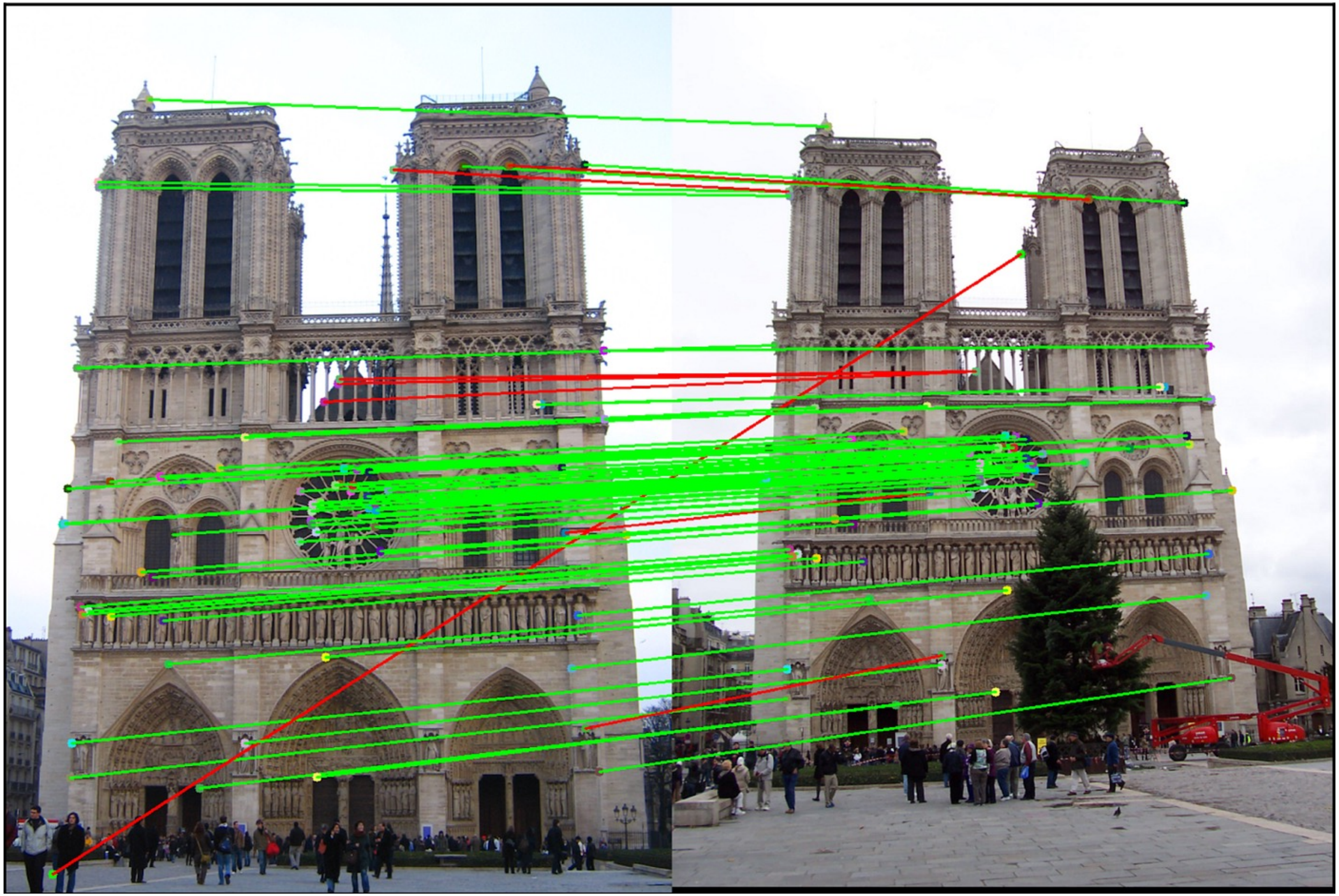


a b
c d

FIGURE 10.25 (a) Original image of size 834 × 1114 pixels, with intensity values scaled to the range [0, 1]. (b) Thresholded gradient of smoothed image. (c) Image obtained using the Marr-Hildreth algorithm. (d) Image obtained using the Canny algorithm. Note the significant improvement of the Canny image compared to the other two.

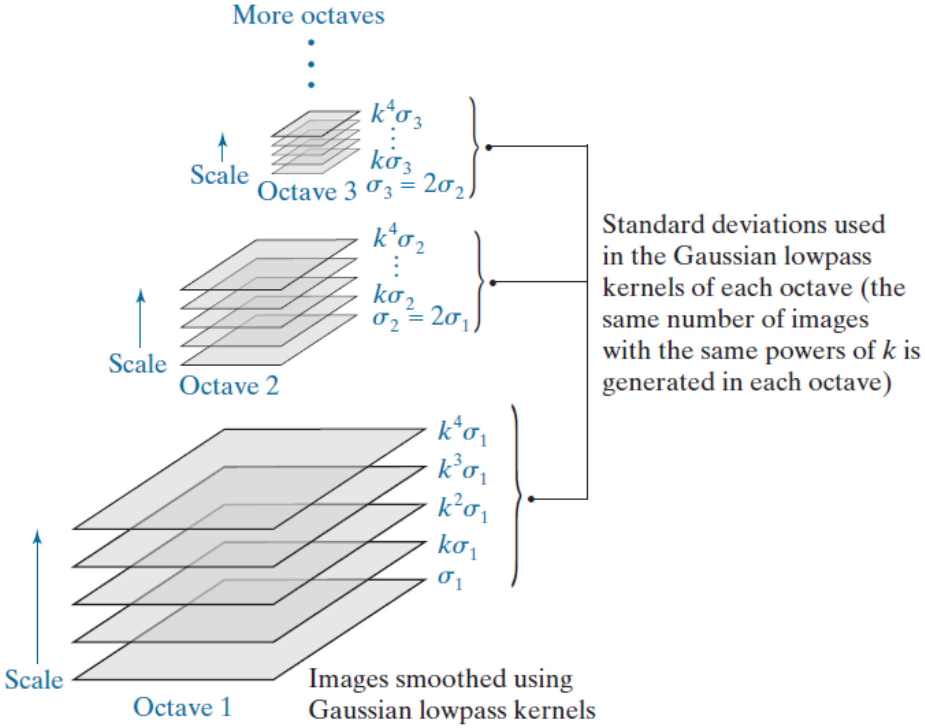
- Noise reduction
- Enhancement
- Restoration
- Segmentation

- Classification/Detection
- Regression

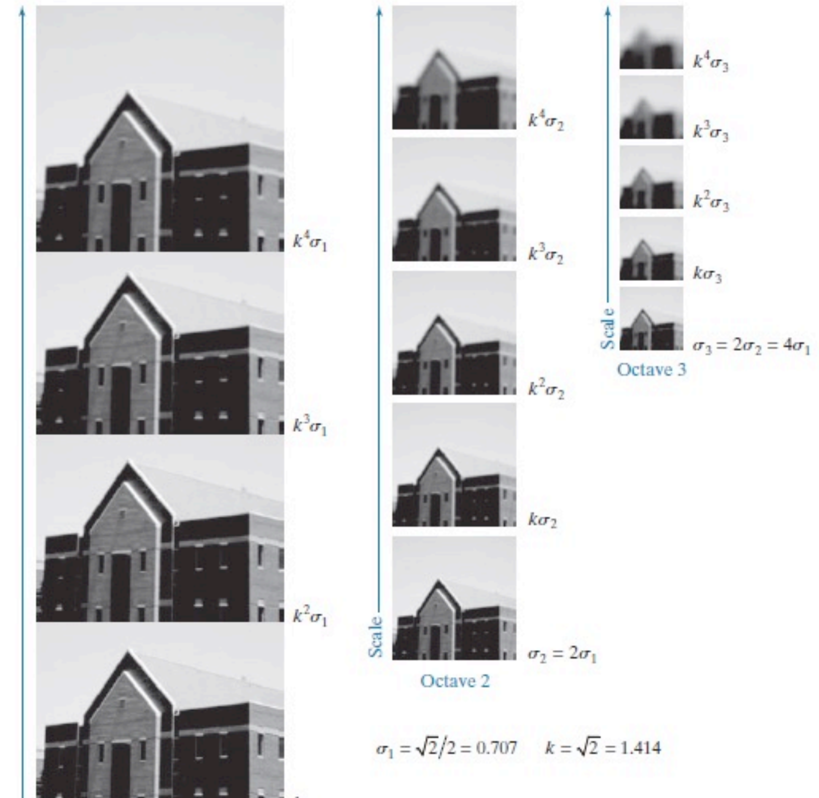


https://medium.com/@loboateresa/understanding-structure-from-motion-algorithms-fc034875fd0c

$s = 2$

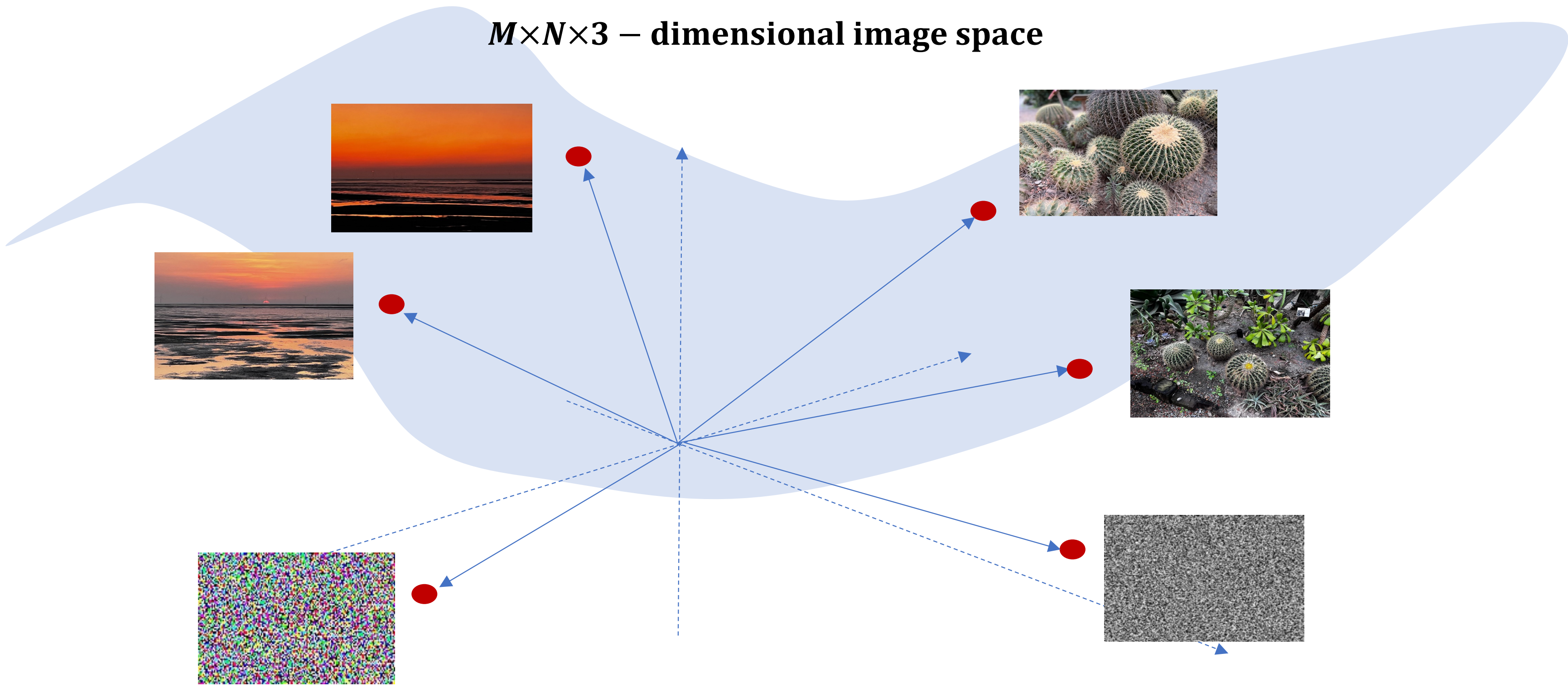


- The first image of a new octave can be obtained directly by down-sampling the third image of the previous octave (named **octave image**) by 2.

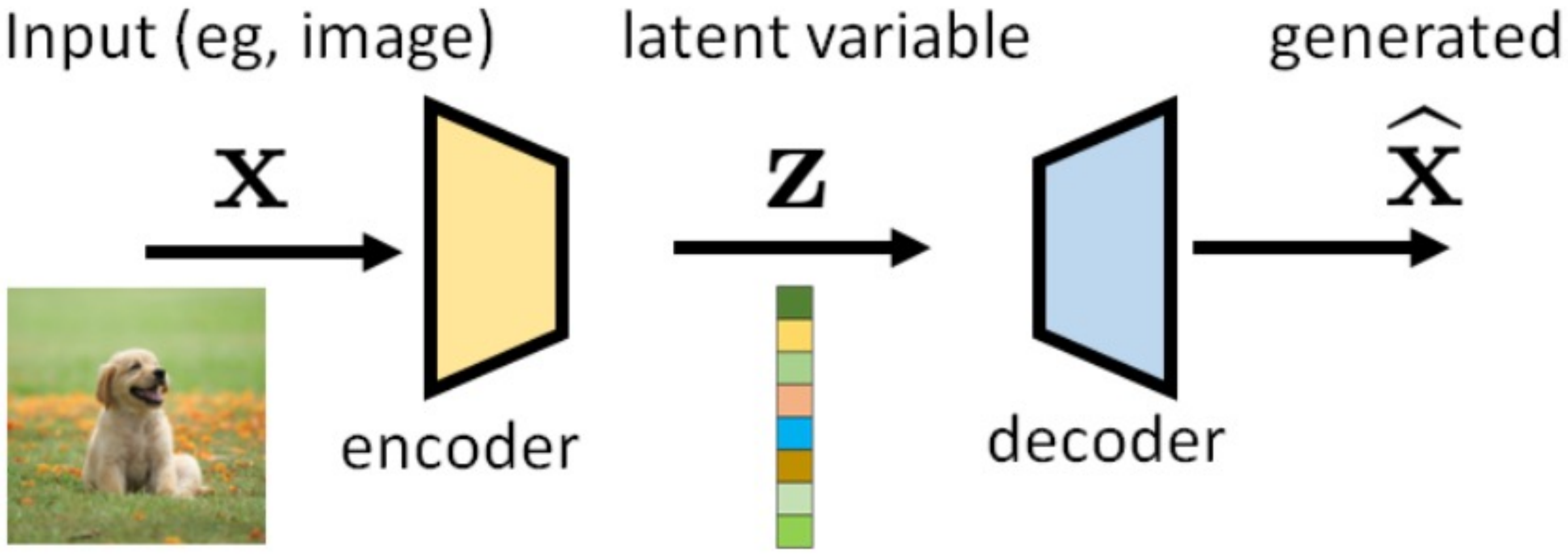


Octave	Scale 1	2	3	4	5
1	0.707	1.000	1.414	2.000	2.828
2	1.414	2.000	2.828	4.000	5.657
3	2.828	4.000	5.657	8.000	11.314

Hidden Image Manifold

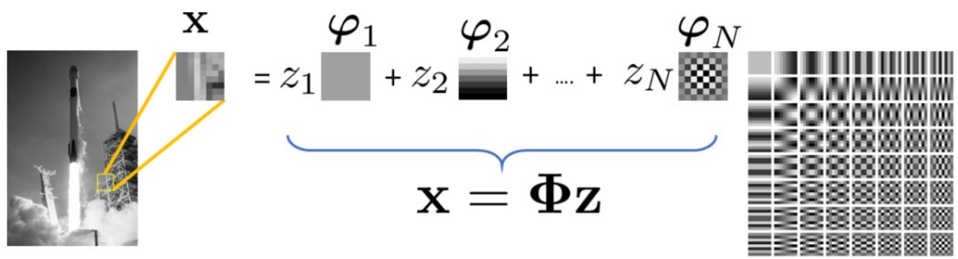


- Build a model to represent the hidden image manifold.



The latent variable $\mathbf{z}$ has two special roles in this setup. With respect to the input, the latent variable encapsulates the information that can be used to describe $\mathbf{x}$. The encoding procedure could be a lossy process, but our goal is to preserve the important content of $\mathbf{x}$ as much as we can. With respect to the output, the latent variable serves as the "seed" from which an image $\widehat{\mathbf{x}}$ can be generated. Two different $\mathbf{z}$'s should in theory give us two different generated images.

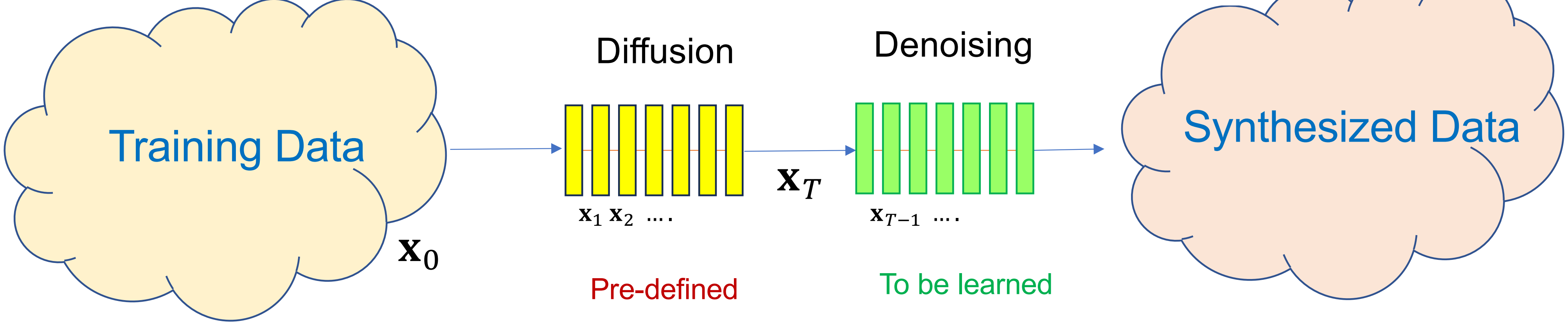
Example 1.1. Getting a latent representation of an image is not an alien thing. Back in the time of JPEG compression (which is arguably a dinosaur), we used discrete cosine transform (DCT) basis functions $\boldsymbol{\varphi}_n$ to encode the underlying image/patches of an image. The coefficient vector $\mathbf{z} = [z_1, \ldots, z_N]^T$ is obtained by projecting the image $\mathbf{x}$ onto the space spanned by the basis, via $z_n = \langle \boldsymbol{\varphi}_n, \mathbf{x} \rangle$. So, given an image $\mathbf{x}$, we can produce a coefficient vector $\mathbf{z}$. From $\mathbf{z}$, we can use the inverse transform to recover (i.e. decode) the image.



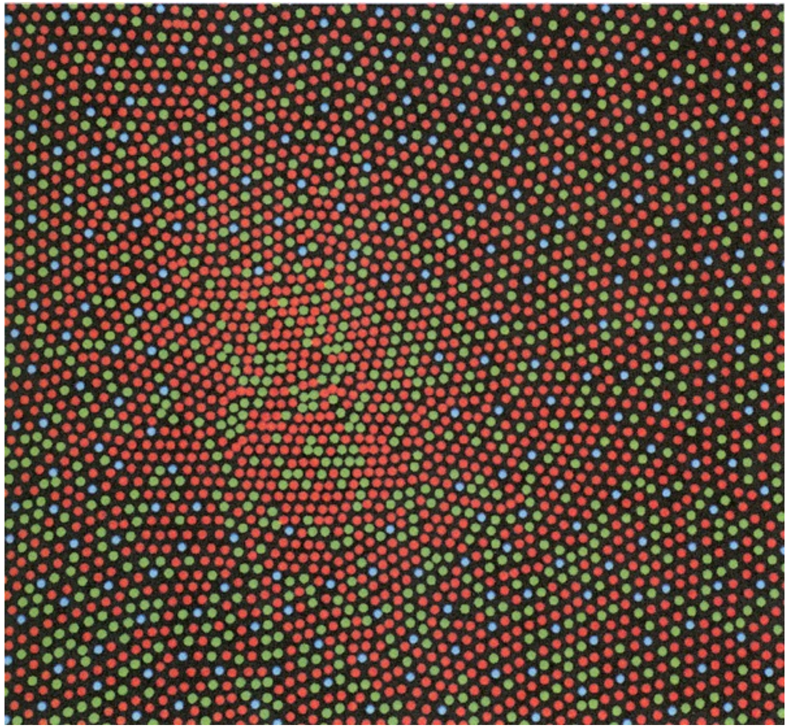
In this example, the coefficient vector $\mathbf{z}$ is the latent variable. The encoder is the DCT transform, and the decoder is the inverse DCT transform.

DPM (Diffusion Probabilistic Model)

$$p_\theta(\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_T) := p(\mathbf{x}_T) \prod_{t=1}^{T} p_\theta(\mathbf{x}_{t-1} \mid \mathbf{x}_t)$$



$$q(\mathbf{x}_{(0\ldots T)}) = q(\mathbf{x}_0) \prod_{t=1}^{T} q(\mathbf{x}_t | \mathbf{x}_{t-1}) \qquad p_\theta(\mathbf{x}_{(0\ldots T)}) = p(\mathbf{x}_T) \prod_{t=1}^{T} p_\theta(\mathbf{x}_{t-1} | \mathbf{x}_t)$$



Color

Fall 2024

Yi-Ting Chen

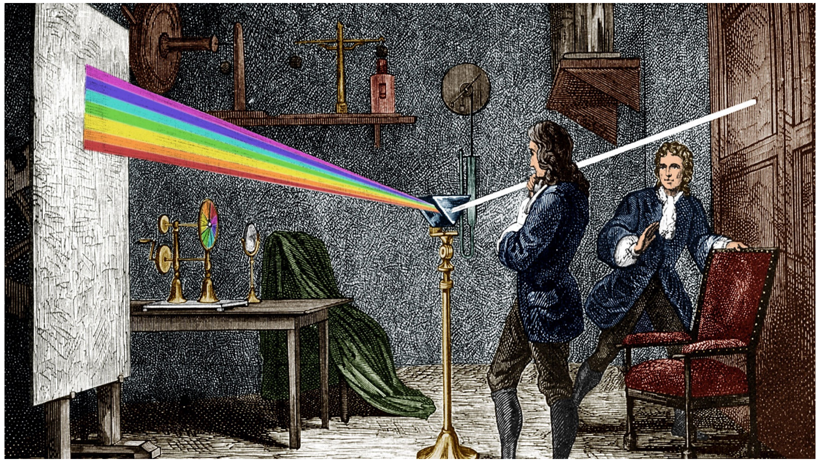
From Mark Fairchild, *Color Appearance Models*, Wiley.

Emanation theory

- Empedocles (5th century BC) – the “eye is like a lantern”
- Galen (130-200 AD)
  - physiologist
  - the brain is “an organ where all sensations arrive, and where all mental images, and all intelligent ideas arise.
  - rays are discharged in the direction of the object, interact with the object, and return to the eye
  - His views remained influential for about 1500 years

Spectrum of white light

- Newton's experiment with prism (1730)
- Concluded that white light is a mixture of colored lights
- Removed color from perceived object and placed it in rays reflected from the object



https://www.livescience.com/physics-mathematics/refraction-is-then-all-there-is-to-it-how-isaac-newtons-experiments-revealed-the-mystery-of-light

Visible Light

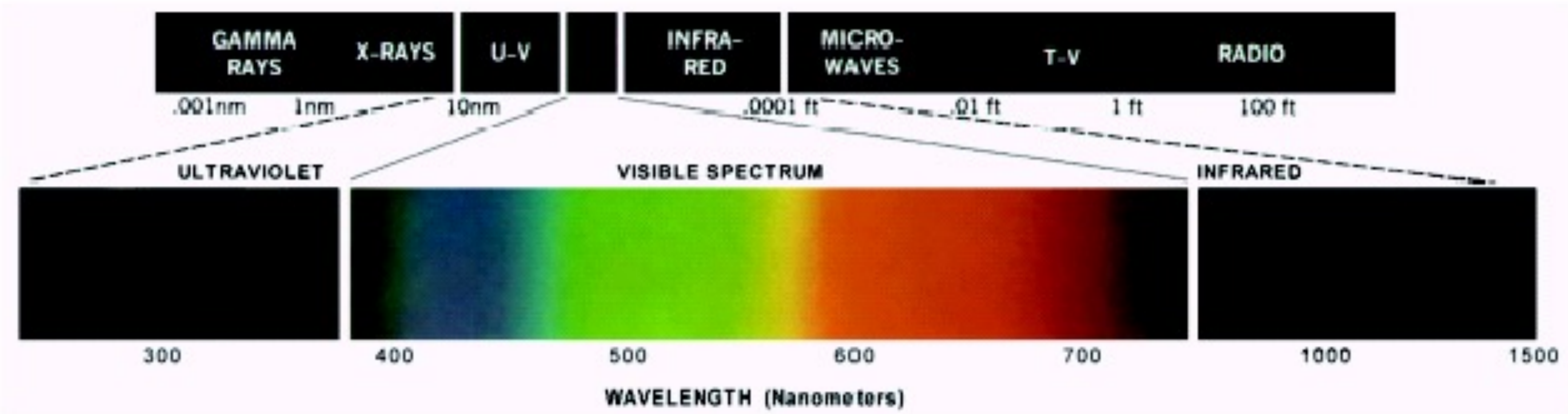


FIGURE 6.2 Wavelengths comprising the visible range of the electromagnetic spectrum. (Courtesy of the General Electric Co., Lamp Business Division.)

Trichromacy

- Color mixtures
  - Artisans and scientists experimented with them from ancient times
  - Are properties due to materials or due to perception?
- Thomas Young proposed trichromatic theory in 1801
  - "As it is almost impossible to conceive each sensitive point of the retina to contain an infinite number of particles, each capable of vibrating in perfect unison with every possible undulation, it becomes necessary to suppose the number limited; for instance to the three principal colours, red, yellow, and blue, that each of the particles is capable of being put in motion more or less forcibly by undulations differing less or more from perfect unison."

Credit: Prof. Jan Allebach ECE638

Trichromacy (cont.)

- Young's ideas were ignored for over 50 years
- Helmholtz revived them in his *Handbook of Physiological Optics* first published from 1856 to 1866

Helmholtz's three spectral sensivity curves (1924)

1. The eye is provided with three distinct sets of nervous fibres. Stimulation of the first excites the sensation of red, stimulation of the second the sensation of green, and stimulation of the third the sensation of violet.

2. Objective homogeneous light excites these three kinds of fibres in various degrees, depending on its wave-length. The red-sensitive fibres are stimulated most by light of longest wave-length, and the violet-sensitive fibres by light of shortest wave-length. But this does not mean that each colour of the spectrum does not stimulate all three kinds of fibres, some feebly and others strongly; on the contrary, in order to explain a series of phenomena, it is necessary to assume that that is exactly what does happen. Suppose tnat the colours of the spectrum are plotted horizontally in Fig. 21 in their natural sequence, from red to violet, the three curves may be taken to indicate something like the degree of excitation of the three kinds of fibres, No. 1 for the red-sensitive fibres, No. 2 for the green-sensitive fibres, and No. 3 for the violet-sensitive fibres.

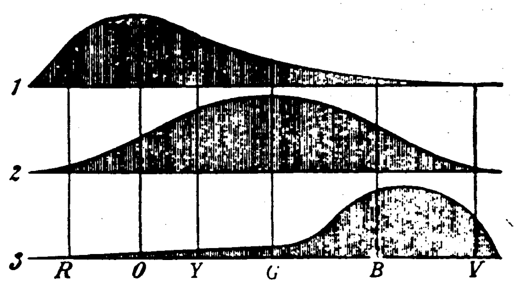
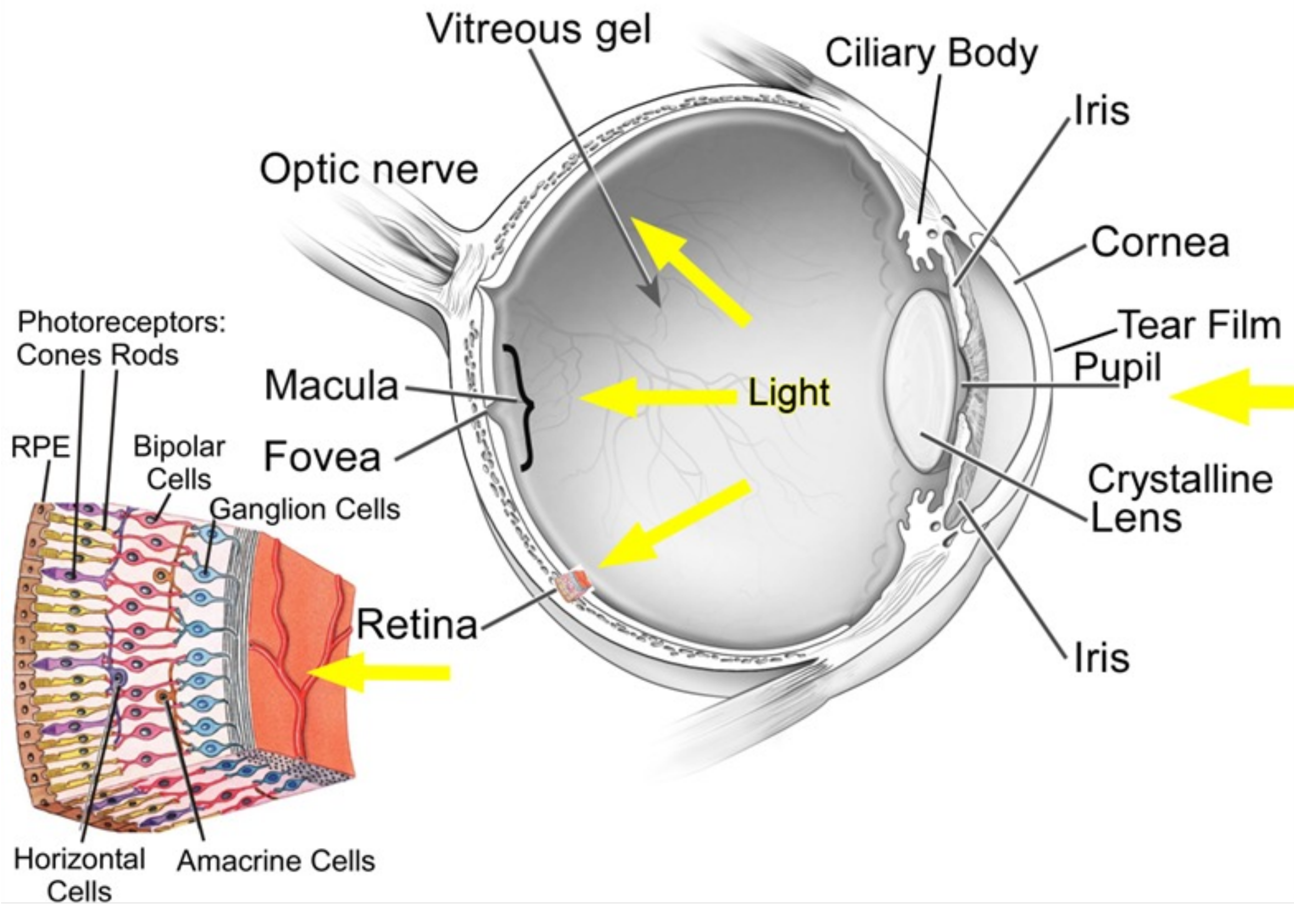
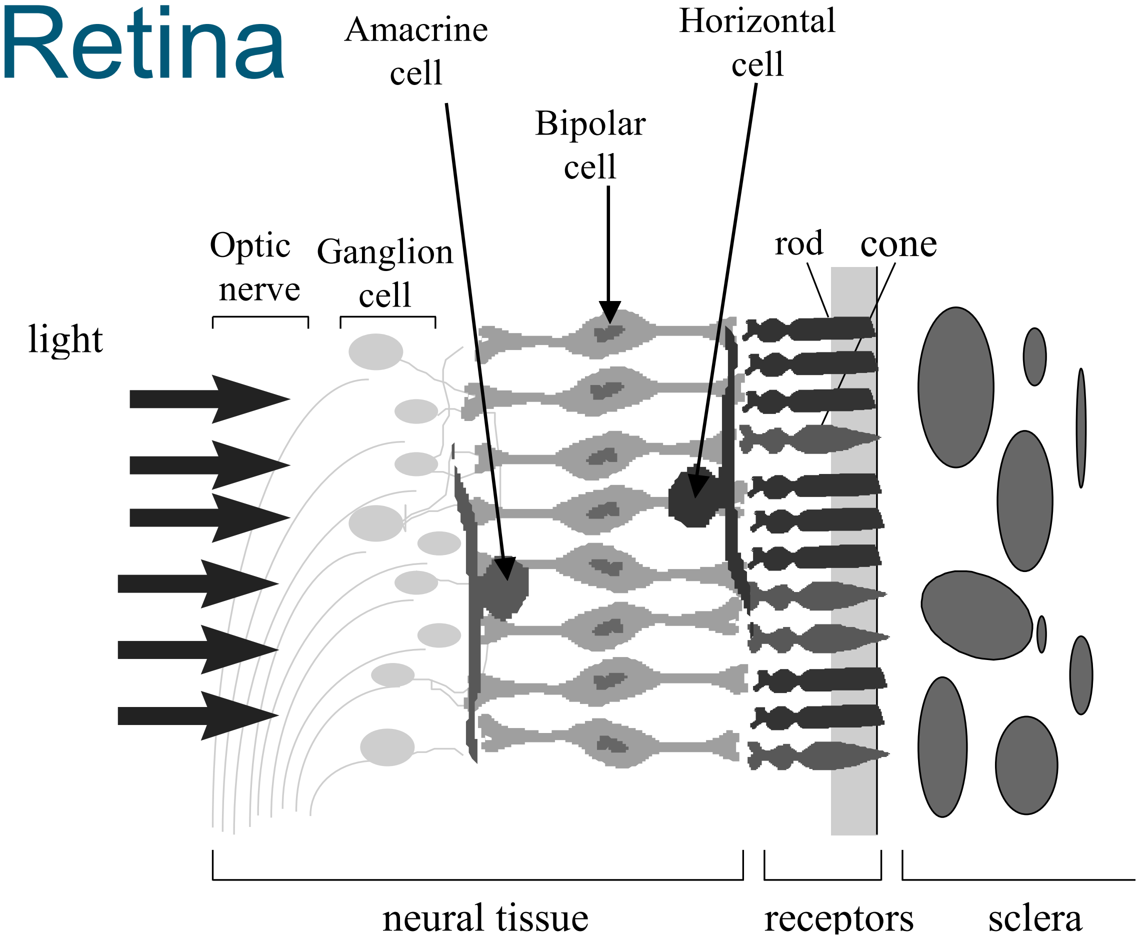
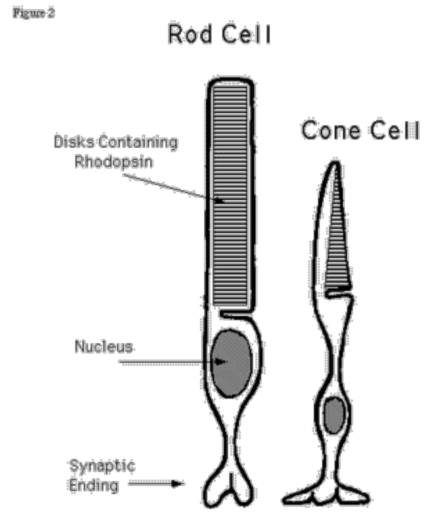


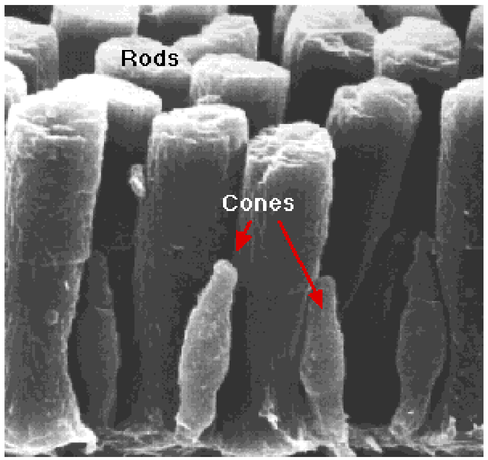
Fig. 21.


Vitreous gel
Ciliary Body
Iris
Optic nerve
Cornea
Photoreceptors:
Cones Rods
Tear Film
Pupil
Macula
Light
RPE
Bipolar
Cells
Fovea
Crystalline
Lens
Ganglion Cells
Retina
Iris
Horizontal
Cells
Amacrine Cells

Retina



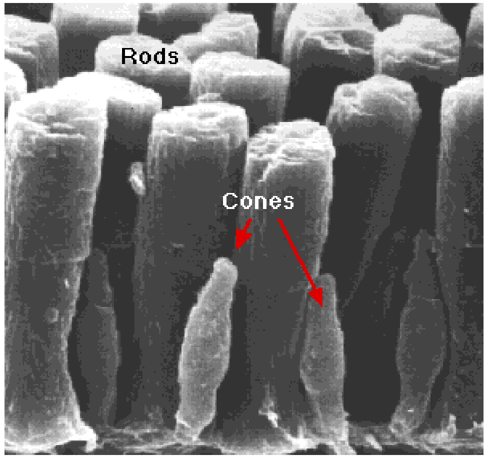


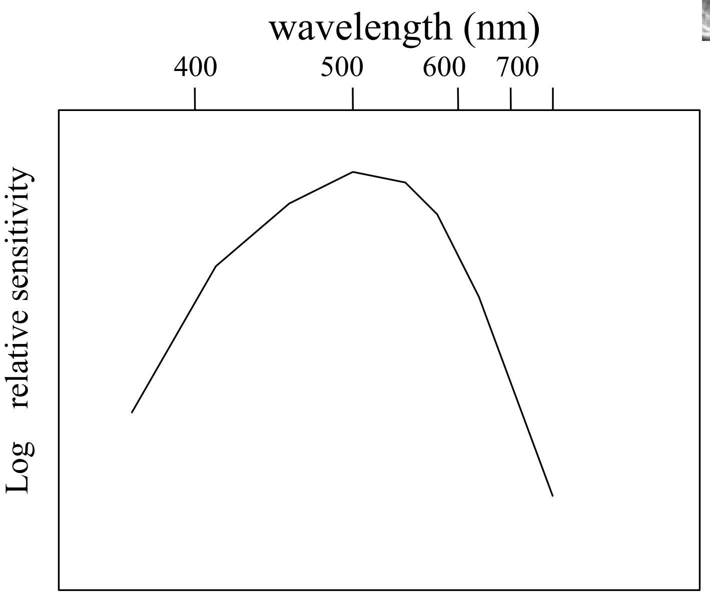


http://users.rcn.com/jkimball.ma.ultranet/BiologyPages/V/Vision.html

Rod

- sensitive to low levels of illumination (scotopic vision)
- not involved in color vision
- 75~150 million
- general, overall picture
- slower response
- about 25 times more sensitive than cones

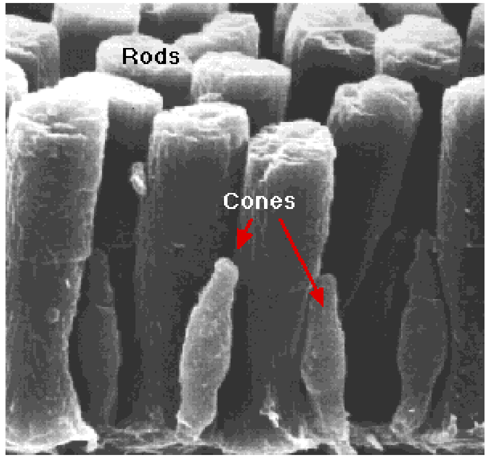


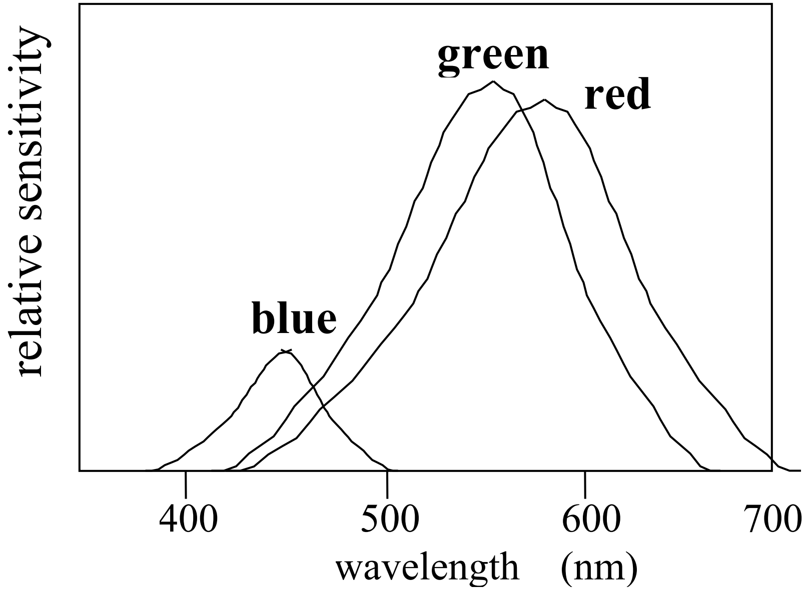


Credit: Prof. Sheng-Jyh Wang NYCU EE

Cones

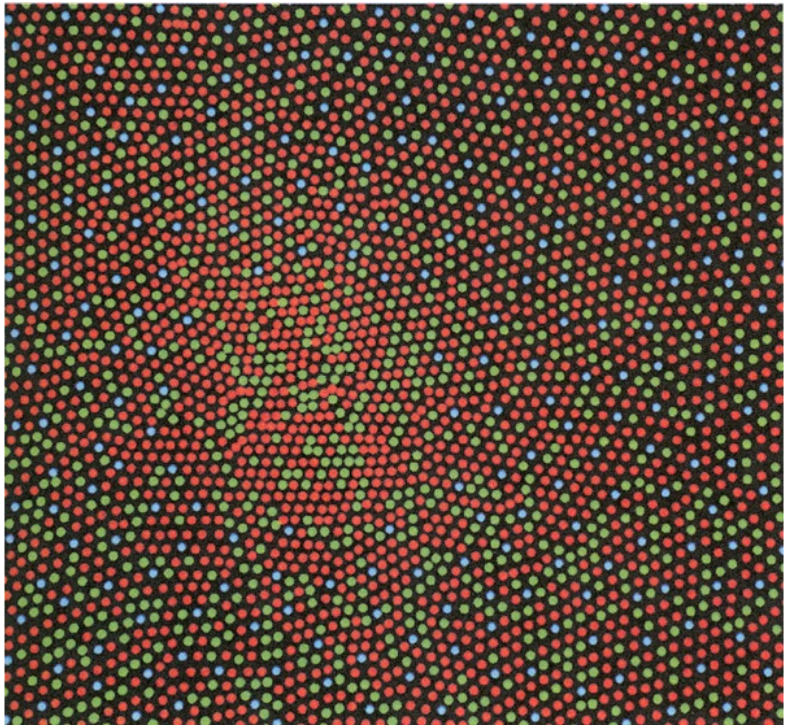
- for high levels of illumination (photopic vision)
- color vision (trichromacy)
- ~ 6.5 million
- high density in fovea
- faster response





Credit: Prof. Sheng-Jyh Wang NYCU EE

Simulated retinal mosaic

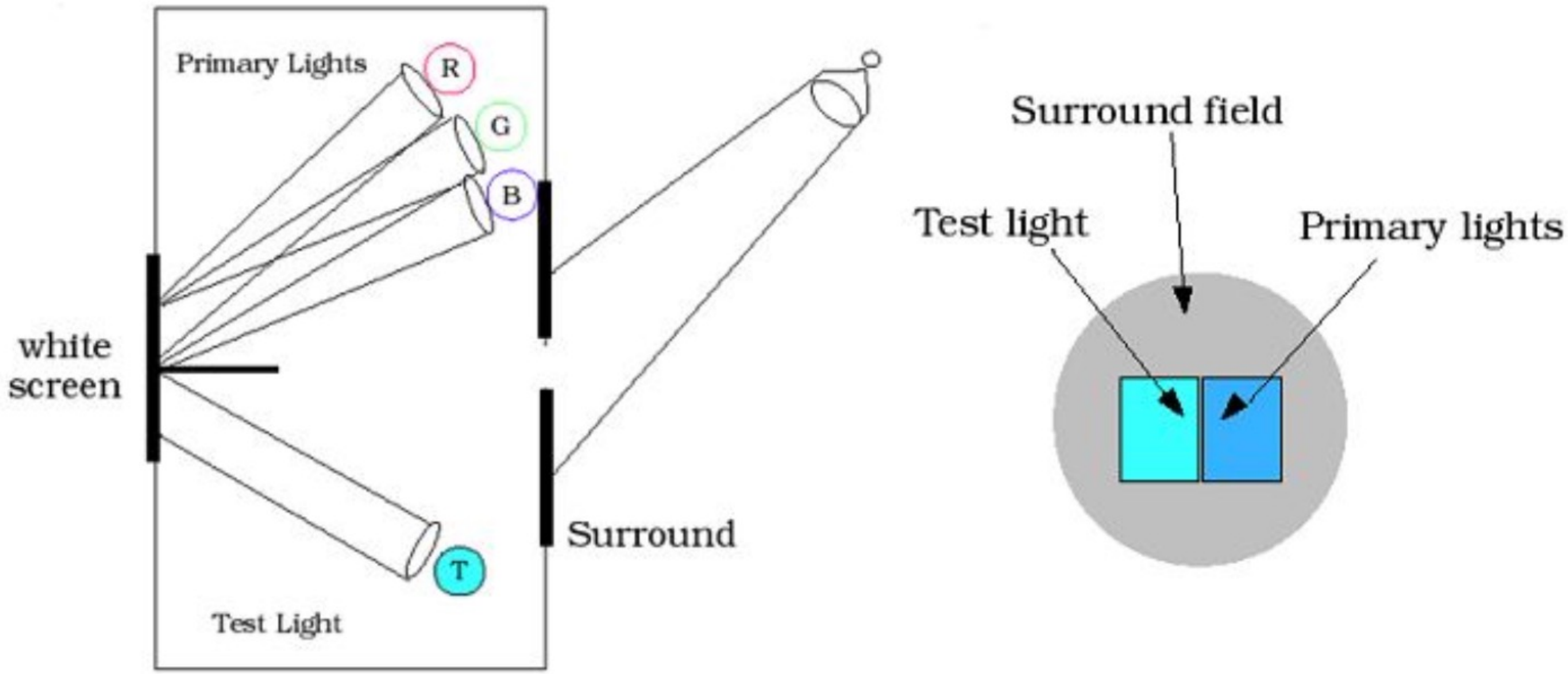


From Mark Fairchild, *Color Appearance Models*, Wiley.

If we cannot conclusively establish physiological properties of the human eye, how can we quantitatively work with color?

Answer: we treat the human visual system (HVS) as a block box, and characterize it by its response to color stimuli.

Color matching experiment



Color matching experiment - procedure

- Test stimulus C_T is fixed
- Observer individually adjusts strengths of the three match stimuli C_1, C_2, C_3 to achieve a visual match between the two sides of the split field
- Mixture is assumed to be additive, i.e. radiant power of mixture C_M in any wavelength interval (λ_1, λ_2) is sum of radiant powers of the three match stimuli in the same interval

"…over a wide range of conditions of observation, many color stimuli can be matched in color completely by additive mixtures of three fixed primary stimuli whose radiant powers have been suitably adjusted"

Grassman's laws of color matching (1853)

- Definitions: $C_A \doteq C_B$ means " C_A matches C_B "

  $C_A + C_B$ is an additive mixture of C_A and C_B

- Symmetry: $C_A \doteq C_B \Leftrightarrow C_B \doteq C_A$
- Transitivity: $C_A \doteq C_B \text{ and } C_B \doteq C_C \Rightarrow C_A \doteq C_C$

Grassman's laws (cont.)

- Proportionality: $C_A \doteq C_B \Rightarrow \alpha C_A \doteq \alpha C_B$

  where α is a positive factor that scales the radiant power without changing relative spectral distribution

- Additivity: Any two of $C_A \doteq C_B, C_C \doteq C_D, C_A + C_C \doteq C_B + C_D$

  implies that $C_A + C_D \doteq C_B + C_C$

Credit: Prof. Jan Allebach ECE638

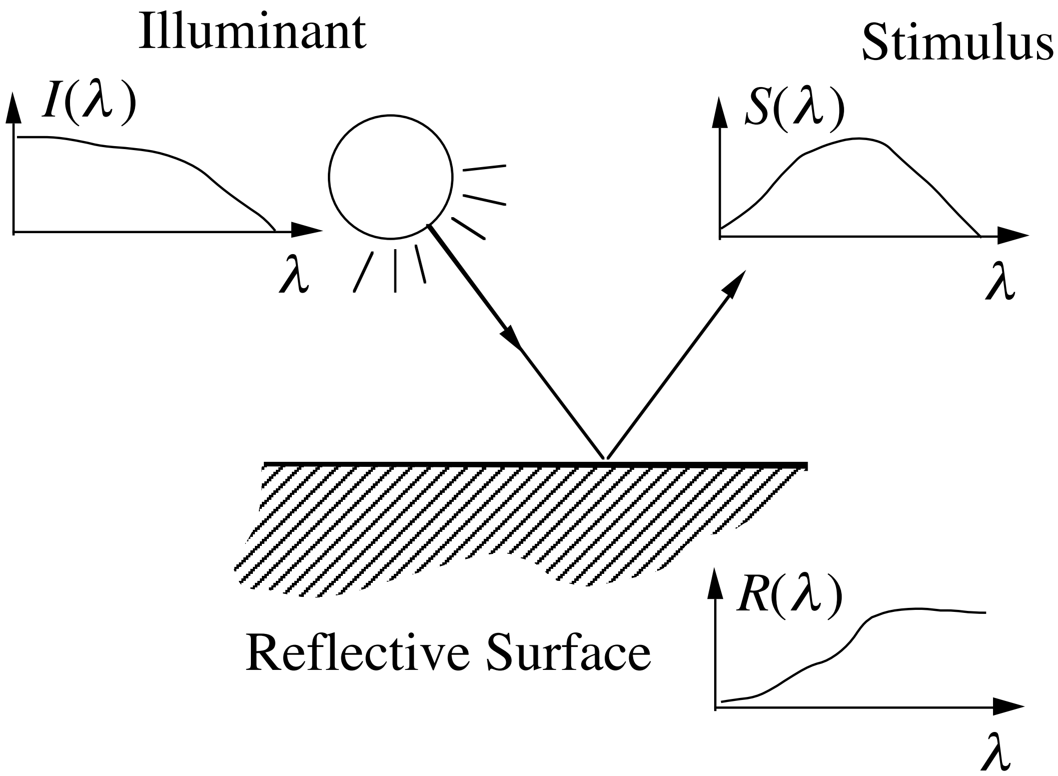
Limitations of Grassman's laws

- Grassman's laws provide a basic description of the results of color matching experiments
- But they don't provide:
  - general basis for understanding why the laws hold
  - a theory that can easily describe a wider range of situations

Trichromatic model

- Assume simple characterizations for
  - illuminant
  - optical properties of surfaces
  - interaction of illuminant with surface
  - response of HVS to visual stimulus
- All are based on spectral representations

Spectral Representation of Colors



Interaction of illuminant and surface

- At each wavelength, characterize surface by ratio of power in reflected light to that incident on surface

$$R(\lambda) \equiv \frac{S(\lambda)}{I(\lambda)}$$

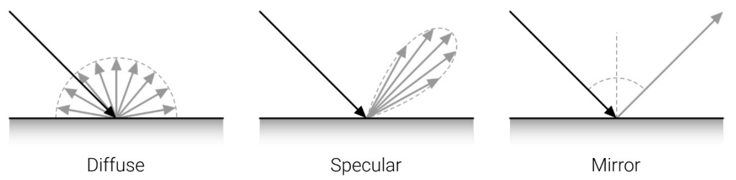
- Reflectance $0 \le R(\lambda) \le 1$

- Absorptance $A(\lambda) = 1 - R(\lambda)$

  - Absorptance includes any light that is not reflected from front surface

Limitations of interaction model – angular dependence

- Does not account for dependence of spectral reflectance on
  - angle between incident light and surface normal
  - angle between reflected light and surface normal
- More general model includes both specular and body reflectance terms
- Complete characterization requires bi-directional reflectance function (BRDF)



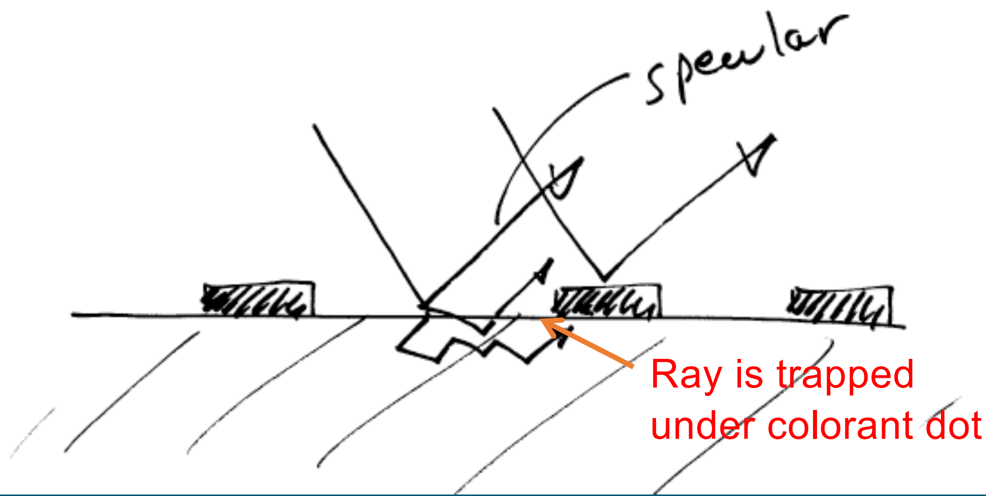
- Typical BRDFs have a **diffuse** and a **specular** component
- The diffuse (=constant) component scatters light uniformly in all directions
- This leads to shading, i.e., smooth variation of intensity wrt. surface normal
- The specular component depends strongly on the outgoing light direction

Limitations of interaction model – scattering

- Model is only valid for:
  - bulk (macroscopic) behavior
  - Microscopic behavior in which there is no lateral scattering
- More complex scattering models exist

Scattering example – Yule-Nielsen effect

- Lateral scattering of light within paper causes light rays that enter white paper substrate to be trapped under colorant dots
- Print appears darker than predicted by fractional area coverage of colorant



Limitations of interaction model – fluorescence

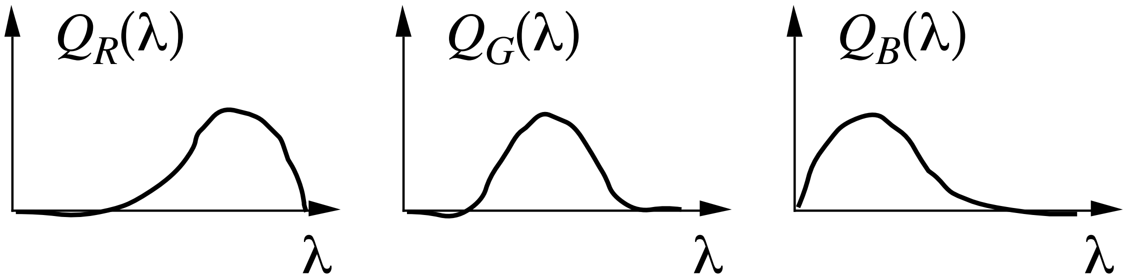
- Model assumes that reflected light at any wavelength depends only on light incident at that wavelength
- Does not describe interaction of light with surfaces that fluoresce
  - papers and fabrics containing whiteners or brighteners

Trichromatic sensor model

$$R_S = \int S(\lambda)Q_R(\lambda)d\lambda$$

$$G_S = \int S(\lambda)Q_G(\lambda)d\lambda$$

$$B_S = \int S(\lambda)Q_B(\lambda)d\lambda$$



$Q_R(\lambda), Q_G(\lambda), Q_B(\lambda)$ are spectral response functions that characterize the sensor

Credit: Prof. Jan Allebach ECE638

Trichromatic sensor model (cont.)

- With suitable choices for the spectral response functions $Q_R(\lambda), Q_G(\lambda), Q_B(\lambda)$, this model applies to the HVS, as well as color capture devices, such as digital cameras and scanners.
- The 3-tuple response vector (R_S, G_S, B_S) represents the system output for color capture devices and an internal signal for the HVS. The subscript ‘S’ refers to the particular stimulus, in this case $S(\lambda)$

Credit: Prof. Jan Allebach ECE638

Color matching condition

Two stimuli will match to a human viewer if and only if they have identical tristimulus (vector) values, i.e.

$$C_1 \doteq C_2 \Leftrightarrow (R_1, G_1, B_1) = (R_2, G_2, B_2)$$

where

$$R_1 = \int S_1(\lambda) Q_R(\lambda) \mathrm{d}\lambda \qquad R_2 = \int S_2(\lambda) Q_R(\lambda) \mathrm{d}\lambda$$

$$G_1 = \int S_1(\lambda) Q_G(\lambda) \mathrm{d}\lambda \qquad G_2 = \int S_2(\lambda) Q_G(\lambda) \mathrm{d}\lambda$$

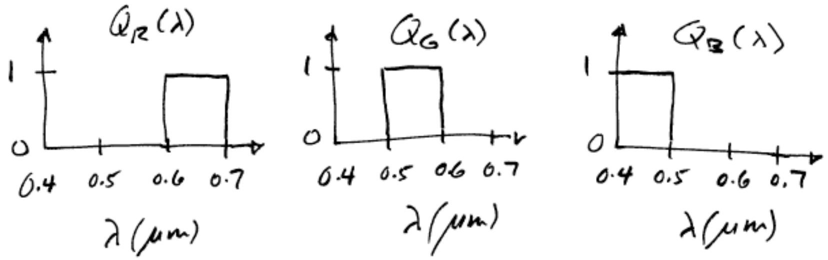
$$B_1 = \int S_1(\lambda) Q_B(\lambda) \mathrm{d}\lambda \qquad B_2 = \int S_2(\lambda) Q_B(\lambda) \mathrm{d}\lambda$$

and $S_1(\lambda)$ and $S_2(\lambda)$ are the spectral power distributions corresponding to the stimuli C_1 and C_2

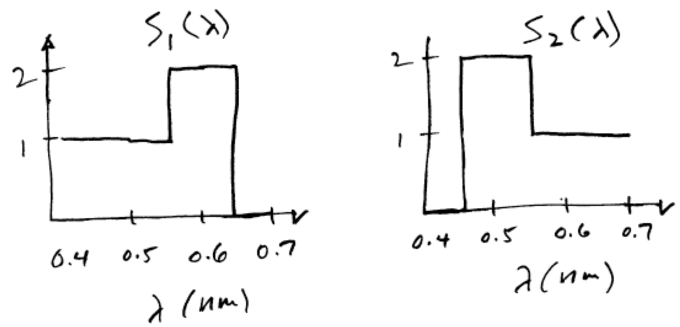
Note: These statements only hold when the stimuli are viewed under identical conditions with the viewer in identical states of adaptation.

Example 1

- Ideal block sensor



- Find response to two different stimuli

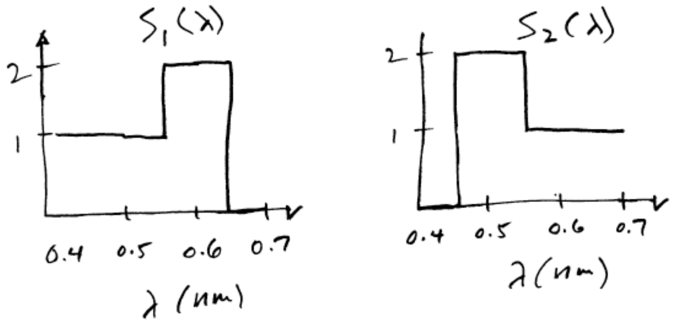


Example 1 (cont.)

- Sensor responses to both stimuli are identical

$$(R_1, G_1, B_1) = (R_2, G_2, B_2) = (0.1, 0.15, 0.1)$$

- What are the implications of this fact?
  - their spectra are quite different



  - but sensor cannot distinguish between them

Metamerism

- Stimuli $S_1(\lambda)$ and $S_2(\lambda)$ are said to be *metameric* with respect to the sensor $\left(Q_R(\lambda), Q_G(\lambda), Q_B(\lambda)\right)$ if they elicit identical responses
- Metamerism is both a blessing and a curse for color imaging systems designers

The blessing

- Consider for the moment if there were no metamerism
  - i.e. every pair of distinct spectral stimuli $S_1(\lambda)$ and $S_2(\lambda)$ yielded distinct responses (R_1, G_1, B_1) and (R_2, G_2, B_2)
- What are the implications for color imaging systems design?

  Faithful color reproduction would require exact replication of the spectrum of the original object or scene

- Can a trichromatic sensor system achieve this kind of performance?

  No, the space of all spectral distributions is infinite-dimensional. The space of all trichromatic sensor responses is 3-D

The curse

- Consider two different sensors

$$\left(V_R(\lambda), V_G(\lambda), V_B(\lambda)\right) \neq \left(Q_R(\lambda), Q_G(\lambda), Q_B(\lambda)\right)$$

- and two different stimuli $S_1(\lambda) \neq S_2(\lambda)$

- Suppose that $(R_1^V, G_1^V, B_1^V) = (R_2^V, G_2^V, B_2^V)$

- But $(R_1^Q, G_1^Q, B_1^Q) \neq (R_2^Q, G_2^Q, B_2^Q)$

What are the implications of this situation…

- If sensor V is that of the HVS, whereas Q is that of a digital camera?
- The camera cannot faithfully reproduce color as seen by the human viewer.
- How about if sensor V is that of the HVS under one illuminant, whereas Q is that of the HVS under another illuminant?

Role of illuminant

- Consider the response of sensor V to an object with reflectance $R(\lambda)$ observed under $I(\lambda)$ illuminant
- The stimulus is then $S(\lambda) = R(\lambda)I(\lambda)$
- and the response is given by

$$R = \int R(\lambda)I(\lambda)V_R(\lambda)\mathrm{d}\lambda$$

$$G = \int R(\lambda)I(\lambda)V_G(\lambda)\mathrm{d}\lambda$$

$$B = \int R(\lambda)I(\lambda)V_B(\lambda)\mathrm{d}\lambda$$

Role of illuminant (cont.)

- By grouping the illuminant spectral density with the sensor response functions, we obtain a new effective sensor V^I with response

$$R = \int R(\lambda) V_R^I(\lambda) \mathrm{d}\lambda$$

$$G = \int R(\lambda) V_G^I(\lambda) \mathrm{d}\lambda$$

$$B = \int R(\lambda) V_B^I(\lambda) \mathrm{d}\lambda$$

to the surface with reflectance $R(\lambda)$ viewed under an equal energy $E(\lambda) \equiv 1$ illuminant

where $V_i^I(\lambda) = I(\lambda) V_i(\lambda),\ i = R, G, B$

Now consider again the question posed earlier

- We have one sensor V (the HVS)
- two different surfaces with reflectances $R_1(\lambda)$ and $R_2(\lambda)$
- two different illuminants $I(\lambda)$and $J(\lambda)$
- Under the first illuminant, the two surfaces appear identical, i.e.

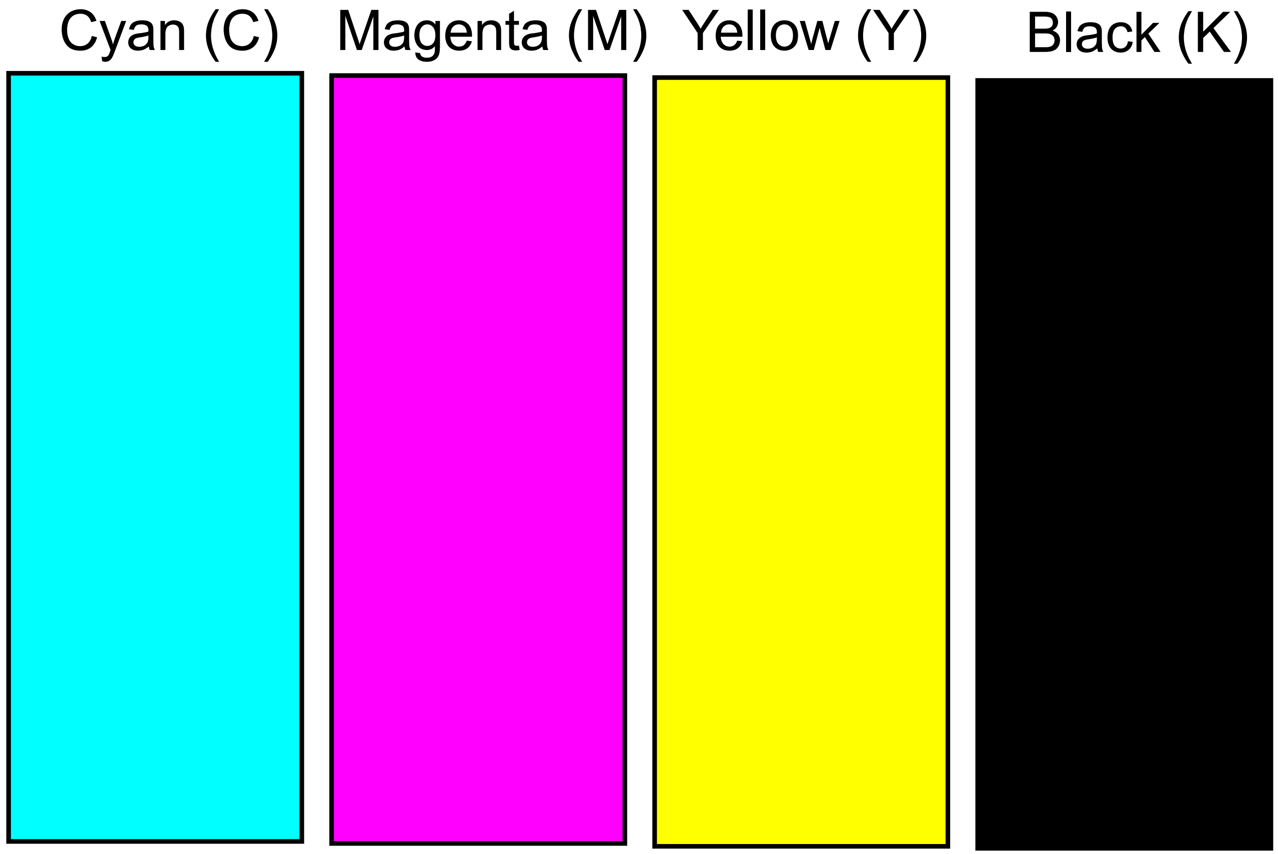
$$(R_1^{V^I}, G_1^{V^I}, B_1^{V^I}) = (R_2^{V^I}, G_2^{V^I}, B_2^{V^I})$$

- Under the second illuminant, they do not; so

$$(R_1^{V^J}, G_1^{V^J}, B_1^{V^J}) \neq (R_2^{V^J}, G_2^{V^J}, B_2^{V^J})$$

- This situation can be a serious problem for color process control and color imaging systems

CMYK

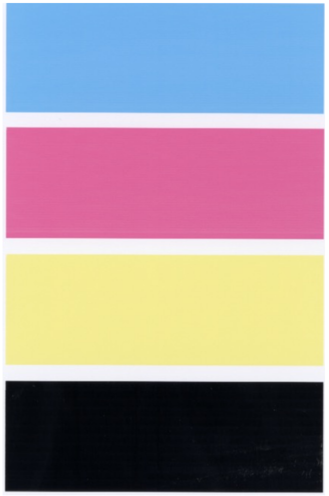


Credit: Prof. Jan Allebach ECE638

Different Color Imaging Systems

Olympus C3000
Digital Camera

Heidelberg Scanner



Credit: Prof. Jan Allebach ECE638

Linearity of sensor response

- Recall again our trichromatic sensor model

$$R = \int S(\lambda) Q_R(\lambda)\,\mathrm{d}\lambda$$

$$G = \int S(\lambda) Q_G(\lambda)\,\mathrm{d}\lambda$$

$$B = \int S(\lambda) Q_B(\lambda)\,\mathrm{d}\lambda$$

- It follows directly from the linearity of the integral, that the trichromatic system is linear

$$S_1(\lambda) \rightarrow (R_1, G_1, B_1) \text{ and } S_2(\lambda) \rightarrow (R_2, G_2, B_2)$$

$$\Rightarrow aS_1(\lambda) + bS_2(\lambda) \rightarrow (aR_1 + bR_2, aG_1 + bG_2, aB_1 + bB_2)$$

Response to monochromatic stimuli

- From the linearity of the sensor system, it follows that the response to any stimulus can be expressed in terms of the response to monochromatic stimuli

$$S(\lambda) = \delta(\lambda - \lambda_0) \Rightarrow$$

$$\begin{bmatrix} R^{\lambda_0} \\ G^{\lambda_0} \\ B^{\lambda_0} \end{bmatrix} = \begin{bmatrix} \int \delta(\lambda - \lambda_0) Q_R(\lambda) \mathrm{d}\lambda \\ \int \delta(\lambda - \lambda_0) Q_G(\lambda) \mathrm{d}\lambda \\ \int \delta(\lambda - \lambda_0) Q_B(\lambda) \mathrm{d}\lambda \end{bmatrix} = \begin{bmatrix} Q_R(\lambda_0) \\ Q_G(\lambda_0) \\ Q_B(\lambda_0) \end{bmatrix}$$

- $\begin{bmatrix} Q_R(\lambda_0) \\ Q_G(\lambda_0) \\ Q_B(\lambda_0) \end{bmatrix}$ is the “impulse response” of the system

Principle of superposition

- For an arbitrary stimulus $S(\lambda)$, we can write

$$S(\lambda) = \int S(\lambda_0)\delta(\lambda - \lambda_0)\mathrm{d}\lambda_0$$

- Each component of this stimulus generates a scaled version of the impulse response

$$S(\lambda_0)\delta(\lambda - \lambda_0) \xrightarrow[\text{Sensor System}]{} S(\lambda_0)\begin{bmatrix} Q_R(\lambda_0) \\ Q_G(\lambda_0) \\ Q_B(\lambda_0) \end{bmatrix}$$

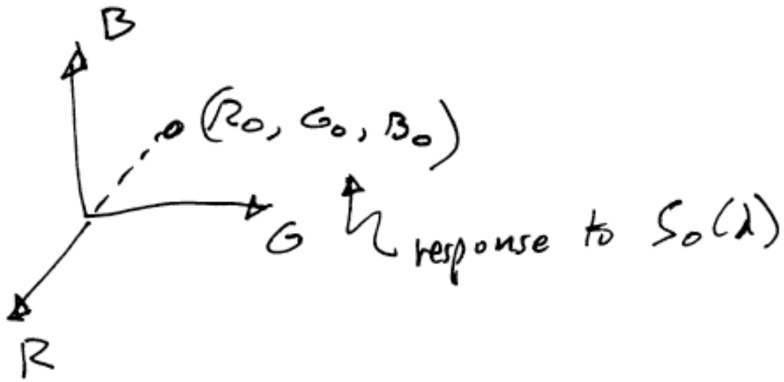
Principle of superposition (cont.)

- We sum responses to stimuli at each wavelength to get total response to the stimulus

$$\int S(\lambda_0)\delta(\lambda-\lambda_0)\mathrm{d}\lambda_0 \xrightarrow[\text{Sensor System}]{} \int S(\lambda_0)\begin{bmatrix} Q_R(\lambda_0) \\ Q_G(\lambda_0) \\ Q_B(\lambda_0) \end{bmatrix}\mathrm{d}\lambda_0 = \begin{bmatrix} R^S \\ G^S \\ B^S \end{bmatrix}$$

Geometric interpretation of the sensor response

- The sensor response is a 3-tuple; so the response to each stimulus may be viewed as a point in Euclidean three space that we call the sensor space



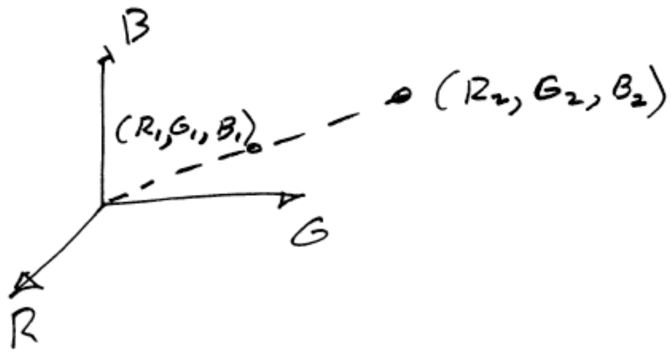
- Note that the axes need not be perpendicular for this representation to be meaningful

Effect of scaling the stimulus

- If we scale the stimulus by a constant α , the response will scale by the same amount

$$S_2(\lambda) = \alpha S_1(\lambda) \Rightarrow \left(R_2, G_2, B_2\right) = \left(\alpha R_1, \alpha G_1, \alpha B_1\right)$$

- In the sensor space, the response point will shift along a straight line as we vary α



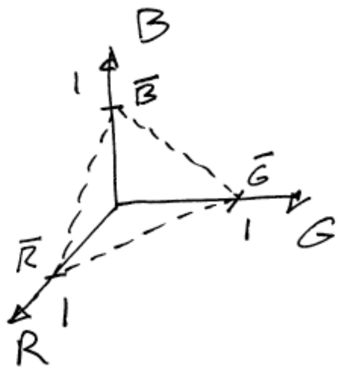
Sensor chromaticity diagram

- Since scaling the stimulus by a constant does not modify the relative spectral power at each wavelength, we intuitively expect that such changes will only affect the lightness, and not the saturation or hue of the resulting color
- Thus, if we are only interested in studying how the saturation and hue vary with the spectral shape of the stimulus, it is sufficient to consider the point of intersection of the sensor vector with a fixed plane in the sensor space

Credit: Prof. Jan Allebach ECE638

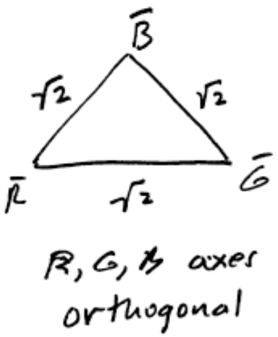
Plane of equal lightness (brightness)

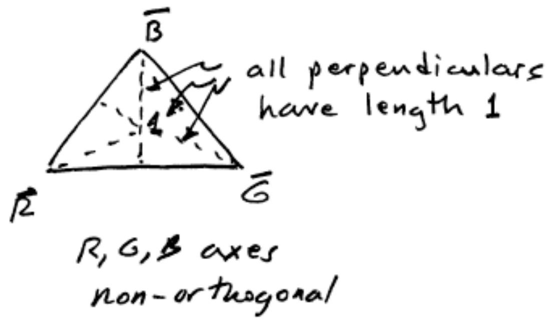
- Intuitively, we also expect that all points in the sensor space for which $R + B + G = \kappa$ for some constant κ, will appear equally light or bright (in reality, this is only approximately true)
- Thus we choose the plane $R + B + G = 1$ for our sensor chromaticity diagram



Orthogonal vs. non-orthogonal axes

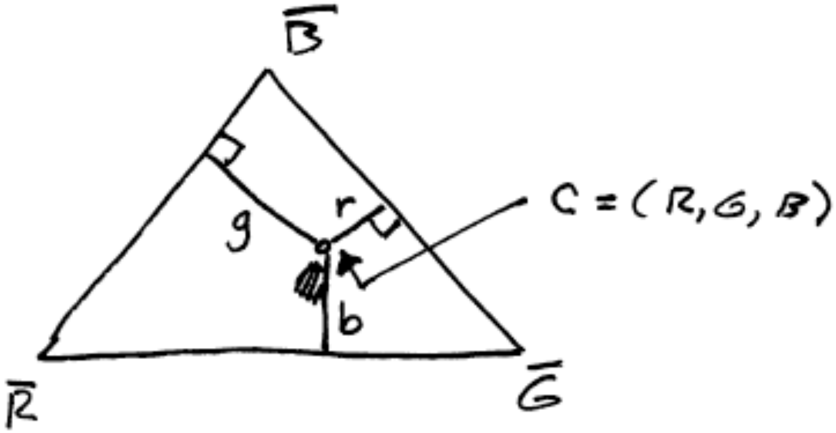
- When the (R,G,B) axes are orthogonal, the triangle is equilateral with sides $\sqrt{2}$
- It is of interest to choose a particular set of non-orthogonal axes for which we have an equilateral triangle with all perpendiculars having length 1





Chromaticity diagram

- In this case, the lengths of the perpendiculars (r,g,b) illustrated below can be shown to satisfy



$$r = \frac{R}{R+G+B}$$

$$g = \frac{G}{R+G+B}$$

$$b = \frac{B}{R+G+B}$$

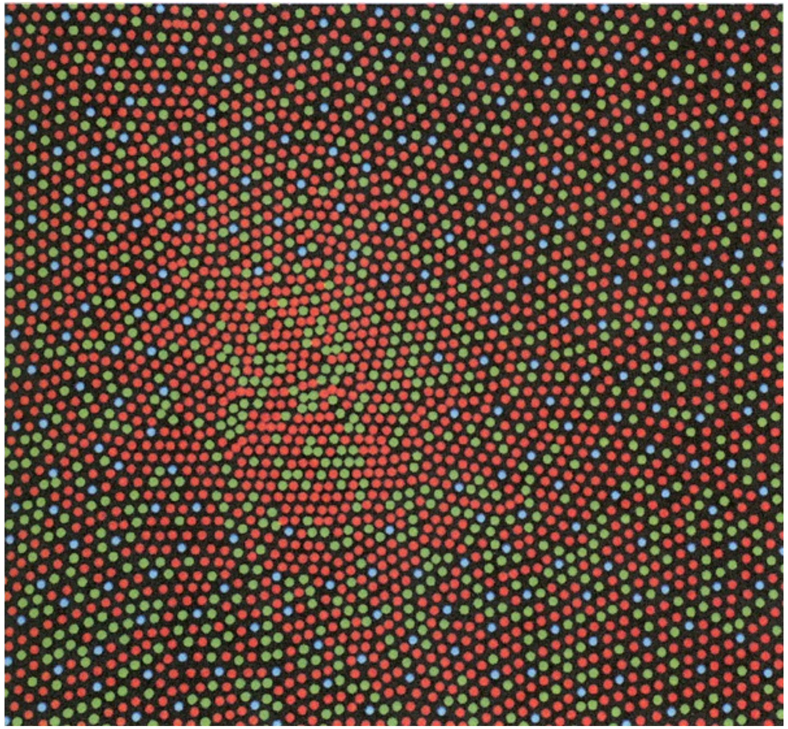
$$r + g + b = 1$$

Credit: Prof. Jan Allebach ECE638

What is color?

Color is a sensation experienced by a human being in response to a visual stimulus.

Simulated retinal mosaic

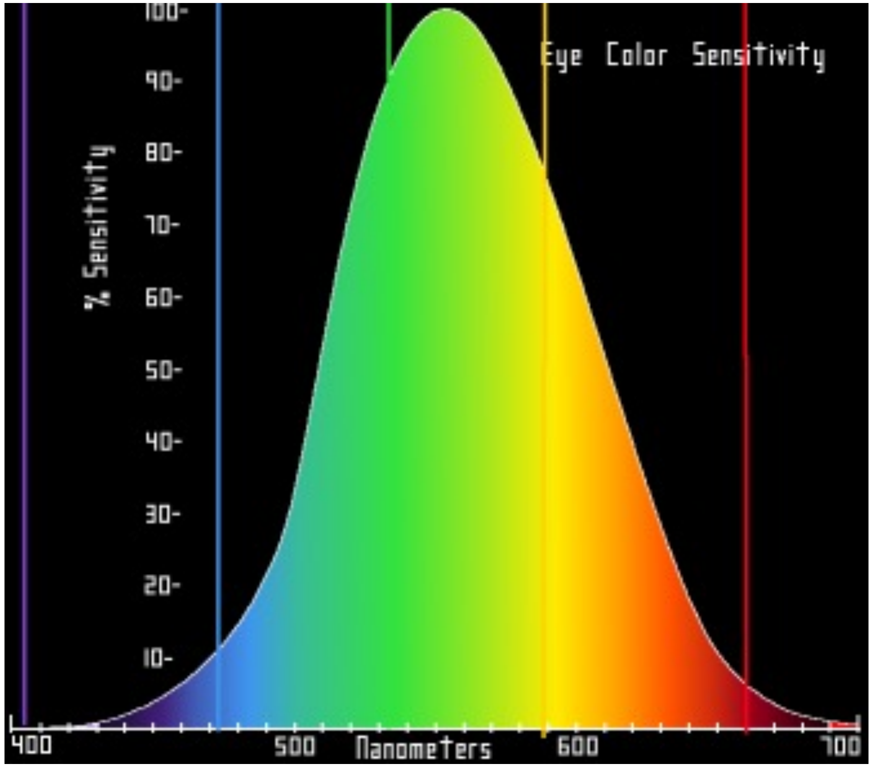


From Mark Fairchild, *Color Appearance Models*, Wiley.

The human retina indeed has a higher proportion of **L-cones** (red-sensitive) compared to **M-cones** (green-sensitive).

- However, this abundance of L-cones does not directly translate to how we perceive brightness and detail.

Human Color Sensitivity



Despite having more L-cones, the human eye's overall **sensitivity to light intensity (luminance)** peaks in the green part of the spectrum (~555 nm). This is because **M-cones** and **L-cones** collectively contribute more to luminance perception, with green being the dominant factor.

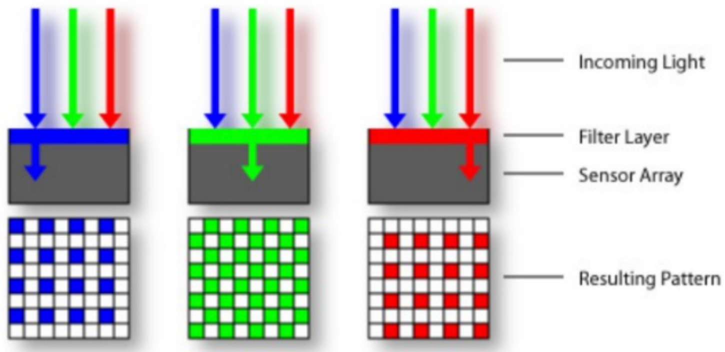
http://scienceblogs.com/builtonfacts/2013/01/29/the-human-eye-optimized-for-sunlight-maybe/

Color Filter Arrays

- To measure color, pixels are arranged in a color array, e.g., Bayer RGB pattern
- Missing colors at each pixel are interpolated from the neighbors (demosaicing)

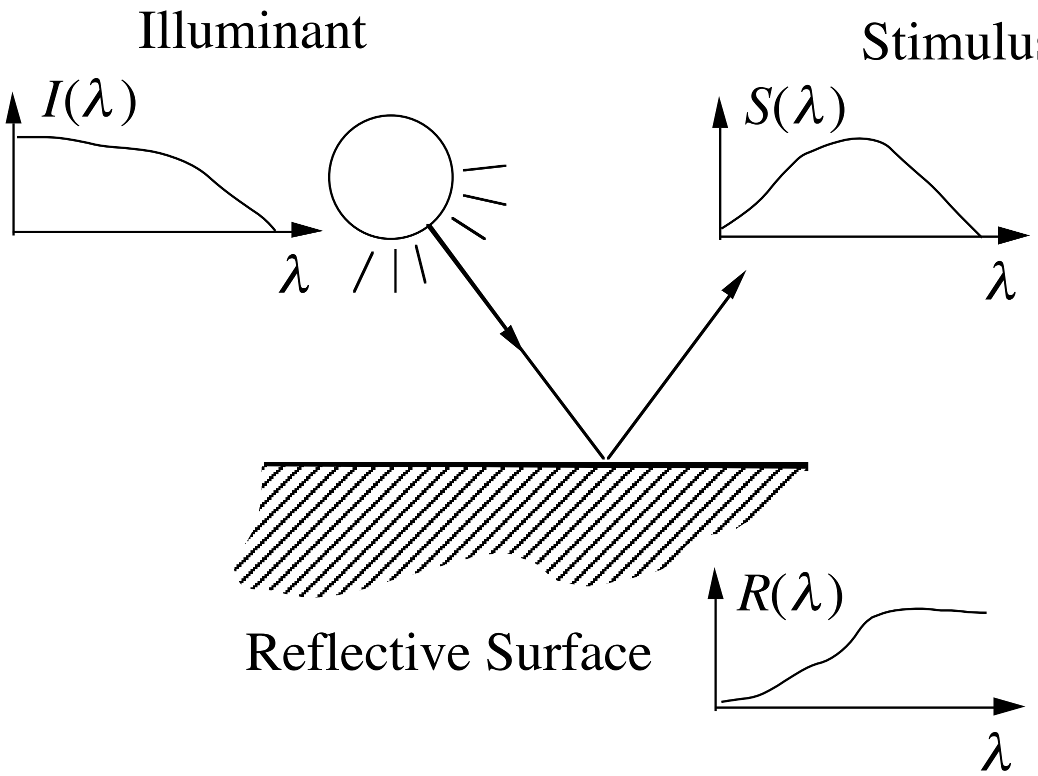
G	R	G	R
B	G	B	G
G	R	G	R
B	G	B	G

Bayer RGB Pattern



Why there are more green channels?

Spectral Representation of Colors

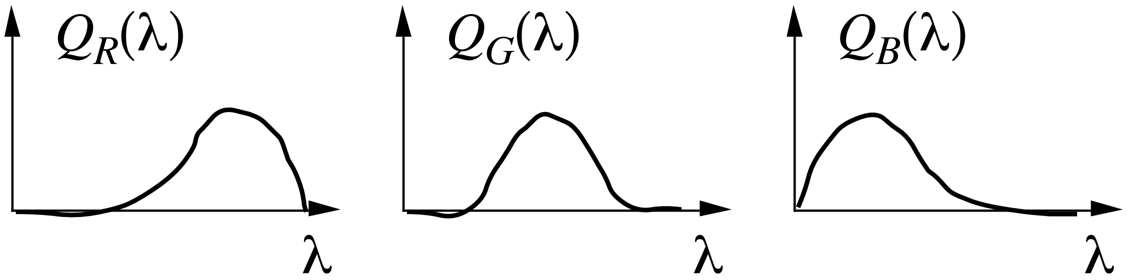


Trichromatic sensor model

$$R_S = \int S(\lambda) Q_R(\lambda) d\lambda$$

$$G_S = \int S(\lambda) Q_G(\lambda) d\lambda$$

$$B_S = \int S(\lambda) Q_B(\lambda) d\lambda$$



$Q_R(\lambda), Q_G(\lambda), Q_B(\lambda)$ are spectral response functions that characterize the sensor

Notation

- Use upper case symbols for sensor response

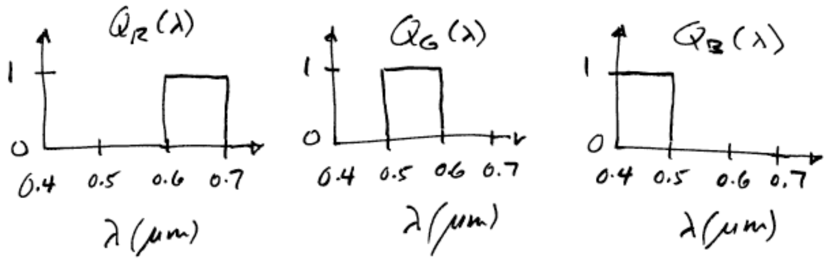
$$\vec{C} = (R, G, B)$$

- Use lower case symbols for corresponding chromaticity coordinates

$$\vec{c} = (r, g, b)$$

Effect of shape of senor response functions on sensor performance

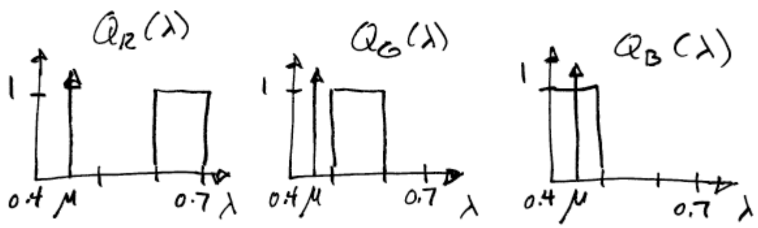
- Consider again the ideal block sensor



- Intuitively, we might expect that this would be a good sensor
  - full coverage of spectral band at uniform sensitivity
  - no overlap of response from different channels $\Rightarrow$ minimize cross-talk or "confusion"

Spectral locus

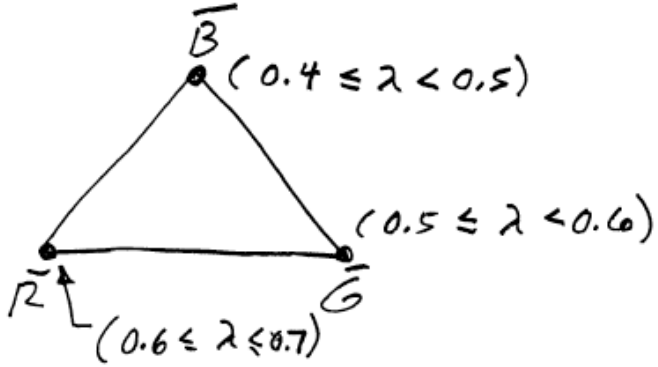
- Recall that the response of a sensor to an arbitrary stimulus can be expressed in terms of the response to monochromatic stimuli at all wavelengths μ
- The resulting curve in the chromaticity diagram is called the spectral locus **S**
- Consider case $0.4 \le \mu < 0.5$



$$(R,G,B) = (0,0,1), \quad 0.4 \le \mu < 0.5$$
$$(r,g,b) = (0,0,1),$$

Spectral locus for block sensor

$$\mathbf{S}=\begin{cases}(0,0,1), & 0.4 \le \lambda < 0.5 \\ (0,1,0), & 0.5 \le \lambda < 0.6 \\ (1,0,0), & 0.6 \le \lambda \le 0.7\end{cases}$$



- Is this a good sensor?

  Sensor cannot distinguish between different monochromatic stimuli between 0.4 and 0.5 μm, or between 0.5 and 0.6 μm, or between 0.6 and 0.7 μm

- How do we fix this problem?

Mixture of two stimuli

- Consider two stimuli and the responses of a trichromatic sensor to these stimuli

$$S_1(\lambda) \rightarrow (R_1, G_1, B_1) \text{ and } S_2(\lambda) \rightarrow (R_2, G_2, B_2)$$

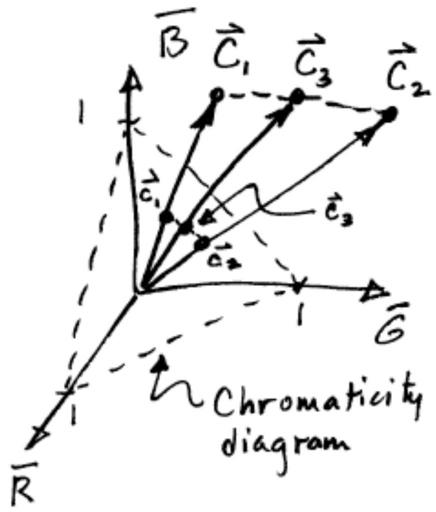
- Now suppose we create a third stimulus as a mixture of these two stimuli with mixture $0 \le \alpha \le 1$ parameter $S_3(\lambda) = \alpha S_1(\lambda) + (1-\alpha) S_2(\lambda), \quad 0 \le \alpha \le 1$
- Then by linearity of the sensor, we have that

$$\begin{bmatrix} R_3 \\ G_3 \\ B_3 \end{bmatrix} = \alpha \begin{bmatrix} R_1 \\ G_1 \\ B_1 \end{bmatrix} + (1-\alpha) \begin{bmatrix} R_2 \\ G_2 \\ B_2 \end{bmatrix}$$

Geometric interpretation of mixture

- As α ranges from 0 to 1, (R_3, G_3, B_3) traces a line in the sensor space connecting (R_2, G_2, B_2) with (R_1, G_1, B_1)

- We observe analogous behavior in the chromaticity diagram



$$\vec{C}_i = (R_i, G_i, B_i), \quad i = 1,2,3$$
$$\vec{c}_i = (r_i, g_i, b_i),$$

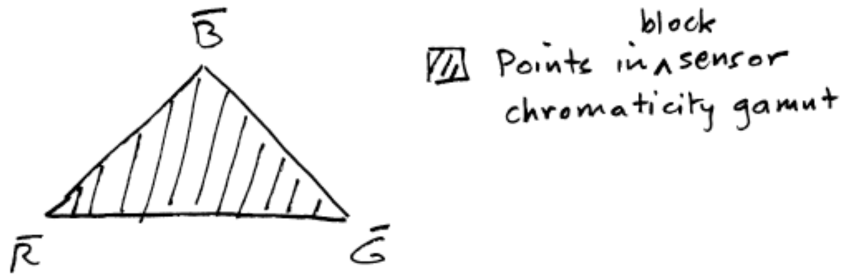
Block sensor response to mixture stimuli

- Any three stimuli $S_1(\lambda), S_2(\lambda), S_3(\lambda)$ possibly, but not necessarily monochromatic, with support confined to the ranges $\lambda \in [0.4, 0.5), [0.5, 0.6), [0.6, 0.7]$, respectively, will correspond to chromaticity coordinates $\vec{c}_1, \vec{c}_2, \vec{c}_3$ at the vertices of the chromaticity diagram
- The chromaticity coordinate $\vec{c}_4$ of any stimulus $S_4(\lambda)$ that is a mixture of two or three of the stimuli $S_1(\lambda), S_2(\lambda), S_3(\lambda)$ will lie within the convex hull formed by the chromaticity coordinates $\vec{c}_1, \vec{c}_2, \vec{c}_3$

Credit: Prof. Jan Allebach ECE638

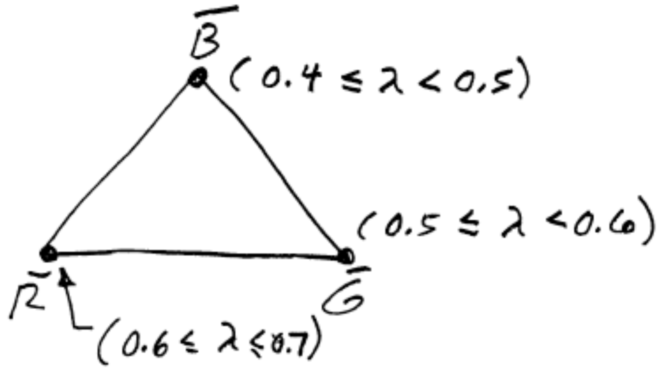
Chromaticity gamut of block sensor

- We define the chromaticity gamut for a sensor to be the set of all points in the chromaticity space that correspond to the response of that sensor to some real stimulus
- From the foregoing, we conclude that the chromaticity gamut of the block sensor fills the entire chromaticity diagram



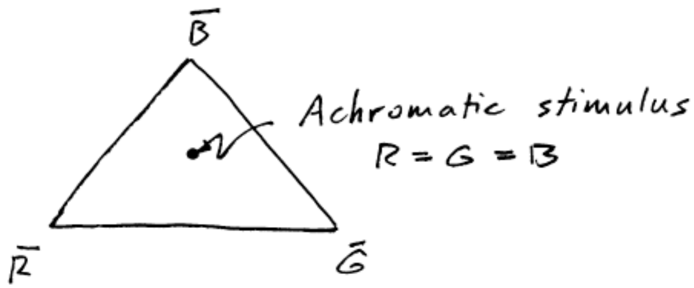
Interpretation of regions of chromaticity diagram

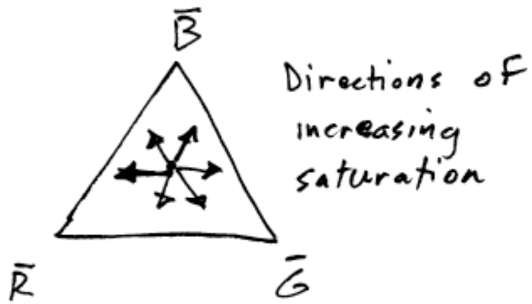
- We have seen that stimuli confined to a narrow spectral band will excite only one channel, and will yield a chromaticity point at one of the three vertices of the chromaticity diagram
- As we increase wavelength from 0.4 to 0.7 μm, we switch from the blue to the green to the red vertices



Interpretation (cont.)

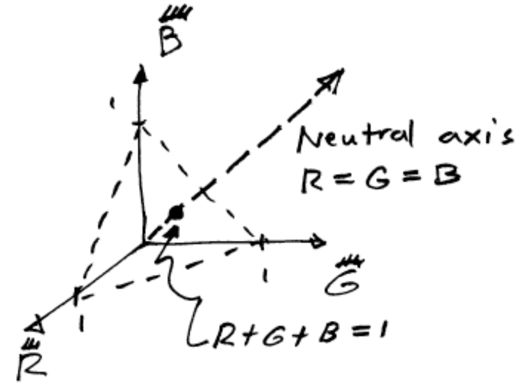
- A stimulus which equally excites all three channels will correspond to a chromaticity coordinate at the center of the diagram. Visually, this color should look achromatic
- As we move from this point outward, the stimulus response is concentrated in one or two channels $\Rightarrow$ the color should appear more spectrally pure or more saturated

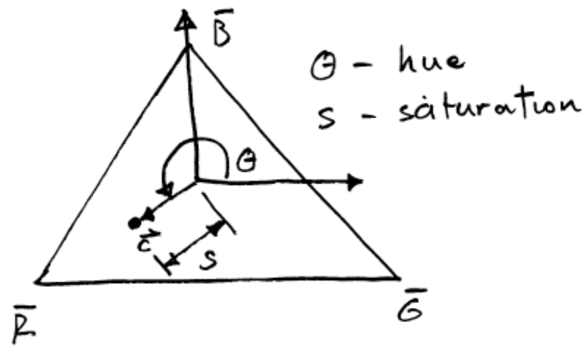




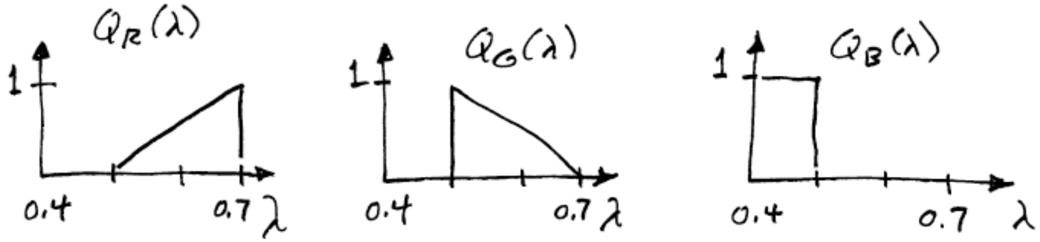
Polar coordinate representation of color

- This suggests a polar coordinate interpretation of color
- Origin of system is at center of chromaticity diagram – corresponding to intersection of neutral axis in RGB sensor space with chromaticity plane
- Angle of chromaticity coordinate with respect to horizontal axis is a correlate of hue
- Distance from origin is a correlate of saturation





Sensor with response overlap in two channels



- Response for $0.4 \le \lambda < 0.5$ is same as before
- Consider response at $\lambda = 0.55$

$$R = 0.25$$
$$G = 0.75$$
$$B = 0.00$$

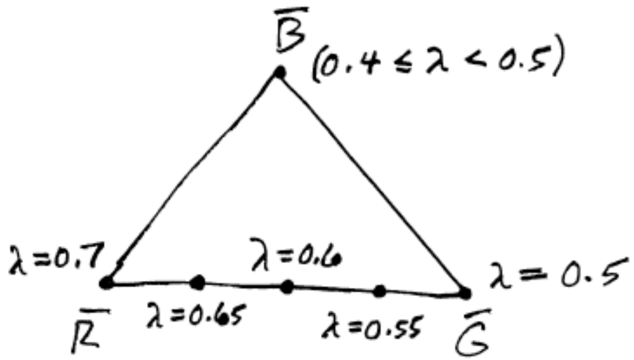
$$r = \frac{R}{R+G+B} = \frac{0.25}{0.25+0.75+0.00} = 0.25$$

Similarly, $g = 0.75$

$b = 0.00$

Spectral locus for sensor with response overlap in two channels

λ	(r,g,b)
[0.4,0.5)	(0,0,1)
0.50	(0,1,0)
0.55	(0.25,0.75,0.0)
0.60	(0.5,0.5,0.0)
0.65	(0.75,0.25,0.0)
0.70	(1,0,0)



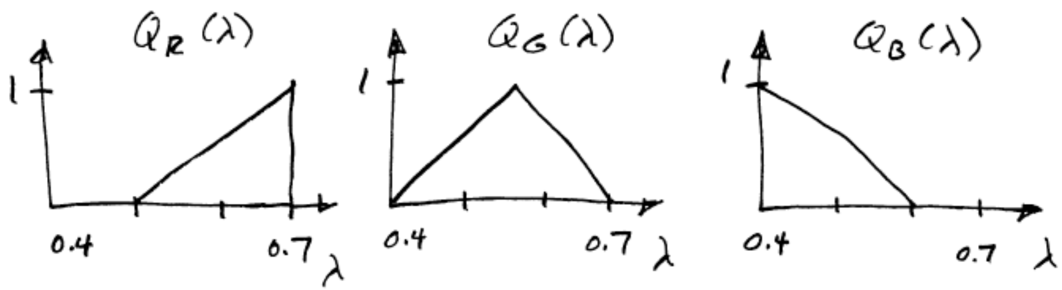
- Each wavelength between 0.5 and 0.7 μm corresponds to a unique chromaticity coordinate
- What is the gamut for this sensor?

Credit: Prof. Jan Allebach ECE638

Chromaticity gamut for sensor with two channel overlap

- Chromaticity gamut is same as that for block sensor

Sensor with three channel overlap



- Response at 0.45 μm

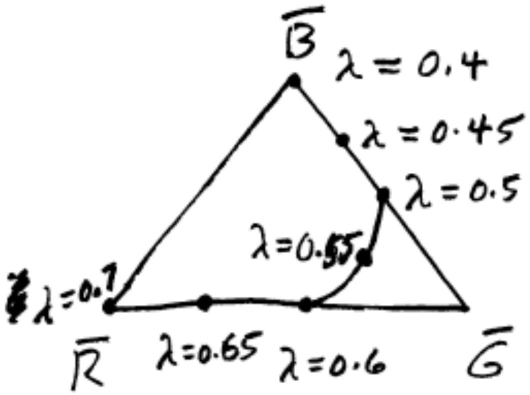
$$(R,G,B) = (0, 1/3, 3/4)$$
$$R + G + B = 13/12$$
$$(r,g,b) = (0, 4/13, 9/13)$$
$$= (0.0, 0.31, 0.69)$$

- Response at 0.55 μm

$$(R,G,B) = (1/4, 1, 1/4)$$
$$R + G + B = 3/2$$
$$(r,g,b) = (1/6, 2/3, 1/6)$$
$$= (0.167, 0.667, 0.167)$$

Spectral locus for sensor with 3 channel overlap

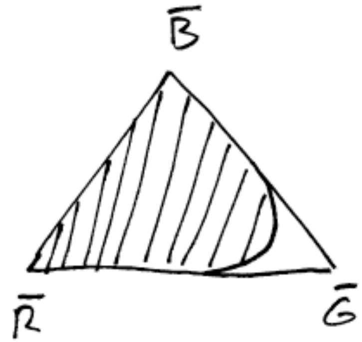
λ	(r,g,b)
0.40	(0,0,1)
0.45	(0.0,0.31,0.69)
0.50	(0.0,0.57,0.43)
0.55	(0.17,0.67,0.17)
0.60	(0.43,0.57,0.0)
0.65	(0.69,0.31,0.0)
0.70	(1,0,0)



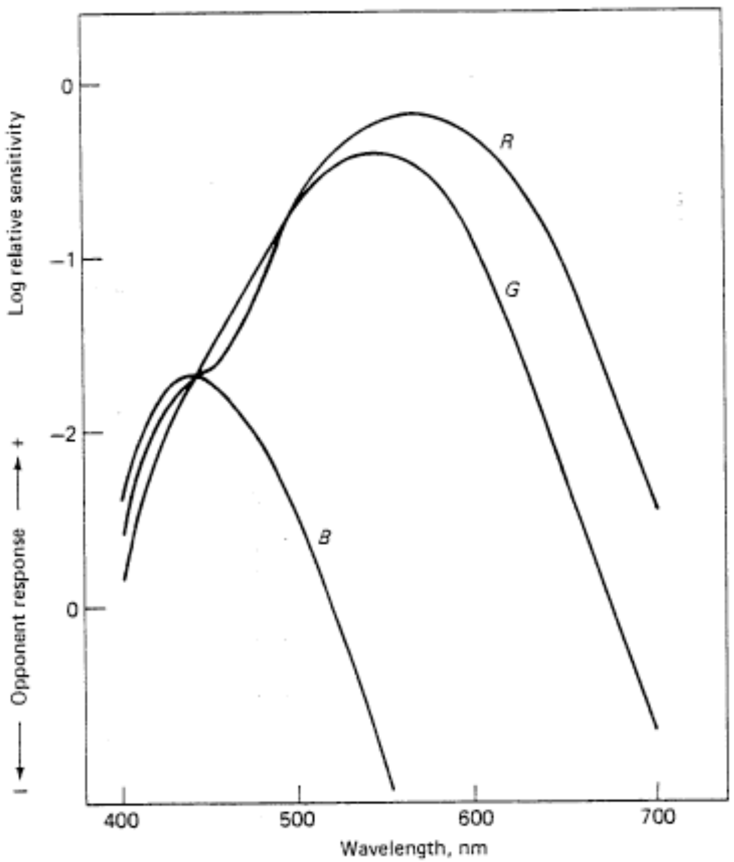
- Now every wavelength corresponds to a unique chromaticity coordinate
- What is the downside of the using this sensor?

Chromacity gamut for 3 channel overlap sensor

- The set of all realizable chromaticities is indicated by the shaded region
- Because it is impossible to excite only the green sensor by itself, we cannot get to the $\overline{G}$ vertex
- Thus we see that while overlap of sensor responses is desirable from the standpoint of yielding a unique chromaticity coordinate for each distinct wavelength, it also limits the sensor gamut



Cone sensitivity functions for the HVS



- reproduced from Boynton, 1992, p. 153.
- data is due to Smith and Pokorny, 1975.

Limitations of trichromatic theory

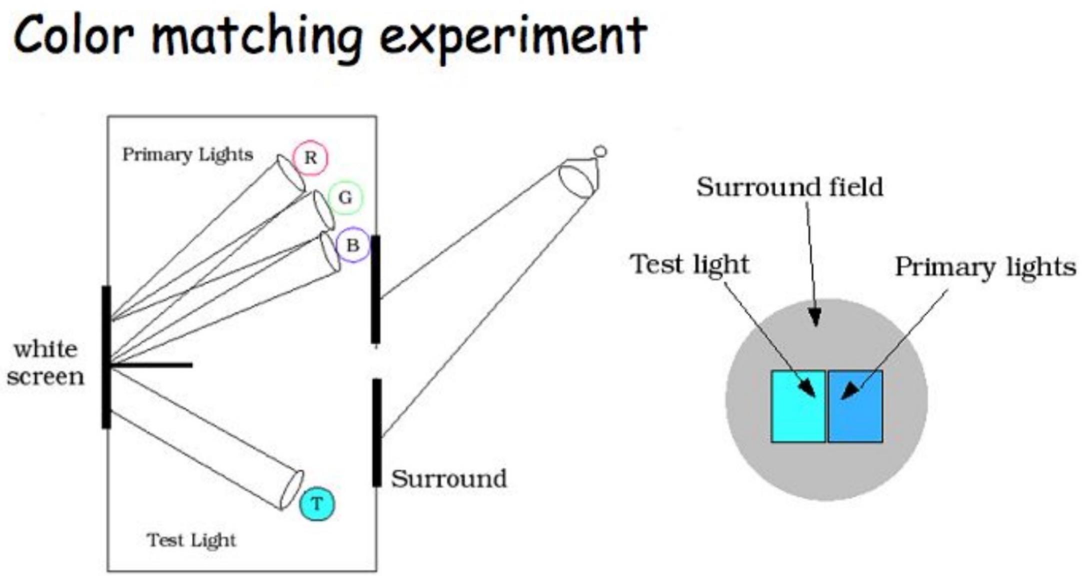
- Does not yield a uniform color space
- Fails to account for color opponency
- Does not predict color appearance

Limitations of Grassman's laws

- Grassman's laws provide a basic description of the results of color matching experiments
- But they don't provide:
  - general basis for understanding why the laws hold
  - a theory that can easily describe a wider range of situations

Primaries

- In order to understand color matching experiment, we need to develop a spectral model for the match stimulus



Additive mixture of primaries

- Consider a mixture of three primaries, each of which is described by its spectral density

$$P(\lambda) = p_R P_R(\lambda) + p_G P_G(\lambda) + p_B P_B(\lambda)$$

- The amounts of the three primaries are given by the constants (p_R, p_G, p_B)
- For convenience, we identify the primaries as R,G, and B. However, at this point, their corresponding spectral densities are completely arbitrary

Sensor response to additive primary mixture

$$R_P = \int P(\lambda) Q_R(\lambda) \mathrm{d}\lambda$$

$$G_P = \int P(\lambda) Q_G(\lambda) \mathrm{d}\lambda$$

$$B_P = \int P(\lambda) Q_B(\lambda) \mathrm{d}\lambda$$

- Consider R channel **response** only

$$\begin{aligned}
R_P &= \int \left[p_R P_R(\lambda) + p_G P_G(\lambda) + p_B P_B(\lambda) \right] Q_R(\lambda) \mathrm{d}\lambda \\
&= p_R \int P_R(\lambda) Q_R(\lambda) \mathrm{d}\lambda + \\
&\quad p_G \int P_G(\lambda) Q_R(\lambda) \mathrm{d}\lambda + \\
&\quad p_B \int P_B(\lambda) Q_R(\lambda) \mathrm{d}\lambda
\end{aligned}$$

Sensor response (cont.)

- The G and B channel **responses** can be expressed similarly
- Define

$$\vec{C}_P = \begin{bmatrix} R_P \\ G_P \\ B_P \end{bmatrix}, \quad \vec{p} = \begin{bmatrix} p_R \\ p_G \\ p_B \end{bmatrix}$$

$$\mathbf{A} = \left[a_{ij}\right], \quad a_{ij} = \int P_j(\lambda) Q_i(\lambda) \mathrm{d}\lambda, \quad i,j = R,G,B$$

- Then we can write $\vec{C}_P = \mathbf{A}\vec{p}$

Interpretation of the matrix A

$$\mathbf{A} = \left[a_{ij}\right], \quad a_{ij} = \int P_j(\lambda) Q_i(\lambda)\,\mathrm{d}\lambda, \quad i, j = R, G, B$$

- The i-th row is the response of channel i to all three primaries.
- The j-th column is the response of all three channels to primary j.
- For color matching, it is important that A be nonsingular (invertible)
- The following three statements are equivalent:
  - A is nonsingular.
  - The 3-tuple response of one channel to all three primaries cannot be written as a linear combination of the 3-tuple responses of the other two channels to all three primaries.
  - The 3-tuple response of all three channels to one primary cannot be written as a linear combination of the 3-tuple responses of all three channels to the other two primaries.
- In this case, we say that the primaries are (visually) independent.

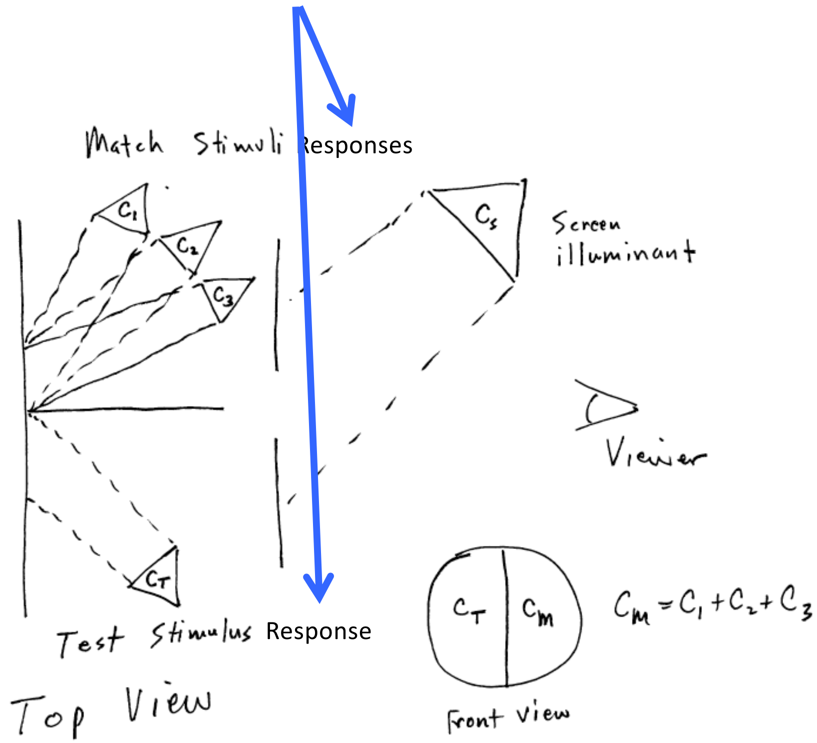
Solution to color matching experiment

- $\vec{C}_T$ **response** to test stimulus
- $\vec{C}_P$ **response** to match stimulus
- Match condition

$$\vec{C}_T = \vec{C}_P = \mathbf{A}\vec{p}$$

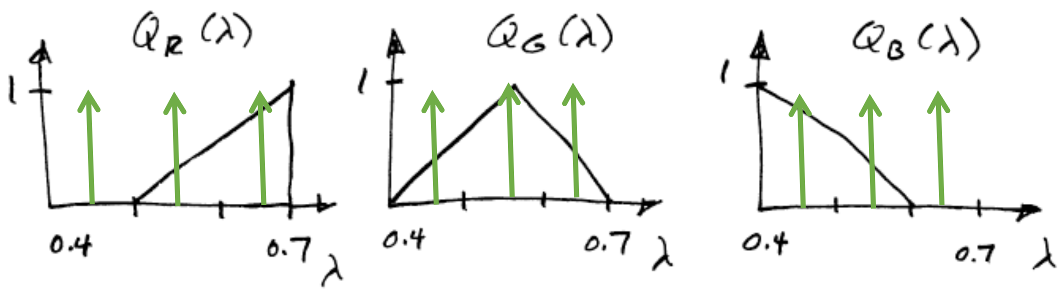
- Amount of each primary required for match

$$\vec{p} = \mathbf{A}^{-1}\vec{C}_T$$



Example 1: three channel overlap sensor and monochromatic primaries

- Sensor response functions



- Primaries

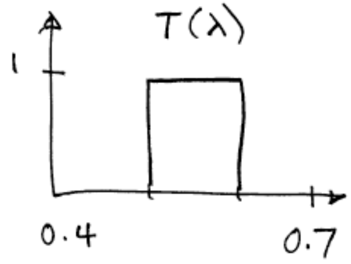
$$P_R(\lambda) = \delta(\lambda - 0.65)$$
$$P_G(\lambda) = \delta(\lambda - 0.55)$$
$$P_B(\lambda) = \delta(\lambda - 0.45)$$

- Primary response matrix

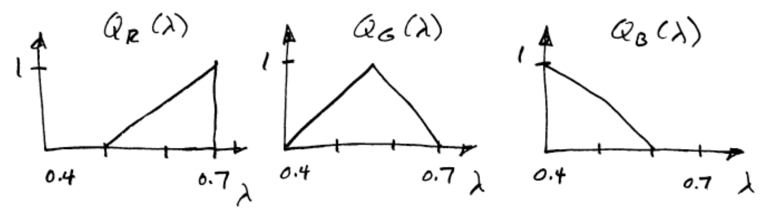
$$\mathbf{A} = \begin{bmatrix} 0.75 & 0.25 & 0.0 \\ 0.33 & 1.0 & 0.33 \\ 0.0 & 0.25 & 0.75 \end{bmatrix}$$

Example 1 (cont.)

- Test stimulus



- Sensor response functions and response to test stimulus



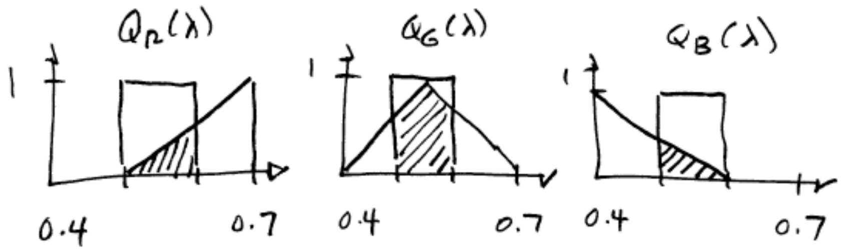
$$\vec{C}_T = \begin{bmatrix} 0.025 \\ 0.088 \\ 0.025 \end{bmatrix}$$

- Match amount of primaries

$$\vec{p} = \mathbf{A}^{-1}\vec{C}_T = \begin{bmatrix} 1.524 & -0.429 & 0.191 \\ -0.571 & 1.286 & -0.571 \\ 0.191 & -0.429 & 1.524 \end{bmatrix} \begin{bmatrix} 0.025 \\ 0.088 \\ 0.025 \end{bmatrix} = \begin{bmatrix} 0.0051 \\ 0.0846 \\ 0.0051 \end{bmatrix}$$

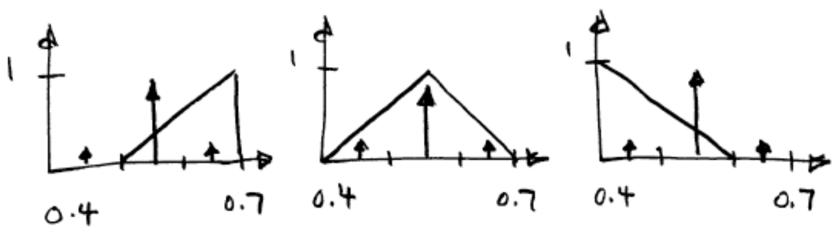
Interpretation for Example 1

- Response to test stimulus



$$\vec{C}_T = \begin{bmatrix} 0.025 \\ 0.088 \\ 0.025 \end{bmatrix}$$

- Amount of primaries required to achieve a match



$$\vec{p} = \begin{bmatrix} 0.0051 \\ 0.0846 \\ 0.0051 \end{bmatrix}$$

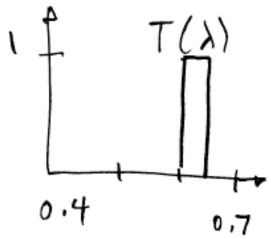
Impulses are scaled up by 10x

Interpretation (cont.)

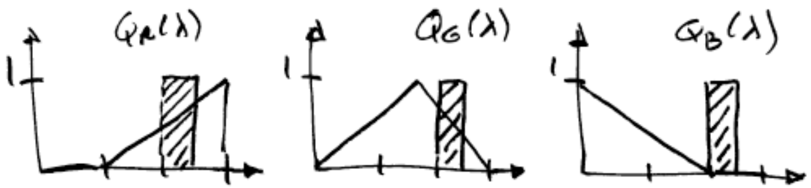
- Since primary at 0.55 μm, excites all three channels, might expect to use it alone.
- However, it doesn't provide same amount of excitation of R and B, relative to excitation of G, as does test stimulus.
- Therefore, we add a small amount of primaries at 0.45 and 0.65 μm to boost response of R and B.
- However, G channel also responds to these primaries; so we must decrease amount of primary at 0.55 μm to restore response in G channel to proper level.
- But this also decreases excitation of R and B…
- So we solve a set of simultaneous equations to get answer.

Example 2: Same sensor and primaries, but new test stimulus

- Test stimulus



- Response to test stimulus

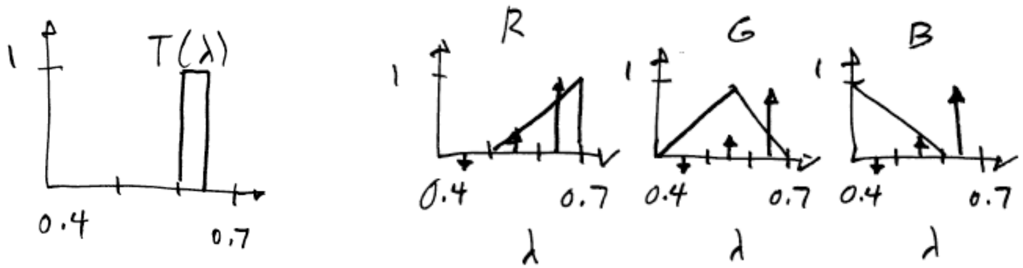


$$\vec{C}_T = \begin{bmatrix} 0.0625 \\ 0.0500 \\ 0.0000 \end{bmatrix}$$

Example 2 (cont.)

- Match amount of primaries

$$\vec{p} = \mathbf{A}^{-1}\vec{C}_T = \begin{bmatrix} 1.5214 & -0.4274 & 0.1880 \\ -0.5641 & 1.2821 & -0.5641 \\ 0.1880 & -0.4274 & 1.5214 \end{bmatrix} \begin{bmatrix} 0.0625 \\ 0.0500 \\ 0.0000 \end{bmatrix} = \begin{bmatrix} 0.0737 \\ 0.0288 \\ -0.0096 \end{bmatrix}$$



Impulses are scaled up by 10x

- Why is the amount of the primary at 0.45 μm negative?
- What is the physical meaning of this fact?

Interpretation for example 2

- Test stimulus primarily stimulates R channel, but also stimulates G channel too.
- Ratio of excitation of R channel relative to that of G channel by primary at 0.65 μm (0.75:0.33) is greater than that for test stimulus (0.0625:0.0500) $\Rightarrow$ we need to use some of primary at 0.55 μm to boost response of G channel.
- But, primary at 0.55 μm also excites B channel, whereas the test stimulus provides no excitation of B channel.
- So we need a negative amount of the primary at 0.45 μm to cancel the unwanted response of the B channel.

Physical meaning of the negative primary

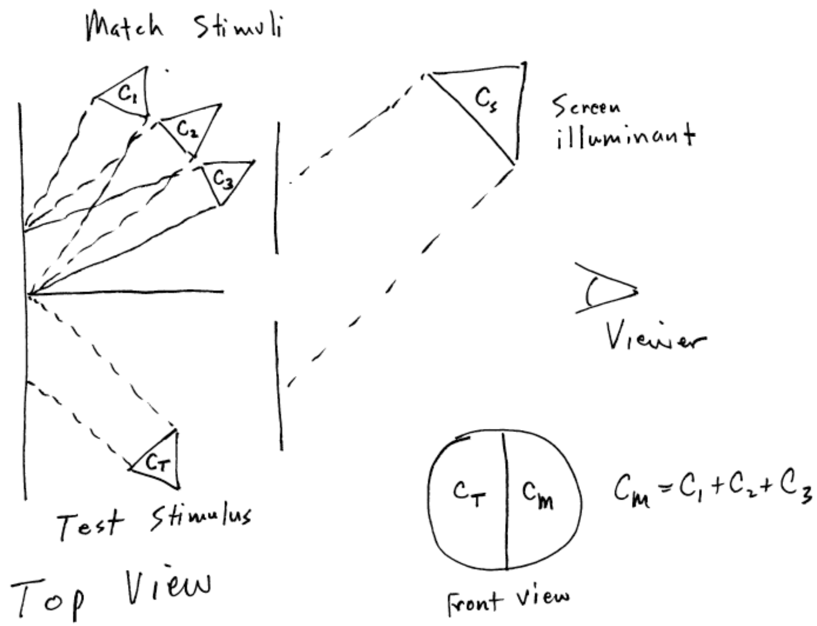
- Match is achieved by adding the negative primary to the test stimulus

$$T(\lambda) + 0.0096\delta(\lambda - 0.45) \doteq$$
$$0.0737\delta(\lambda - 0.65) + 0.0288\delta(\lambda - 0.55)$$

- This will desaturate the very red test stimulus

Application to color matching experiment

- We move the match stimulus C_3 to the lower side of the diagram.
- It will desaturate the test stimulus C_T
- Then we can achieve a match.



Transformation between sets of primaries

- Consider two sets of primaries $[P_R(\lambda), P_G(\lambda), P_B(\lambda)]$ and $[P_X(\lambda), P_Y(\lambda), P_Z(\lambda)]$ and a test stimulus $T(\lambda)$
- Suppose we know that primary amounts $\vec{p}_{RGB} = [p_R, p_G, p_B]^T$ yield a match with the RGB primaries to the test stimulus.
- Can we find the match amount $\vec{p}_{XYZ} = [p_X, p_Y, p_Z]^T$ for the XYZ primaries to the same stimulus from a knowledge of $\vec{p}_{RGB} = [p_R, p_G, p_B]^T$ without knowing $T(\lambda)$?
- The answer is "yes" (under certain conditions on the primaries).

Color matching functions

- We previously showed that we could express the response of a sensor to an arbitrary stimulus in terms of the response of that sensor to monochromatic stimuli at all wavelengths.
- We similarly can express the 3-tuple primary amounts required to match any stimulus in terms of the primary amounts required to match all monochromatic stimuli.

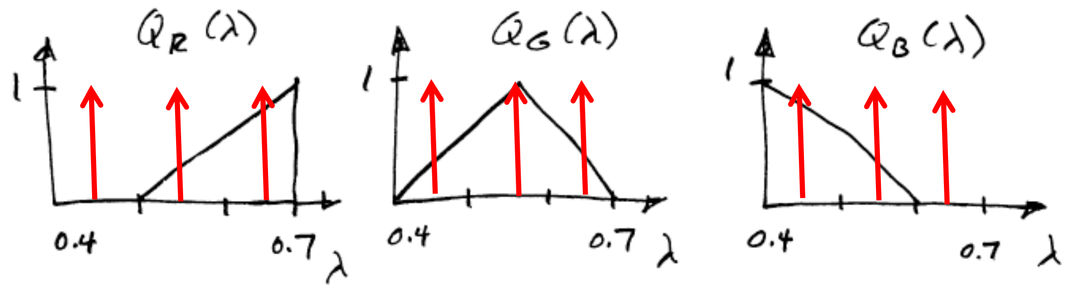
$$S(\lambda_0)\delta(\lambda-\lambda_0) \underset{\text{Sensor System}}{\longrightarrow} S(\lambda_0)\begin{bmatrix} Q_R(\lambda_0) \\ Q_G(\lambda_0) \\ Q_B(\lambda_0) \end{bmatrix}$$

Color matching functions (cont.)

- Given primaries $\left[P_R(\lambda), P_G(\lambda), P_B(\lambda)\right]$, let $\left[\bar{r}(\mu), \bar{g}(\mu), \bar{b}(\mu)\right]$ denote the amount of these primaries required to match the stimulus $\delta(\lambda-\mu)$ for each fixed wavelength μ
- Thus, we have $\vec{p} = \mathbf{A}^{-1}\vec{C}_T$
  - where $\vec{p} = \left[\bar{r}(\mu), \bar{g}(\mu), \bar{b}(\mu)\right]^{\mathrm{T}}$,
  - the matrix $\mathbf{A}^{-1}$ is given as before,
  - and $\vec{C}_T = \left[Q_R(\mu), Q_G(\mu), Q_B(\mu)\right]^{\mathrm{T}}$
- Combining these results, we obtain $\begin{bmatrix} \bar{r}(\mu) \\ \bar{g}(\mu) \\ \bar{b}(\mu) \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} Q_R(\mu) \\ Q_G(\mu) \\ Q_B(\mu) \end{bmatrix}$

Example: three channel overlap sensor and monochromatic primaries

- Sensor response functions



- Primaries

$$P_R(\lambda) = \delta(\lambda - 0.65)$$
$$P_G(\lambda) = \delta(\lambda - 0.55)$$
$$P_B(\lambda) = \delta(\lambda - 0.45)$$

- Primary response matrix

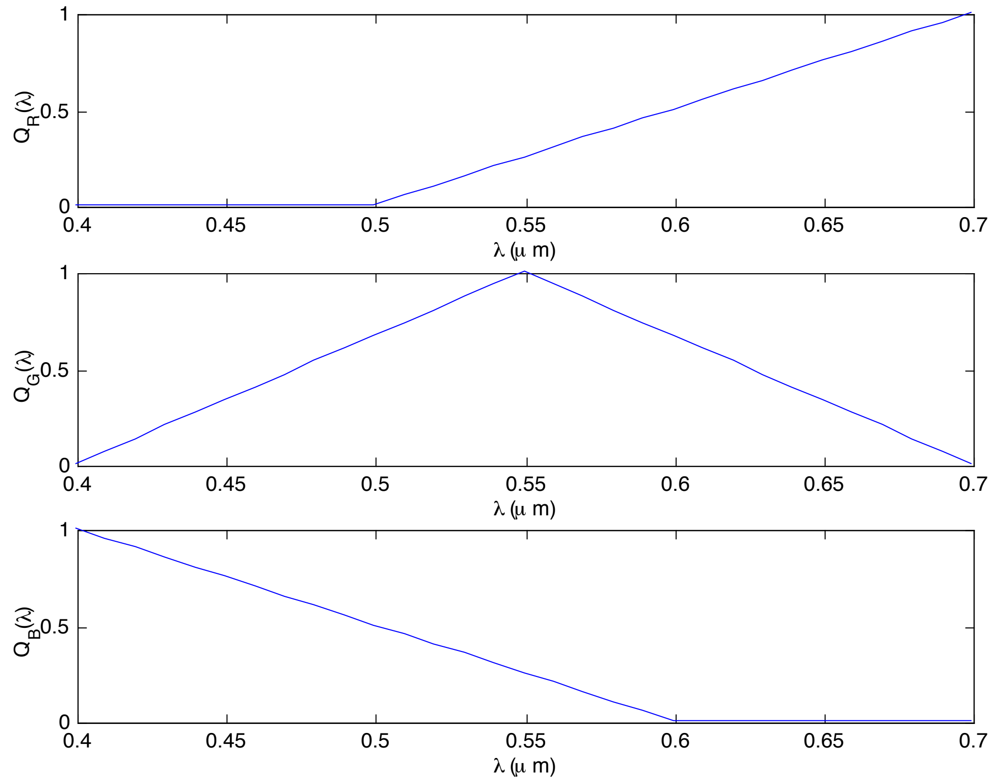
$$\mathbf{A} = \begin{bmatrix} 0.75 & 0.25 & 0.0 \\ 0.33 & 1.0 & 0.33 \\ 0.0 & 0.25 & 0.75 \end{bmatrix}$$

Computation of color matching functions

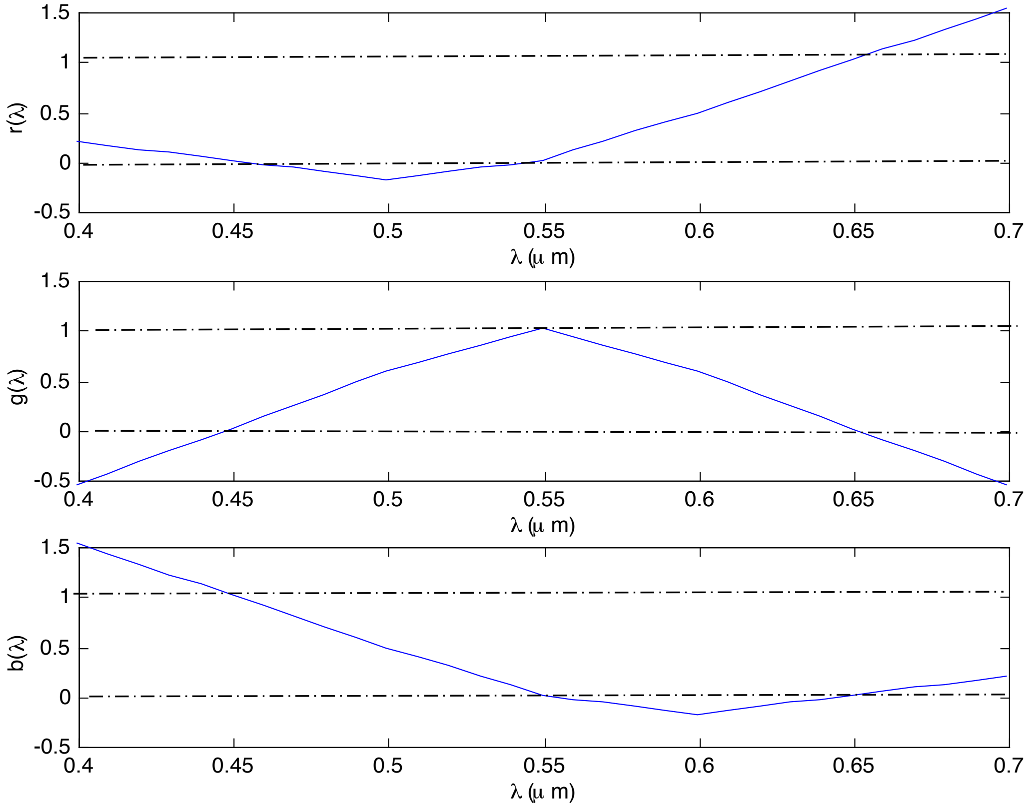
$$\begin{bmatrix} \bar{r}(\mu) \\ \bar{g}(\mu) \\ \bar{b}(\mu) \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} Q_R(\mu) \\ Q_G(\mu) \\ Q_B(\mu) \end{bmatrix}$$

$$= \begin{bmatrix} 1.521 & -0.427 & 0.188 \\ -0.564 & 1.282 & -0.564 \\ 0.188 & -0.427 & 1.521 \end{bmatrix} \begin{bmatrix} Q_R(\mu) \\ Q_G(\mu) \\ Q_B(\mu) \end{bmatrix}$$

Three channel overlap sensor response functions



Color matching functions for three channel overlap sensor



Note that at wavelength 0.65 μ m,

$$\begin{bmatrix} \bar{r}(0.65) \\ \bar{g}(0.65) \\ \bar{b}(0.65) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Use of Color Matching Functions (1/2)

- Consider an arbitrary stimulus $T(\lambda)$
- Let us examine it at a specific wavelength μ
- So we have $T_\mu = T(\mu)\delta(\lambda - \mu)$
- Then, we can match this monochromatic stimulus with primary amounts

$$\vec{p} = \mathbf{A}^{-1}\vec{C}_T \qquad \vec{p}^{\,T_\mu}_{RGB} = \begin{bmatrix} p_R^{T_\mu} \\ p_G^{T_\mu} \\ p_B^{T_\mu} \end{bmatrix} = \begin{bmatrix} T(\mu)(\bar{r}(\mu) \\ T(\mu)(\bar{g}(\mu) \\ T(\mu)(\bar{b}(\mu) \end{bmatrix}$$

Use of color matching functions (2/2)

- Now we form the superposition of the stimulus over all wavelengths

$$T(\lambda) = \int T(\mu)\delta(\lambda - \mu)\mathrm{d}\mu$$

- By linearity of the sensor response, we can express the amount of the primaries $\left[P_R(\lambda), P_G(\lambda), P_B(\lambda)\right]$ required to match $T(\lambda)$ as

$$\vec{p}_{RGB}^{\,T} = \begin{bmatrix} p_R^T \\ p_G^T \\ p_B^T \end{bmatrix} = \begin{bmatrix} \int T(\mu)\bar{r}(\mu)\mathrm{d}\mu \\ \int T(\mu)\bar{g}(\mu)\mathrm{d}\mu \\ \int T(\mu)\bar{b}(\mu)\mathrm{d}\mu \end{bmatrix}$$

Sensor analogy

$$\vec{p}_{RGB}^{\,T} = \begin{bmatrix} p_R^T \\ p_G^T \\ p_B^T \end{bmatrix} = \begin{bmatrix} \int T(\mu)\bar{r}(\mu)\mathrm{d}\mu \\ \int T(\mu)\bar{g}(\mu)\mathrm{d}\mu \\ \int T(\mu)\bar{b}(\mu)\mathrm{d}\mu \end{bmatrix}$$

- Note that the color matching functions $\left[\bar{r}(\mu), \bar{g}(\mu), \bar{b}(\mu)\right]$ play the role of sensor response functions with respect to the stimulus $T(\mu)$
- Rather than yielding sensor responses to that stimulus, they yield amounts of the primaries $\left[P_R(\lambda), P_G(\lambda), P_B(\lambda)\right]$ required to match that stimulus when viewed by the sensor with response functions $\left[Q_R(\mu), Q_G(\mu), Q_B(\mu)\right]$

Relation between sensor and color matching functions

$$\vec{p}_{RGB}^{\,T} = \begin{bmatrix} p_R^T \\ p_G^T \\ p_B^T \end{bmatrix} = \begin{bmatrix} \int T(\mu)\bar{r}(\mu)\mathrm{d}\mu \\ \int T(\mu)\bar{g}(\mu)\mathrm{d}\mu \\ \int T(\mu)\bar{b}(\mu)\mathrm{d}\mu \end{bmatrix}$$

- Where does the sensor come into play here?

$$\begin{bmatrix} \bar{r}(\mu) \\ \bar{g}(\mu) \\ \bar{b}(\mu) \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} Q_R(\mu) \\ Q_G(\mu) \\ Q_B(\mu) \end{bmatrix}$$

Sensor response to monochromatic stimulus

Relation between sensor and color matching functions (cont.)

- Sensor subspace

$$\mathbf{A} = \left[a_{ij}\right], \quad a_{ij} = \int P_j(\lambda) Q_i(\lambda) \mathrm{d}\lambda, \quad i, j = R, G, B$$

Sensor response to primaries

- Since color matching functions are a linear combination of the sensor response functions, the color matching function observer sees colors the same way as does the original sensor

$$\begin{bmatrix} \bar{r}(\mu) \\ \bar{g}(\mu) \\ \bar{b}(\mu) \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} Q_R(\mu) \\ Q_G(\mu) \\ Q_B(\mu) \end{bmatrix}$$

Review: Primary mixture and sensor response

$$P(\lambda) = p_R P_R(\lambda) + p_G P_G(\lambda) + p_B P_B(\lambda)$$

$$R_P = \int P(\lambda) Q_R(\lambda) \mathrm{d}\lambda$$

$$G_P = \int P(\lambda) Q_G(\lambda) \mathrm{d}\lambda$$

$$B_P = \int P(\lambda) Q_B(\lambda) \mathrm{d}\lambda$$

$$\vec{C}_P = \begin{bmatrix} R_P \\ G_P \\ B_P \end{bmatrix}, \quad \vec{p} = \begin{bmatrix} p_R \\ p_G \\ p_B \end{bmatrix}$$

$$\mathbf{A} = \left[a_{ij} \right], \quad a_{ij} = \int P_j(\lambda) Q_i(\lambda) \mathrm{d}\lambda, \quad i, j = R, G, B$$

$$\vec{C}_P = \mathbf{A}\vec{p}$$

Review: Match equation

- Response to test stimulus

$$R_T = \int T(\lambda) Q_R(\lambda) \mathrm{d}\lambda$$

$$G_T = \int T(\lambda) Q_G(\lambda) \mathrm{d}\lambda$$

$$B_T = \int T(\lambda) Q_B(\lambda) \mathrm{d}\lambda$$

$$\vec{C}_T = \begin{bmatrix} R_T \\ G_T \\ B_T \end{bmatrix}$$

- Match condition

$$\vec{C}_P = \vec{C}_T \Rightarrow \vec{C}_T = \mathbf{A}\vec{p}$$

- Match amounts of primaries

$$\vec{p} = \mathbf{A}^{-1}\vec{C}_T$$

Review: Color matching functions

- Given primaries $\left[P_R(\lambda), P_G(\lambda), P_B(\lambda)\right]$, let $\left[\bar{r}(\mu), \bar{g}(\mu), \bar{b}(\mu)\right]$ denote the amount of these primaries required to match the stimulus $\delta(\lambda - \mu)$ for each fixed wavelength μ
- Thus, we have $\vec{p} = \mathbf{A}^{-1}\vec{C}_T$
  - where $\vec{p} = \left[\bar{r}(\mu), \bar{g}(\mu), \bar{b}(\mu)\right]^{\mathrm{T}}$,
  - the matrix $\mathbf{A}^{-1}$ is given as before,
  - and $\vec{C}_T = \left[Q_R(\mu), Q_G(\mu), Q_B(\mu)\right]^{\mathrm{T}}$
- Combining these results, we obtain $\begin{bmatrix} \bar{r}(\mu) \\ \bar{g}(\mu) \\ \bar{b}(\mu) \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} Q_R(\mu) \\ Q_G(\mu) \\ Q_B(\mu) \end{bmatrix}$

Review: Computation of match amounts for an arbitrary stimulus

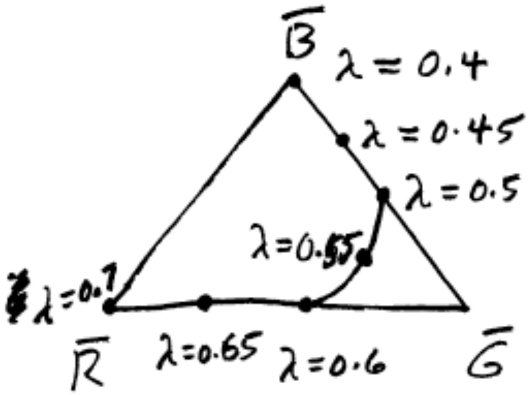
- Consider arbitrary stimulus $T(\lambda)$
- Match amounts are given by

$$\vec{p}^{\,T}_{RGB} = \begin{bmatrix} p^T_R \\ p^T_G \\ p^T_B \end{bmatrix} = \begin{bmatrix} \int T(\mu)\bar{r}(\mu)\mathrm{d}\mu \\ \int T(\mu)\bar{g}(\mu)\mathrm{d}\mu \\ \int T(\mu)\bar{b}(\mu)\mathrm{d}\mu \end{bmatrix}$$

- This looks like a sensor

Review: Spectral locus for sensor with 3 channel overlap

λ	(r,g,b)
0.40	(0,0,1)
0.45	(0.0,0.31,0.69)
0.50	(0.0,0.57,0.43)
0.55	(0.17,0.67,0.17)
0.60	(0.43,0.57,0.0)
0.65	(0.69,0.31,0.0)
0.70	(1,0,0)



- Now every wavelength corresponds to a unique chromaticity coordinate
- What is the downside of the using this sensor?

Credit: Prof. Jan Allebach ECE638

Chromaticity coordinates for primaries

- In same way that we did for the sensor response, we can define chromaticities $\left[r^T, g^T, b^T\right]$ corresponding to primary amounts $\left[p_R^T, p_G^T, p_B^T\right]$ required to match a given stimulus $T(\lambda)$ as viewed by a given sensor $[Q_R(\mu), Q_G(\mu), Q_B(\mu)]$

$$r^T = \frac{p_R^T}{p_R^T + p_G^T + p_B^T}$$

$$g^T = \frac{p_G^T}{p_R^T + p_G^T + p_B^T}$$

$$b^T = \frac{p_B^T}{p_R^T + p_G^T + p_B^T}$$

Spectral locus in primary chromaticity space

- For the special case of a monochromatic stimulus with wavelength λ, the primary amounts are given by the color matching functions evaluated at that wavelength.
- In this case, when plotted as a function of λ, the chromaticity coordinates yield the spectral locus.

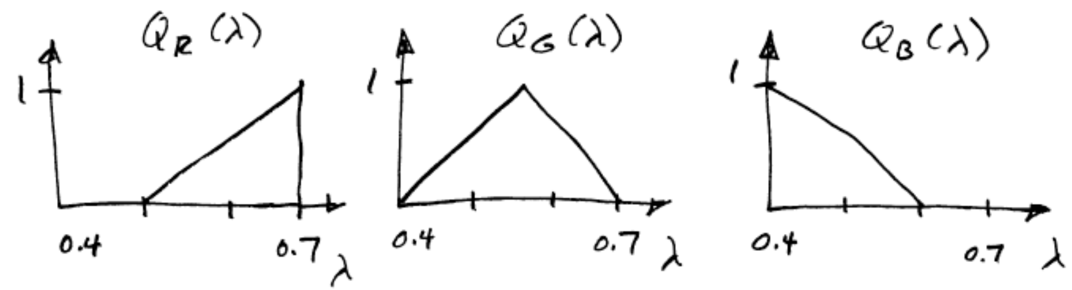
$$r(\lambda) = \frac{\bar{r}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$g(\lambda) = \frac{\bar{g}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

$$b(\lambda) = \frac{\bar{b}(\lambda)}{\bar{r}(\lambda) + \bar{g}(\lambda) + \bar{b}(\lambda)}$$

Example: three channel overlap sensor and monochromatic primaries

- Sensor response functions



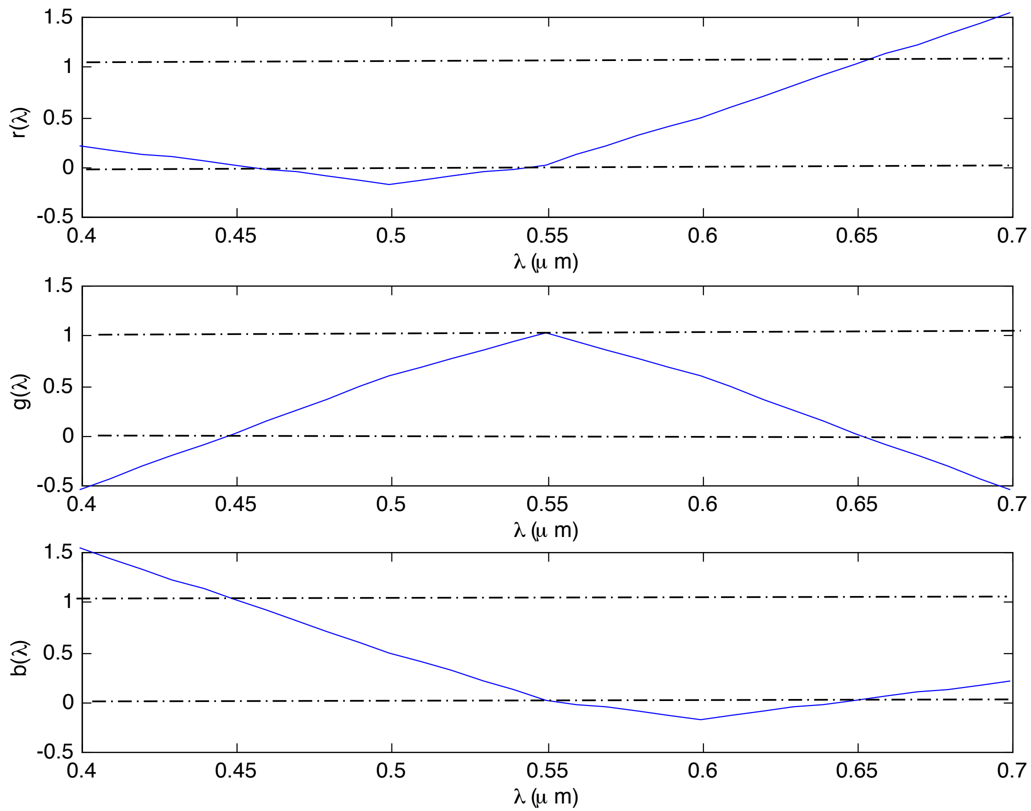
- Primaries

$$P_R(\lambda) = \delta(\lambda - 0.65)$$
$$P_G(\lambda) = \delta(\lambda - 0.55)$$
$$P_B(\lambda) = \delta(\lambda - 0.45)$$

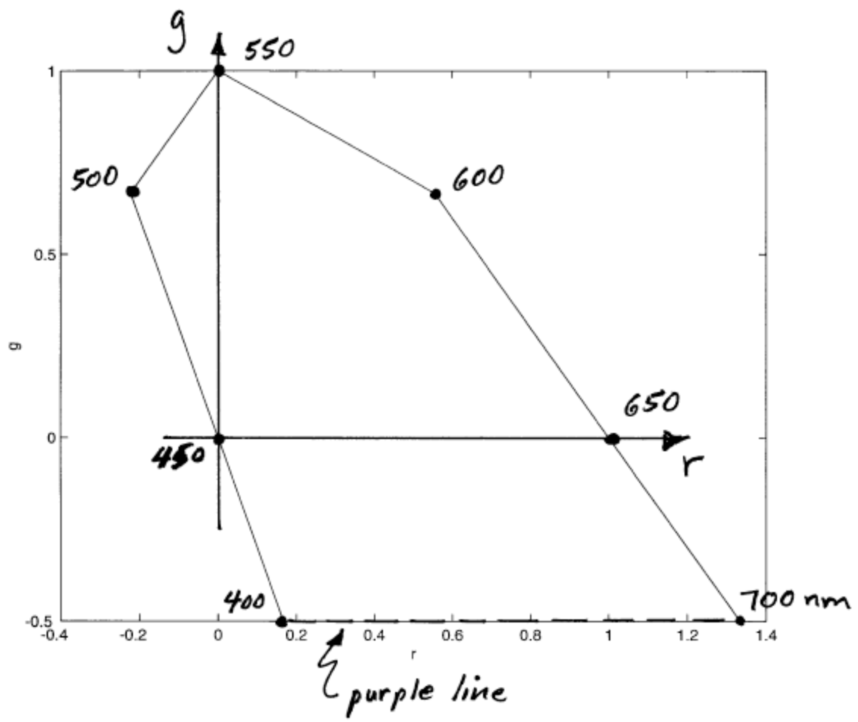
- Primary response matrix

$$\mathbf{A} = \begin{bmatrix} 0.75 & 0.25 & 0.0 \\ 0.33 & 1.0 & 0.33 \\ 0.0 & 0.25 & 0.75 \end{bmatrix}$$

Color matching functions

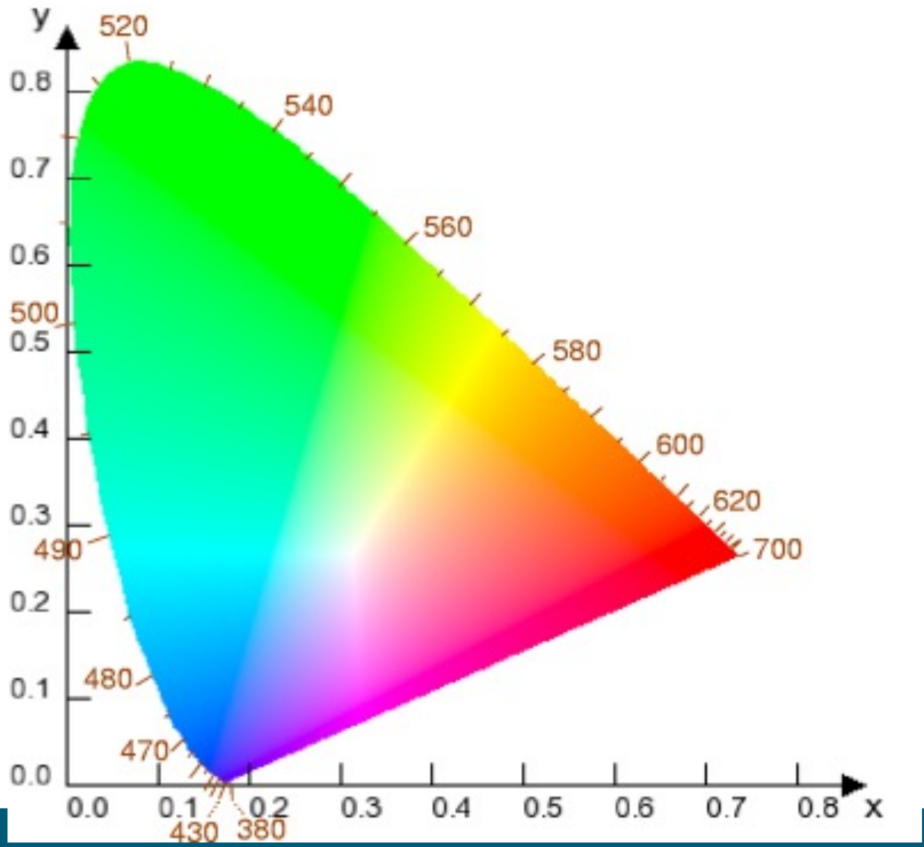


Spectral locus



- Note coordinates (1,0,0), (0,1,0), and (0,0,1) that occur at wavelengths 650, 550, and 450 nm, respectively.

Compare to common CIE (x, y) chromaticity diagram



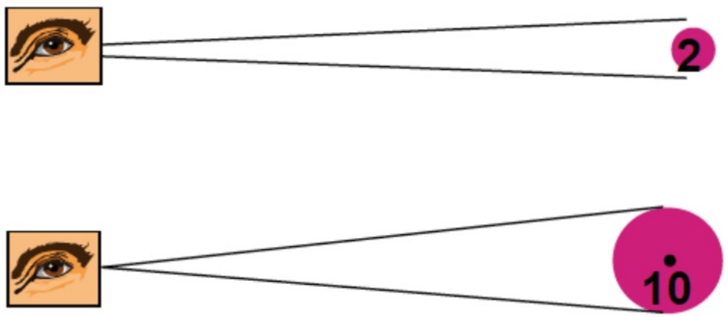
What is the CIE?*

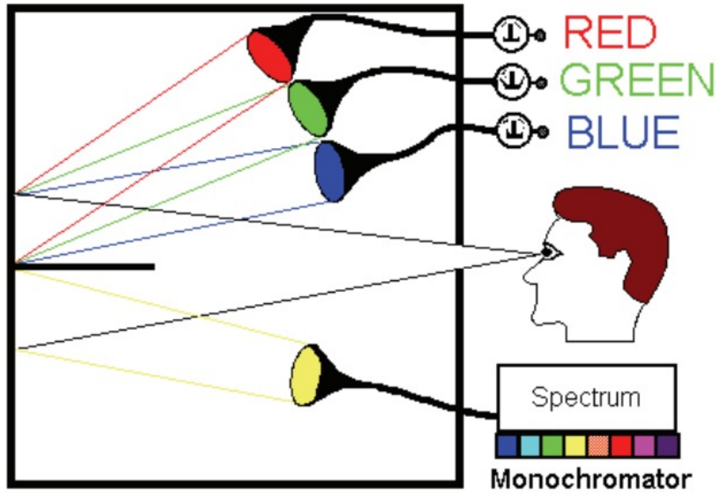
- International standards organization
- Commission Internationale de l'Eclairage (International Commission on Illumination)
- an organization devoted to international cooperation and exchange of information among its member countries on all matters relating to the science and art of lighting.
- Formed in 1913. Predecessor organization formed in 1903.
- http://members.eunet.at/cie/

*This material is largely taken from Wyszecki and Stiles.

Standard Observer

- 2^o viewing angle (1931)
- 10^o viewing angle (1964)



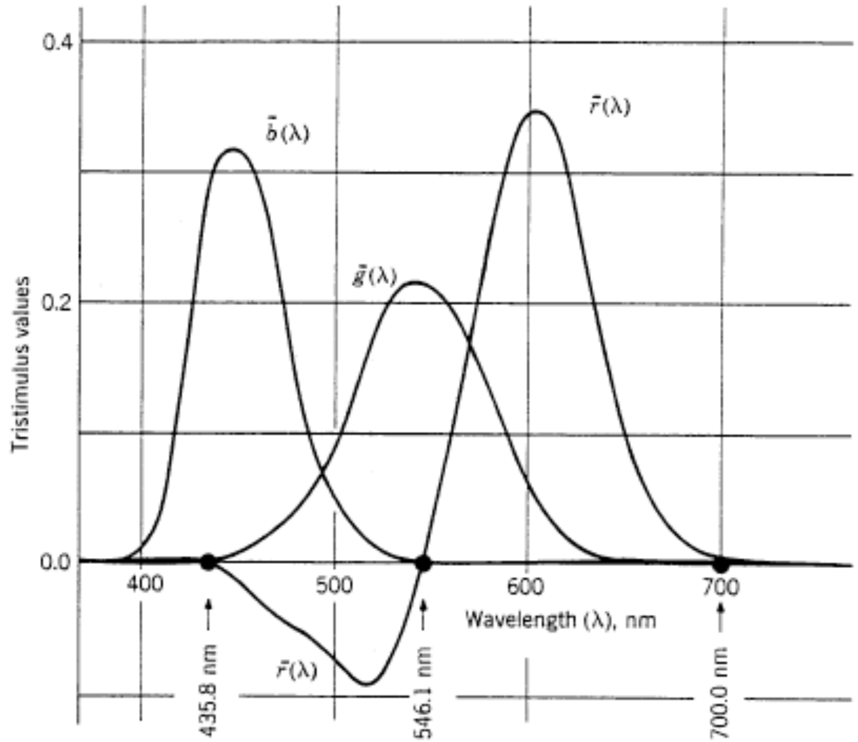


CIE 1931 standard RGB observer

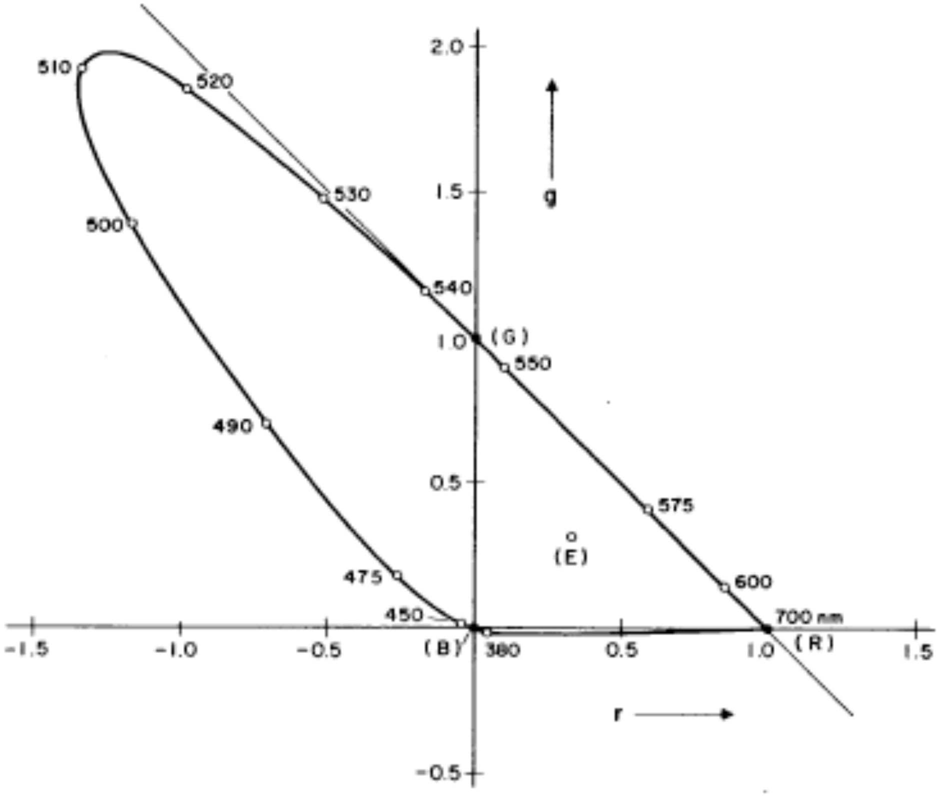
- Observer consists of color matching functions corresponding to monochromatic primaries
- Primaries
  - R – 700 nm
  - G – 546.1 nm
  - B – 435.8 nm
- Chosen to place chromaticity of equal energy stimulus E at center of (r-g) chromaticity diagram, i.e. at (0.333,0.333)

  $\Rightarrow$ that areas under color matching functions are identical.
- Based on observations in a 2 degree field of view using color matching method discussed earlier.

Color matching functions for 1931 CIE standard RGB observer

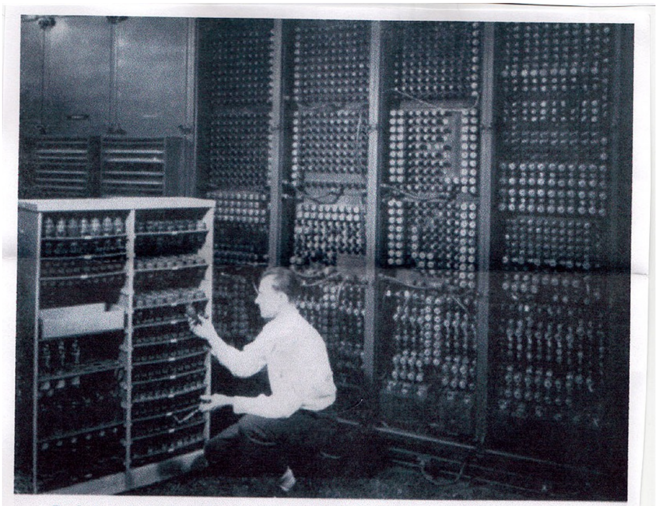


Chromaticity diagram for 1931 CIE standard RGB observer



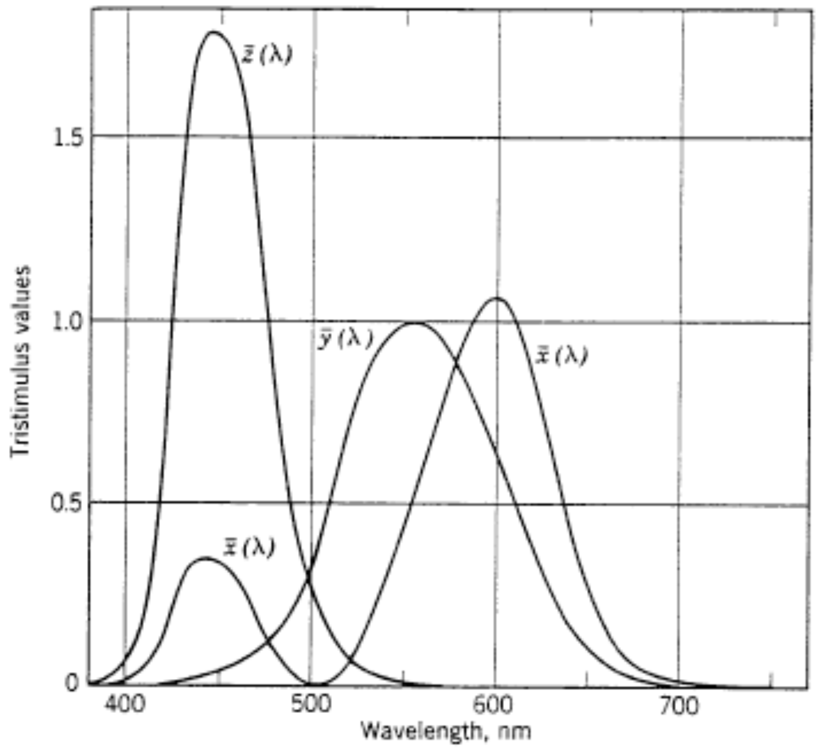
Why non-negative color matching functions?

- The 30-ton ENIAC computer with 19,000 vacuum tubes and 1,500 relays was placed in service in 1945.
- IBM introduced the 5150 PC in 1981.
- Color scientists were worried about calculation errors, especially those due to working with negative numbers.

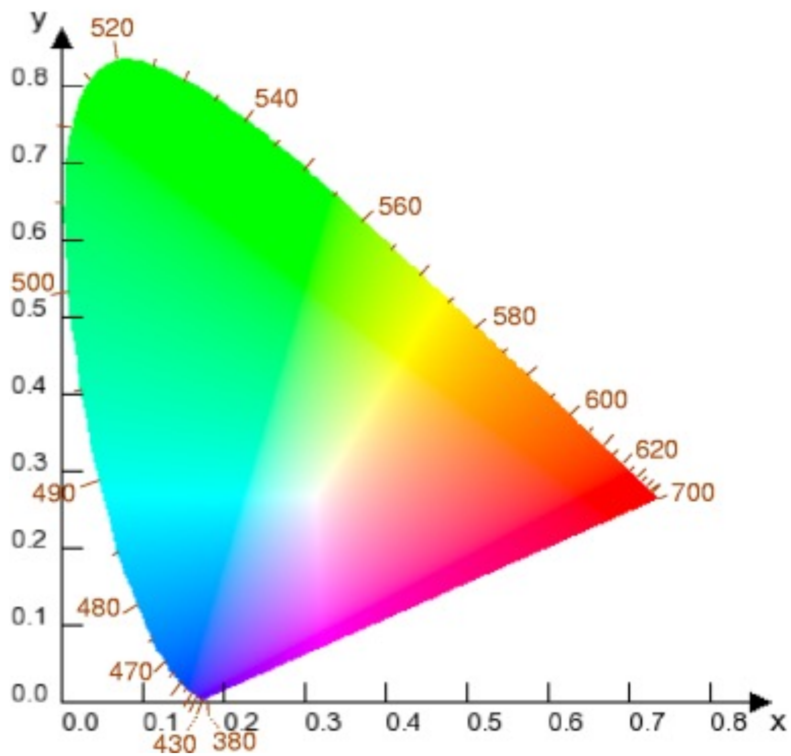


Replacing a bad tube meant checking among ENIAC's 19,000 possibilities.

Color matching functions for 1931 CIE standard XYZ observer



Chromaticity diagram for 1931 CIE standard XYZ observer



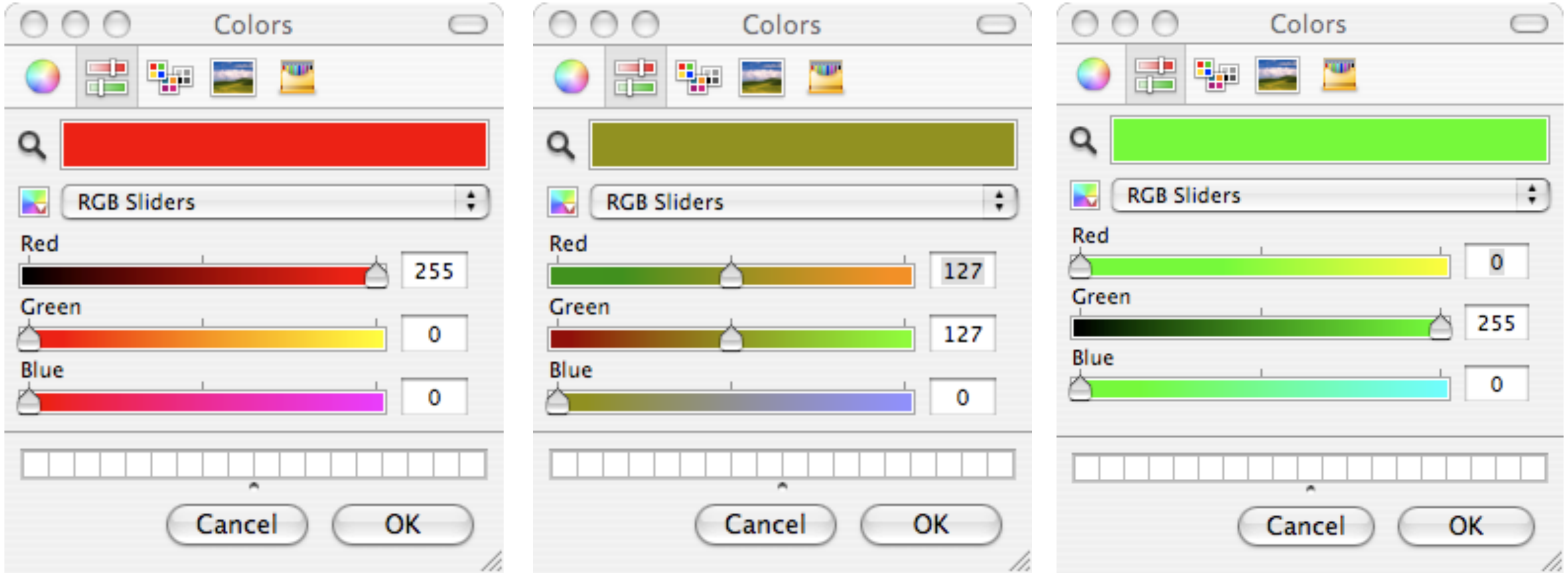
Limitations of trichromatic theory

- Does not yield a uniform color space
- Fails to account for color opponency
- Does not predict color appearance

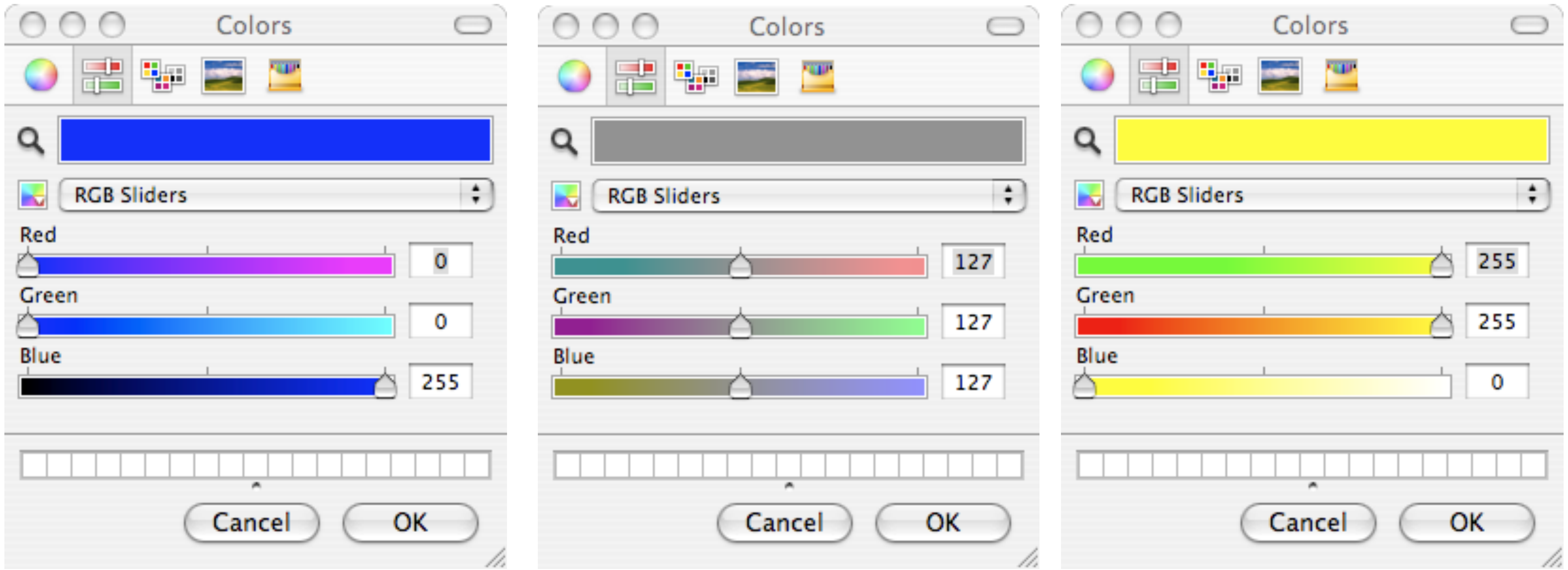
Opponent stage

- Trichromatic theory provides the basis for understanding whether or not two spectral power distributions will appear the same to an observer when viewed under the same conditions.
- However, the trichromatic theory will tell us nothing about the appearance of a stimulus.
- In the early 1900’s, Ewald Hering observed some properties of color appearance
  - Red and green never occur together – there is no such thing as a reddish green, or a greenish red
  - If I add a small amount of blue to green, it looks bluish-green. If I add more blue to green, it becomes cyan.
  - In contrast, if I add red to green, the green becomes less saturated. If I add enough red to green, the color appears gray, blue, or yellow
  - If I add even more red to green, the color appears red, but never reddish green

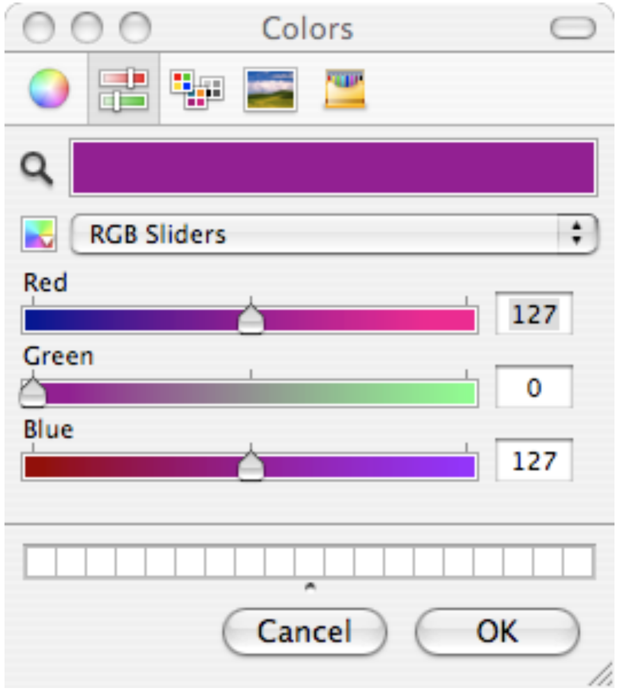
Red-green color opponency

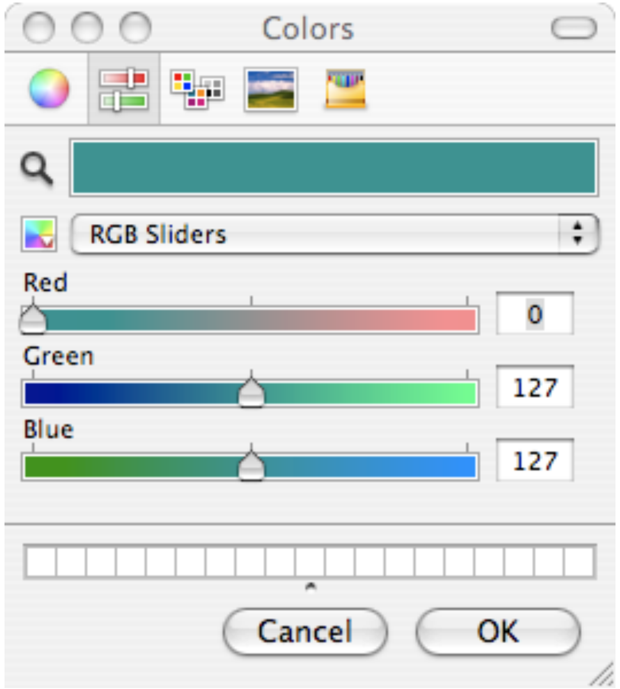


Blue-yellow color opponency



Red-blue and green-blue combinations

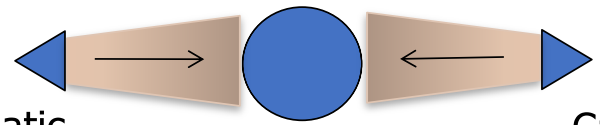




Opponent stage (cont.)

- Hering postulated that there existed two kinds of neural pathways in the visual system
  - Red-Green pathway fires fast if there is a lot of red, fires slowly if there is a lot of green
  - Blue-Yellow pathway fires fast if there is a lot of blue, fires slowly if there is a lot of yellow
- Hering provided no experimental evidence for his theory; and it was ignored for over 50 years

Hue Cancellation



- Observer looks at patch & makes two observations (no $S_C(\lambda)$ yet)
  1) Reddish or greenish (or neither)
  2) Bluish or yellowish (or neither)

Hue Cancellation (cont.)

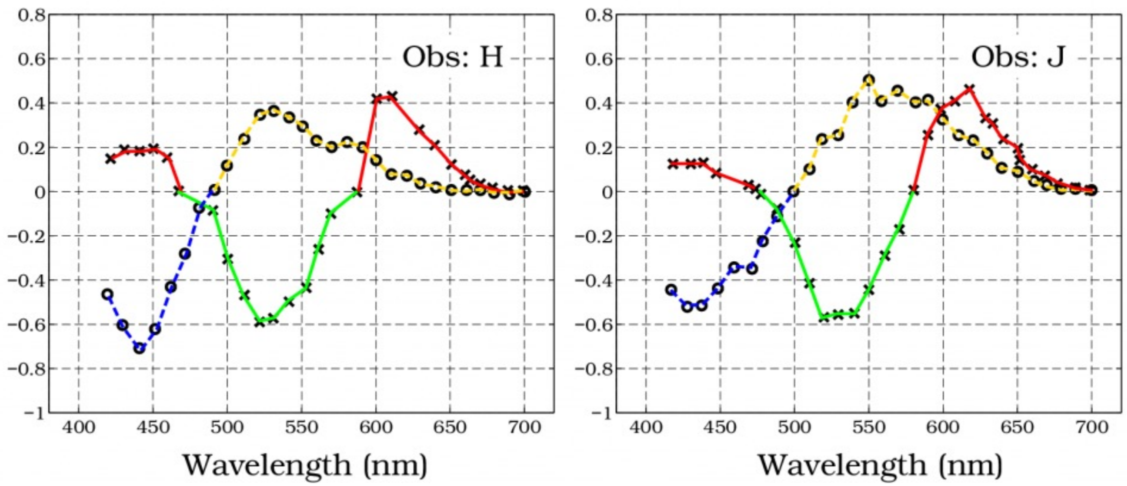
- Do two experiments separately

1) a. If subject said reddish, add enough green $S_C^G(\lambda)$
   to cancel reddish appearance
   b. If subject said greenish, add enough red $S_C^R(\lambda)$
   to cancel greenish appearance

2) Perform similar experiment for blue-yellow

Experimental evidence for opponency

- Hurvitch and Jameson hue cancellation experiment (1955)

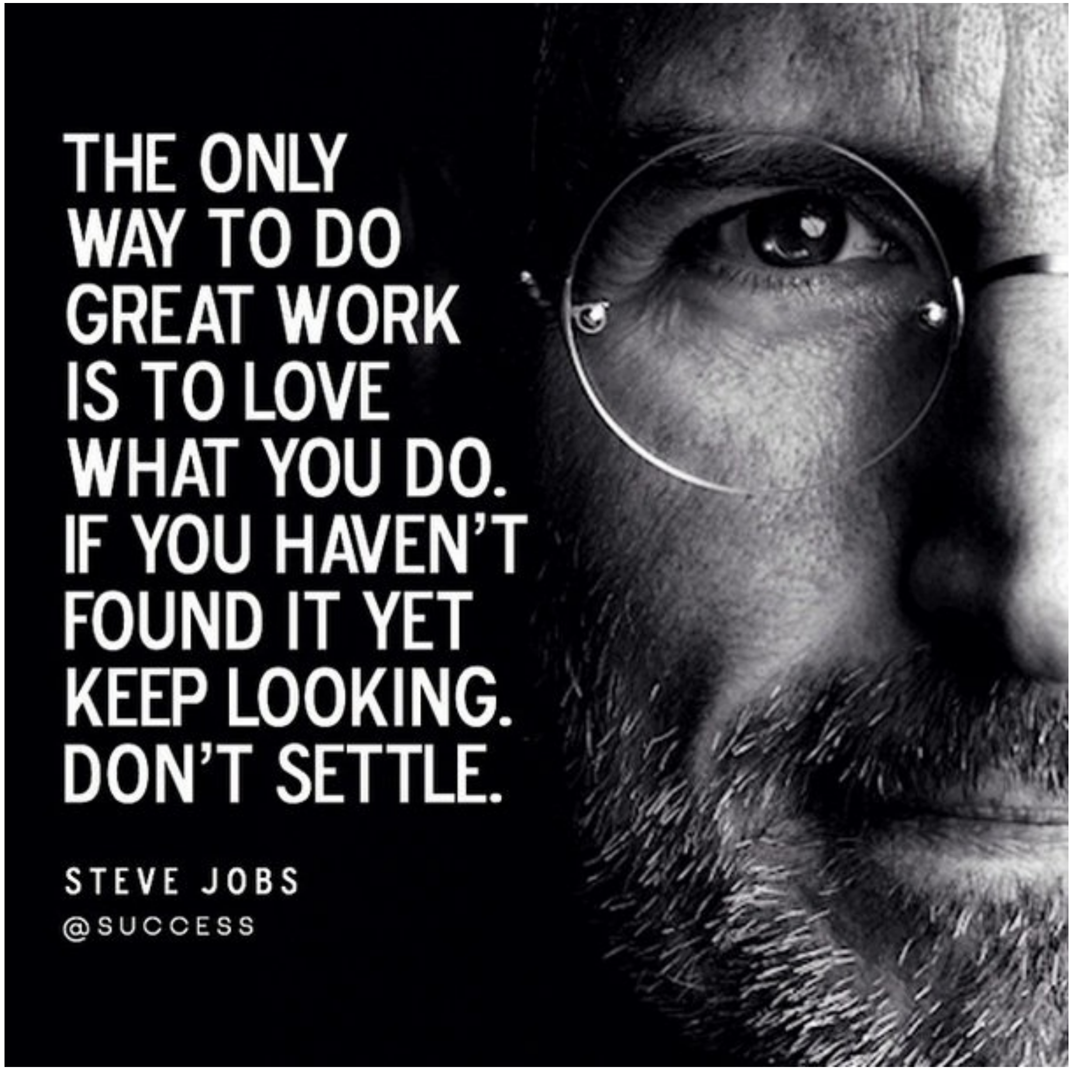


Left and right plots show data for two different observers.
Cross show cancellation of red-green appearance.
Closed circles show cancellation of blue-yellow appearance.

Measurements from the hue cancellation experiment. An observer is presented with a monochromatic test light. If the light appears red then some amount of a green canceling light is added to cancel the redness. If the light appears green, then a red canceling light is added to cancel the greenness. The horizontal axis of the graph measures the wavelength of the monochromatic test light, and the vertical axis measures the relative intensity of the canceling light.

Reference

https://foundationsofvision.stanford.edu/chapter-9-color/


THE ONLY
WAY TO DO
GREAT WORK
IS TO LOVE
WHAT YOU DO.
IF YOU HAVEN'T
FOUND IT YET
KEEP LOOKING.
DON'T SETTLE.
STEVE JOBS
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