Assignment 2

Numerical Methods, 2024 Spring

Due on Apr 10

Note: You should explain how you obtain your solution in your submission. If you use MATLAB or any other software to compute your results, you should provide your code or describe your solving process. This is a good practice for you to explain things in a logical, organized, and concise way! **Please hand in your assignment with clear photos or scans to the E3 website.**

1. (20%) Use Gaussian elimination with partial pivoting to solve the following equations(given as the augmented matrix). Are any row interchanges needed?

$$\left[\begin{array}{ccc|c} 3 & 1 & -4 & 7 \\ -2 & 3 & 1 & -5 \\ 2 & 0 & 5 & 10 \end{array}\right]$$

2. (20%) A system of two equations can be solved by graphing the two lines and finding where they intersect. (Graphing three equations could be done, but locating the intersection of the three planes is difficult.) Graph this system; you should find the intersection at (6, 2).

$$\begin{aligned} 0.1x + 51.7y &= 104, \\ 5.1x - 7.3y &= 16, \end{aligned}$$

   (a) Now, solve using three significant digits of precision and no row interchanges. Compare the answer to the correct value.

   (b) Repeat part (a) but do partial pivoting.

   (c) Repeat part (a) but use scaled partial pivoting. Which of part (a) or (b) does this match, if any?

3. (20%) Given system A:

$$A = \begin{bmatrix} 2 & -1 & 3 & 2 \\ 2 & 2 & 0 & 4 \\ 1 & 1 & -2 & 2 \\ 1 & 3 & 4 & -1 \end{bmatrix}$$

   Find the LU equivalent of matrix A that has 2's in each diagonal position of L rather than 1's.

4. (20%) Solve this system with the Jacobi method. First rearrange to make it diagonally dominant if possible. Use [0, 0, 0] as the starting vector. How many iterations to get the solution accurate to five significant digits?

$$\begin{bmatrix} 7 & -3 & 4 & 6 \\ -3 & 2 & 6 & 2 \\ 2 & 5 & 3 & -5 \end{bmatrix}$$

5. (20%) Repeat Problem 4 with the Gauss-Seidel method. Are fewer iterations required?

6. (25%) This 2×2 matrix is obviously singular and is almost diagonally dominant. If the right-hand-side vector is [0, 0], the equations are satisfied by any pair where x = y.

$$\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

   (a) What happens if you use the Jacobi method with these starting vectors: [1, 1], [1, -1], [-1, 1], [2, 5], [5, 2]?

   (b) What happens if the Gauss-Seidel method is used with the same starting vectors as in part (a)?

   (c) If the elements whose values are -2 in the matrix are changed slightly, to -1.99, the matrix is no longer singular but is almost singular. Repeat parts (a) and (b) with these new matrix.