Assignment 4

Numerical Methods, 2024 Spring

Due on May 27

Note: You should explain how you obtain your solution in your submission. If you use MATLAB or any other software to compute your results, you should provide your code or describe your solving process. This is a good practice for you to explain things in a logical, organized, and concise way! **Please hand in your assignment with clear photos or scans to the E3 website.**

1. (20pts) The following ordinary difference table is for $f(x) = x + \dfrac{sin(x)}{3}$. Use it to find

   (a) $f'(0.72)$ from a cubic polynomial.

   (b) $f'(1.33)$ from a quadratic.

   (c) $f'(0.50)$ from a fourth-degree polynomial.

   In each part, choose the best starting i-value.

i	x_i	f_i	Δf_i	$\Delta^2 f_i$	$\Delta^3 f_i$	$\Delta^4 f_i$
0	0.30	0.3985	0.2613	-0.0064	-0.0022	0.0003
1	0.50	0.6598	0.2549	-0.0086	-0.0018	0.0004
2	0.70	0.9147	0.2464	-0.0104	-0.0014	0.0005
3	0.90	1.1611	0.2360	-0.0118	-0.0010	
4	1.10	1.3971	0.2241	-0.0128		
5	1.30	1.6212	0.2113			
6	1.50	1.8325				

2. (20pts) Use the method of undetermined coefficients to obtain the formulas for $f''(x)$, $f'''(x)$ and $f^{(4)}(x)$ at x_0 using five evenly spaced points from x_2 to x_{-2}, together with their error terms.

3. (20pts) Simpson's $\frac{1}{3}$ rule, although based on passing a quadratic through three evenly spaced points, actually gives the exact answer if $f(x)$ is a cubic. The implication is that the area under any cubic between $x = a$ and $x = b$ is identical to the area of a parabola that matches the cubic at $x = a$, $x = b$, and $x = \dfrac{a+b}{2}$. Prove this.

4. (20pts) Compute the integral of $f(x) = \dfrac{sin(x)}{x}$ between $x = 0$ and $x = 1$ using Simpson's $\frac{1}{3}$ rule with $h = 0.5$ and then with $h = 0.25$. (Remember that the limit of $\dfrac{sin(x)}{x}$ at $x = 0$ is 1.) From these two results, extrapolate to get a better result. What is the order of the error after the extrapolation? Compare your answer with the true answer.

5. (20pts) Evaluate the following integral, and compare your answers to the analytical solution. Use $h = 0.1$ in both directions in parts (a) and (b),

   (a) Using the trapezoidal rule in both directions.

   (b) Using Simpson's $\frac{1}{3}$ rule in both directions.

   (c) Using Gaussian quadrature, three-term formulas in both directions.

$$\int_{-0.2}^{1.4} \int_{0.4}^{2.6} e^x sin(2y)\, dydx$$

6. (20pts) Please use Monte Carlo Integration to compute the double integral of $f(x, y) = (x - 1)^2 + \dfrac{y^2}{16}$ where $R = [-2, 3] \times [-1, 2]$.

$$\iint_R f(x, y)\, dy\, dx$$