Assignment 1

Numerical Methods, 2024 Spring

Due on Mar 28

Note: You should explain how you obtain your solution in your submission. If you use MATLAB or any other software to compute your results, you should provide your code or describe your solving process. This is a good practice for you to explain things in a logical, organized, and concise way! **Please hand in your assignment with clear photos or scans to the E3 website.**

1. (20%) The function $f(x) = x * \sin\frac{(x-2)}{(x-1)}$ has many zeros, especially near $x = 1$ where the function is discontinuous. Find the four zeros nearest to $x = 0.95$ by bisection, correct to five significant figures. How can you find good starting intervals?

2. (20%) Repeat Problem 1 but use the secant method. How many fewer iterations are required?

3. (20%) This polynomial obviously has roots at $x = 2$ and at $x = 4$; one is a double root, the other is a triple root:

$$\begin{aligned} p(x) &= (x-2)^3(x-4)^2 \\ &= x^5 - 14x^4 + 76x^3 - 200x^2 + 256x - 128 \end{aligned}$$

   (a) Which root can you get with bisection? Which root can't you get?

   (b) Repeat part (a) with the secant method.

   (c) If you begin with the interval [1, 5], which root will you get with (1) bisection, (2) the secant method, (3) false position?

4. (20%) Use Muller's method to find roots of these equations.

   (a) $4x^3 - 3x^2 + 2x - 1 = 0$, root near $x = 0.6$.

   (b) $x^2 + e^x = 5$, roots near $x = 1, x = -2$.

5. (20%) Most functions can be rearranged in several ways to give $x = g(x)$ with which to begin the fixed-point method. For $f(x) = e^x - 2x^2$, one $g(x)$ is

$$x = \pm\sqrt{\frac{e^x}{2}}$$

(a) Show that this converges to the root near 1.5 if the positive value is used and to the root near -0.5 if the negative is used.

(b) There is a third root 2.6. Show that we do not converge to this root even though values near to the root are used to begin the iterations. Where does it converge if $x_0 = 2.5$? If $x_0 = 2.7$?

(c) Find another rearrangement that does converge correctly to the third root.

6. (20%) Solve the following system of nonlinear equations using Newton's method.

$$y = cos^2(x)$$
$$x^2 + y^2 - x = 2$$