Assignment 3

Numerical Methods, 2024 Spring

Due on May 08

Note: You should explain how you obtain your solution in your submission. If you use MATLAB or any other software to compute your results, you should provide your code or describe your solving process. This is a good practice for you to explain things in a logical, organized, and concise way! **Please hand in your assignment with clear photos or scans to the E3 website.**

1. (20pts)

   (a) Construct the divided-difference table from these data:

x	-0.2	0.3	0.7	-0.3	0.1
$f(x)$	1.23	2.34	-1.05	6.51	-0.06

   (b) Use the divided-difference table to interpolate for $f(0.4)$ with the first three points.

   (c) Repeat (b) but use the best set of three points. Which points should be used?

2. (20pts) Fit the function below with a natural cubic spline that matches to $f(x)$ at five evenly spaced points in [-1, 1]. Use end conditions 3 and 4 to plot the spline curve together with $f(x)$. Which end condition gives the best fit to the function?

$$f(x) = \begin{cases} 0, & -1 < x < -0.5 \\ 1 - |2x|, & -0.5 < x < 0.5 \\ 0, & 0.5 < x < 1 \end{cases}$$

3. (20pts) If these four points are connected in order by straight lines, a zigzag line is created:

$$(0, 0), (1, 0.3), (2, 1.7), (3, 1.5).$$

   (a) Using the two interior points as controls, find the cubic Bezier curve. Plot this together with the zigzag line

   (b) If the second and third points (the control points) are moved, the Bezier curve will change. If these are moved vertically, where should they be located so that the Bezier curve passes through all of the original four points?

4. (20pts)The function whose values are tabulated below is $z = x + e^y$. Construct the B-spline surface from the rectangular array of 16 points nearest to (2.8, 0.54) and find z(2.8, 0.54).

$x\backslash y$	0.2	0.4	0.5	0.7	0.9
1.3	2.521	2.792	2.949	3.314	3.760
2.5	3.721	3.992	4.149	4.514	4.960
3.1	4.321	4.592	4.749	5.114	5.560
4.7	5.921	6.192	6.349	6.714	7.160
5.5	6.721	6.992	7.149	7.514	7.960

5. (20pts) The equation of a plane is z = ax + by + c. We can fit experimental data to such a plane using the least-squares technique. Here are some data for $z = f(x, y)$

x	0.40	1.2	3.4	4.1	5.7	7.2	9.3
y	0.70	2.1	4.0	4.9	6.3	8.1	8.9
z	0.031	0.933	3.058	3.349	4.870	5.757	8.921

   (a) Develop the normal equations to fit the (x, y) data to a plane.

   (b) Use these equations to fit $z = ax + by + c$.

   (c) What is the sum of the squares of the deviations of the points from the plane?

6. (20pts) Find the first few terms of the Chebyshev series for $cos(x)$ by rewriting the Maclaurin series in terms of the $T(x)$'s and collecting terms. Convert this to a power series in x. Compare the error of both the Chebyshev series and the power series after truncating each to the fourth degree.

7. (20pts)Find the Fourier coefficients for $f(x) = x^2 - 1$ if it is periodic and one period extends from $x = -1$ to $x = 2$.