SECI1013: DISCRETE STRUCTURES

SESSION 2023/2024 – SEMESTER 1

ASSIGNMENT 2 (CHAPTER 2)

INSTRUCTIONS:

a. This assignment must be conducted in a group. Please clearly write the group members' names & matric numbers on the front page of the submission.
b. Solutions for each question must be readable and neatly written on plain A4 paper. Every step or calculation should be properly shown. Failure to do so will result in the rejection of the submission of the assignment.
c. This assignment has 13 questions (100 marks), contributing 5% of overall course marks.

STRUCTURES:

1. Chapter 2 Part 1: Relation [50 Marks]
2. Chapter 2 Part 2: Function [30 Marks]
3. Chapter 2 Part 3: Recurrence Relation [20 Marks]

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Q1. Relation

1. Given A = {2, 3, 4, 5, 6, 7, 8} and R a relation over A. Draw the directed graph of R after realising that xRy iff x−y = 3n for some n ∈ Z. Find all possible equivalence relations for R.

   (5 marks)

2. Let A = {1, 2, 3} and B = {9, 8, 7}.
   Let R: A to B. For all (a, b) ∈ A × B, and given a R b ⇔ a+b is an even number,

   a. Determine R and R^{-1}.
   b. Draw arrow diagrams for both.
   c. Describe R−1 in words.

   (10 marks)

3. Let $A = \{1, 2, 3, 4, 5\}$, and let R be the relation on A that has the matrix (given below)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

   Construct the digraph of R, and list in-degrees and out-degrees of all vertices.

   (6 marks)

4. Given A = {0, 1, 2, 3, 4}, and
   R = {(0, 0),(0, 1),(0, 3),(0, 4),(1, 0),(1,1), (1,2), (2,1), (2, 2),(2,3), (3, 0),(3, 2),(3,3),(3,4),(4,0),(4,3),(4,4)}. Draw the relation graph and find is R reflexive, symmetric, or transitive?

   (12 marks)

5. Relation R in the set A = {1, 2, 3…13, 14} defined as R = {(x, y): 3x − y = 0}, Determine whether the relation is
   a. Reflexive
   b. Symmetric
   c. Transitive

   Support your answer with the reason.

(9 marks)

6. Suppose that the given is a relation matrix for R and S,

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

   Using Boolean Arithmetric, Find
   a. RS
   b. SR

(8 marks)

Q2. Function

7. What is the different between Relation and Function?

(2 Marks)

8. If A = {2, 3, 4, 5}, then write whether each of the following relations on set A is a function or not. Give reasons also.
   (i) {(2, 3), (3, 4), (4, 5), (5, 2)}
   (ii) {(2, 4), (3, 4), (5, 4), (4, 4)}
   (iii) {(2, 3), (2, 4), (5, 4)}
   (iv) {(2, 3), (3, 5), (4, 5)} (v) {(2, 2), (2, 3), (4, 4), (4, 5)}

(8 marks)

9. Given the relation of $R = \{(x, y) | y = x + 5\ , x\ is\ \mathbb{Z}^{+}\ less\ than\ 6\}$. Depict this relationship using roster form. Write down the domain and the range.

(3 marks)

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10. In the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

(v) $f = R \rightarrow R, f(x) = 1 - 2x$

(vi) $f = R \rightarrow R, f(x) = 5x^2 - 1$

(vii) $f = R \ \rightarrow R, f(x) = x^4$

(viii) $f = R \rightarrow R, f(x) = \left(\frac{x-2}{x-3}\right)$

(8 marks)

11. Given the following functions, find the function $f(g(x))$ and find the value of the function if $x = \{0, 1, 2, 3\}$

(ix) $f(x) = 3x - 1\,; g(x) = x^2 - 1$

(x) $f(x) = x^2\,; g(x) = 5x - 6$

(xi) $f(x) = x - 1\,; g(x) = x^3 + 1$

(9 marks)

Q3. Recurrence Relation

12. Solve the recurrence relation given;

(xii) $a_n = 6a_{n-1} - 9a_{n-2}$; initial conditions $a_0 = 1 \; and \; a_1 = 6$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$;
initial conditions $a_0 = 2\,, a_1 = 5 \; and \; a_2 = 15$

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$
initial conditions $a_0 = 1\,, a_1 = -2 \; and \; a_2 = -1$

(12 marks)

13. A sequence $a_1, a_2, a_3, a_4, \ldots$ is given by

$$a_{n+1} = 5a_n - 3 \; ; \; a_1 = k$$

where k is a non-zero constant.

(i) Find the value of a_4 in terms of k .

(ii) Given that a_4= 7 , determine the value of k .

(8 marks)

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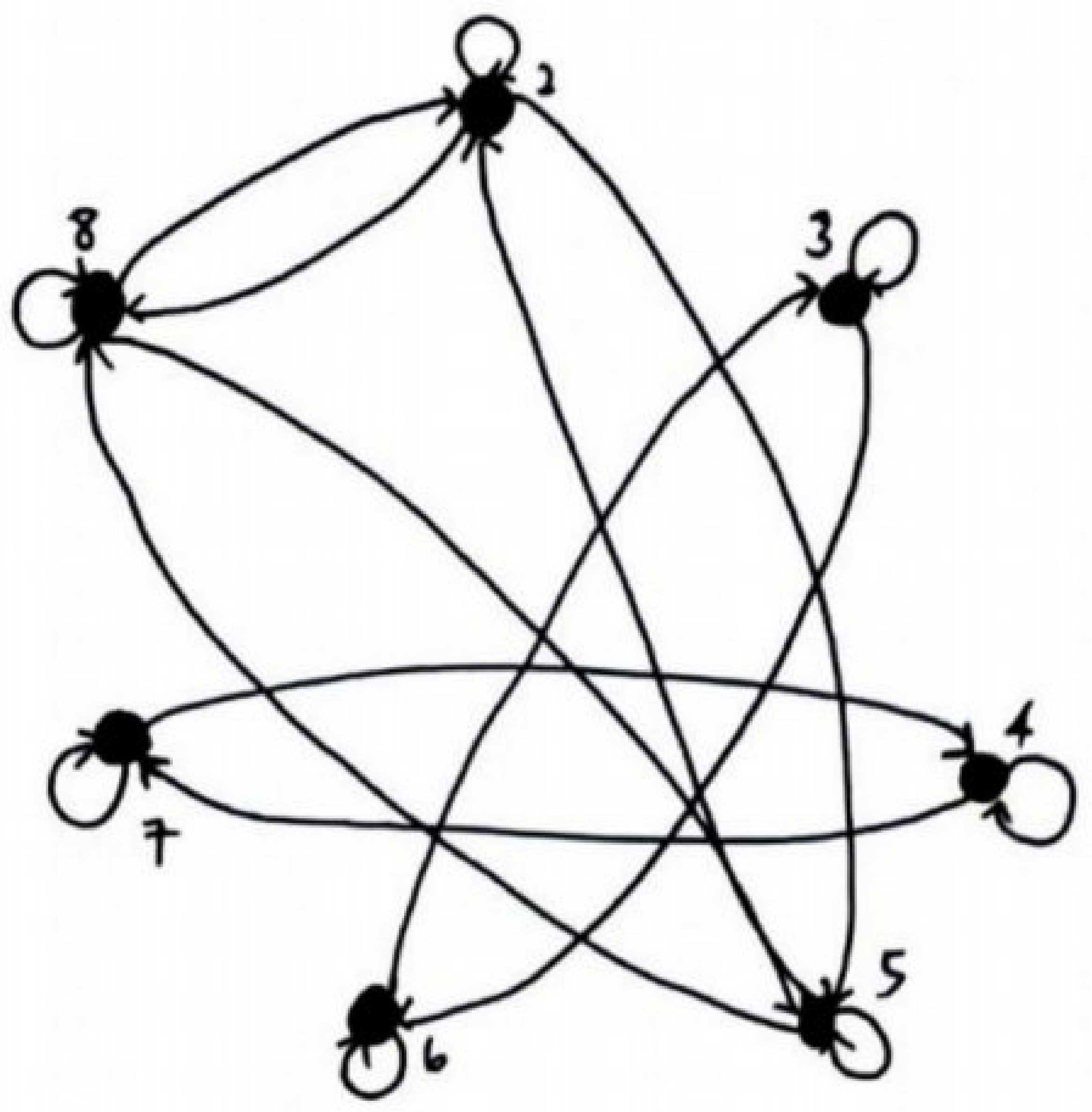
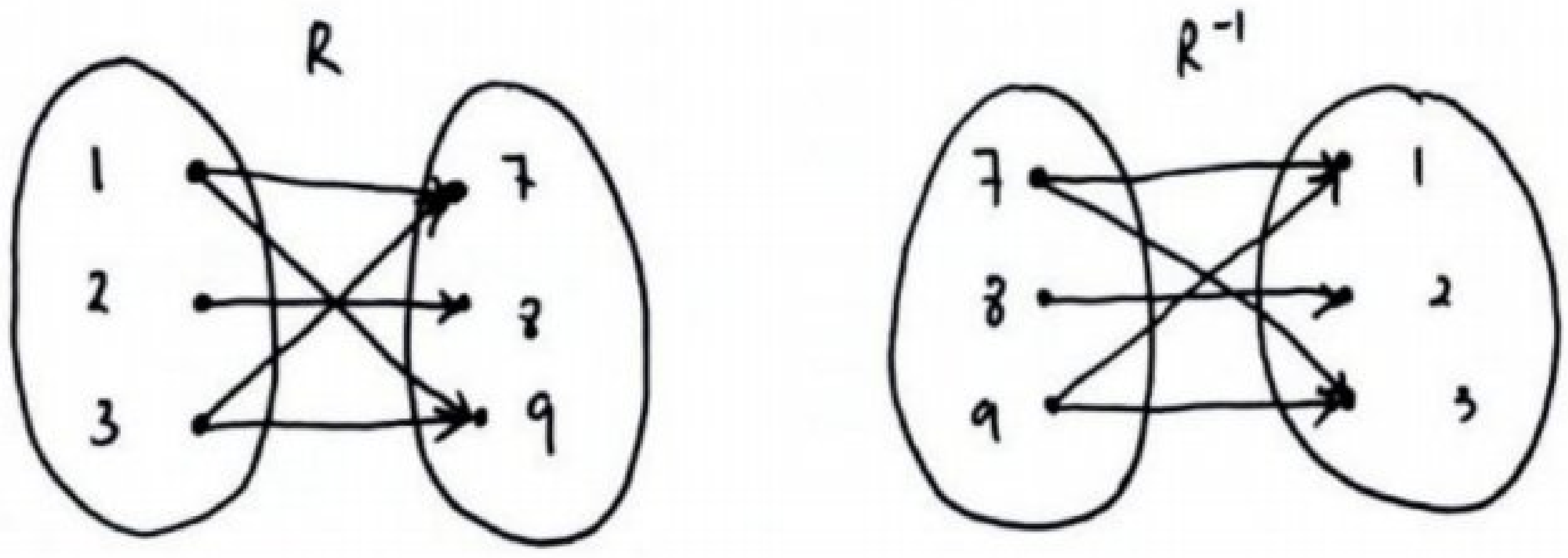
1. $R=\{(2,2),(2,5),(2,8),(3,3),(3,6),(4,4),(4,7),(5,2),(5,5),$
$(5,8),(6,3),(6,6),(7,4),(7,7),(8,2),(8,5),(8,8)\}$



2. (a) $R=\{(1,7),(1,9),(2,8),(3,9),(3,7)\}$
$R^{-1}=\{(7,1),(9,1),(8,2),(9,3),(7,3)\}$

(b)



(c) R^{-1} is inverse function of R. $R^{-1} \in (b,a)$, $R \in (a,b)$.
R^{-1} is is a function from B to A, $bRa \leftrightarrow b+a$ is even number.

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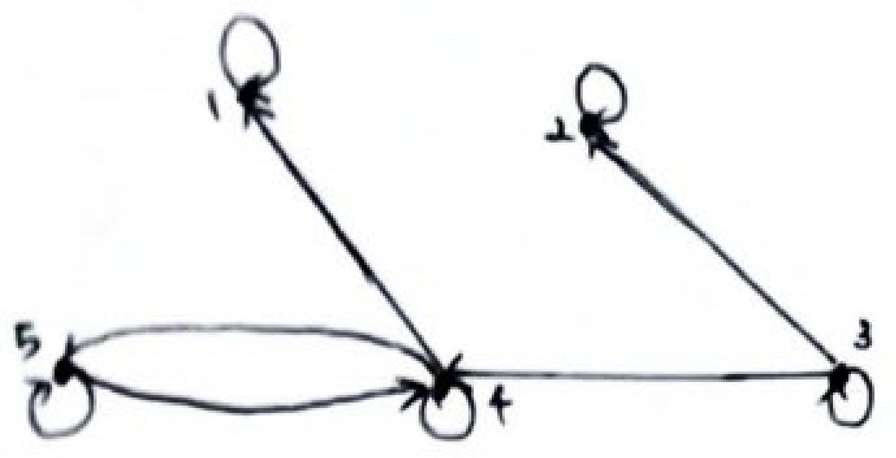
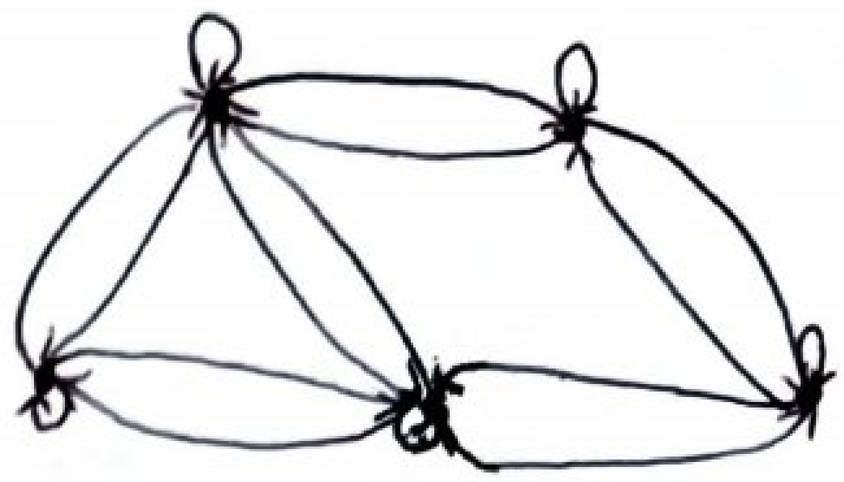
3. $R = \{(1,1), (2,2), (3,2), (3,3), (3,4), (4,1), (4,4), (4,5), (5,4), (5,5)\}$



	1	2	3	4	5
in-degrees	2	2	1	3	2
out-degrees	1	1	3	3	2

4.



* R is reflexive. $\forall x (x,x) \in R$ for $\forall x\, x \in A$
* R is symmetric, $\forall x,y \in A, (x,y) \in R \to (y,x) \in R$
* R is not transitive, $(0,1)$ and $(1,2) \in R$ but $(0,2) \notin R$.

5. $R = \{(1,3), (2,6), (3,9), (4,12)\}$

(a)

R is not reflexive. $\forall x\ (x,x) \notin R$, there is no any $(1,1), (2,2)$ in R

(b) R is not symmetric. $\forall x,y \in A,\ (x,y) \in R \to (y,x) \notin R$.
There have $(1,3) \in R$ but $(3,1) \notin R$.

(c) R is not transitive. $(1,3), (3,9) \in R$ but $(1,9) \notin R$, $\forall x,y \in A$

6. (a) RS

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$(0 \wedge 1) \vee (0 \wedge 0) \vee (1 \wedge 0) \vee (1 \wedge 0) = 0$

$(0 \wedge 0) \vee (0 \wedge 1) \vee (1 \wedge 1) \vee (1 \wedge 0) = 1$

(b) SR

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$(1 \wedge 0) \vee (0 \wedge 1) \vee (0 \wedge 0) \vee (1 \wedge 0) = 0$

$(1 \wedge 0) \vee (0 \wedge 1) \vee (0 \wedge 0) \vee (1 \wedge 0) = 0$

7. - Relations are a group of ordered pairs from one set of objects to another set of objects whereas functions are relations that connect one set of inputs to another set of outputs.

- In relation, the elements in domain can be assigned to more than one values in codomain whereas in function, the elements in domain can only be assigned to one value in codomain.

8 i. Is a function, because each element in domain is assigned to only one element in codomain.

ii. Is a function, because each element in domain is assigned to only one element in codomain.

iii. Not a function, because the elements (3 and 4) in domain are not assigned to any element in codomain.

iv. Not a function, because the element (2) in domain is assigned to more than one element in codomain (2 and 3).

9. $R = \{(1,6),(2,7),(3,8),(4,9),(5,10)\}$
Domain $= \{1,2,3,4,5\}$
Range $= \{6,7,8,9,10\}$

iv. $f(x) = 1-2x$

if $x = \{-1, 0, 1\}$

$f(-1) = 1-2(-1)$

$= 3$

$f(0) = 1-2(0)$

$= 1$

$f(1) = 1-2(1)$

$= -1$

- Is one-to-one function, because each element in domain has a unique value in range.
- Is onto function, because the range of the function includes positive and negative values.

$\therefore$ The function $f(x) = 1-2x$ is a bijective function.

vi. $f(x) = 5x^2 - 1$

if $x = \{-1, 0, 1\}$

$f(-1) = 5(-1)^2 - 1$

$= 4$

$f(0) = 5(0)^2 - 1$

$= -1$

$f(1) = 5(1)^2 - 1$

$= 4$

- Is not one-to-one function, because the elements (-1 and 1) in domain is assigned to same element in range (4).
- Is not onto function, because the range does not include negative values.

$\therefore$ The function $f(x) = 5x^2 - 1$ is not a bijective function.

vii. $f(x)=x^4$

if $x=\{-1,0,1\}$

$f(-1)=(-1)^4$

$=1$

$f(0)=(0)^4$

$=0$

$f(1)=(1)^4$

$=1$

- Is not one-to-one function, because the elements (-1 and 1) in domain is assigned to same element in range (1).
- Is not onto function, because the range does not include negative values.

$\therefore$ The function $f(x)=x^4$ is not a bijective function.

viii. $f(x)=\left(\frac{x-2}{x-3}\right)$

if $x=\{-2.5,0,2.5\}$

$f(-2.5)=\frac{(-2.5)-2}{(-2.5)-3}$

$=\frac{-4.5}{-5.5}$

$=\frac{9}{11}$

$f(0)=\frac{(0)-2}{(0)-3}$

$=\frac{-2}{-3}$

$=\frac{2}{3}$

$f(2.5)=\frac{(2.5)-2}{(2.5)-3}$

$=\frac{0.5}{-0.5}$

$=-1$

- Is one-to-one function, because each element in domain has a unique value in range.
- Is onto function, because the range of the function includes positive and negative values.

$\therefore$ The function $f(x)=\left(\frac{x-2}{x-3}\right)$ is a bijective function.

viii. $f(g(x)) = 3(x^2-1)-1$
$= 3x^2-4$
$x=0, f(g(0)) = 3(0)^2-4$
$= -4$
$x=1, f(g(1)) = 3(1)^2-4$
$= -1$
$x=2, f(g(2)) = 3(2)^2-4$
$= 8$
$x=3, f(g(3)) = 3(3)^2-4$
$= 23$

x. $f(g(x)) = (5x-6)^2$
$= 25x^2-60x+36$
$x=0, f(g(0)) = 25(0)^2-60(0)+36$
$= 36$
$x=1, f(g(1)) = 25(1)^2-60(1)+36$
$= 1$
$x=2, f(g(2)) = 25(2)^2-60(2)+36$
$= 16$
$x=3, f(g(3)) = 25(3)^2-60(3)+36$
$= 81$

xi. $f(g(x)) = (x^3+1)-1$
$= x^3$
$x=0, f(g(0)) = (0)^3$
$= 0$
$x=1, f(g(1)) = (1)^3$
$= 1$
$x=2, f(g(2)) = (2)^3$
$= 8$
$x=3, f(g(3)) = (3)^3$
$= 27$

12 xii. $a_n = 6a_{n-1} - 9a_{n-2}$; $a_0=1, a_1=6$

$a_2 = 6(6)-9(1)$
$= 27$

$a_3 = 6(27)-9(6)$
$= 108$

$a_4 = 6(108)-9(27)$
$= 405$

$a_5 = 6(405)-9(108)$
$= 1458$

$\therefore 1, 6, 27, 108, 405, 1458, \ldots$

xiii. $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$; $a_0=2, a_1=5, a_2=15$

$a_3 = 6(15)-11(5)+6(2)$
$= 47$

$a_4 = 6(47)-11(15)+6(5)$
$= 147$

$a_5 = 6(147)-11(47)+6(15)$
$= 455$

$a_6 = 6(455)-11(147)+6(47)$
$= 1395$

$\therefore 2, 5, 15, 47, 147, 455, 1395, \ldots$

xiv. $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$; $a_0=1, a_1=-2, a_2=-1$

$a_3 = -3(-1)-3(-2)+(1)$
$= 10$

$a_4 = -3(10)-3(-1)+(-2)$
$= -29$

$a_5 = -3(-29)-3(10)+(-1)$
$= 56$

$a_6 = -3(56)-3(-29)+(10)$
$= -71$

$\therefore 1, -2, -1, 10, -29, 56, \ldots$

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13) $\because a_{n+1} = 5a_n - 3$; $a_1 = k$

$a_4 = 5a_3 - 3$

$= 5(5a_2 - 3) - 3$

$= 25a_2 - 18$

$= 25(5a_1 - 3) - 18$

$= 125a_1 - 93$

$= 125(k) - 93$

$= 125k - 93$

$\therefore a_4 = 125k - 93$

$7 = 125k - 93$

$125k = 100$

$k = 0.8$