

Chapter 7: Probability

7.1 Probability

No:

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$$P(A) = \frac{n(A)}{n(S)}$$

* find

given/known

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad \left. \vphantom{\frac{P(A \cap B)}{P(B)}} \right\} \text{conditional probability}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad \left. \vphantom{P(A \cap B)} \right\} \text{Addition Rule}$$

↖ A effect B

↖ empty set

$$A \cap B = \emptyset$$

$$P(A \cap B) = 0$$

$$P(A \cup B) = P(A) + P(B)$$

mutually exclusive

$$0 \leq P(A) \leq 1$$

$$P(A') = 1 - P(A)$$

$$\hookrightarrow @ \bar{A} @ A^c$$

properties

↖ A no effect B

$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cup B) = P(A) + P(B) - [P(A) \cdot P(B)] \quad \left. \vphantom{P(A) \cdot P(B)} \right\} \text{independent}$$

tools \leftarrow venn
table
tree

7.1 Probability

1. $S = \{1, 2, 3, 4, 5, 6\}$

(a) $A = \{1, 2, 3\}$

$$P(A) = \frac{3}{6} = \frac{1}{2}$$

(b) $B = \{ \}$

$$P(B) = 0$$

(c) $C = \{2, 4, 6\}$

$$P(C) = \frac{3}{6} = \frac{1}{2}$$

(d) $D = \{1, 3, 5\}$

$$P(D) = \frac{3}{6} = \frac{1}{2}$$

(e) $E = \{1, 2, 3, 6\}$

$$P(E) = \frac{4}{6} = \frac{2}{3}$$

2.

	H	H'	Total
G	5	20	25
B	7	8	15
Total	12	28	40

(a) $P(H') = \frac{28}{40} = \frac{7}{10}$

(b) $P(B \cap H) = \frac{7}{40}$

(c) $P(G \cap H') = \frac{20}{40} = \frac{1}{2}$

7.2 Permutation vs combinations

Permutations

- order is important
- $AB \neq BA$
- arrange, position

arrange $\begin{cases} \text{all (different)}: n! @ {}^n P_n \\ \text{some (different)}: {}^n P_r \\ \text{all (some identical)}: \frac{n!}{r_1! r_2!} \end{cases}$

Combinations

- order is Not important
- $AB = BA$
- select, pick, group

15(a) number of ways = $8!$
= 40320

(b) number of ways = ${}^8 P_4$
= 1680

16(a) number of ways = $5!$
= 120

(b) $\frac{1}{1} - \frac{4!}{1} - -$

number of ways = $1 \times 4!$
= 24

17(a) $\begin{matrix} 9 & 10 & 10 & 10 \\ \swarrow & \downarrow & \downarrow & \downarrow \\ 1,2,3,4 & 5,6,7,8,9,10 & 0,1,2,3,4,5,6,7,8,9 & \end{matrix}$
number of ways = $9 \times 10 \times 10 \times 10$
= 9000

(b) number of ways = $9 \times 9 \times 8 \times 7$
= 4536

$\frac{9 \times 9 P_3}{\text{bcs 第一个用掉一个 number}}$
= 4536

18. $\frac{5 P_3}{-}$

number of ways = $5 P_3$
= 60

(a) $\frac{4 P_2}{-} \frac{1}{3}$

number of ways = $4 P_2 \times 1$
= 12