$y = f(x)$

$\frac{d}{dx}(y) = f'(x)$

(Differentiate y or f with respect to x.)

① First Principal $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

② Derivative of f at $x=a$ $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$

③ Differentiability

f is differentiable at $x=a$ if $f'(a)$ exists.

$$\boxed{\lim_{x \to a^-} \frac{f(x)-f(a)}{x-a} = \lim_{x \to a^+} \frac{f(x)-f(a)}{x-a}}$$

OR

f is differentiable at $x=a$.

① f is continuous at $x=a$ ← $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$

② $\lim_{x \to a^-} f'(x) = \lim_{x \to a^+} f'(x)$

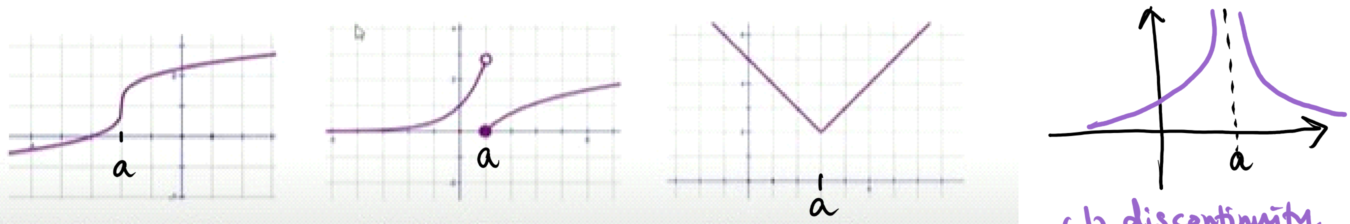
* f is differentiable at $x=a$. → f is continuous at $x=a$.
* f is not continuous at $x=a$. → f is not differentiable at $x=a$.
* f is continuous at $x=a$. (?) → f may differentiable at $x=a$. / → f may not differentiable at $x=a$.

(at $x=a$)

How a Function can Fail to be Differentiable?

Remember that the derivative is the measure of the slope of the tangent line.

Therefore, if the tangent line has an undefined slope, is not in the domain of the derivative function, or if the graph is discontinuous or has a corner point then the derivative may be undefined.



(a) A vertical tangent line

(b) a discontinuity (jump)

(c) a corner point

(d) discontinuity (infinite)

④ Rules of Differentiation

* Power Rule

$y = x^n$

$\frac{dy}{dx} = nx^{n-1}$

Up Level Power Rule

$y = [f(x)]^n$

$\frac{dy}{dx} = n[f(x)]^{n-1} \cdot f'(x)$

* Product Rule

$y = u(x) \cdot v(x)$

$\frac{dy}{dx} = uv' + vu'$

* Quotient Rule

$y = \frac{u(x)}{v(x)}$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

* Miss Choo's Rule ?!

Sometimes you need to expand/simplify the function first before the differentiation.

* **Exponential Function** $y = a^{f(x)}$

e.g. $y = 2^x$

$y = e^{\sin x}$

$y = 10^{\sqrt{x}}$

$$\frac{dy}{dx} = f'(x) \cdot a^{f(x)} \cdot \ln a$$

Rules of Log.

1. $\ln(ab) = \ln a + \ln b$
2. $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$
3. $\ln a^p = p \ln a$

* **Logarithmic Function** $y = \ln[f(x)]$

$$\frac{dy}{dx} = \frac{1}{f(x)} \cdot f'(x)$$

* **Trigonometric Function**

$$\frac{d}{dx}(\sin ax) = \cos ax \cdot a$$

$$\frac{d}{dx}(\cos ax) = -\sin ax \cdot a$$

$$\frac{d}{dx}(\tan ax) = \sec^2 ax \cdot a$$

$$\frac{d}{dx}(\cot ax) = -\operatorname{cosec}^2 ax \cdot a$$

$$\frac{d}{dx}(\sec ax) = \sec ax \tan ax \cdot a$$

$$\frac{d}{dx}(\operatorname{cosec} ax) = -\operatorname{cosec} ax \cot ax \cdot a$$

note:

$y = \sin^2 3x$

$\Downarrow$

$y = (\sin 3x)^2$

* **Parametric Equations** $\boxed{x = f(t) \;,\; y = g(t)}$

(Chain Rule)

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \quad , \quad \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \times \frac{dt}{dx}$$