Lecture 2

Example 8

Let $P(x) = x^3 + 3x^2 + x - 1$

$R = P(-1)$

$R = (-1)^3 + 3(-1)^2 + (-1) - 1$

$R = 0$

Since $R = 0$, $\therefore (x+1)$ is a factor of $P(x)$.

$P(x) = (x+1)(Ax^2 + Bx + C)$ → L-Division method

$P(x) = (x+1)(x^2 + 2x - 1)$ ← cannot be factorised.

$$x+1 \overline{)\, x^3 + 3x^2 + x - 1}$$

$x^2 + 2x - 1$

$x^3 + x^2$

$2x^2 + x - 1$

$2x^2 + 2x$

$-x - 1$

$-x - 1$

0

$x^3 + 3x^2 + x - 1 = 0$

$(x+1)(x^2 + 2x - 1) = 0$

$x + 1 = 0$ or $x^2 + 2x - 1 = 0$

$x = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{4 - 4(1)(-1)}}{2(1)}$$

$$x = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$x = -1 \pm \sqrt{2}$$

$\therefore$ The roots are $x = -1$, $x = -1 + \sqrt{2}$ and $x = -1 - \sqrt{2}$. ✱

Exercise

Show that $x^4+3x^3+5x^2+x-10$ has only two real factors.

Let $P(x) = x^4 + 3x^3 + 5x^2 + x - 10$

* How to know values 1 and -2 ? Get Hint from calculator !!

$R = P(1)$
$R = 1+3+5+1-10$
$R = 0$

$R = P(-2)$
$R = 16 - 24 + 20 - 2 - 10$
$R = 0$

Since $R=0$, $\therefore (x-1)$ and $(x+2)$ are the factors of $P(x)$.

$P(x) = x^4 + 3x^3 + 5x^2 + x - 10$
$P(x) = (x-1)(x+2)(Ax^2+Bx+C)$ ← Find A, B, C using comparison method
$P(x) = (x^2+x-2)(Ax^2+Bx+C)$

Compare

x^4: $x^4 = Ax^4$
$A = 1$

constant: $-10 = -2C$
$C = 5$

x^3: $3x^3 = Bx^3 + Ax^3$
$3 = B + A$
$3 = B + 1$
$B = 2$

$P(x) = (x-1)(x+2)(x^2+2x+5)$

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(5)}}{2}$$

$$x = \frac{-2 \pm \sqrt{-16}}{2}$$

$$x = \frac{-2 \pm 4i}{2}$$

$x = -1 \pm 2i$ (not real)

$\therefore (x-1)$ and $(x+2)$ are two real factors of $P(x)$. #.