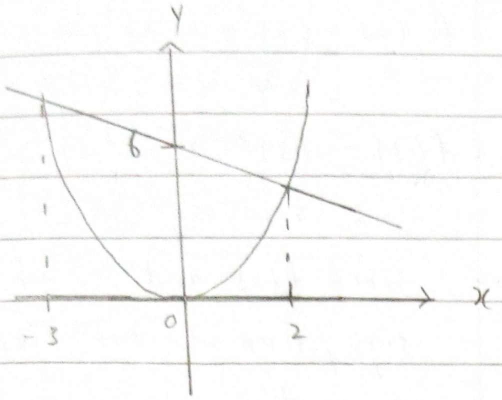
1(i) $y_1 = x^2$

x-intercept:
when $y = 0$,
$x = \pm 0$

$y_2 = 6 - x$

y-intercept:
when $x = 0$,
$y = 6 - 0$
$= 6$

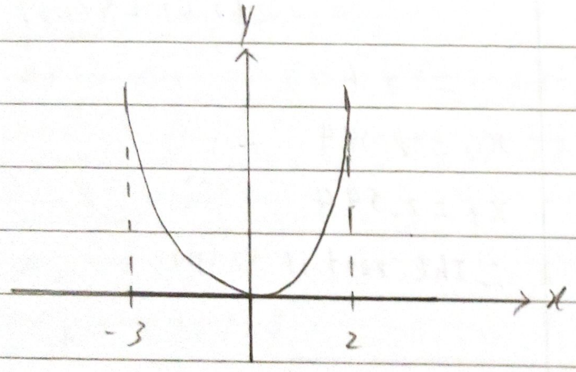
$y_1 = y_2$
$x^2 = 6 - x$
$x^2 + x - 6 = 0$
$(x+3)(x-2) = 0$
$x = -3, x = 2$



(ii) $y = x^2 + x - 6$

$(x+3)(x-2) = 0$

$x = -3, x = 2$



$\therefore$ Roots : -3 and 2

(iii) The intersection points are actually the roots of the equation.

2. $e^x + x - 2 = 0$

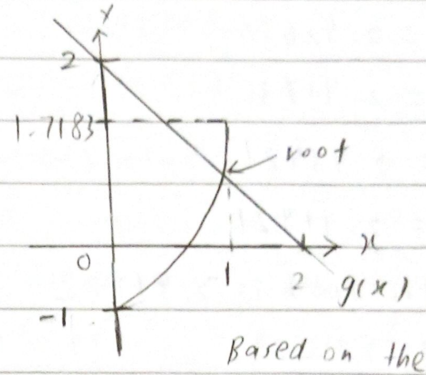
$e^x = 2 - x$

$f(x) = e^x \quad g(x) = 2 - x$

i) root $= 0, 1$

when $x = 1$
$y_1 = e^1 + 1 - 2$
$= 1.7183$

when $x = 0$,
$y_1 = e^0 + 0 - 2$
$= -1$



Based on the graph, the root is between 0 and 1.

ii) $f(x) = e^x + x - 2$

$f(0) = e^0 + 0 - 2$
$= -1$

$f(1) = e^1 + 1 - 2$
$= 1.7183$

$\therefore$ Since $f(0)$ and $f(1)$ have different signs, then the root lies between 0 and 1.

3. $f(x) = e^{2x} + 4x - 5$

$e^{2x} + 4x - 5 = 0$

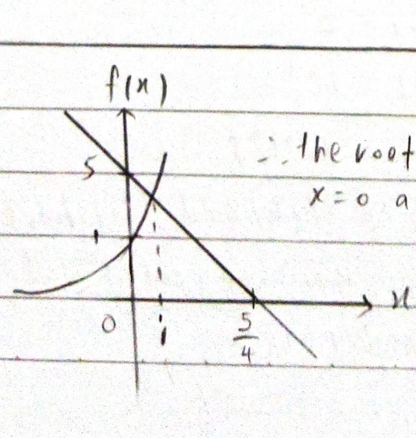
$g(x) = e^{2x} \quad h(x) = -4x + 5$

when $x = 0$,
$g(0) = e^{2(0)}$
$= 1$

$h(0) = -4(0) + 5$
$= 5$

when $y = 0$,
$e^{2x} = 0$

$-4x + 5 = 0$
$-4x = -5$
$x = \frac{5}{4}$



$\therefore$ the root lies between $x = 0$ and $x = \frac{5}{4}$.

4 (a) $x^3-5x-2=0$; $x_0=2$

$f(x)=x^3-5x-2$

$f'(x)=3x^2-5$

$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$

$=2-\frac{(2)^3-5(2)-2}{3(2)^2-5}$

$=2.57143$

$x_2=2.42680$

$x_3=2.41430$

$x_4=2.41421$

$x_5=2.41421$

$\therefore$ The root is 2.4142.

(b) $\ln x=2-x$; $x_0=2$

$\ln x+x-2=0$

$f(x)=\ln x+x-2$

$f'(x)=\frac{1}{x}+1$

$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$

$=2-\frac{\ln 2+2-2}{\frac{1}{2}+1}$

$=1.53790$

$x_2=1.55700$

$x_3=1.55715$

$x_4=1.55715$

$\therefore$ The root is 1.5572.

5. $x^3-2x^2=4$

$x^3-2x^2-4=0$

$x=2$ and $x=3$

$f(x)=x^3-2x^2-4$

$f(2)=(2)^3-2(2)^2-4$

$=-4$

$f(3)=(3)^3-2(3)^2-4$

$=5$

$\therefore$ Since $f(2)$ and $f(3)$ have different sign, then the root lies between 2 and 3.

$f'(x)=3x^2-4x$ Assume $x_0=2.5$,

$x_1=x_0-\frac{f(x_0)}{f'(x_0)}$

$=2.5-\frac{(2.5)^3-2(2.5)^2-4}{3(2.5)^2-4(2.5)}$

$=2.6$

$x_2=2.594$

$x_3=2.594$

$\therefore$ The root is 2.59.

6. $xe^x=1$

$xe^x-1=0$

$x=0$ and $x=1$

$f(x)=xe^x-1$

$f(0)=0$

$f(1)=1e^1-1$

$=1.7183$

$\therefore$ Since $f(0)$ and $f(1)$ have different sign, then the root lies between 0 and 1.7183.

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$f(x) = xe^x - 1$ $\quad u = x \quad v = e^x$

$f'(x) = x(e^x) + e^x(1) - 1 \quad u' = 1 \quad v' = e^x$

$= xe^x + e^x$

Assume $x_0 = 0.5$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 0.5 - \frac{0.5e^{0.5} - 1}{e^{0.5}(0.5+1)}$

$= 0.5710$

$x_2 = 0.5671$

$x_3 = 0.5671$

8. $3x + \cos x - 3 = 0$

$f(x) = 3x + \cos x - 3 \quad f'(x) = 3 - \sin x$

$x_0 = 0.5$

$x_1 = 0.5 - \frac{3(0.5) + \cos(0.5) - 3}{3 - \sin(0.5)}$

$= 0.74693$

$x_2 = 0.75789$

$x_3 = 0.75791$

$x_4 = 0.75791$

$\therefore x = 0.7579$

7. $x\ln x - \ln x - 1 = 0 \quad u = x \quad v = \ln x$

$f(x) = x\ln x - \ln x - 1 \quad u' = 1 \quad v' = \frac{1}{x}$

$f'(x) = x\left(\frac{1}{x}\right) + \ln x - \frac{1}{x}$

$= 1 + \ln x - \frac{1}{x}$

$x_0 = 1.3$

$x_1 = 1.3 - \frac{1.3\ln 1.3 - \ln 1.3 - 1}{1 + \ln 1.3 - \frac{1}{1.3}}$

$= 3.16824$

$x_2 = 2.35174$

$x_3 = 2.24268$

$x_4 = 2.23998$

$\therefore x = 2.24$ (3s.f.)

9. $x = \sqrt[3]{15}$

$x^3 = 15$

$x^3 - 15 = 0$

$f(x) = x^3 - 15$

$f'(x) = 3x^2$

$f(0) = 0^3 - 15 = -15 < 0$

$f(1) = 1^3 - 15 = -14 < 0$

$f(2) = 2^3 - 15 = -7 < 0$

$f(3) = 3^3 - 15 = 12 > 0$

There is a root between 2 and 3.

$x_0 = 2.5$

$x_1 = 2.5 - \frac{2.5^3 - 15}{3(2.5)^2}$

$= 2.4479$

$x_2 = 2.4663$

$x_3 = 2.4663$

$\therefore x = 2.466$