

Rotation of rigid body

- ① Average angular velocity

$$\omega_{ave} = \frac{\theta_f - \theta_i}{t_f - t_i}$$

- ② Instantaneous angular velocity

$$\omega_{ins} = \frac{d\theta}{dt}$$

↑
rad/s
↑
 $\frac{d}{dt}(\theta)$

- ③ Average angular acceleration

$$\alpha_{ave} = \frac{\omega_f - \omega_i}{t_f - t_i}$$

↑
rad/s²

- ④ Instantaneous angular acceleration

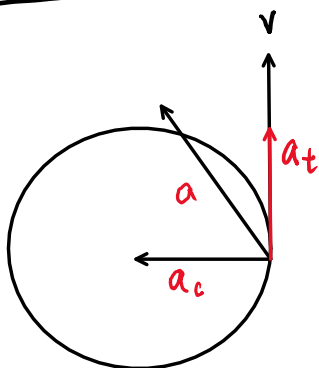
$$\alpha_{ins} = \frac{d\omega}{dt}$$

① $V = r\omega$

② $a = r\alpha$ → $a_t = r\alpha$

③ $a_c = r\omega^2$ → $a_c = \frac{v^2}{r}$

④ $s = r\theta$



$$|\vec{a}| = \sqrt{a_t^2 + a_c^2}$$

① $v = u + at$

$\omega = \omega_0 + \alpha t$

$$\omega = \omega_0 + \alpha t$$

$$\textcircled{2} v^2 = u^2 + 2as$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\textcircled{3} s = ut + \frac{1}{2}at^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

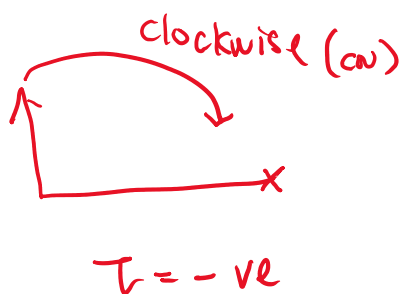
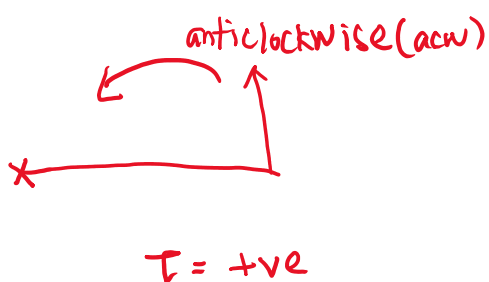
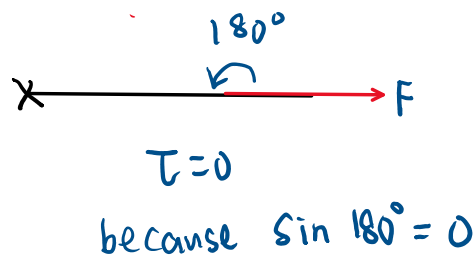
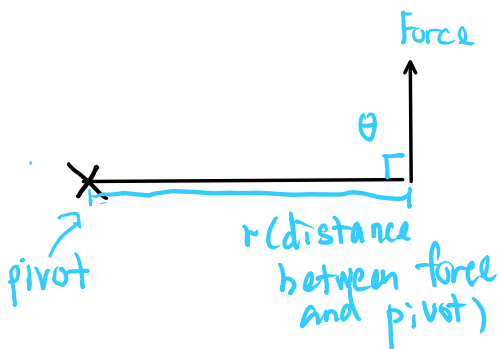
$$\textcircled{4} s = \frac{1}{2}(u+v)t$$

$$\theta = \frac{1}{2}(\omega_0 + \omega)t$$

① Torque

$$\tau = r F \sin \theta$$

(Nm) (m) (N) angle between force and distance



① 平衡 Translation equilibrium

$$\sum F_{\text{net}} = 0$$

$$\downarrow \quad \searrow$$
$$\sum F_x = 0 \quad \sum F_y = 0$$

② rotational equilibrium

$$\sum \tau = 0$$

$$\sum \tau_{\text{acw}} - \sum \tau_{\text{cw}} = 0$$

$$\sum \tau_{\text{acw}} = \sum \tau_{\text{cw}}$$

① Moment of Inertia

affected by

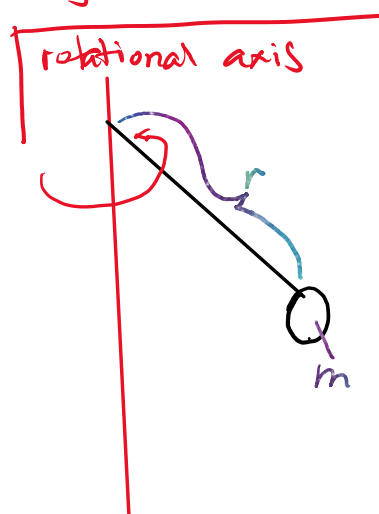
- shape
- body
- mass
- rotational axis

$I = \sum mr^2$

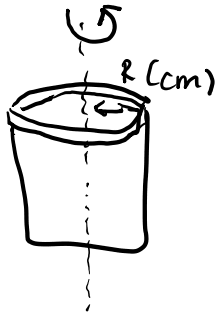
kgm^2

mass object (kg)

distance between object and rotational axis



① Hollow cylindrical



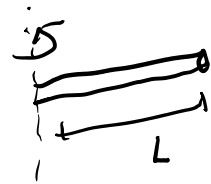
$$I = MR^2$$

② Solid cylindrical



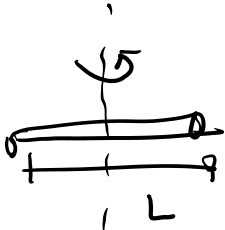
$$I = \frac{1}{2} MR^2$$

③



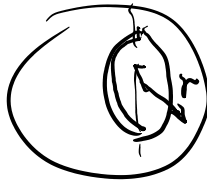
$$I = \frac{1}{3} ML^2$$

④



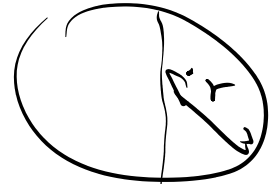
$$I = \frac{1}{12} ML^2$$

⑤ Hollow sphere



$$I = \frac{2}{3} MR^2$$

⑥ Solid sphere



$$I = \frac{2}{5} MR^2$$

① Linear motion $= F = ma$

② rotational motion $=$

$$T = I\alpha \rightarrow \left(\frac{a}{r}\right) \rightarrow a = r\alpha$$

\downarrow \downarrow \downarrow
 Nm kgm^2 $rads^{-2}$

Angular momentum

① Linear motion

$$p = mv$$

② Angular motion

$$L = I\omega$$

② Angular motion

$$L = I \omega$$

\downarrow \downarrow \downarrow
 $\text{kgm}^2\text{s}^{-1}$ kgm^2 rads^{-1}

③ Conservation

$$\sum L_i = \sum L_f$$

$$I_i \omega_i = I_f \omega_f$$

$$m_i r_i^2 \left(\frac{a}{r} \right) = m_f r_f^2 \left(\frac{a}{r} \right)$$