Simple harmonic motion

① Amplitude $x_{max} = A$ meter, m
(max magnitude of displacement from equilibrium position)

② Period, T $T = \frac{1}{f}$ second, s
(time taken for one oscillation)

③ Frequency, f $f = \frac{1}{T}$ Hertz, Hz, s^{-1}
(num. of oscillations in one second)

④ Angular frequency / Angular velocity / Angular speed
$w = \frac{d\theta}{dt}$ $rads^{-1}$
$w = \frac{2\pi}{T}$ (rate of change of angular displacement)
$w = 2\pi f$

General SHM equation

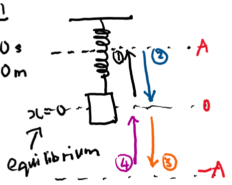
$\boxed{x = A \sin(wt + \phi)}$

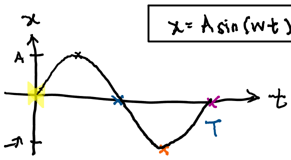
equilibrium position
$t = 0$

displacement from equilibrium position — max displacement / amplitude — angular frequency — time — initial phase angle

Case 1

$t = 0$ s
$x = 0$ m

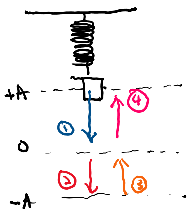


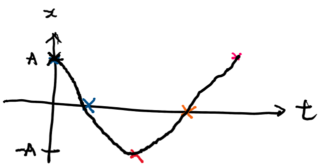


$\boxed{x = A\sin(wt)}$

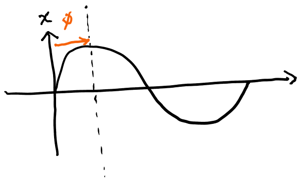
Case 2

$t = 0$ s
$x = +A$ m





$\boxed{x = A\cos(wt)}$

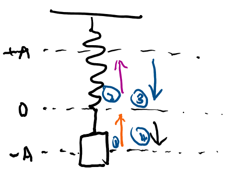


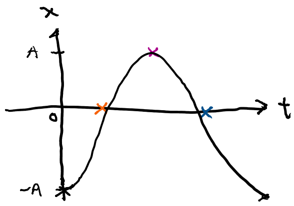
$x = A\sin(wt + \phi)$

$\boxed{x = A\sin(wt + \frac{\pi}{2})}$

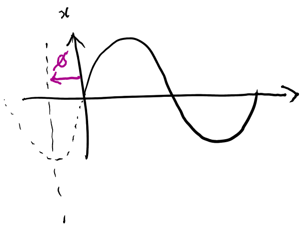
Case 3

$t = 0$ s
$x = -A$





$\boxed{x = -A\cos(wt)}$

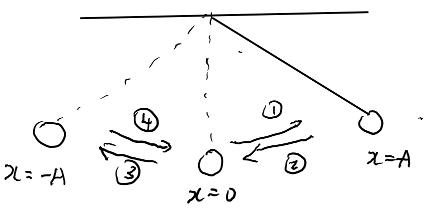


$x = A\sin(wt - \phi)$

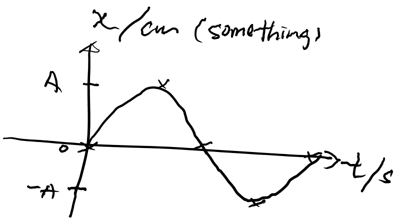
$\boxed{x = A\sin(wt - \frac{\pi}{2})}$

ø = phase angle

$\boxed{\begin{array}{l} \phi \leftarrow (-) \\ \phi \rightarrow (+) \end{array}}$

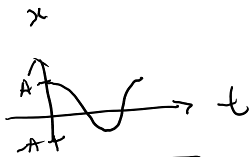


K = 0	Kmax	K = 0
Umax	U = 0	Umax
a max	a = 0	a max



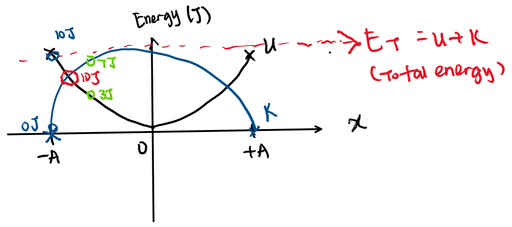
$\boxed{x = A\sin wt}$

start from 0



$\boxed{x = A\cos wt}$

start from +A



relation	max	with t	with x
x	A	$x = A\sin(wt)$	
$v = \frac{dx}{dt}$	Aw	$v = \frac{d}{dt}(A\sin(wt))$ $= A\cos w\, wt$ $v = Aw\cos(wt)$	$v = w\sqrt{A^2 - x^2}$
$a = \frac{dv}{dt}$	$A = \lvert -Aw^2 \rvert$ $= Aw^2$	$a = \frac{d}{dt}(Aw\cos(wt))$ $= Aw - \sin w(wt)$ $a = -Aw^2\sin(wt)$	$a = -w^2 x$
$K = \frac{1}{2}mv^2$	$\frac{1}{2}mA^2w^2$	$K = \frac{1}{2}mv^2$ $= \frac{1}{2}m(Aw\cos(wt))^2$ $K = \frac{1}{2}mA^2w^2\cos^2(wt)$	$K = \frac{1}{2}mv^2(A^2 - x^2)$
u	$\frac{1}{2}mA^2w^2$	$u = \frac{1}{2}mA^2w^2\sin^2(wt)$	$u = \frac{1}{2}mw^2x^2$
$E = K + u$	$\frac{1}{2}mw^2A^2$	$E = \frac{1}{2}mw^2$	$E = \frac{1}{2}mw^2A^2$

Period of SHM

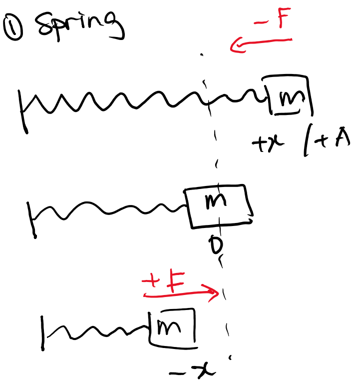
$$\boxed{T = \frac{1}{f}}$$

$$w = \frac{2\pi}{T}$$

$$\boxed{T = \frac{2\pi}{w}}$$

K = spring constant

① Spring



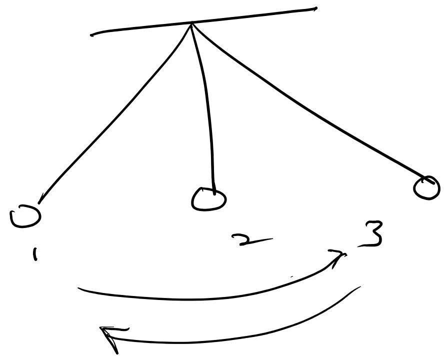
$$\boxed{T = 2\pi\sqrt{\frac{m}{K}}, \; w = \sqrt{\frac{K}{m}}}$$

$$w = \frac{2\pi}{T}$$

$$= \frac{2\pi}{2\pi\sqrt{\frac{m}{K}}}$$

$$w = \sqrt{\frac{K}{m}}$$

② Pendulum



$$\boxed{T = 2\pi\sqrt{\frac{l}{g}}, \; w = \sqrt{\frac{g}{l}}}$$

l = length of spring

$g = 9.81\ ms^{-2}$