3.1 Separable Variables

1(a) $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=4y$

Order : 2

Degree : 1

(b) $\left(\frac{d^2y}{dx^2}\right)^2=\left(1+\frac{dy}{dx}\right)^3$

Order : 2

Degree : 2

2(a) $\frac{dy}{dx}=6x^2y$

$\int\frac{1}{y}dy=\int 6x^2dx$

$\int\frac{1}{y}dy=6\int x^2dx$

$\ln|y|=6\left(\frac{x^3}{3}\right)+c$

$\ln|y|=2x^3+c$

$y=e^{2x^3+c}$

$y=e^{2x^3}\cdot e^c$

$y=Ae^{2x^3}$

(b) $(x+1)\frac{dy}{dx}=3(y+2)$

$\frac{1}{y+2}\frac{dy}{dx}=\frac{3}{(x+1)}$

$\int\frac{1}{y+2}dy=\int\frac{3}{(x+1)}dx$

$\ln|y+2|=3\ln|x+1|$

$y+2=e^{3\ln|x+1|}$

$y=e^{3\ln|x+1|-2}$

(c) $\frac{dv}{dt}=2-v$

$v\frac{dv}{dt}=2$

$v\,dv=2dt$

$\int v\,dv=\int 2\,dt$

$\frac{v^2}{2}=2t$

$v^2=4t$

$v=(2t)^{\frac{1}{2}}$

3(a) $e^{2y}\frac{dy}{dx}=\sec^2x,\ y\left(\frac{\pi}{4}\right)=0$

$\int e^{2y}dy=\int\sec^2x\,dx$

$\frac{e^{2y}}{2}=\tan x+c$

$\frac{e^{2(0)}}{2}=\tan\frac{\pi}{4}+c$

$\frac{1}{2}=1+c$

$c=-\frac{1}{2}$

$\frac{e^{2y}}{2}=\tan x-\frac{1}{2}$

$e^{2y}=2\tan x-1$

$\ln e^{2y}=\ln|2\tan x-1|$

$2y=\ln|2\tan x-1|$

$y=\frac{1}{2}\ln|2\tan x-1|$

(b) $x\frac{dy}{dx}=xy+y,\ y(3)=2$

$x\frac{dy}{dx}=y(x+1)$

$\frac{1}{y}dy=\frac{x+1}{x}dx$

$\int\frac{1}{y}dy=\int 1+\frac{1}{x}dx$

$\ln|y|=x+\ln|x|+c$

$y=e^{x+\ln|x|+c}$

$y=e^x\cdot e^{\ln|x|}\cdot e^c$

$y=Axe^x$

when $x=3,\ y=2$

$2=A(3)e^3$

$2=A$

$A=\frac{2}{3e^3}\quad\therefore y=\frac{2}{3}xe^{x-3}$

3(c) $xy\,dx + (1+x^2)\,dy = 0,\ y(1) = 2$

$xy\,dx = -(1+x^2)\,dy$

$\frac{x}{1+x^2}\,dx = -\frac{1}{y}\,dy$

$\int -\frac{1}{y}\,dy = \frac{1}{2}\int \frac{2x}{1+x^2}\,dx$

$-\ln y = \frac{1}{2}\ln(1+x^2) + c$

$\ln y^{-1} = \ln(1+x^2)^{\frac{1}{2}} + c$

$\frac{1}{y} = (1+x^2)^{\frac{1}{2}} \cdot e^c$

$\frac{1}{y} = A\sqrt{1+x^2}$

when $x = 1,\ y = 2$

$\frac{1}{2} = A\sqrt{2}$

$A = \frac{1}{2\sqrt{2}}$

$\frac{1}{y} = \frac{1\sqrt{1+x^2}}{2\sqrt{2}}$

$y = \frac{2\sqrt{2}}{\sqrt{1+x^2}}$

$= 2\sqrt{\frac{2}{1+x^2}}$

5. $e^y\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x}$

$e^y\frac{dy}{dx} = \frac{e^y}{x} - \frac{1}{x}$

$e^y\frac{dy}{dx} = \frac{e^y - 1}{x}$

$e^y\,dy = \frac{e^y - 1}{x}\,dx$

$\int \frac{e^y}{e^y - 1}\,dy = \int \frac{1}{x}\,dx$

$e^{\ln(e^y - 1)} = e^{\ln x + c}$

$e^y - 1 = x \cdot e^c$

$e^y - 1 = Ax$

$e^y = Ax + 1$

$y = \ln|Ax + 1|$

$\ln 2 = \ln|A + 1|$

$A = 1$

$\therefore y = \ln|x+1|$

4. $\ln x \frac{dy}{dx} = \frac{\tan y}{x}$

$\ln x\,dy = \frac{\tan y}{x}\,dx$ let $u = \ln x$

$\frac{1}{\tan y}\,dy = \frac{1}{x\ln x}\,dx$ $\frac{du}{dx} = \frac{1}{x}$

$\int \frac{\cos y}{\sin y}\,dy = \int \frac{1}{x\ln x}\,dx$ $du = \frac{1}{x}\,dx$

$\ln(\sin y) = \int \frac{1}{u}\,du$

$\ln(\sin y) = \ln(u) + c$

$\ln(\sin y) = \ln(\ln x) + c$

$\sin y = (\ln x) \cdot e^c$

$\sin y = A\ln x$

$y = \sin^{-1}(A\ln x)$