Chapter 8 Random variables

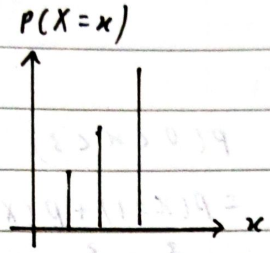
8.1 Discrete Random Variables

Probability Distribution
$P(X=x)$

- table

x			
$P(X=x)$			

- function
- graph



Properties :
① $0 \le P(X=x) \le 1$
② $\sum P(X=x) = 1$

Cumulative Distribution
$F(x)$
$P(X \le x)$

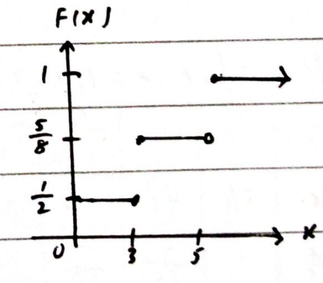
- table

x			
$F(x)$			

- function

$$F(x) = \begin{cases} 0, & x<0 \\ \frac{1}{2}, & 0 \le x < 3 \\ \frac{5}{6}, & 3 \le x < 5 \\ 1, & x \ge 5 \end{cases}$$

- graph



mean, $E(X) = \sum x P(X=x)$
$E(x^2) = \sum x^2 P(X=x)$

$Var(X) = E(x^2) - [E(x)]^2$

standard deviation : $\sqrt{Var(x)}$
× must positive value

median: $F(m) = 0.5$
$P(X \le m) = 0.5$

mode : highest $P(X=x)$

Properties :
① $E(Ax \pm B) = AE(x) \pm B$
② $Var(Ax \pm B) = A^2 var(X)$

8.2 Continuous Random Variable

No: Date:

$f(x) \geq 0$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$ } properties

Probability density $f(x)$ $\xrightarrow{F(x) = \int_{-\infty}^{\infty} f(t)\,dt}$ Cumulative distribution $F(x)$

Cumulative distribution $F(x)$ $\xrightarrow{f(x) = \frac{d}{dx} F(x)}$ Probability density $f(x)$

$$P(a \leq x \leq b) = \int_a^b f(x)\,dx$$
$$= P(a < x < b)$$
$$= P(a \leq x < b)$$
$$= P(a < x \leq b)$$
$$= F(b) - F(a)$$

mean, $E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

median: $F(m) = 0.5 = \int_{-\infty}^{m} f(x)\,dx = \int_{m}^{\infty} f(x)\,dx$

Properties :

$E(Ax \pm B) = AE(x) \pm B$

$Var(Ax \pm B) = A^2 Var(x)$

mode: sketch $f(x)$, find the highest point

$Var(x) = E(x^2) - [E(x)]^2$

Past year no. 4

$$f(x) = \begin{cases} \frac{x^2}{18}, & -c < x < c \\ 0, & \text{elsewhere} \end{cases}$$

(a) Show that $c = 3$

(b) $F(x)$

(c) i) $P(0 \leq x \leq 2)$

ii) median

(a) i

$0 \quad \frac{x^2}{18} \quad 0$

$-\infty \quad -c \quad c \quad \infty$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$
$$\cancel{\int_{-\infty}^{-c} 0\,dx}^{\,0} + \int_{-c}^{c} \frac{x^2}{18}\,dx + \cancel{\int_{c}^{\infty} 0\,dx}^{\,0} = 1$$
$$\left[\frac{x^3}{54}\right]_{-c}^{c} = 1$$
$$\frac{c^3}{54} - \frac{-c^3}{54} = 1$$
$$\frac{2c^3}{54} = 1$$
$$c^3 = 27$$
$$c = 3 \text{ (shown)}$$

(b)

$0 \quad \frac{x^2}{18} \quad 0$

$-3 \quad 3$

$$F(x) = \int_{-\infty}^{x} 0\,dt = 0$$
$$F(x) = \int_{-\infty}^{-3} 0\,dt + \int_{-3}^{x} \frac{t^2}{16}\,dt$$
$$= \left[\frac{t^3}{54}\right]_{-3}^{x}$$
$$= \frac{-x^3}{54} - \frac{(-3)^3}{54}$$
$$= \frac{1}{54}(x^3 + 27)$$
$$F(x) = \int_{-\infty}^{-3} 0\,dt + \int_{-3}^{3} \frac{t^2}{18}\,dt + \int_{3}^{x} 0\,dt$$
$$= F(3)$$
$$= \frac{1}{54}(3^3 + 27)$$
$$= 1$$

$$\therefore F(x) = \begin{cases} 0, & x \leq -3 \\ \frac{1}{54}(x^3 + 27), & -3 < x < 3 \\ 1, & x \geq 3 \end{cases}$$