CHAPTER 4 NOTES

Properties of Transpose

① $(A \pm B)^T = A^T \pm B^T$

② $(A^T)^T = A$

③ $(AB)^T = B^T A^T$

④ $(kA)^T = kA^T$
(where k is a constant.)

Properties of Inverse

① $(A^{-1})^{-1} = A$

② $(A^T)^{-1} = (A^{-1})^T$

③ $(AB)^{-1} = B^{-1}A^{-1}$

④ $|A^{-1}| = \frac{1}{|A|}$

Properties of Determinant

① Determinant of a transpose matrix. $\boxed{|A^T| = |A|}$

Cth: $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 2 \end{vmatrix} = 5$. Find $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 2 \end{vmatrix}$.

Ans: $\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 2 \end{vmatrix} = 5$ ✱

② Determinant of a matrix with zero rows (or columns). $\boxed{|A| = 0}$

Cth: $A = \begin{pmatrix} 2 & 3 & 0 \\ 0 & 0 & 0 \\ -8 & -2 & 1 \end{pmatrix}$

Ans: $|A| = 0$ ✱

③ Determinant of a matrix with a repeated row (or column). $\boxed{|A| = 0}$

Cth: $A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 5 \\ 1 & 2 & 3 \end{pmatrix}$

Ans: $|A| = 0$ ✱

④ Determinant of a matrix which interchange (swap) 2 rows (or columns). $\boxed{|B| = -|A|}$

Cth: Given $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 2 \end{pmatrix}$ and $|A| = 5$.

Find $|B| = \begin{vmatrix} 3 & 4 & 2 \\ 2 & 3 & 5 \\ 1 & 2 & 3 \end{vmatrix}$ and $|C| = \begin{vmatrix} 4 & 3 & 2 \\ 3 & 2 & 5 \\ 2 & 1 & 3 \end{vmatrix}$.

Ans: $|B| = -|A|$

$|B| = -5$ ✱

$|C| = -|B|$

$|C| = -(-5)$

$|C| = 5$ ✱

⑤ Determinant of a triangular matrix. (upper or lower)

Cth: $A = \begin{pmatrix} -1 & 0 & 0 \\ -5 & 5 & 0 \\ 9 & -4 & -4 \end{pmatrix}$. Find $|A|$.

$|A|$ = product of its diagonal entries

Ans: $|A| = (-1)(5)(-4)$

$= 20$. ✱.

⑥ Determinant of the product of 2 matrices. $|AB| = |A||B|$

Cth: $|A| = 3$ and $|B| = -2$. Find $|AB|$.

Ans: $|AB| = |A||B|$

$= 3(-2)$

$= -6$ ✱.

⑦ Determinant of a matrix which contains common factor k in a row (or column). $|B| = k|A|$

Cth: $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 2 \end{pmatrix}$, $|A| = 5$.

$B = \begin{pmatrix} 2 & 4 & 6 \\ 2 & 3 & 5 \\ 3 & 4 & 2 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & 4 & 9 \\ 2 & 6 & 15 \\ 3 & 8 & 6 \end{pmatrix}$. Find $|B|$ and $|C|$.

Ans: $|B| = \begin{vmatrix} 2 & 4 & 6 \\ 2 & 3 & 5 \\ 3 & 4 & 2 \end{vmatrix}$

$|B| = 2\begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 2 \end{vmatrix}$

$= 2(5)$

$= 10$ ✱

$|C| = \begin{vmatrix} 1 & 4 & 9 \\ 2 & 6 & 15 \\ 3 & 8 & 6 \end{vmatrix}$

$= 2 \cdot 3 \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 3 & 4 & 2 \end{vmatrix}$

$= 6(5)$

$= 30$ ✱.

⑧ Determinant of scalar product k with $n \times n$ matrix A.

Cth: Given matrix $A_{3\times3}$ with $|A| = 5$. Find $|2A|$. $|kA| = k^n|A|$

Ans: $|2A| = 2^3|A|$

$= 8(5)$

$= 40$

3 methods to find inverse matrix

(1) Adjoint method

- Minor matrix, $M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix}$
- Cofactor matrix, $C = \begin{pmatrix} +m_{11} & -m_{12} & +m_{13} \\ -m_{21} & +m_{22} & -m_{23} \\ +m_{31} & -m_{32} & +m_{33} \end{pmatrix}$
- Adjoint matrix, $\text{Adj. } A = C^T$
- Determinant, $|A|$
- Inverse matrix, $A^{-1} = \frac{1}{|A|} \text{Adj. } A$.

(2) $AB = kI$

If given $AB = kI$

- find A^{-1}. → $A^{-1} = \frac{1}{k}B$
- find B^{-1}. → $B^{-1} = \frac{1}{k}A$

(3) Elementary Row Operations (ERO)

3 types operations

(i) $R_i \leftrightarrow R_j$

(ii) $R_i^* = \alpha R_i$

(iii) $R_i^* = \alpha R_j + R_i$

$[\, A \mid I \,]$ ← Augmented matrix

⋮

$[\, I \mid A^{-1} \,]$

↑ Inverse matrix