



第九章 形态学图像处理



一、数学形态学历史及基本概念

- **形态学**：通常指生物学中对动植物的形状和结果进行处理的一个分支。
 - **数学形态学**(mathematical morphology, MM)：是根据形态学概念发展而来具有严格数学理论基础的科学，并在图像处理和模式识别领域得到了成功应用。
-



基本思想:

用具有一定**形态的结构元**去度量和提取图像中的对应形状以达到对图像分析和识别的目的

- ◆形态学图像处理表现为一种邻域运算形式;
 - ◆一种特殊定义的邻域称之为“结构元”(Structure Element), 在每个像素位置上它与二值图像对应的区域进行特定的逻辑运算, 逻辑运算的结果为输出图像的相应像素。
 - ◆形态学运算的效果取决于结构元的大小、内容以及逻辑运算的性质。
-



用途是：

简化图像数据，保持它们基本的形状特性，并除去不相干的结构

基本的运算包括：

二值腐蚀和膨胀、二值开闭运算



发展历史

○60年代：孕育和形成

1964诞生，法国学者Serra对铁矿石的岩相进行定量分析，以预测铁矿石的可轧性。同时，Matheron研究了多孔介质的几何结构、渗透性及二者的关系，二者的研究直接导致数学形态学雏形的形成。1966年命名Mathematical Morphology。1968年在法国成立枫丹白露(Fontainebleau)数学形态学研究中心。

○70年代：

1973年，Mathron的《随机集和积分几何》为数学形态学奠定了基础。



○80年代:

1982 由 Serra 主编完成的《Image Analysis and Mathematical Morphology》是里程碑, 表明数学形态学在理论上已趋于完备。此后, 该书的第二版和第三版相继出版。1986, CVGIP (computer vision graphics and image processing) 发表了MM专辑, 使MM的研究呈现新景象。提出基于MM的纹理分析模型系列。

○90年代至今:

在模式识别, 编码, 运动分析, 运动景物描述、放射医学、工业控制等方面取得进展, 及用于数值函数的形态学算子开发等。



形态学图像分析的优点

MM方法比其他空域或频域图像处理方法有一些明显的优势：

- 在恢复处理中，形态滤波可借助先验的几何特征信息，利用形态学算子有效滤除噪声，又可保留图像的原有信息；
 - MM算法易于用并行处理方法有效实现，且硬件实现容易；
 - 基于MM的边缘信息提取优于基于微分的提取算法，也不象微分算法对噪声那样敏感，同时提取的边缘较光滑；
 - 基于MM方法提取的图像骨架较连续，断点少。
-



二、 数学基础

● 集合论的一些基本概念：

一属于、不属于、空集

令 A 是 Z^2 中的一个集合，如果 a 是其中的一个元素，称 a 属于 A ，并记作： $a \in A$ ，否则，称 a 不属于 A ，记为： $a \notin A$ ，如 A 中没有任何元素，称 A 为空集： \emptyset

一子集、并集、交集

$$A \subseteq B, C = A \cup B, C = A \cap B$$

一不相连（互斥）、补集、差集

$$A \cap B = \emptyset, A^c = \{ a \mid a \notin A \}, A - B = \{ c \mid c \in A, c \notin B \} = A \cap B^c$$



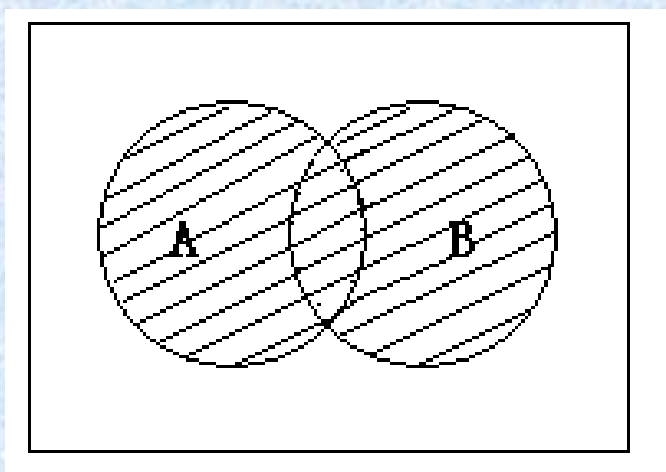
属于、不属于、空集

令 A 是 Z^2 中的一个集合，如果 a 是其中的一个元素，称 a 属于 A ，并记作： $a \in A$ ，否则，称 a 不属于 A ，记为： $a \notin A$ ，如 A 中没有任何元素，称 A 为空集： \emptyset



子集、并集

$$A \subseteq B, C = A \cup B$$

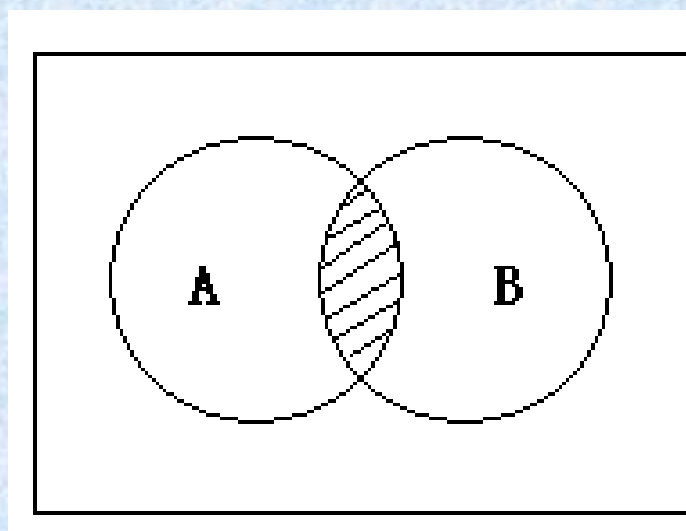


集合的并



交集

$$C = A \cap B$$

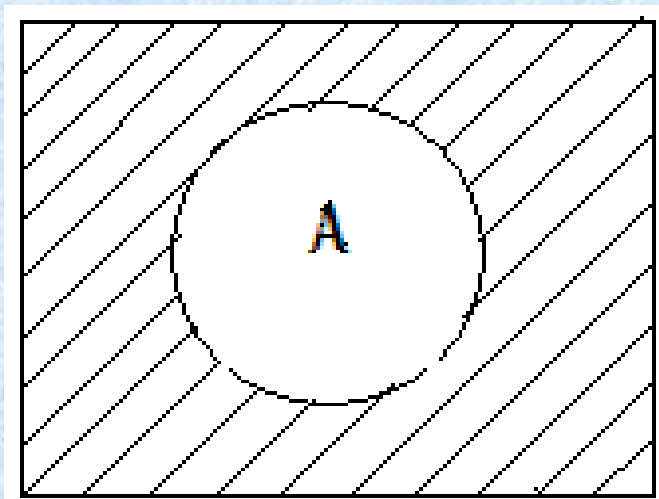


集合的交



不相连（互斥）、补集

$$A \cap B = \emptyset, A^c = \{ a \mid a \notin A \},$$

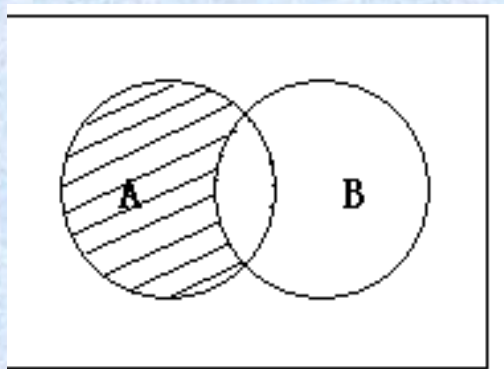


集合的补



差集

$$A - B = \{ c \mid c \in A, c \notin B \} = A \cap B^c$$

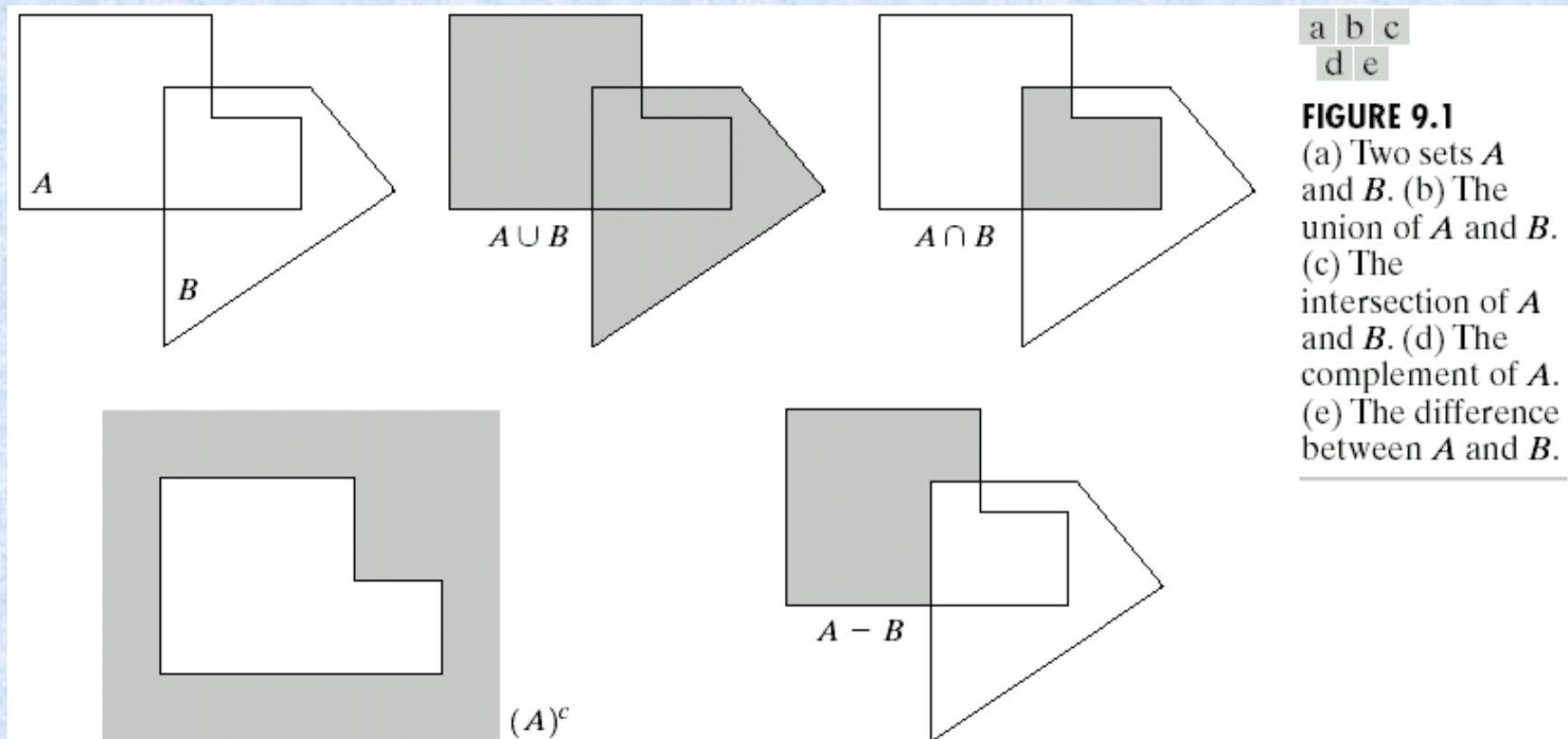


集合的差



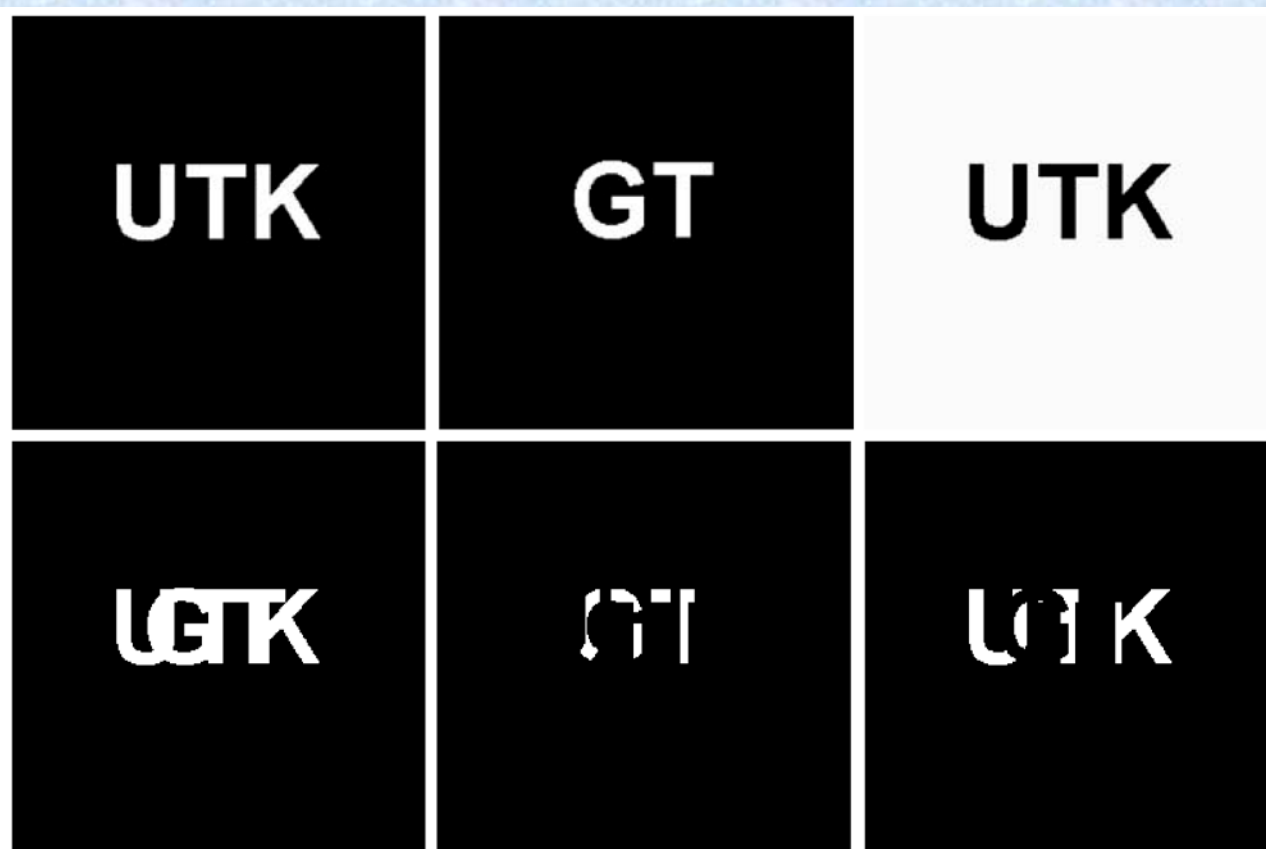
集合关系的图形表示

并、交、补、减





二值图像的逻辑运算



a	b	c
d	e	f

FIGURE 9.3 (a) Binary image A. (b) Binary image B. (c) Complement $\sim A$. (d) Union $A \mid B$. (e) Intersection $A \& B$. (f) Set difference $A \& \sim B$.



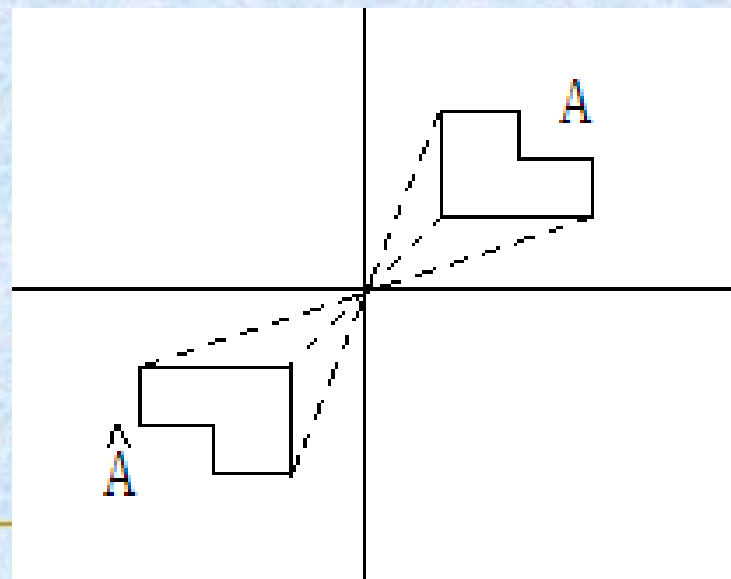
集合的反射

由集合A中所有元素相对于原点的反射元素组成的集合称为集合A的反射，记为 \hat{A} 。

$$\hat{A} = \{x \mid x = -a, a \in A\}$$

其中，x表示集合A中的元素a对应的反射元素。

集合的反射图示 →



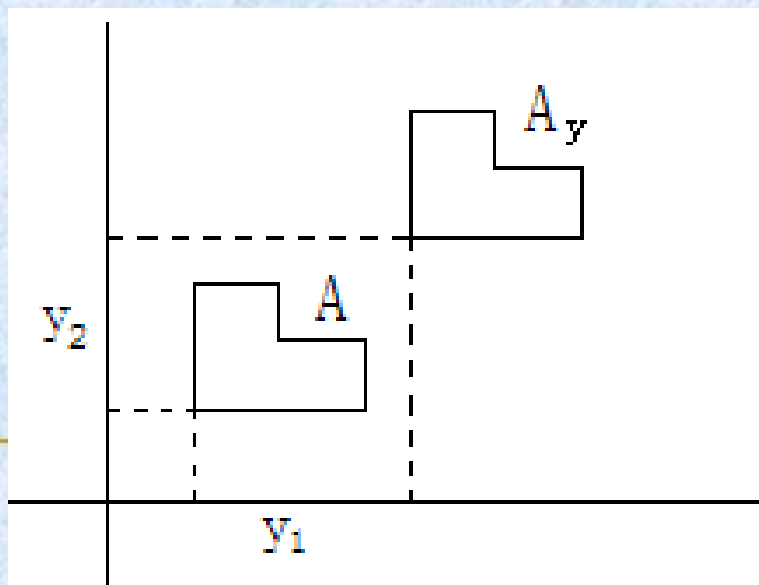


集合的平移

由集合A中所有元素平移 $y=(y_1, y_2)$ 后组成的元素集合称为集合A的平移，记为 $(A)_y$ 。

$$(A)_y = \{x \mid x = a + y, a \in A\}$$

其中， x 表示集合A中的元素 a 平移 y 后形成的元素。



集合的平移图示



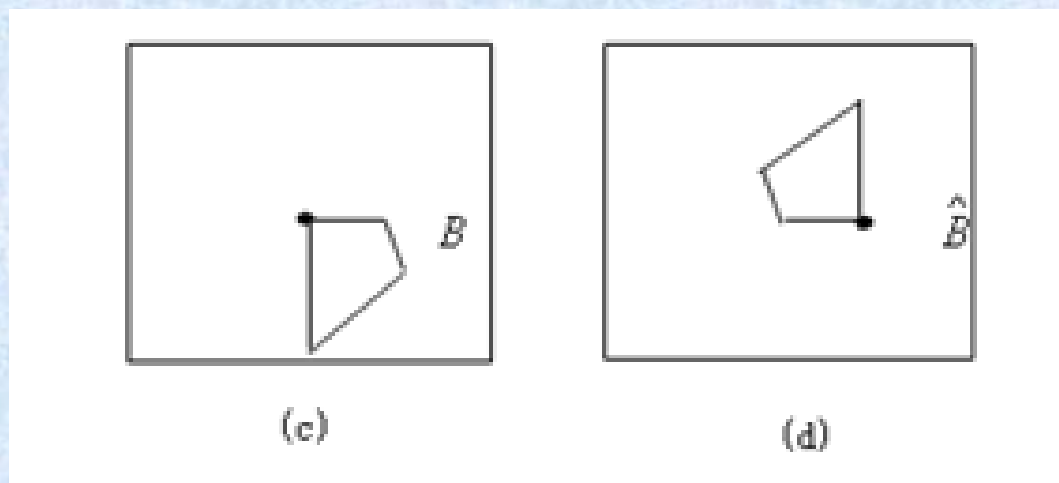
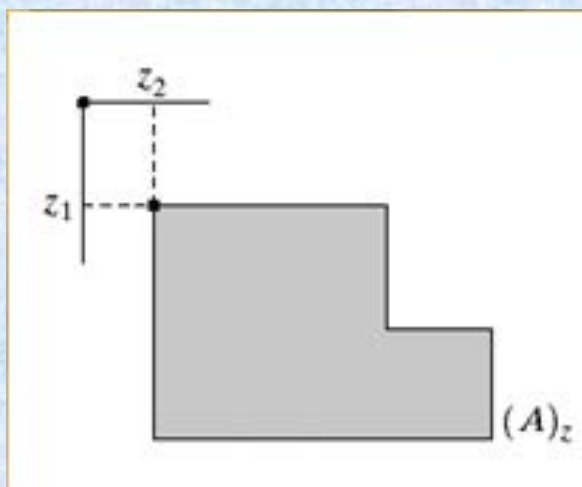
移位、反射

—反射（相对某个中心点）

—移位（相对原点）

$$\hat{B} = \{w \mid w = -b, b \in B\}$$

$$(A)_z = \{c \mid c = a + z, a \in A\}$$





二值图像中的基本逻辑操作

三种最基本的逻辑运算（功能完整的）：与、或、非（补）

p	q	$p \text{ AND } q$ (also $p \cdot q$)	$p \text{ OR } q$ (also $p + q$)	NOT (p) (also \bar{p})
0	0	0	0	1
0	1	0	1	1
1	0	0	1	0
1	1	1	1	0

尽管逻辑操作与集合操作间存在一一对应的关系，但逻辑操作只是针对二值图像。

逻辑操作图形表示

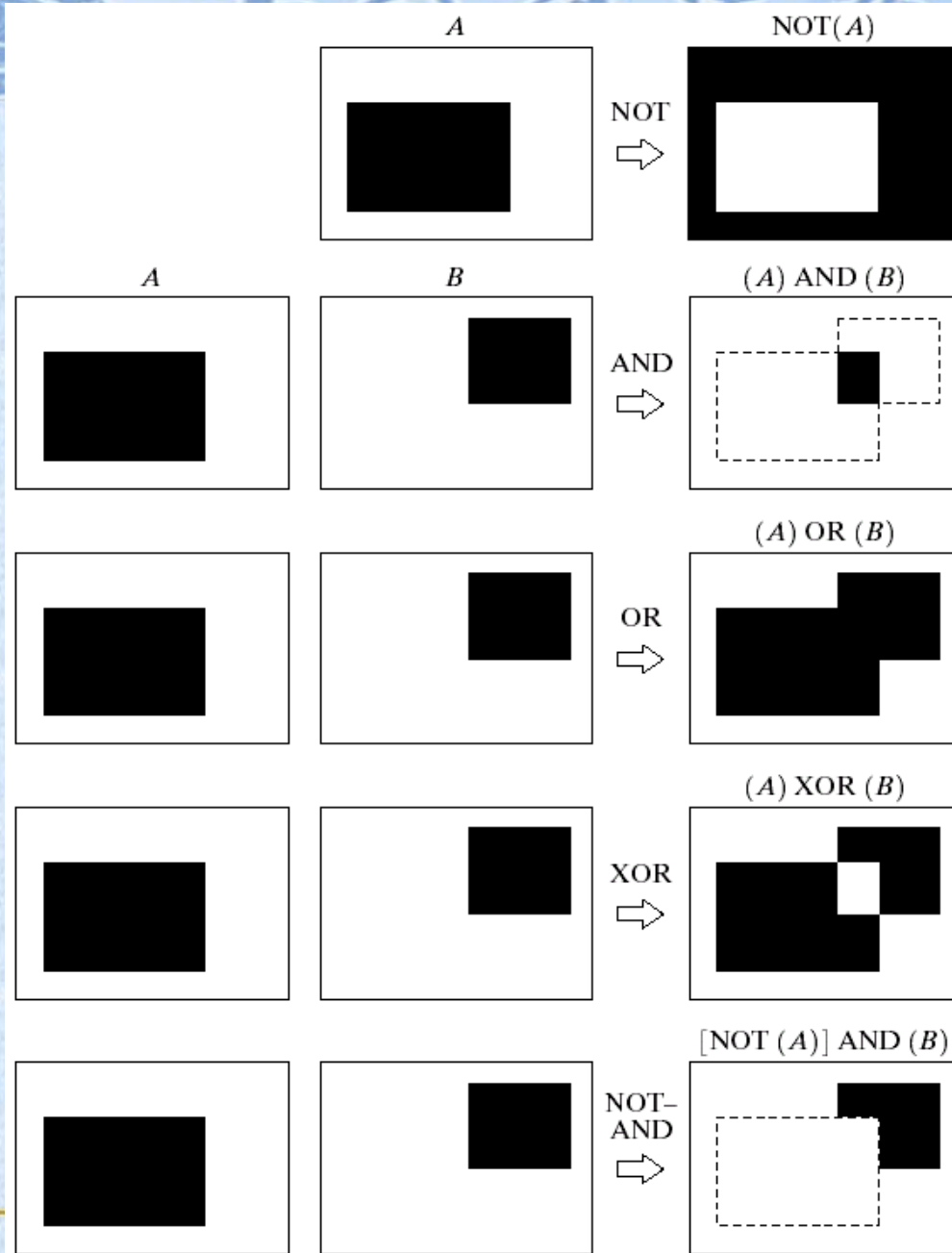
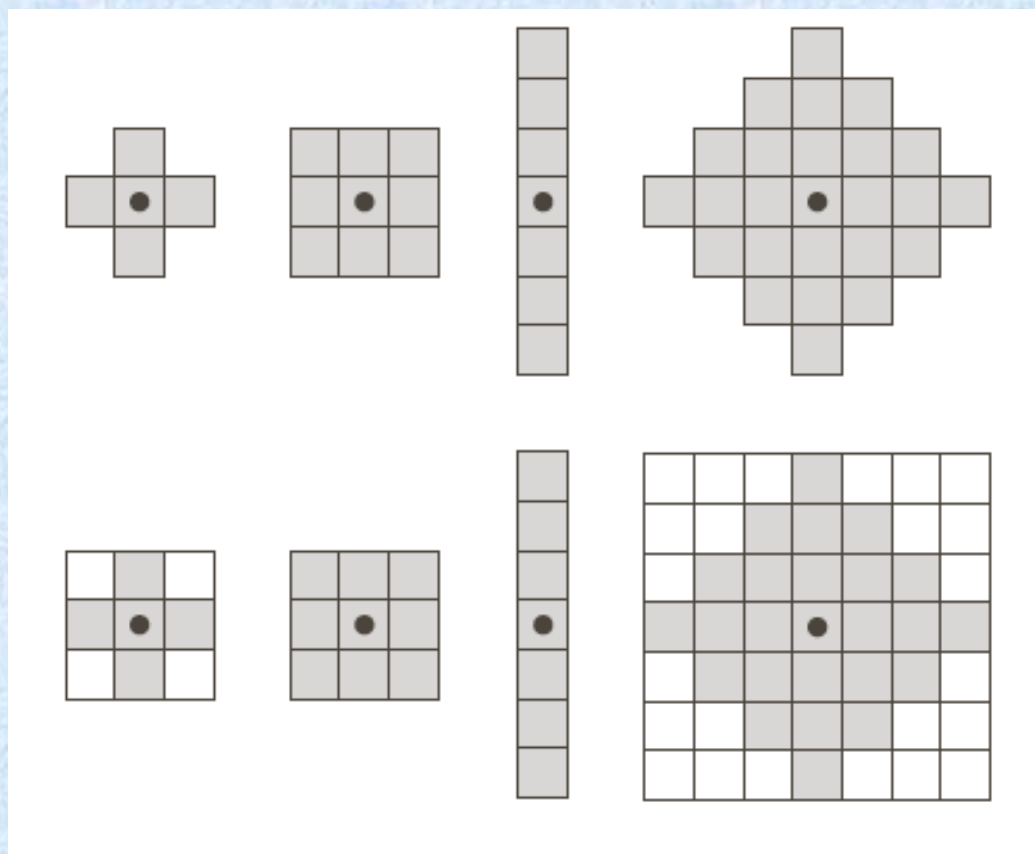


FIGURE 9.3 Some logic operations between binary images. Black represents binary 1s and white binary 0s in this example.



结构元



一般假定原点位于对称中心
考虑空域滤波模板的形状



三、 二值形态学基本运算

- 膨胀 (dilation)
 - 腐蚀 (erosion)
 - 开和闭 (opening and closing)
 - 击中与否变换 (hit-or-miss)
-



(1) 膨胀

- 膨胀：使图像扩大
- A 用 B 来膨胀写作 $A \oplus B$ 定义为：

$$A \oplus B = \{z \mid (\hat{B})_z \cap A \neq \Phi\}$$

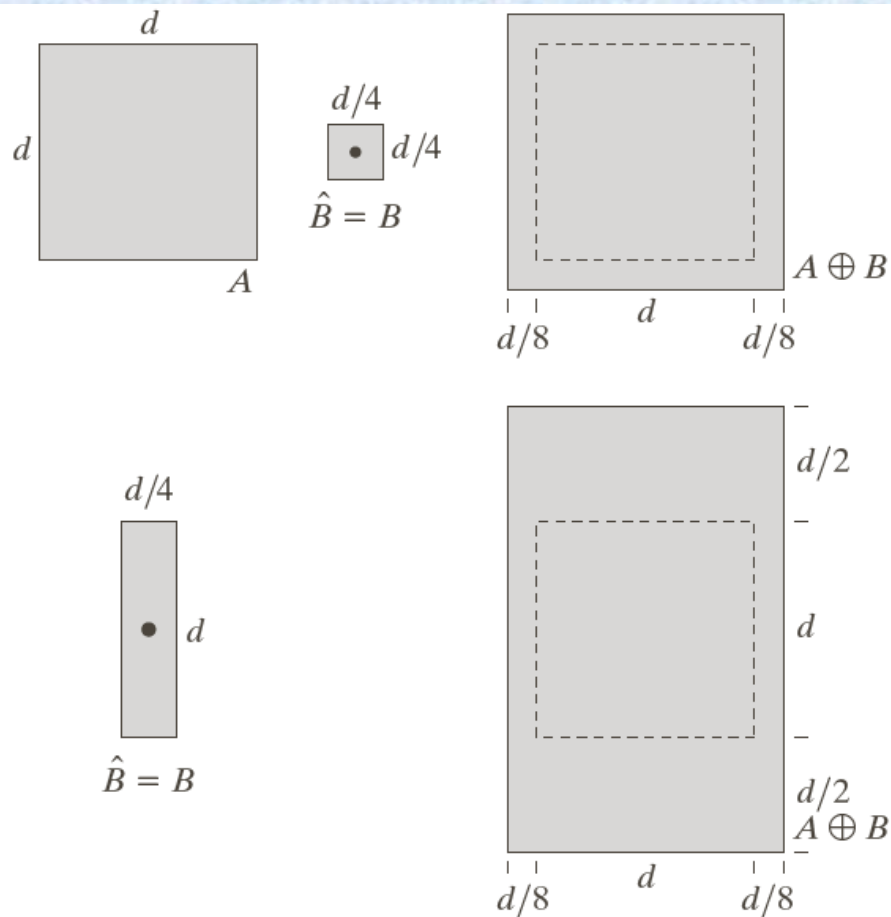
有没有其它的定义？

- 上式表示：
 - B 的反射进行平移与 A 的交集不能为空
- B 的反射： B 相对于自身的映像
- 对 B 的反射进行移位，以便它能滑过集合（图像） A



含义:

每当结构元素 \hat{B} 在目标图像A上平移后，结构元素 \hat{B} 与其覆盖的子图像中至少有一个元素相交时，就将目标图像中与结构元素 \hat{B} 的原点对应的那个位置的像素值置为“1”，否则置为0。

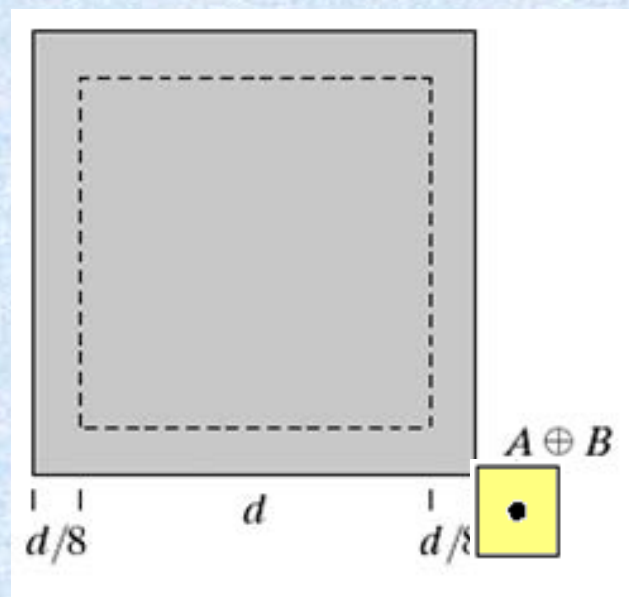
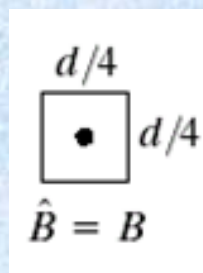
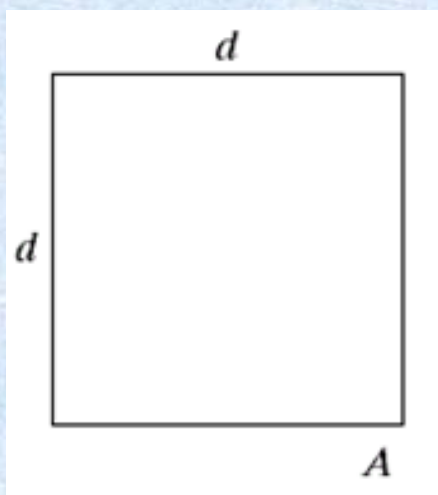


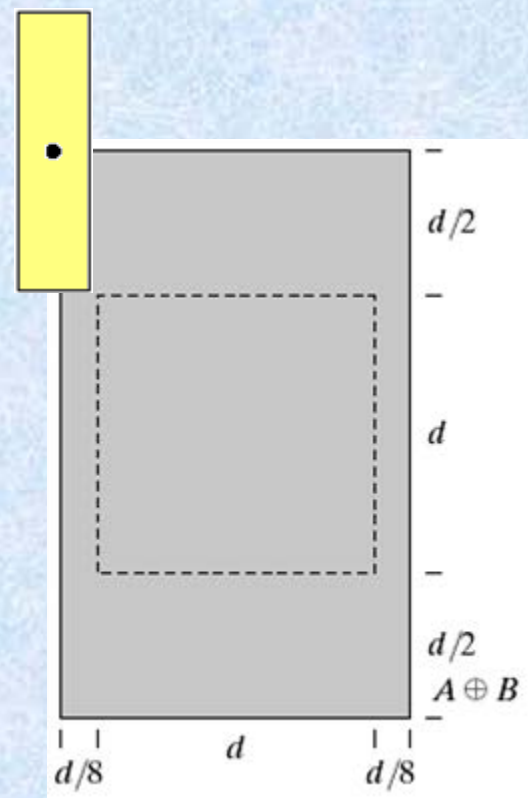
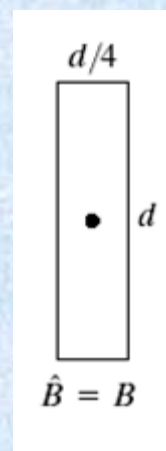
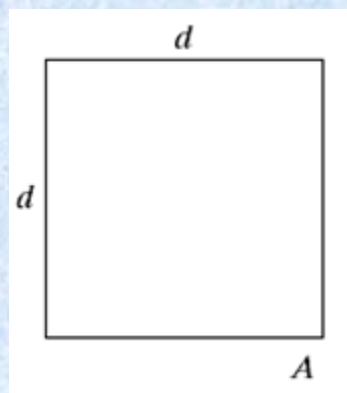
举例：

a	b	c
d	e	

FIGURE 9.6

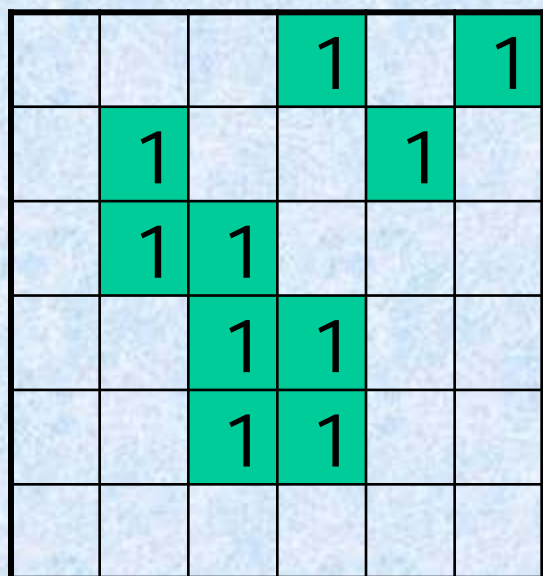
(a) Set A .
(b) Square structuring element (the dot denotes the origin).
(c) Dilation of A by B , shown shaded.
(d) Elongated structuring element.
(e) Dilation of A using this element. The dotted border in (c) and (e) is the boundary of set A , shown only for reference



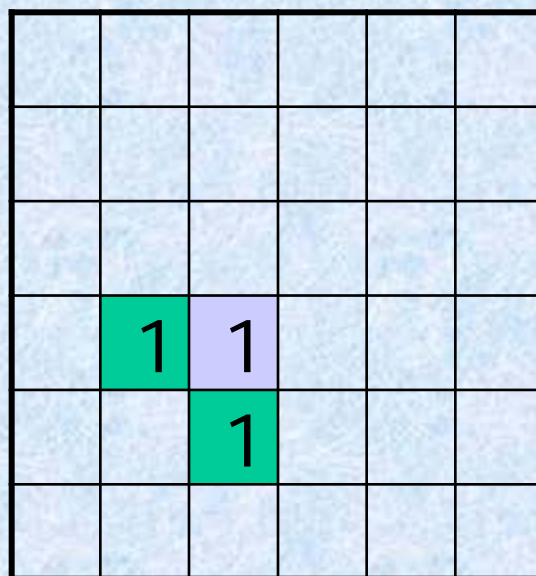




(b)
结构元素
B



(a) 目标图像A



(c) 结构元素 \hat{B}



(d) 膨胀运算结果图像



结构元素形状对膨胀运算结果的影响

当目标图像不变，但所给的结构元素的形状改变时；或结构元素的形状不变，而其原点位置改变时，膨胀运算的结果会发生改变。

膨胀运算会增长或粗化二值图像中的物体，并能连接缝隙和填充小孔



下面给出的是与上图的目标图像相同但结构元素不同时，膨胀运算结果不同的例子。

		1	1		

(b)
结构元素 B

			1		1
	1			1	
	1	1			
		1	1		
		1	1		

(a) 目标图像A

	1	1			

(c) 结构元素 \hat{B}

			1	2	1
	1	2		1	2
	1	1	2		
		1	1	2	
		1	1	2	

(d) 膨胀运算结果图像



下面给出的是与上图的目标图像相同，但仅结构元素的
原点位置改变时，膨胀运算结果不同的例子。

		1			
		1	1		

(b)
结构元素
B

			1		1
	1			1	
	1	1			
		1	1		
		1	1		

(a) 目标图像A

			1	1	
				1	

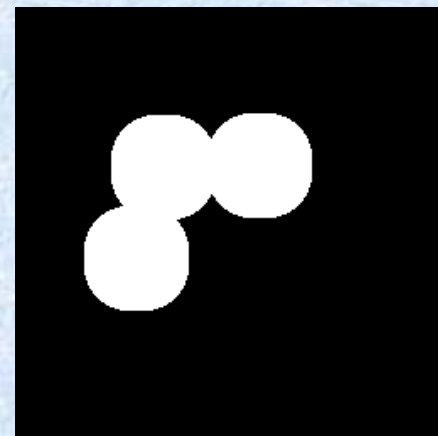
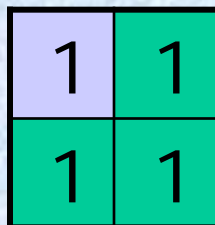
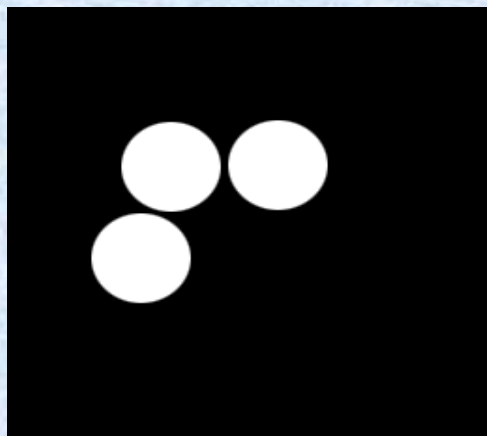
(c) 结构元素 \hat{B}

		2	0	2	0
2	0	2	2	1	2
2	1	0	2	2	
2	2	1	0		
	2	1	1		
	2	2	2		

(d) 膨胀运算结果图像

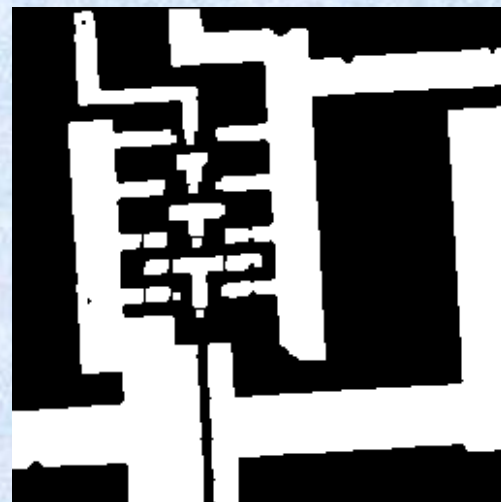
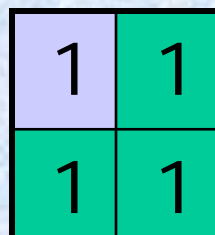
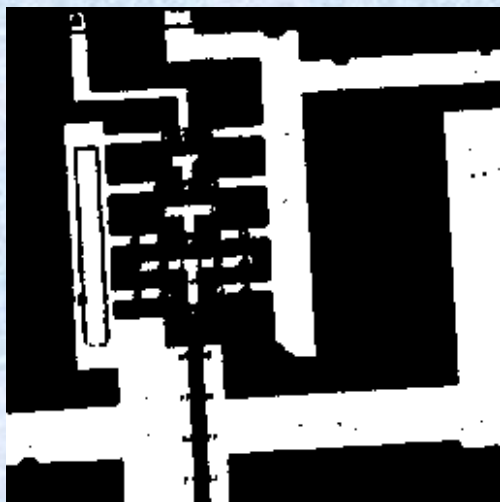


利用膨胀运算将相邻的物体连接起来





利用膨胀运算填充目标区域中的小孔





Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.



0	1	0
1	1	1
0	1	0

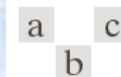


FIGURE 9.7

(a) Sample text of poor resolution with broken characters (see magnified view).
(b) Structuring element.
(c) Dilation of (a) by (b). Broken segments were joined.



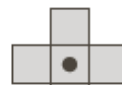
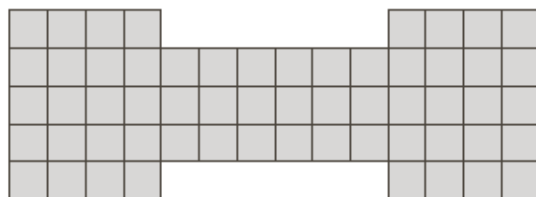
(2) 腐蚀

有没有其它的定义？

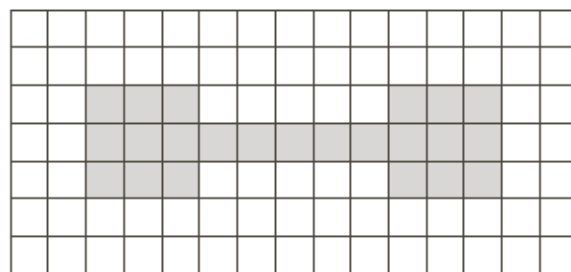
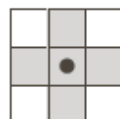
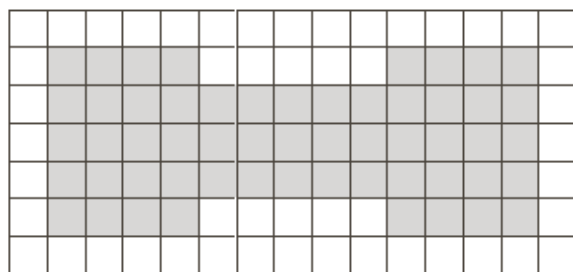
- 腐蚀：使图像缩小
 - A 用 B 来腐蚀写作 $A \ominus B$ ，定义为： $A \ominus B = \{x | (B)_x \subseteq A\}$
 B 移动后完全包含在 A 中时， B 的原点位置的集合
 - 含义：
 - 每当在目标图像 A 中找到一个与结构元素 B 相同的子图像时，就把该子图像中与 B 的原点位置对应的那个像素位置标注为1，图像 A 上标注出的所有这样的像素组成的集合，即为腐蚀运算的结果。
 - 腐蚀运算的实质就是在目标图像中标出那些与结构元素相同的子图像的原点位置的像素。
-

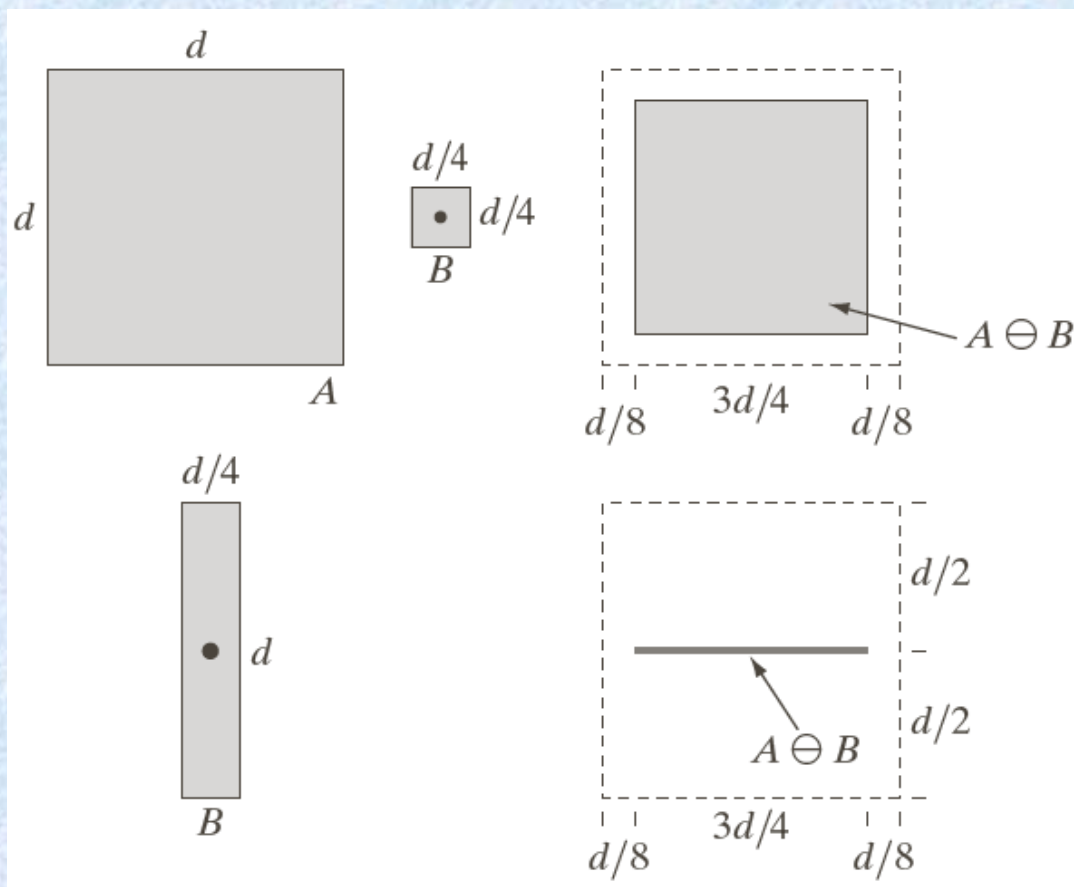


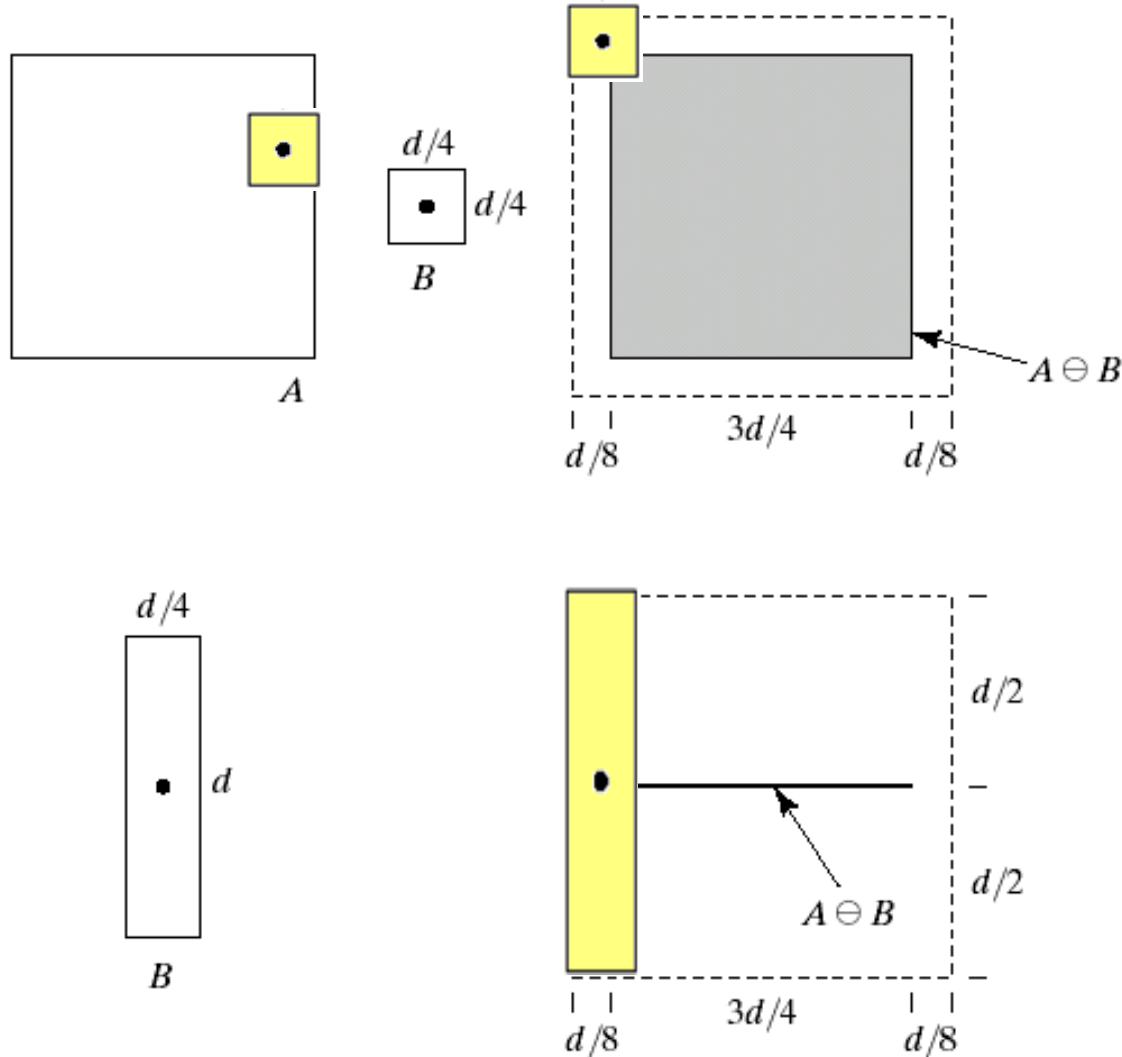
A



B







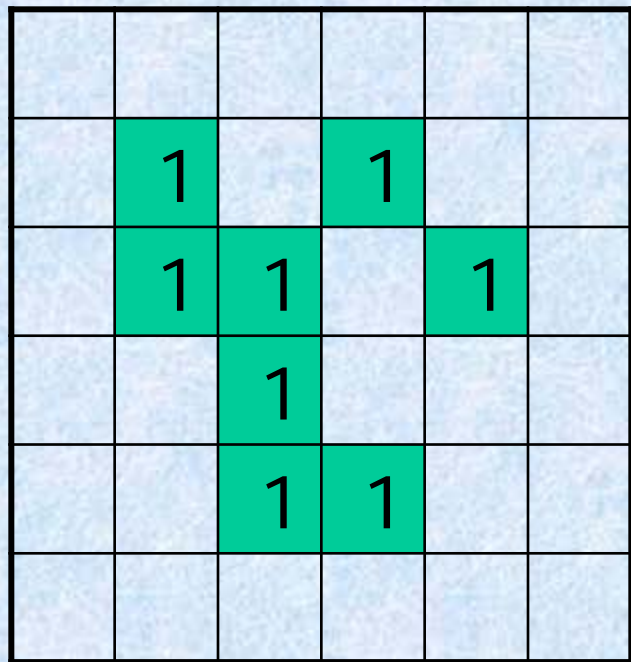
注意：当结构元素在目标图像上平移时，结构元素中的任何元素不能超出目标图像的范围。

a b c
d e

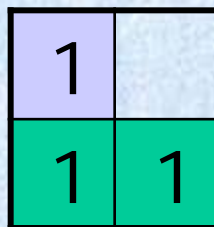
FIGURE 9.6 (a) Set A . (b) Square structuring element. (c) Erosion of A by B , shown shaded. (d) Elongated structuring element. (e) Erosion of A using this element.



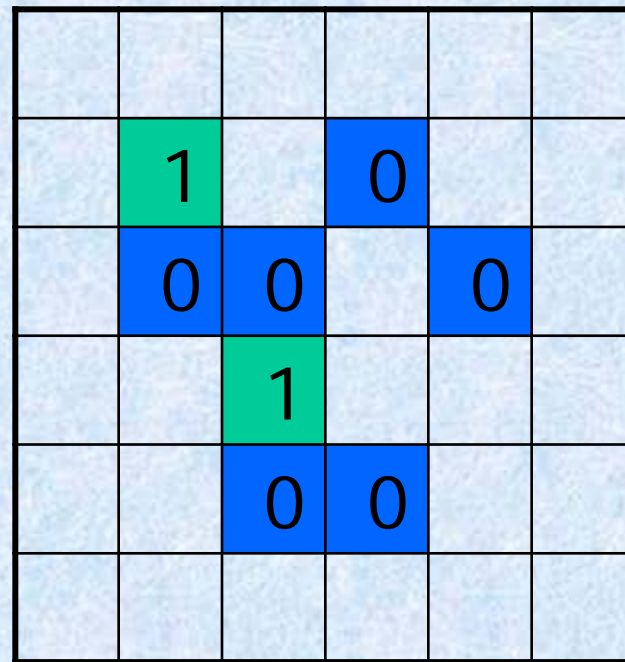
举例：



(a) 目标图像A



(b) 结构元素B



(c) 腐蚀运算结果图像

图 腐蚀运算实例



腐蚀运算的结果不仅与结构元素的形状(矩形、圆形、菱形等)选取有关，而且还与原点位置的选取有关。

腐蚀运算会缩小或细化二值图像中的物体，并能断开连接



			1		
	1			1	
	1	1			
		1	1		
		1	1		

1	1

1	1
1	1

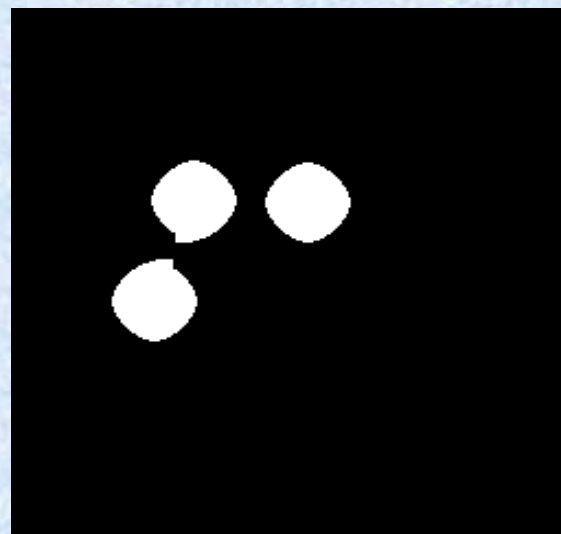
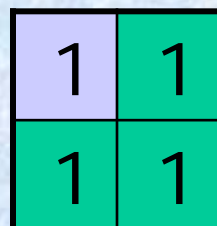
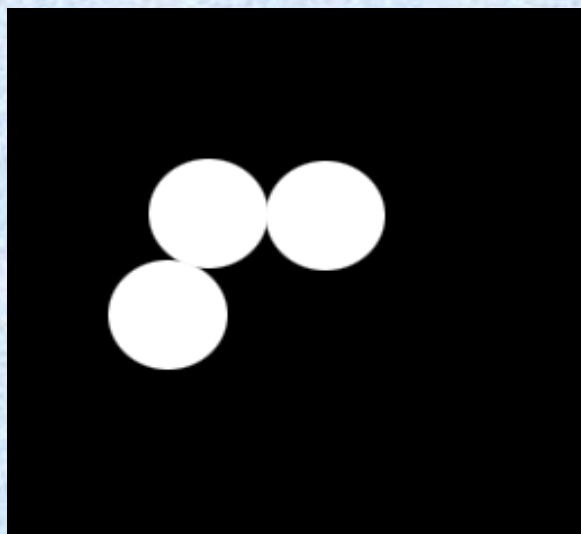
			0		
	0			0	
	1	0			
		1	0		
		1	0		

			0		
	0			0	
	0	0			
		0	1		
		0	0		

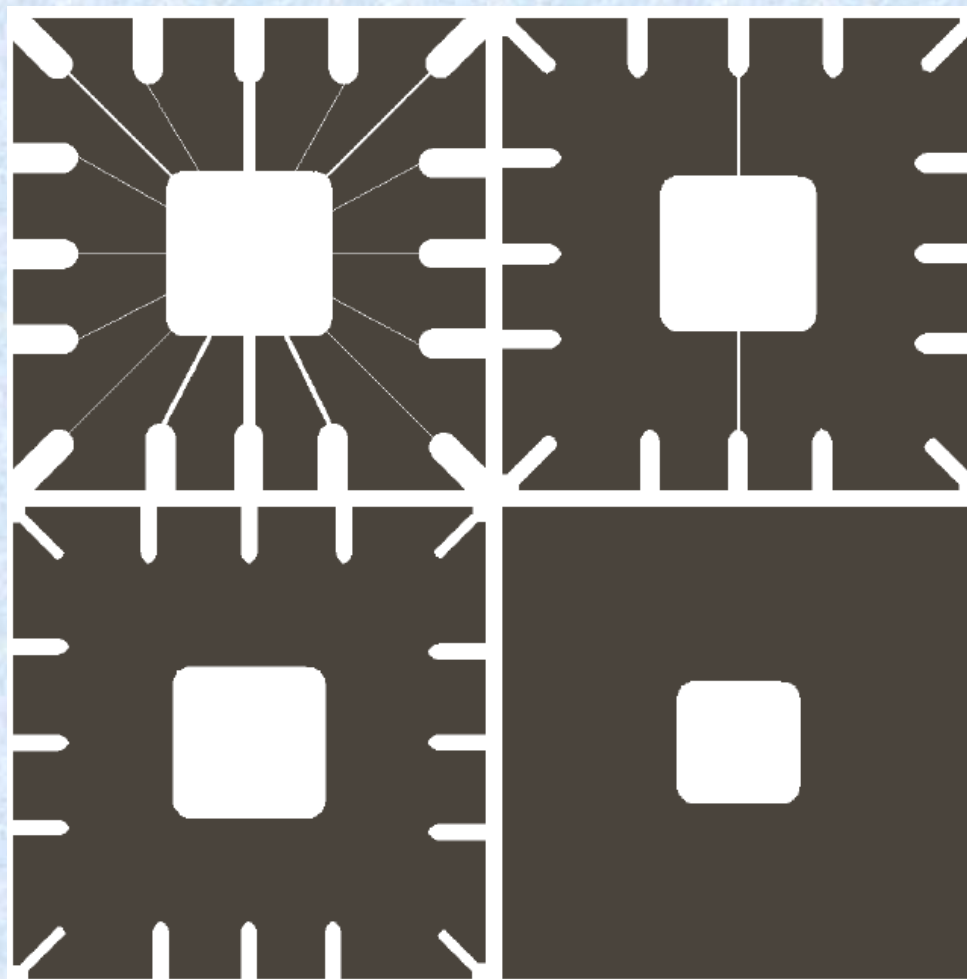
结构元素不同时的腐蚀运算实例



举例：



利用腐蚀算法消除物体之间的粘连示例



a	b
c	d

FIGURE 9.5 Using erosion to remove image components. (a) A 486×486 binary image of a wire-bond mask. (b)–(d) Image eroded using square structuring elements of sizes 11×11 , 15×15 , and 45×45 , respectively. The elements of the SEs were all 1s.



腐蚀运算与膨胀运算的对偶性

膨胀和腐蚀运算的对偶性可分别表示为：

$$(A \oplus B)^c = A^c \ominus \hat{B}$$

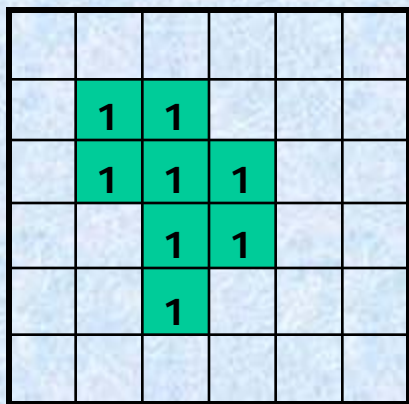
$$(A \ominus B)^c = A^c \oplus \hat{B}$$

也即：

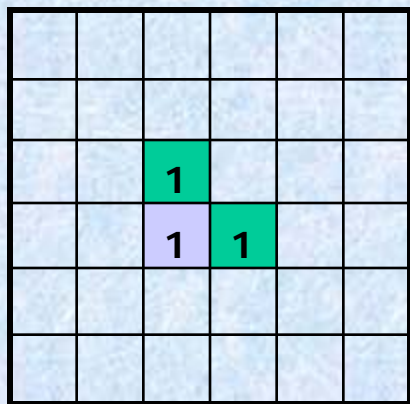
- 对目标图像的膨胀运算，相当于对图像背景的腐蚀运算操作；
- 对目标图像的腐蚀运算，相当于对图像背景的膨胀运算操作。



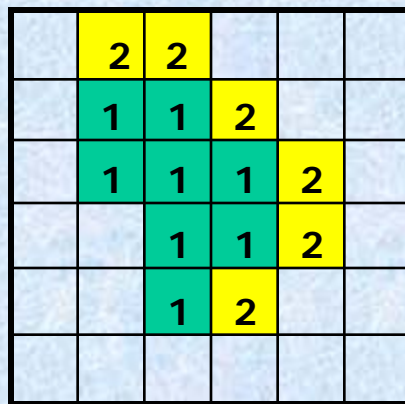
腐蚀运算与膨胀运算的对偶性一示例



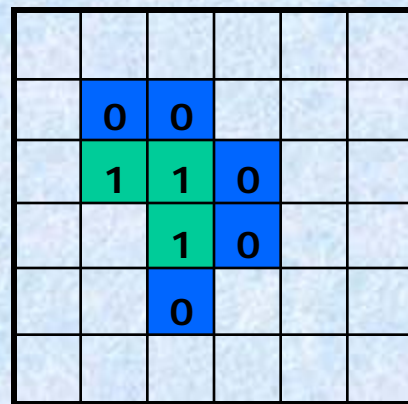
(a) 目标图像 A



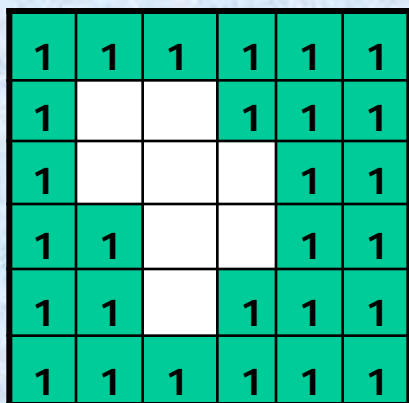
(b) 结构元素 B



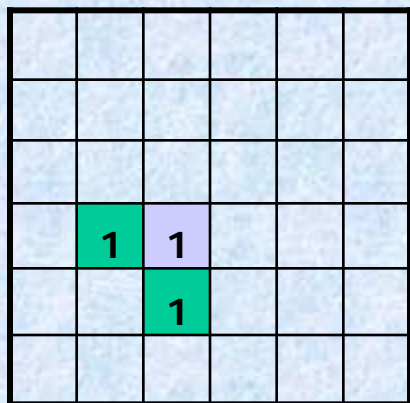
(c) 膨胀 $A \oplus B$



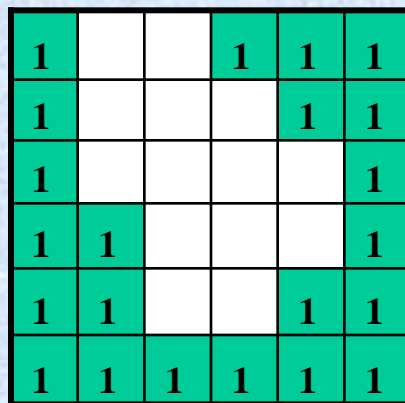
(d) 腐蚀 $A \ominus B$



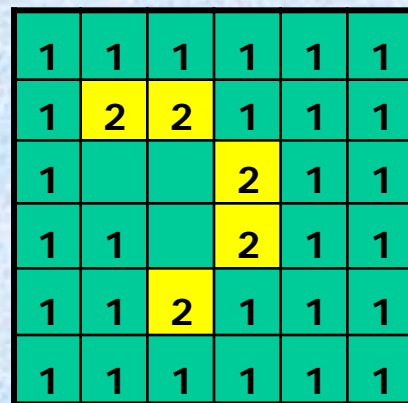
(e) A 的补 A^c



(f) B 的反射 \hat{B}



(g) 腐蚀 $A^c \ominus \hat{B}$



(h) 膨胀 $A^c \oplus \hat{B}$



(3)开操作

- B 对 A 进行的开操作就是先用 B 对 A 腐蚀，然后用 B 对结果进行膨胀。开操作一般会平滑物体的轮廓，断开较窄的狭颈并消除细的突出物

$$A \circ B = (A \ominus B) \oplus B$$



举例：

1	
1	1

1	1
	1

(b) 结构元素B

B的反射 \hat{B}

	1		1		
	1	1		1	
		1			
		1	1		

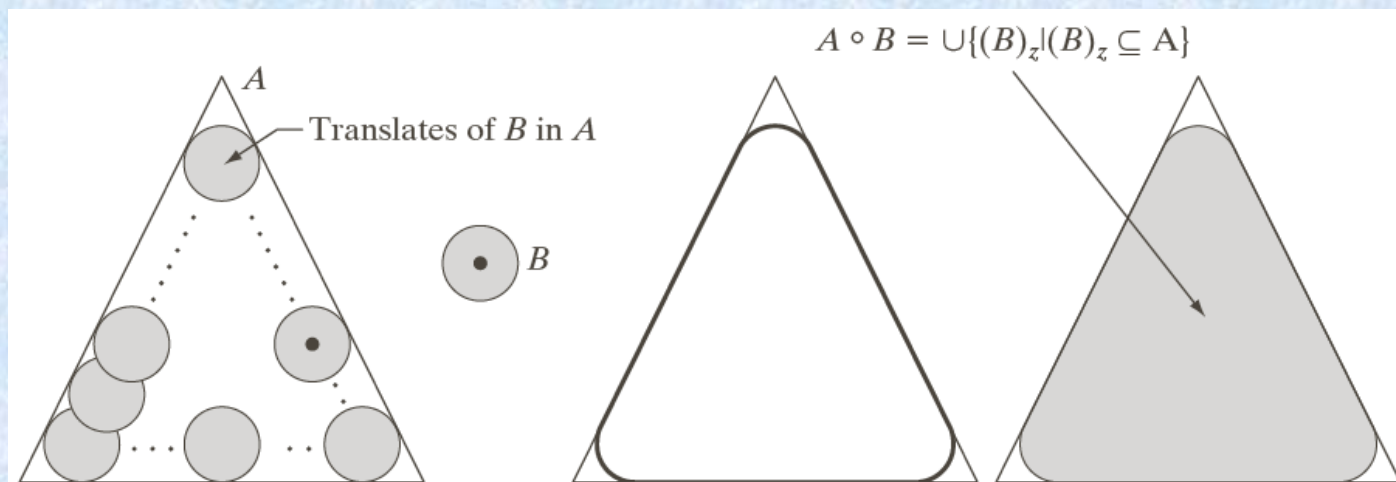
(a) 目标图像A

	1		0		
	0	0		0	
		1			
		0	0		

(c) B对A的腐蚀结果

	1		0		
	1	1		0	
		1			
		1	1		

(d) B对(c)膨胀结果



a b c d

FIGURE 9.8 (a) Structuring element B “rolling” along the inner boundary of A (the dot indicates the origin of B). (b) Structuring element. (c) The heavy line is the outer boundary of the opening. (d) Complete opening (shaded). We did not shade A in (a) for clarity.



(a) 印刷电路板二值图像



(b) 对(a)进行开运算的结果图像

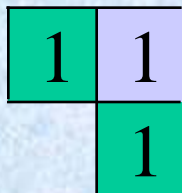
对含噪声的印刷电路板图像进行开运算实例



(4) 闭操作

- B 对 A 进行的闭操作就是先用 B 对 A 膨胀，然后用 B 对结果进行腐蚀。闭操作也会平滑轮廓的一部分，但与开操作相反，它通常会拟合较窄的沟壑，消除小的孔洞，填补轮廓中的断裂

$$A \bullet B = (A \oplus B) \ominus B$$



B的反射 \hat{B}



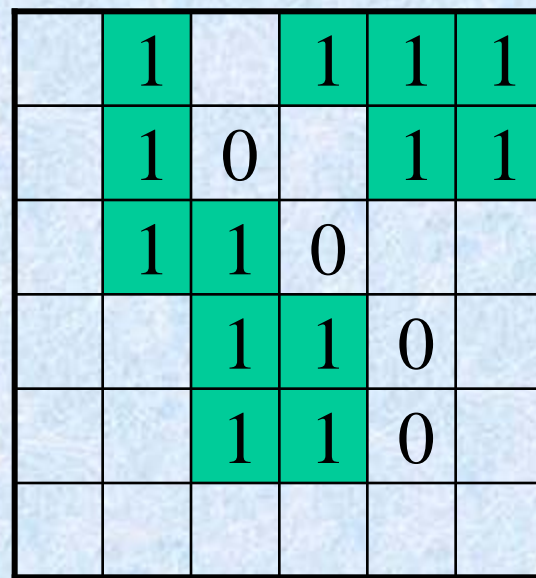
(b) 结构元素 B



(a) 目标图像A



(c) B对A的膨胀结果

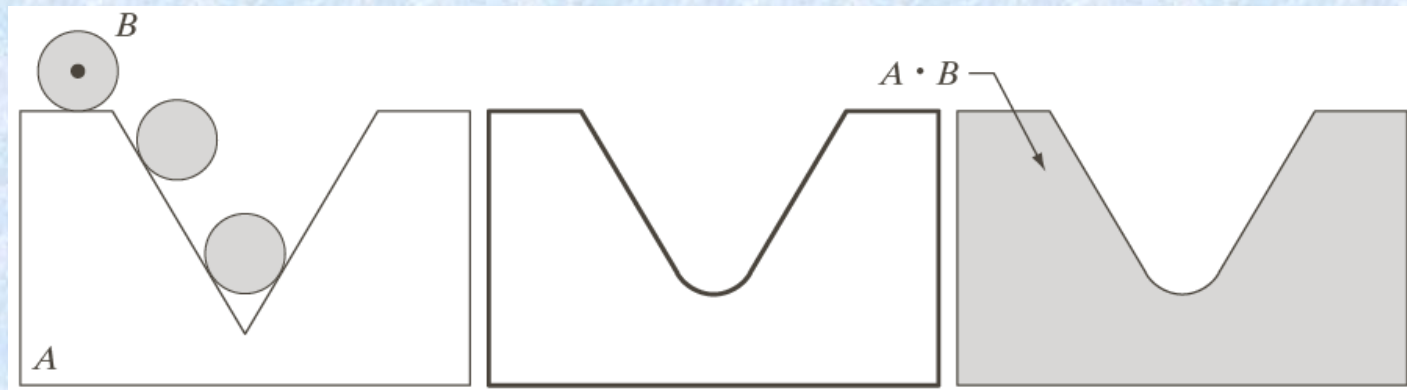


(d) B对(c) 腐蚀结果



(a) 电路板二值图像 (b) 对(a)进行闭运算的结果图像

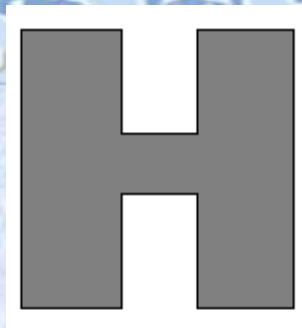
电路板二值图像闭运算实例



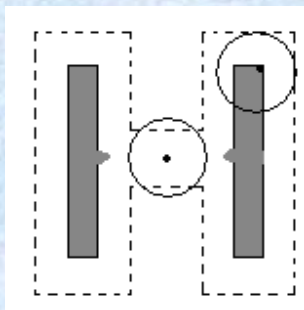
a b c

FIGURE 9.9 (a) Structuring element B “rolling” on the outer boundary of set A . (b) The heavy line is the outer boundary of the closing. (c) Complete closing (shaded). We did not shade A in (a) for clarity.

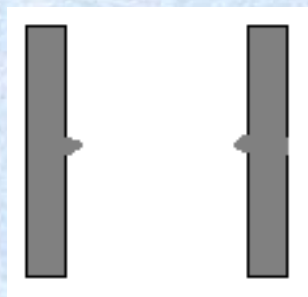
开运算具有磨光图像外边界的作用，而闭运算具有磨光物体内部边界的作用。



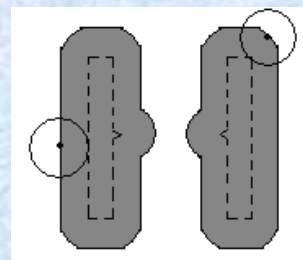
(a) H形原图像



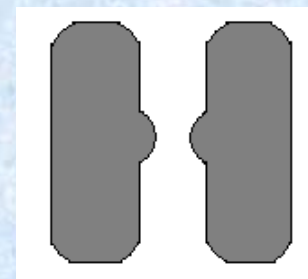
(b) 对图像进行
腐蚀运算



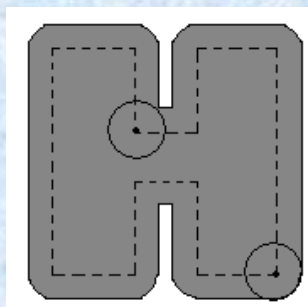
(c) 腐蚀运算结果



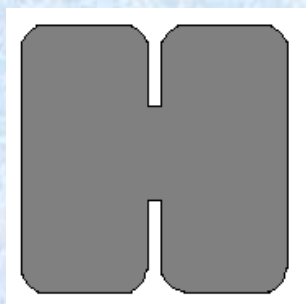
(d) 对图(c)进行
膨胀运算



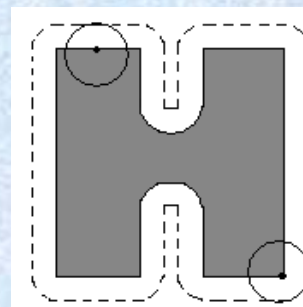
(e) 对H图像开
运算结果



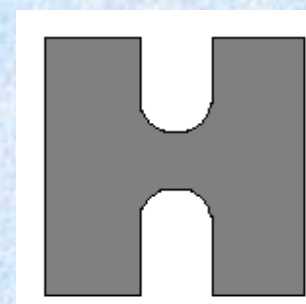
(f) 对图(a)进行
膨胀运算



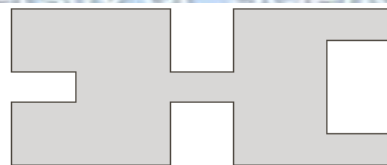
(g) 膨胀运算结果



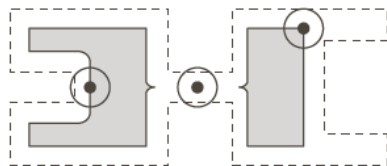
(h) 对图(g)的
腐蚀运算



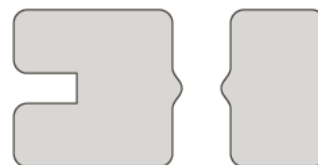
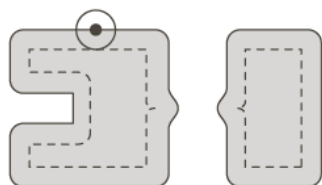
(i) 对H形图像闭
运算结果



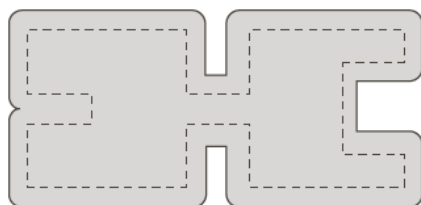
A



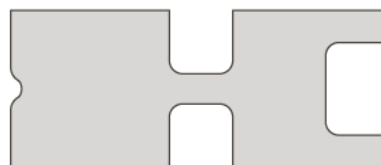
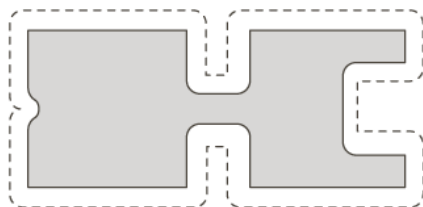
$A \ominus B$



$A \circ B = (A \ominus B) \oplus B$



$A \oplus B$



$A \cdot B = (A \oplus B) \ominus B$

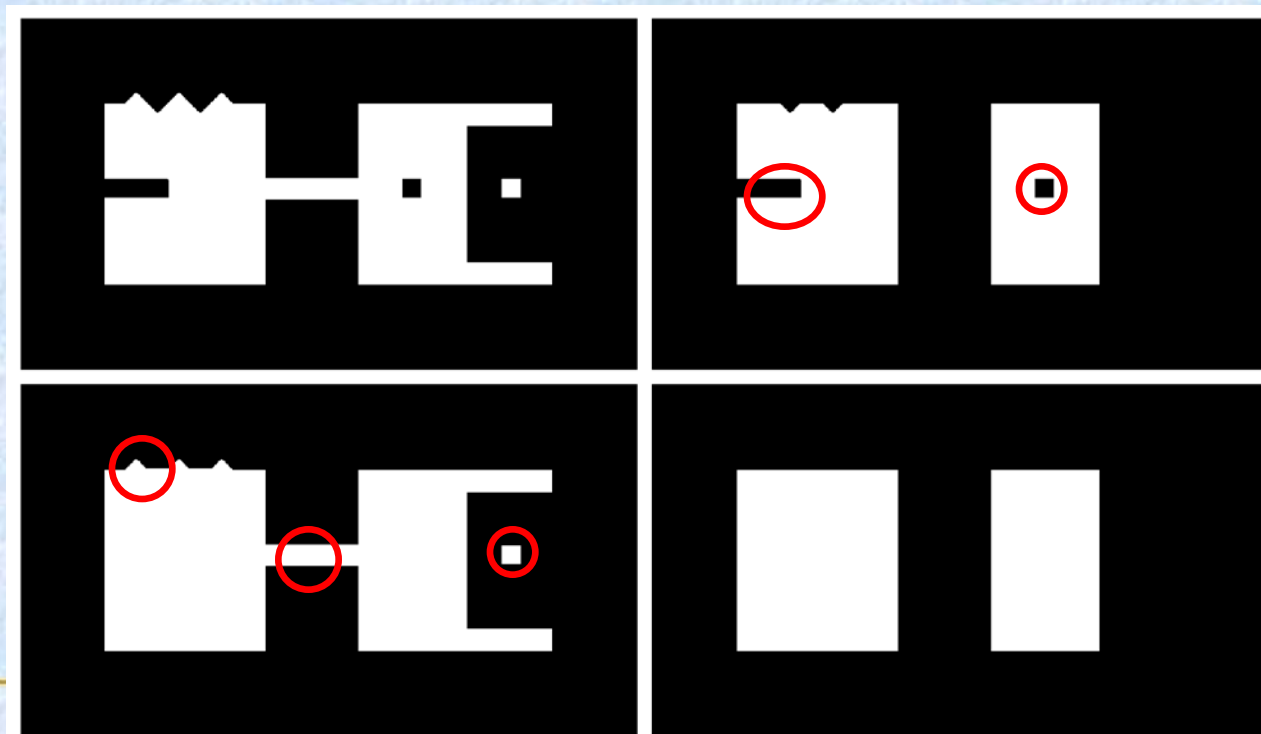


- 开操作的作用:

- 使轮廓平滑，抑制A物体边界的小离散点或尖峰。
- 用来消除小物体、在纤细点处分离物体、平滑较大物体的边界的同时并不明显改变其面积。

- 闭操作的作用:

用来填充物体内部小空洞、连接邻近物体、平滑其边界的同时并不明显改变其面积。



a	b
c	d

FIGURE 9.10

Illustration of opening and closing.

(a) Original image.

(b) Opening.

(c) Closing.

(d) Closing of (b).

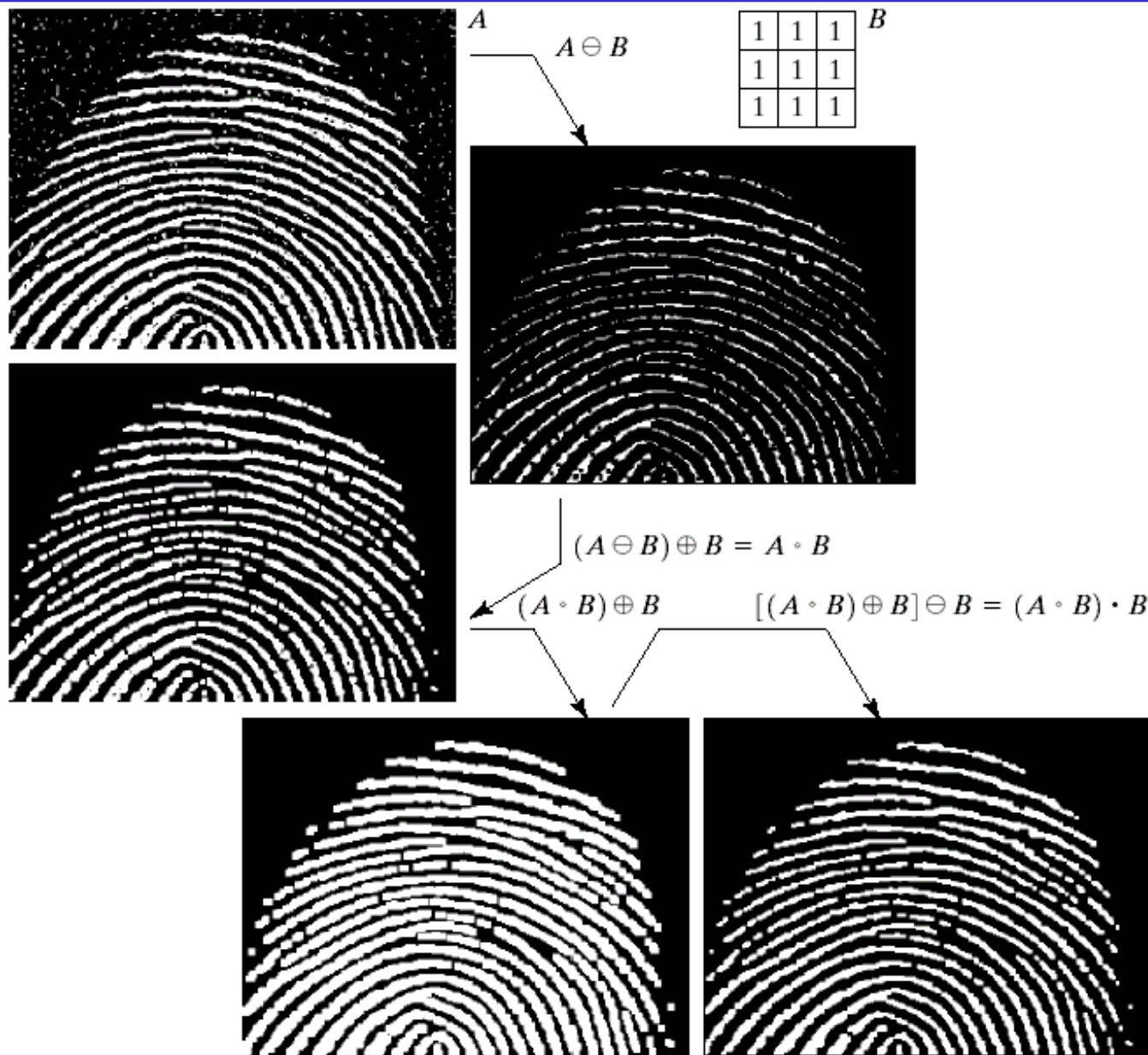


开运算与闭运算对偶性

$$(A \bullet B)^c = A^c \circ \hat{B}$$

$$(A \circ B)^c = A^c \bullet \hat{B}$$

过程：先开后闭，开消除噪声，闭修复开运算造成的指纹断裂。



a	b
d	c
e	f

FIGURE 9.11

(a) Noisy image.
 (c) Eroded image.
 (d) Opening of A .
 (e) Closing of the opening. (Original image for this example courtesy of the National Institute of Standards and Technology.)



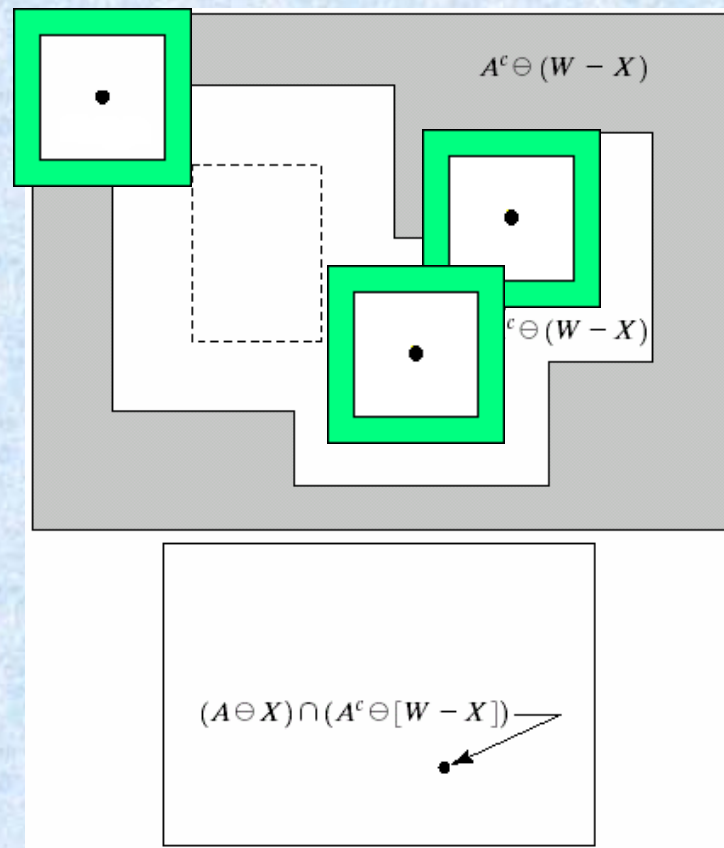
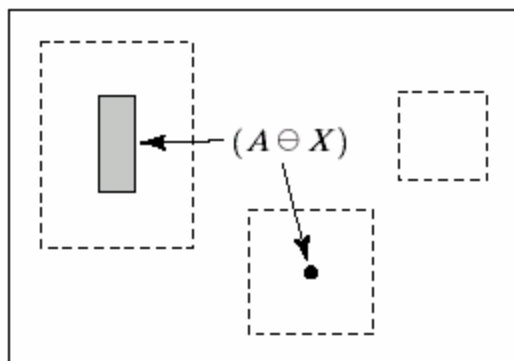
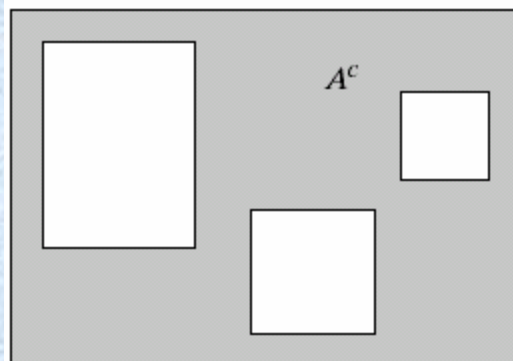
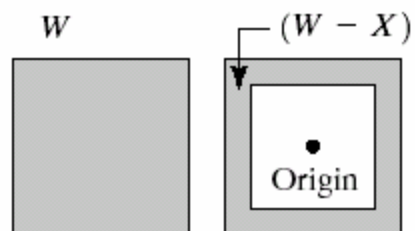
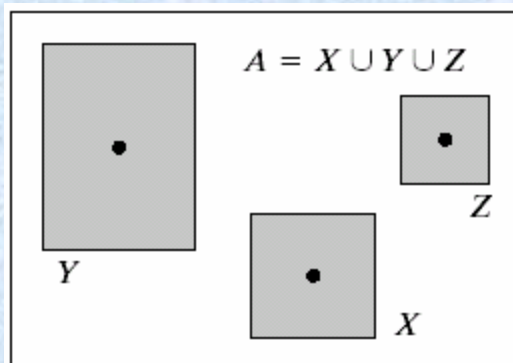
(5) 击中/击不中 形状检测的基本工具

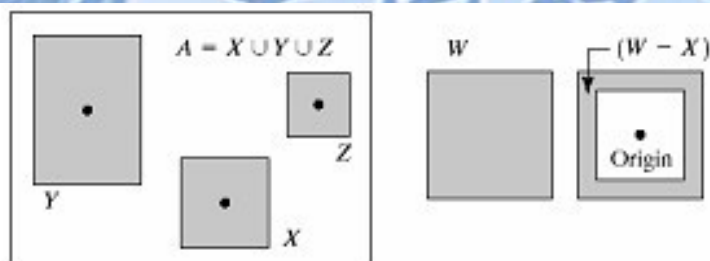
在不同尺寸的图形中检测出想要的形状

$$A * B = (A \ominus B_1) \cap [A^c \ominus B_2] \quad A * B = (A \ominus X) \cap [A^c \ominus (W - X)]$$

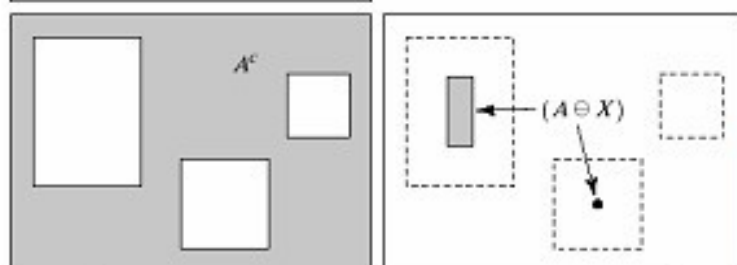
B 表示由 X 和 X 的背景构成的集合

令 $B=(B_1, B_2)$, B_1 是与 B 对象相联系的像素构成的集合。 B_2 是与 B 对象相应背景有关像素的集合。 $B_1 = X, B_2 = (W-X)$



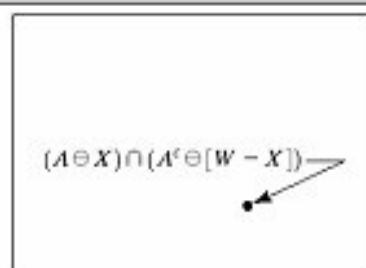
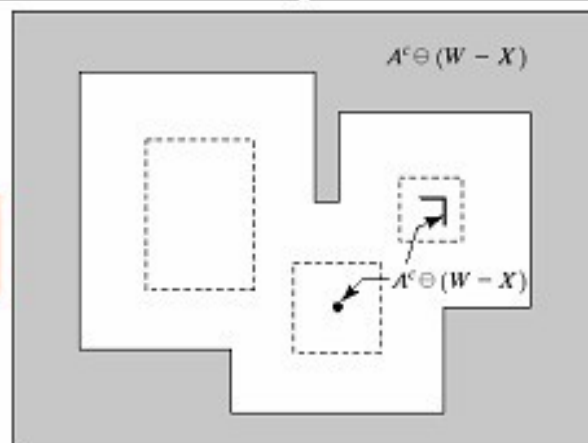


设 X 被包围在一个小窗口 W 中，与 W 有关的 X 的局部背景定义为 $W-X$

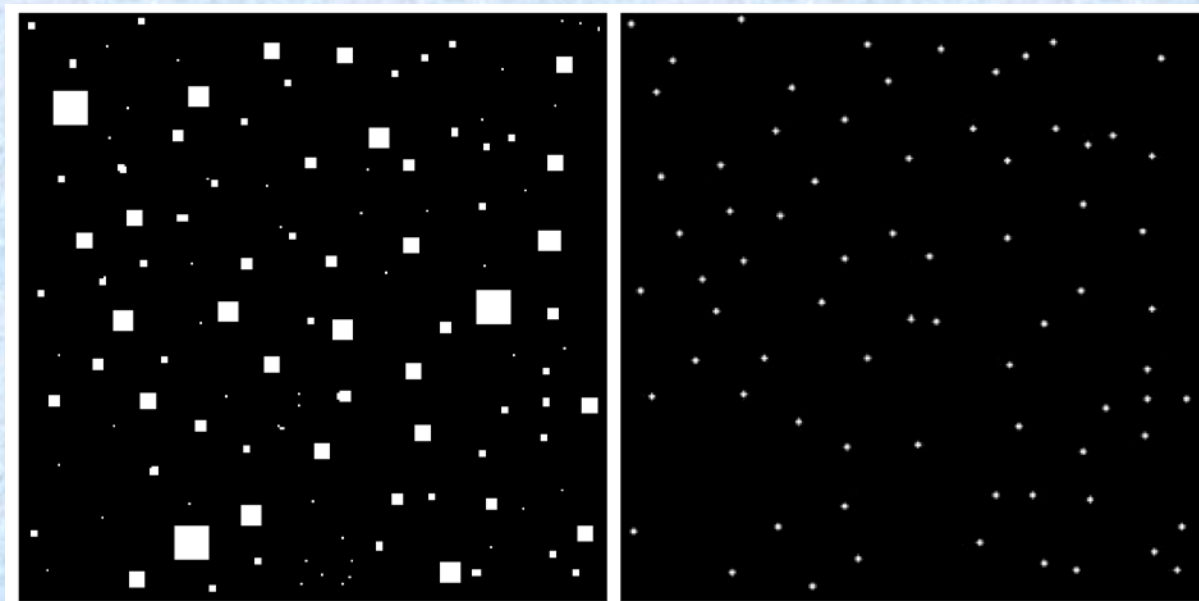


X 对 A 腐蚀的结果

$W-X$ 对 A 的补集
腐蚀的结果



它们的交集，得到 X 的
原点位置。在该位置， X 找到了在 A 中的
匹配（击中）



a b

FIGURE 9.13
(a) Original image. (b) Result of applying the hit-or-miss transformation (the dots shown were enlarged to facilitate viewing).



四、形态学的主要应用

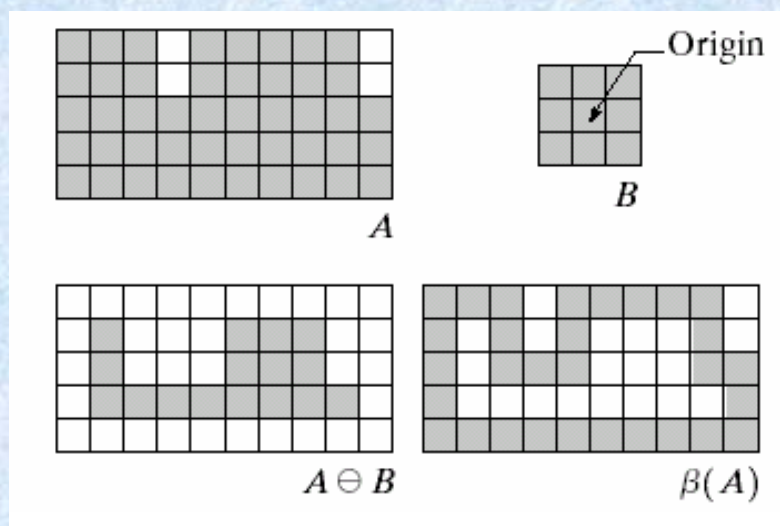
- 处理图像的类型：二值图像
 - 作用：提取对于描绘和表达形状有用的图像成分
 - 主要应用：边界提取、区域填充、连通分量提取、凸壳和细化以及骨架等等
-



边界提取

$$\beta(A) = A - (A \ominus B)$$

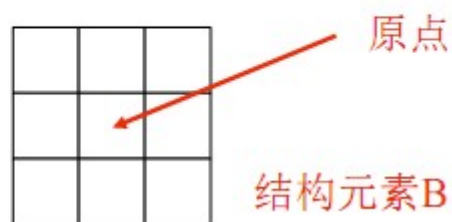
- 设集合 A 的边界表示为 $\beta(A)$ ，选取结构元素 B ，先进行 B 对 A 腐蚀，而后再用 A 减去腐蚀的结果。

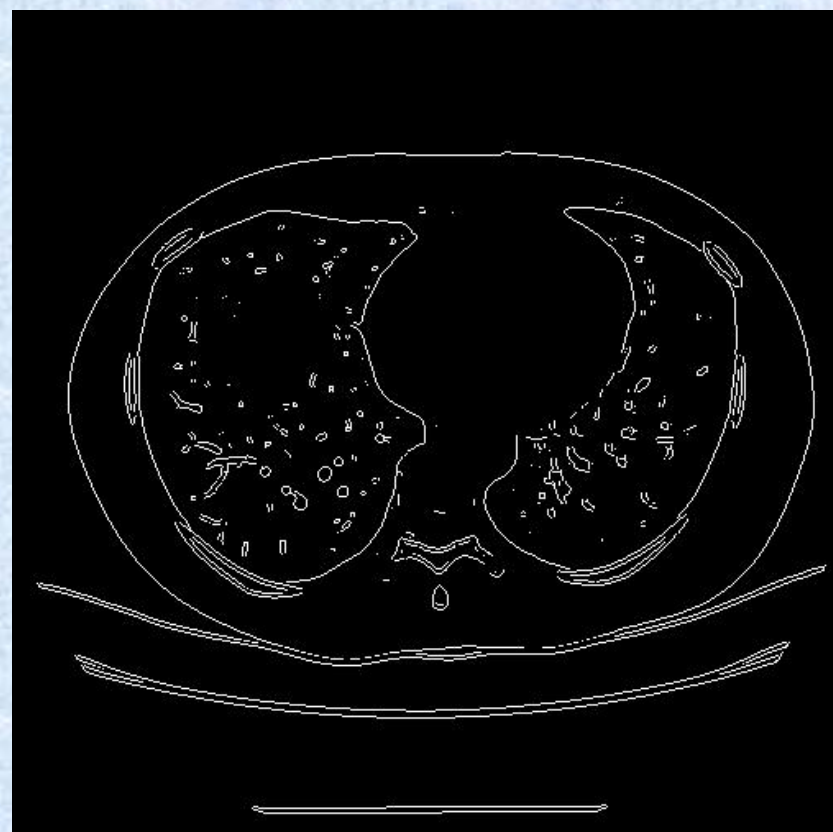
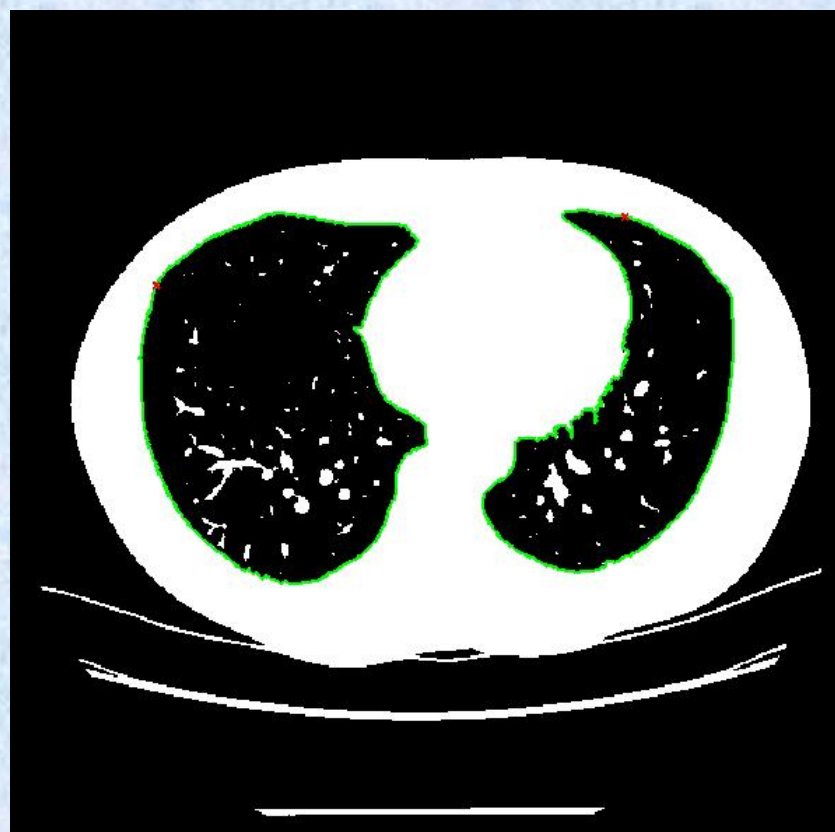


边界宽度是单像素



1表示为白色，0表示为黑色





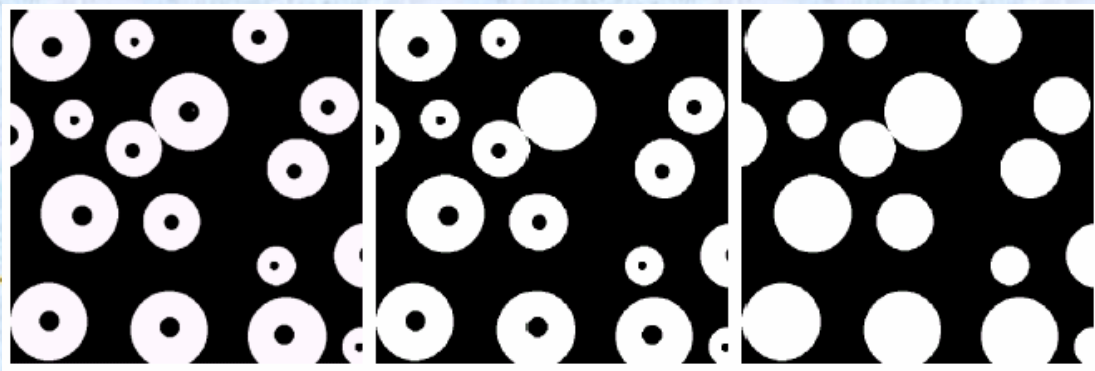
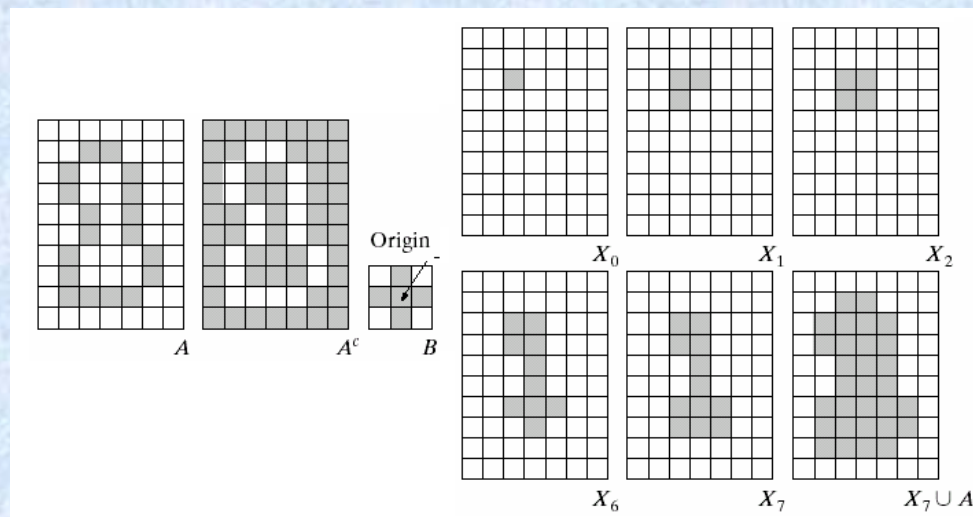


孔洞填充

$$X_k = (X_{k-1} \oplus B) \cap A^c \quad k = 1, 2, 3, \dots$$

迭代k次，直到 $X_k = X_{k-1}$

注意：
要设置初始点



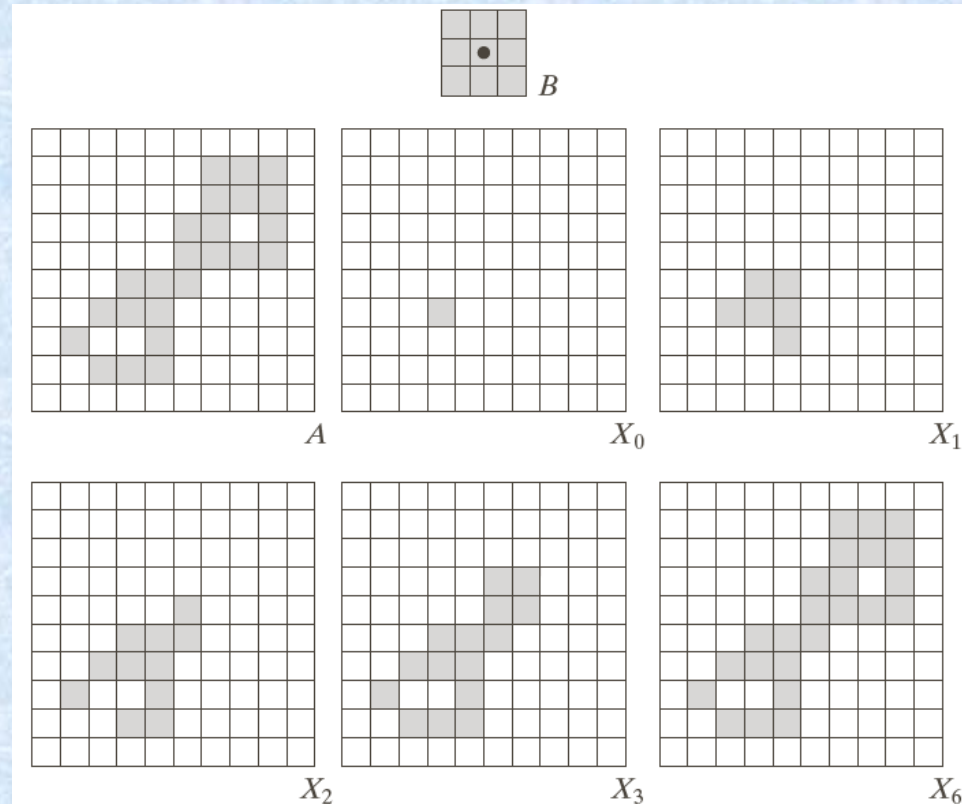


连通分量提取

$$X_k = (X_{k-1} \oplus B) \cap A, k = 1, 2, 3, \dots$$

注意：
要设置
初始点

连通分量的定义？



	a	
b	c	d
e	f	g

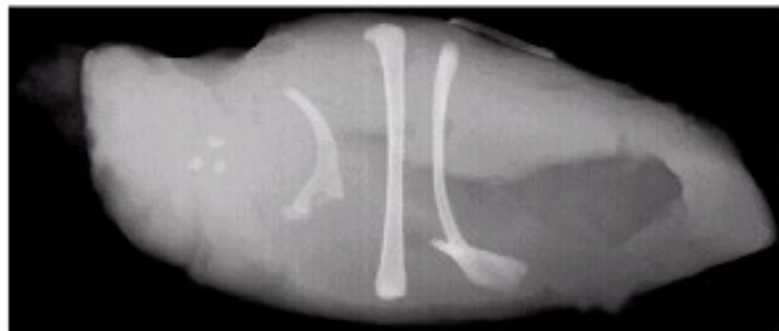
FIGURE 9.17 Extracting connected components. (a) Structuring element. (b) Array containing a set with one connected component. (c) Initial array containing a 1 in the region of the connected component. (d)–(g) Various steps in the iteration of Eq. (9.5-3).



a
b
c d

FIGURE 9.18

(a) X-ray image of chicken filet with bone fragments.
(b) Thresholded image. (c) Image eroded with a 5×5 structuring element of 1's.
(d) Number of pixels in the connected components of (c). (Image courtesy of NTB Elektronische Geraete GmbH, Diepholz, Germany, www.ntbxray.com.)



含有碎骨的鸡胸X光图像

使用阈值将骨头从背景中
提取出来

Connected component	No. of pixels in connected comp
01	11
02	9
03	9
04	39
05	133
06	1
07	1
08	743
09	7
10	11
11	11
12	9
13	9
14	674
15	85

消除细节，对阈值处理后的图像进行腐蚀，
保留大尺寸物体

提取连通分量，识别大尺寸
对象，其中4个具有最大尺寸



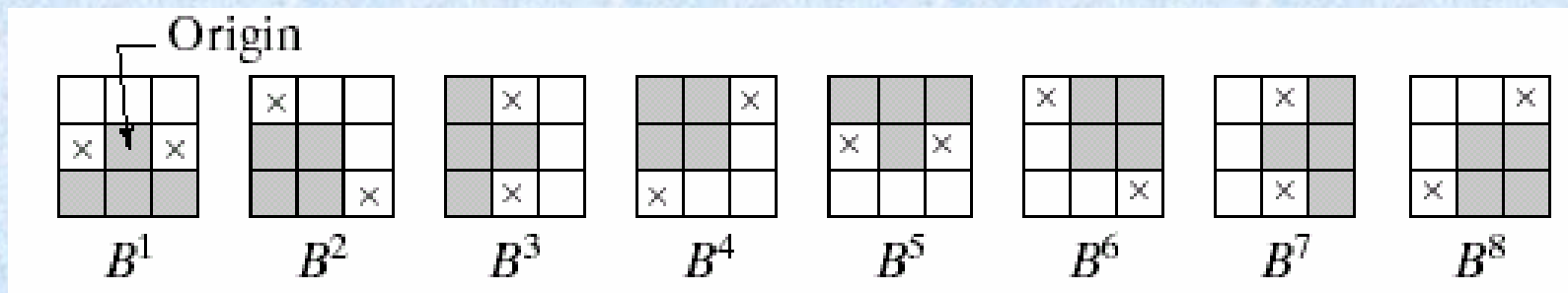
细化

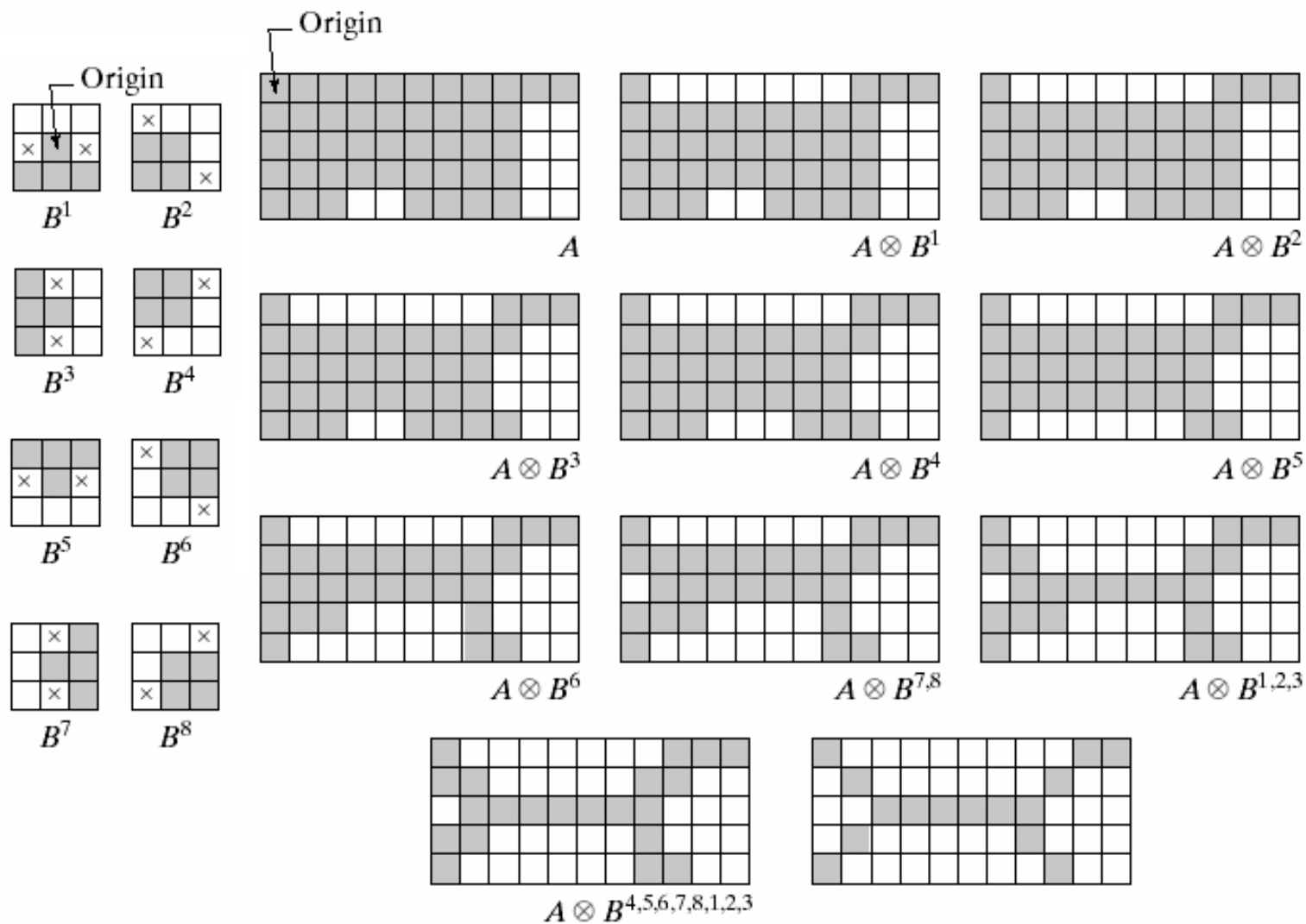


$$A \otimes B = A - (A * B) = A \cap (A * B)^c$$

$$\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$$

$$A \otimes B = (((\dots((A \otimes B^1) \otimes B^2) \dots) \otimes B^n)$$







粗化

$$A \bullet B = A \cup (A * B)$$

注意：
结构体和细化时一样，但0,1要互换

粗化是细化的对偶，可以通过对集合求补后进行细化，最后对细化的过程再进行求补得到

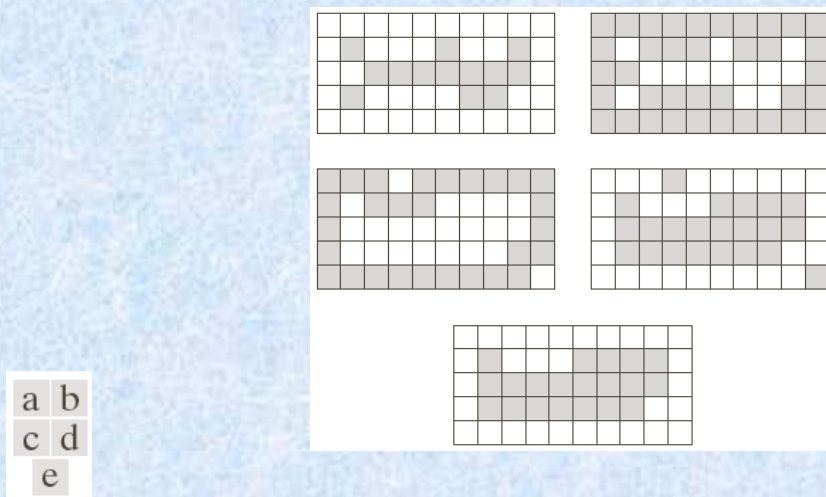


FIGURE 9.22 (a) Set A. (b) Complement of A. (c) Result of thinning the complement of A. (d) Thickened set obtained by complementing (c). (e) Final result, with no disconnected points.



凸壳

凸集：任意两点直线段都在集合中

凸壳：任何集合S的凸壳H是包含S的最小凸集

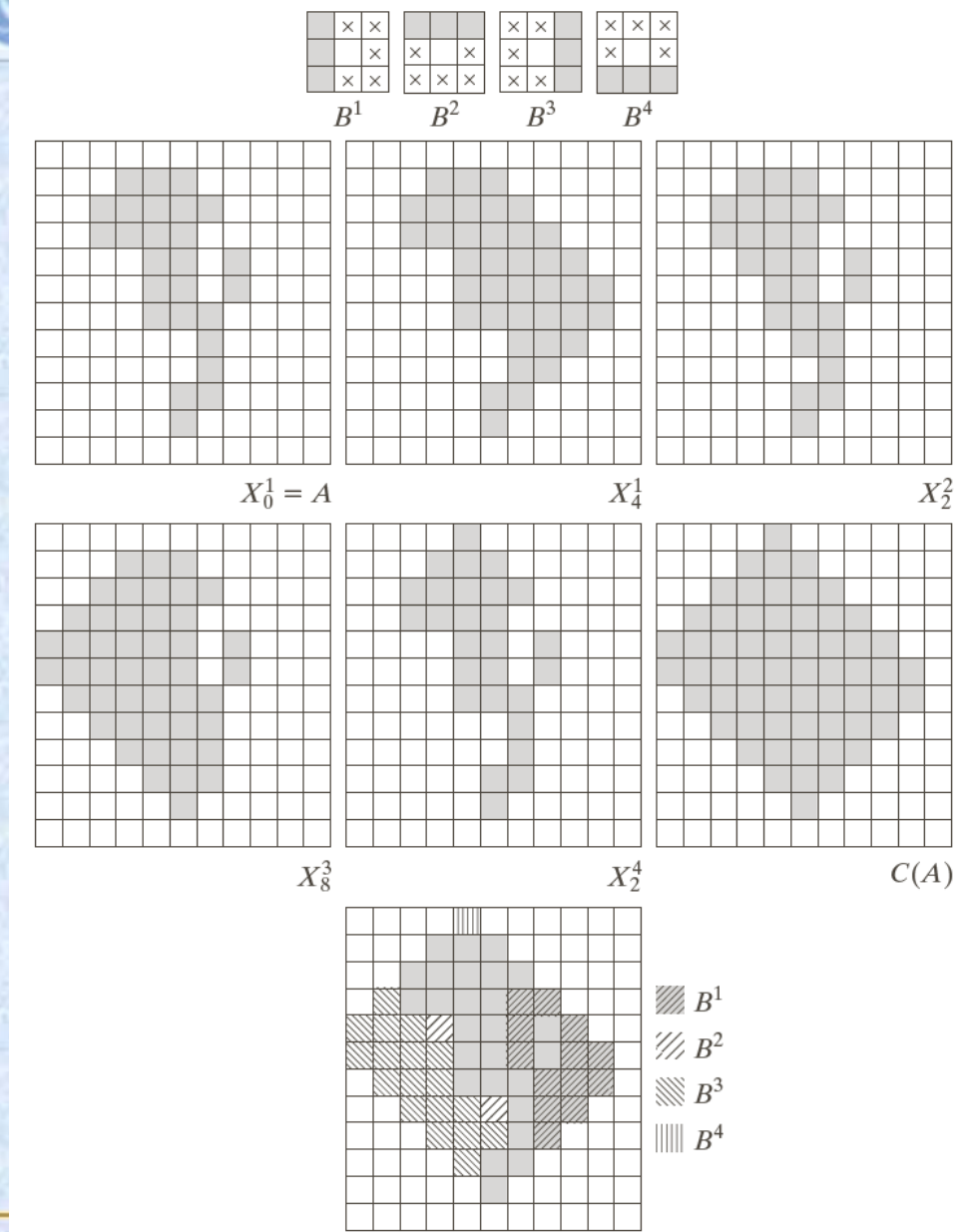
凸缺： $H-S$

$$X_k^i = (X_{k-1} * B^i) \cup A^c \quad k = 1, 2, 3, \dots i = 1, 2, 3, 4$$

$$\text{停止条件 } X_k^i = X_{k-1}^i$$

令 $D^i = X_k^i$ 则A的凸集为：

$$C(A) = \bigcup D^i$$



	a	
b	c	d
e	f	g
	h	

FIGURE 9.19

(a) Structuring elements. (b) Set A . (c)–(f) Results of convergence with the structuring elements shown in (a). (g) Convex hull. (h) Convex hull showing the contribution of each structuring element.



最小凸集

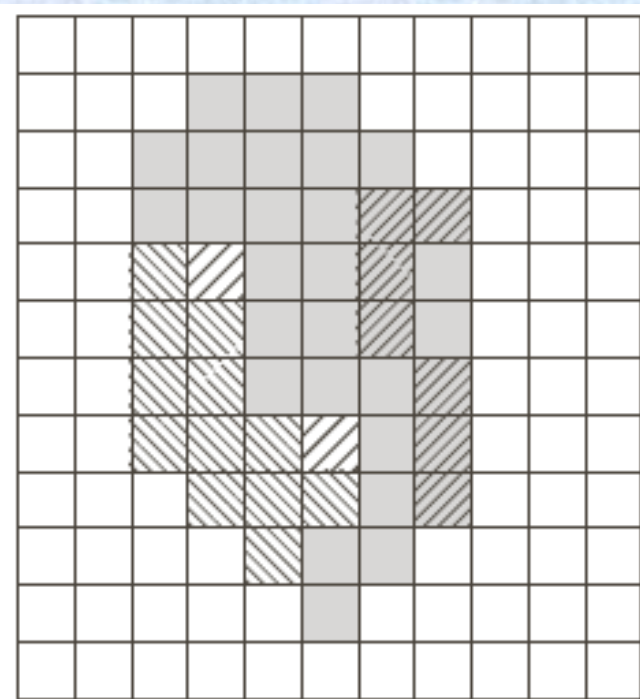


FIGURE 9.20
Result of limiting
growth of the
convex hull
algorithm to the
maximum
dimensions of the
original set of
points along the
vertical and
horizontal
directions.

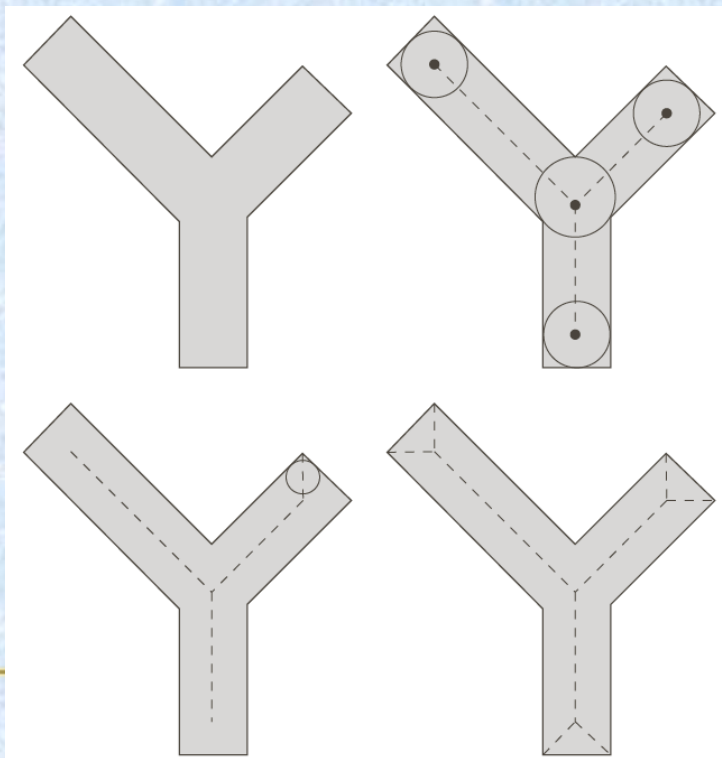


骨架

定义:

(a) 如果 z 是 $S(A)$ 的一个点, 并且 $(D)_z$ 是 A 内以 z 为中心的最大圆盘, 则不存在包含 $(D)_z$ 且位于 A 内的更大圆盘 (不必以 z 为中心)。圆盘 $(D)_z$ 称为最大圆盘

(b) 圆盘 $(D)_z$ 在两个或多个不同的位置与 A 的边界接触



a	b
c	d

FIGURE 9.23

(a) Set A .

(b) Various positions of maximum disks with centers on the skeleton of A .

(c) Another maximum disk on a different segment of the skeleton of A .

(d) Complete skeleton.



骨架可表示为

$$S(A) = \bigcup S_k(A), k = 0, 1, 2 \dots K$$

其中

$$S_k(A) = (A \ominus kB) - (A \ominus kB) \circ B$$

$$K = \max \{k \mid A \ominus kB \neq \Phi\}$$

可以通过骨架的子集来重建集合

$$A = \bigcup (S_k(A) \oplus kB), k = 0, 1, 2 \dots K$$

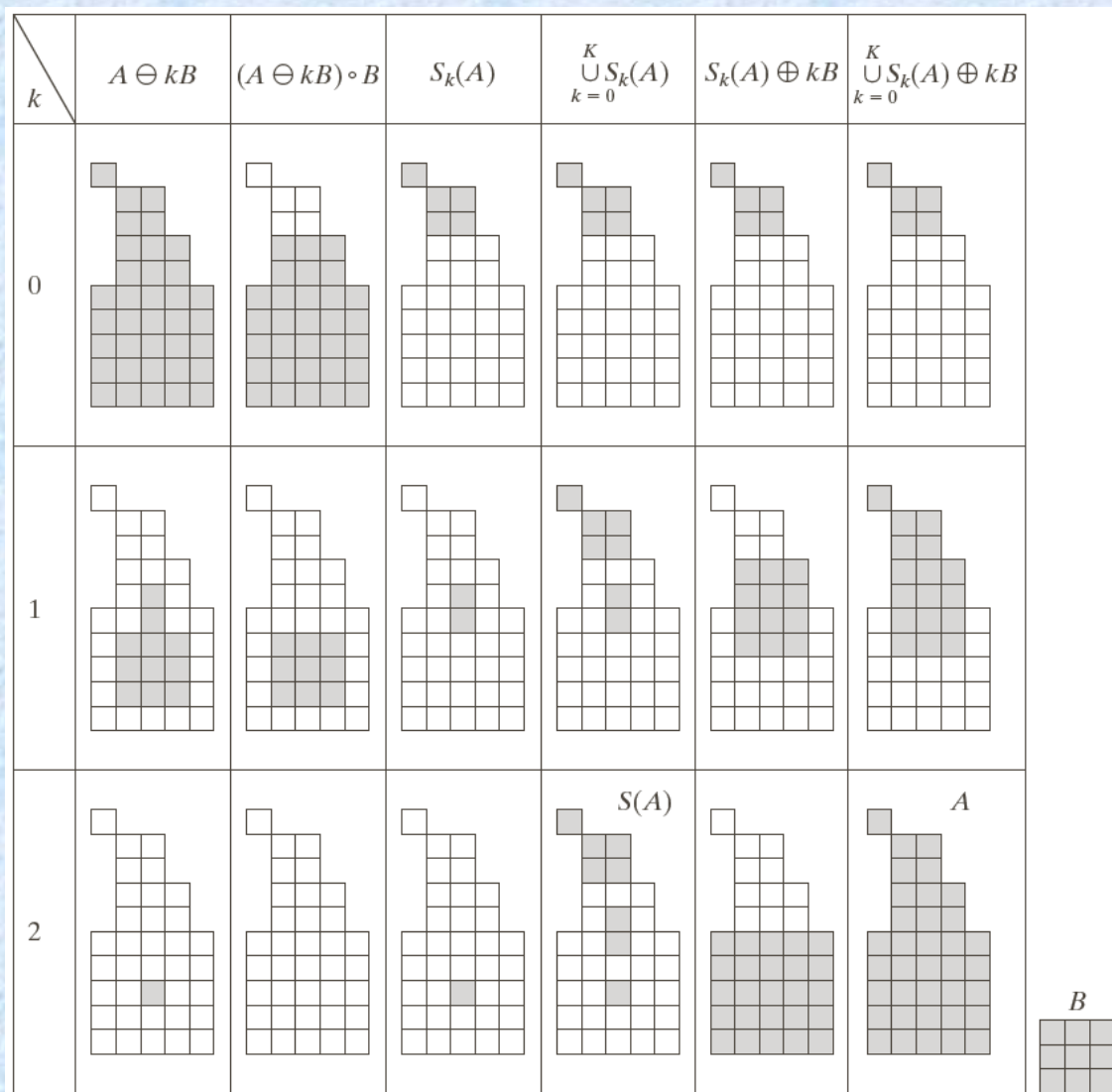


FIGURE 9.24
Implementation
of Eqs. (9.5-11)
through (9.5-15).
The original set is
at the top left, and
its morphological
skeleton is at the
bottom of the
fourth column.
The reconstructed
set is at the
bottom of the
sixth column.



剪裁

剪裁是对细化和骨架算法的补充，因为这些过程会保留某些寄生分量，因为需要用后处理来清楚这些寄生分量

$$X_1 = A \otimes \{B\}$$

$$X_2 = \bigcup (X_1 * B^k)$$

$$X_3 = (X_2 \oplus H) \cap A$$

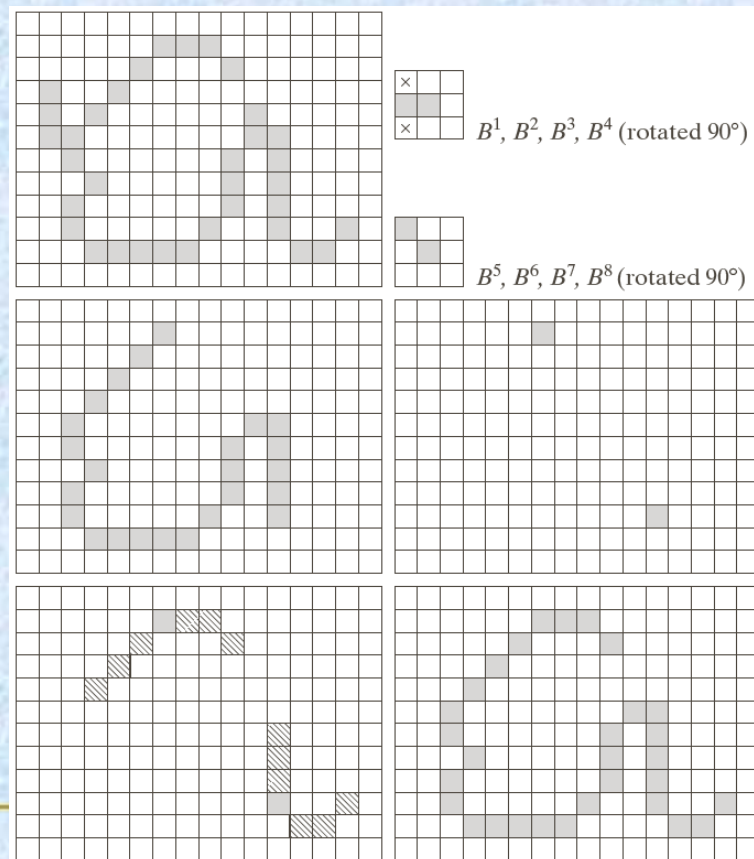


FIGURE 9.25
(a) Original image. (b) and (c) Structuring elements used for deleting end points. (d) Result of three cycles of thinning. (e) End points of (d). (f) Dilation of end points conditioned on (a). (g) Pruned image.



五、形态学重建

形态学重建涉及两幅图像和一个结构元。一幅是标记，它包含变换的起始点，另一幅是模板，它约束该变换。结构元一般是各向同性

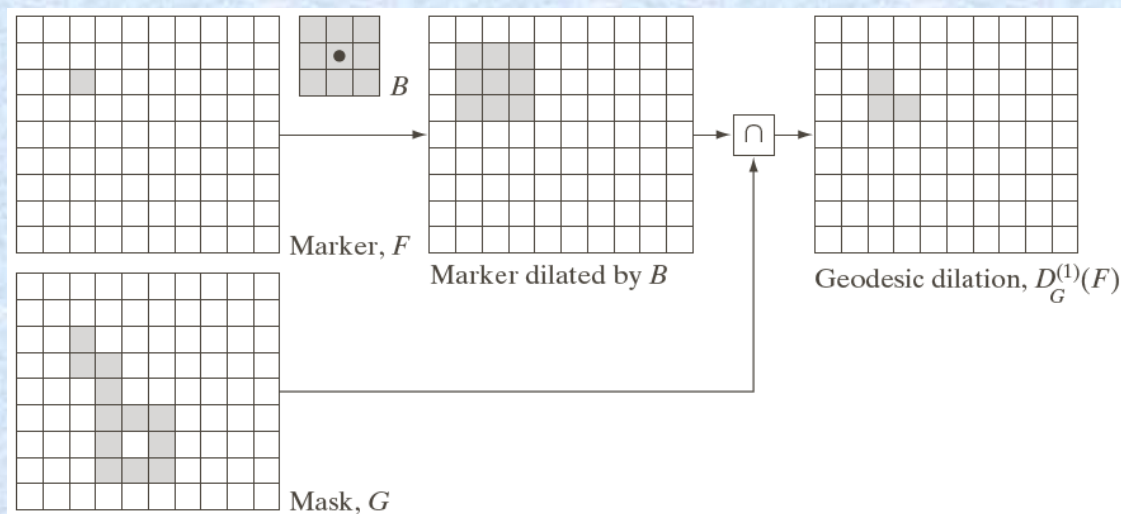


测地膨胀

$$D_G^{(1)}(F) = (F \oplus B) \cap G$$

$D_G^{(1)}(F)$ 表示大小为1的标记图像关于模板的测地膨胀

$$D_G^{(n)}(F) = D_G^{(1)}[D_G^{(n-1)}(F)]$$



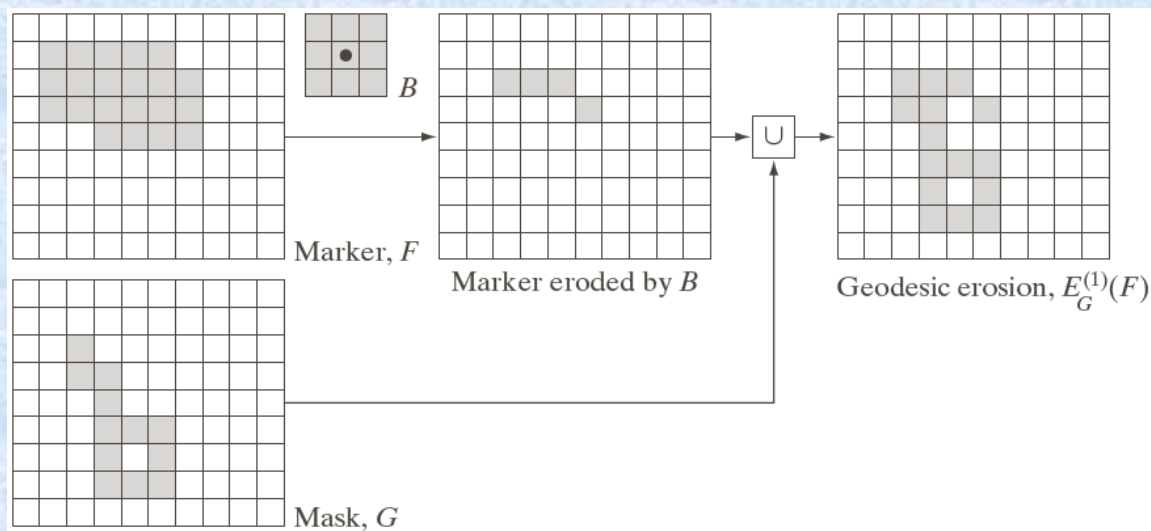


测地腐蚀

$$E_G^{(1)}(F) = (F \ominus B) \cup G$$

$E_G^{(1)}(F)$ 表示大小为1的标记图像关于模板的测地腐蚀

$$E_G^{(n)}(F) = E_G^{(1)}[E_G^{(n-1)}(F)]$$





利用膨胀的形态学重建

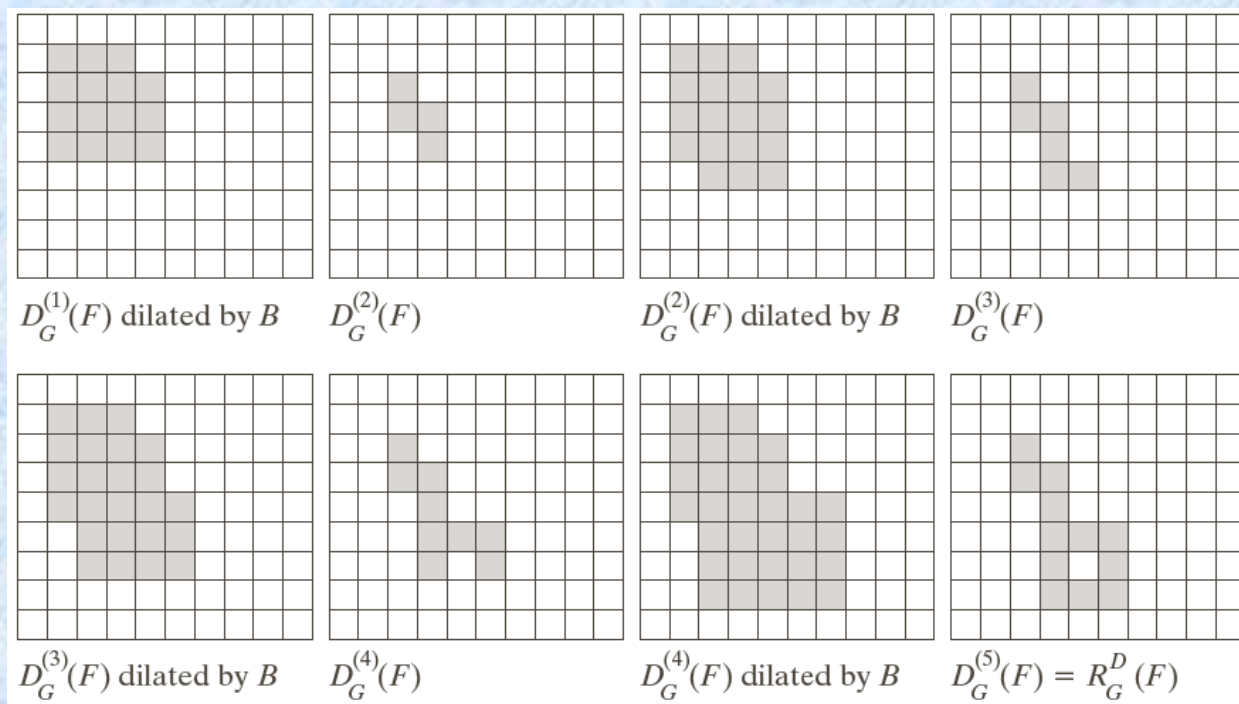
$$R_G^D(F) = D_G^{(k)}(F)$$

迭代k次，直到 $D_G^{(k)}(F) = D_G^{(k+1)}(F)$

利用腐蚀的形态学重建

$$R_G^E(F) = E_G^{(k)}(F)$$

迭代k次，直到 $E_G^{(k)}(F) = E_G^{(k+1)}(F)$



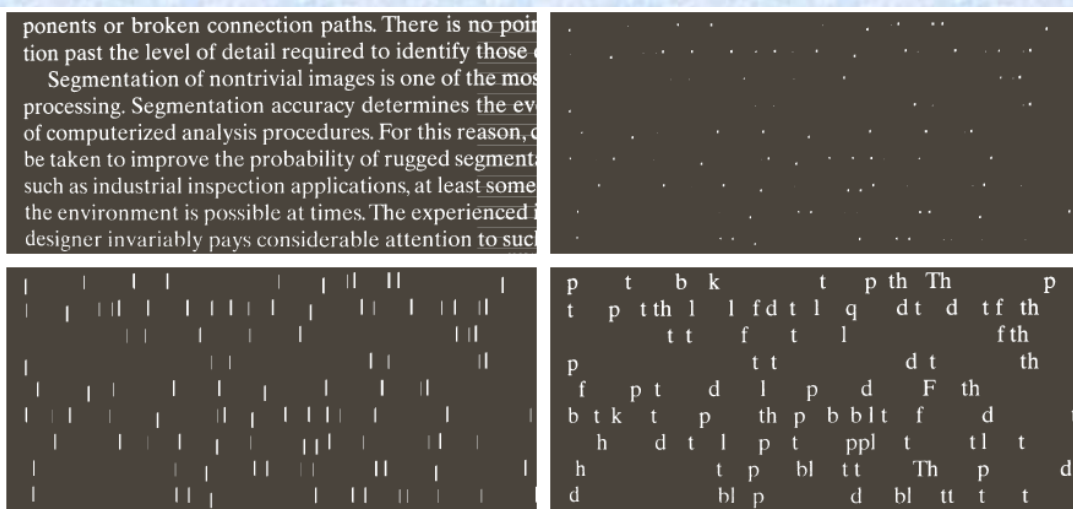
a	b	c	d
e	f	g	h

FIGURE 9.28
Illustration of
morphological
reconstruction by
dilation. F , G , B
and $D_G^{(1)}(F)$ are
from Fig. 9.26.



重建开操作

$$O_R^{(n)} = R_F^D [(F \ominus nB)]$$



a b
c d

FIGURE 9.29 (a) Text image of size 918×2018 pixels. The approximate average height of the tall characters is 50 pixels. (b) Erosion of (a) with a structuring element of size 51×1 pixels. (c) Opening of (a) with the same structuring element, shown for reference. (d) Result of opening by reconstruction.

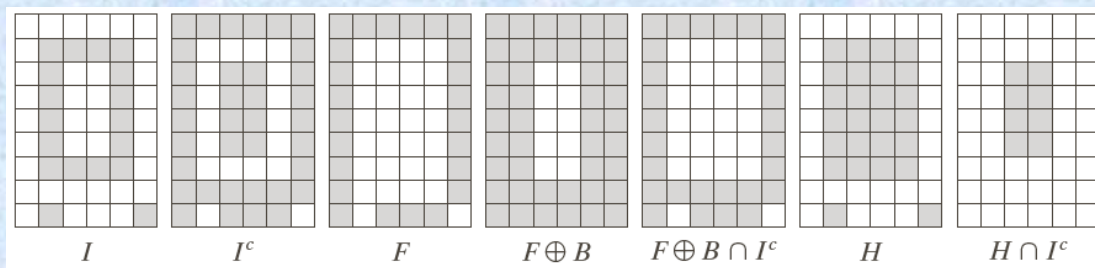
直接使用开操作有什么问题？



填充孔洞

$$F(x, y) = \begin{cases} 1 - I(x, y), & x, y \text{ 在 } I \text{ 的边界上} \\ 0, & \text{其它} \end{cases}$$

$$H = [R_{I^c}^D(F)]^C$$



a b c d e f g

FIGURE 9.30
Illustration of
hole filling on a
simple image.

不需要设置起始点



a	b
c	d

FIGURE 9.31

(a) Text image of size 918×2018 pixels. (b) Complement of (a) for use as a mask image. (c) Marker image. (d) Result of hole-filling using Eq. (9.5-29).

ponents or broken connection paths. There is no point past the level of detail required to identify those

Segmentation of nontrivial images is one of the most difficult tasks in image processing. Segmentation accuracy determines the effectiveness of computerized analysis procedures. For this reason, considerable effort can be taken to improve the probability of rugged segmentation, such as industrial inspection applications, at least some of the time. The experienced designer invariably pays considerable attention to such

ponents or broken connection paths. There is no point past the level of detail required to identify those

Segmentation of nontrivial images is one of the most difficult tasks in image processing. Segmentation accuracy determines the effectiveness of computerized analysis procedures. For this reason, considerable effort can be taken to improve the probability of rugged segmentation, such as industrial inspection applications, at least some of the time. The experienced designer invariably pays considerable attention to such

ponents or broken connection paths. There is no point past the level of detail required to identify those

Segmentation of nontrivial images is one of the most difficult tasks in image processing. Segmentation accuracy determines the effectiveness of computerized analysis procedures. For this reason, considerable effort can be taken to improve the probability of rugged segmentation, such as industrial inspection applications, at least some of the time. The experienced designer invariably pays considerable attention to such



边界清除

$$F(x, y) = \begin{cases} I(x, y), & x, y \text{ 在 } I \text{ 的边界上} \\ 0, & \text{其它} \end{cases}$$

$$X = I - R_I^D(F)$$

ponents or broken connection paths. There is no position past the level of detail required to identify those

Segmentation of nontrivial images is one of the most difficult processing. Segmentation accuracy determines the effectiveness of computerized analysis procedures. For this reason, great care must be taken to improve the probability of rugged segmentation. Such as industrial inspection applications, at least some of the time the environment is possible at times. The experienced designer invariably pays considerable attention to suc

a b

FIGURE 9.32

Border clearing.
(a) Marker image.
(b) Image with no objects touching the border. The original image is Fig. 9.29(a).



六、小结

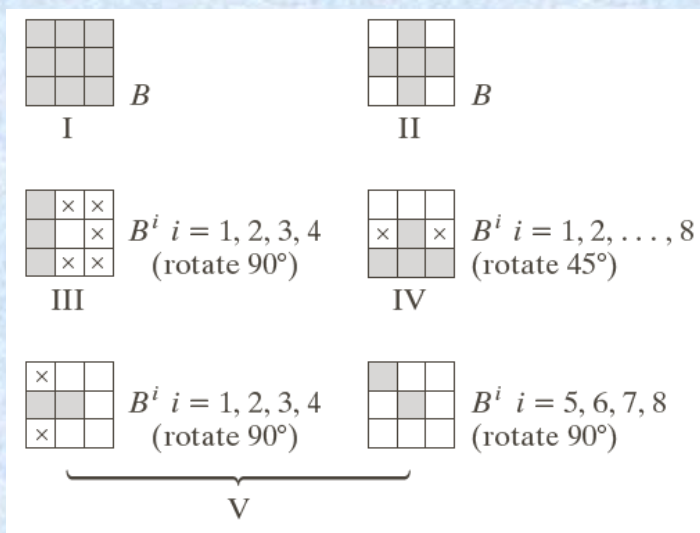


FIGURE 9.33 Five basic types of structuring elements used for binary morphology. The origin of each element is at its center and the \times 's indicate "don't care" values.



Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Translation	$(B)_z = \{w w = b + z, \text{ for } b \in B\}$	Translates the origin of B to point z .
Reflection	$\hat{B} = \{w w = -b, \text{ for } b \in B\}$	Reflects all elements of B about the origin of this set.
Complement	$A^c = \{w w \notin A\}$	Set of points not in A .
Difference	$A - B = \{w w \in A, w \notin B\}$ $= A \cap B^c$	Set of points that belong to A but not to B .
Dilation	$A \oplus B = \{z (\hat{B}_z) \cap A \neq \emptyset\}$	“Expands” the boundary of A . (I)
Erosion	$A \ominus B = \{z (B)_z \subseteq A\}$	“Contracts” the boundary of A . (I)
Opening	$A \circ B = (A \ominus B) \oplus B$	Smooths contours, breaks narrow isthmuses, and eliminates small islands and sharp peaks. (I)

(Continued)

TABLE 9.1

Summary of morphological operations and their properties.



Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Closing	$A \bullet B = (A \oplus B) \ominus B$	Smooths contours, fuses narrow breaks and long thin gulfs, and eliminates small holes. (I)
Hit-or-miss transform	$A \otimes B = (A \ominus B_1) \cap (A^c \ominus B_2)$ $= (A \ominus B_1) - (A \oplus \hat{B}_2)$	The set of points (coordinates) at which, simultaneously, B_1 found a match (“hit”) in A and B_2 found a match in A^c
Boundary extraction	$\beta(A) = A - (A \ominus B)$	Set of points on the boundary of set A . (I)
Hole filling	$X_k = (X_{k-1} \oplus B) \cap A^c$; $k = 1, 2, 3, \dots$	Fills holes in A ; X_0 = array of 0s with a 1 in each hole. (II)
Connected components	$X_k = (X_{k-1} \oplus B) \cap A$; $k = 1, 2, 3, \dots$	Finds connected components in A ; X_0 = array of 0s with a 1 in each connected component. (I)
Convex hull	$X_k^i = (X_{k-1}^i \otimes B^i) \cup A$; $i = 1, 2, 3, 4$; $k = 1, 2, 3, \dots$; $X_0^i = A$; and $D^i = X_{\text{conv}}^i$	Finds the convex hull $C(A)$ of set A , where “conv” indicates convergence in the sense that $X_k^i = X_{k-1}^i$. (III)
Thinning	$A \otimes B = A - (A \otimes B)$ $= A \cap (A \otimes B)^c$ $A \otimes \{B\} =$ $((\dots((A \otimes B^1) \otimes B^2)\dots) \otimes B^n)$ $\{B\} = \{B^1, B^2, B^3, \dots, B^n\}$	Thins set A . The first two equations give the basic definition of thinning. The last equations denote thinning by a sequence of structuring elements. This method is normally used in practice. (IV)
Thickening	$A \odot B = A \cup (A \otimes B)$ $A \odot \{B\} =$ $((\dots(A \odot B^1) \odot B^2 \dots) \odot B^n)$	Thickens set A . (See preceding comments on sequences of structuring elements.) Uses IV with 0s and 1s reversed.
Skeletons	$S(A) = \bigcup_{k=0}^K S_k(A)$ $S_k(A) = \bigcup_{k=0}^K \{(A \ominus kB) - [(A \ominus kB) \odot B]\}$ Reconstruction of A : $A = \bigcup_{k=0}^K (S_k(A) \oplus kB)$	Finds the skeleton $S(A)$ of set A . The last equation indicates that A can be reconstructed from its skeleton subsets $S_k(A)$. In all three equations, K is the value of the iterative step after which the set A erodes to the empty set. The notation $(A \ominus kB)$ denotes the k th iteration of successive erosions of A by B . (I)

(Continued)



Operation	Equation	Comments (The Roman numerals refer to the structuring elements in Fig. 9.33.)
Pruning	$X_1 = A \otimes \{B\}$ $X_2 = \bigcup_{k=1}^8 (X_1 \circledast B^k)$ $X_3 = (X_2 \oplus H) \cap A$ $X_4 = X_1 \cup X_3$	<p>X_4 is the result of pruning set A. The number of times that the first equation is applied to obtain X_1 must be specified. Structuring elements V are used for the first two equations. In the third equation H denotes structuring element I.</p>
Geodesic dilation of size 1	$D_G^{(1)}(F) = (F \oplus B) \cap G$	F and G are called the <i>marker</i> and <i>mask</i> images, respectively.
Geodesic dilation of size n	$D_G^{(n)}(F) = D_G^{(1)}[D_G^{(n-1)}(F)];$ $D_G^{(0)}(F) = F$	
Geodesic erosion of size 1	$E_G^{(1)}(F) = (F \ominus B) \cup G$	
Geodesic erosion of size n	$E_G^{(n)}(F) = E_G^{(1)}[E_G^{(n-1)}(F)];$ $E_G^{(0)}(F) = F$	
Morphological reconstruction by dilation	$R_G^D(F) = D_G^{(k)}(F)$	<p>k is such that</p> $D_G^{(k)}(F) = D_G^{(k+1)}(F)$
Morphological reconstruction by erosion	$R_G^E(F) = E_G^{(k)}(F)$	<p>k is such that</p> $E_G^{(k)}(F) = E_G^{(k+1)}(F)$
Opening by reconstruction	$O_R^{(n)}(F) = R_F^D[(F \ominus nB)]$	$(F \ominus nB)$ indicates n erosions of F by B .
Closing by reconstruction	$C_R^{(n)}(F) = R_F^E[(F \oplus nB)]$	$(F \oplus nB)$ indicates n dilations of F by B .
Hole filling	$H = [R_F^D(F)]^c$	H is equal to the input image I , but with all holes filled. See Eq. (9.5-28) for the definition of the marker image F .
Border clearing	$X = I - R_I^D(F)$	X is equal to the input image I , but with all objects that touch (are connected to) the boundary removed. See Eq. (9.5-30) for the definition of the marker image F .



七、灰度级形态学

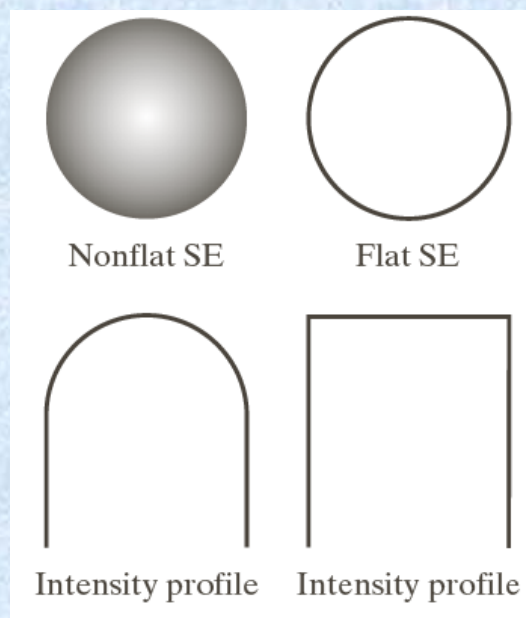
二值图像中目标对象的像素集可用集合表示，但灰度图像不能，需要扩展膨胀、腐蚀、开操作和闭操作等运算

膨胀操作的结果：

- (a) 如果结构元素的值都为正，那么输出图像会更亮
 - (b) 暗的细节部分的变化取决于结构元素的值和形状
-



灰度结构元



平坦结构元:

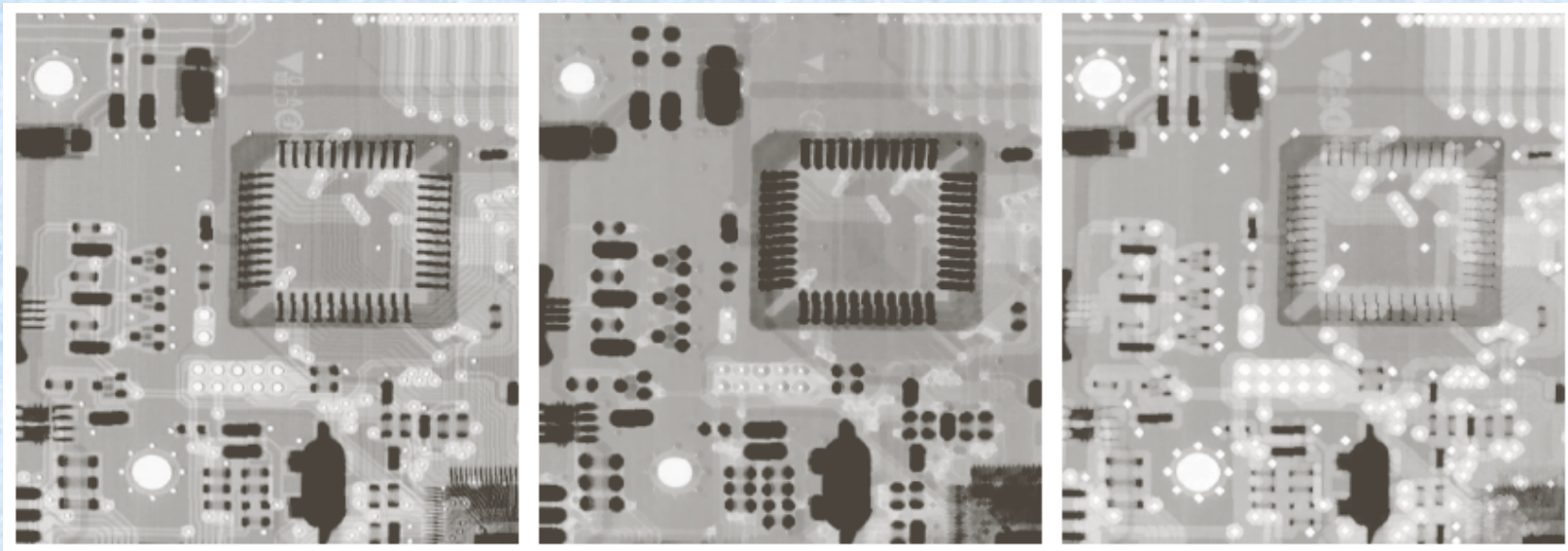
灰度腐蚀: $[f \ominus b](x, y) = \min\{f(x + s, y + t)\}, (s, t) \in b$

灰度膨胀: $[f \oplus b](x, y) = \max\{f(x - s, y - t)\}, (s, t) \in b$

注意结构元b的反射



比较空域滤波中的最大值和最小值滤波



a b c

FIGURE 9.35 (a) A gray-scale X-ray image of size 448×425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)



非平坦结构元:

灰度腐蚀:

$$[f \ominus b_N](x, y) = \min\{f(x + s, y + t) - b_N(s, t)\}, (s, t) \in b_N$$

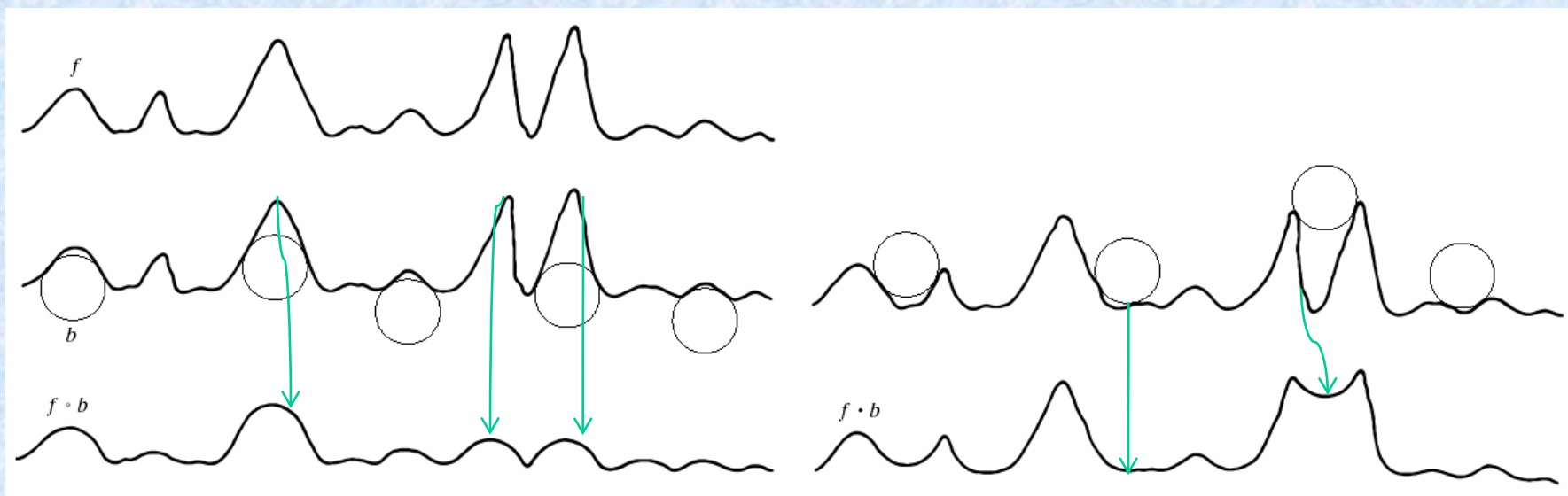
灰度膨胀:

$$[f \oplus b_N](x, y) = \max\{f(x - s, y - t) + b_N(s, t)\}, (s, t) \in b_N$$



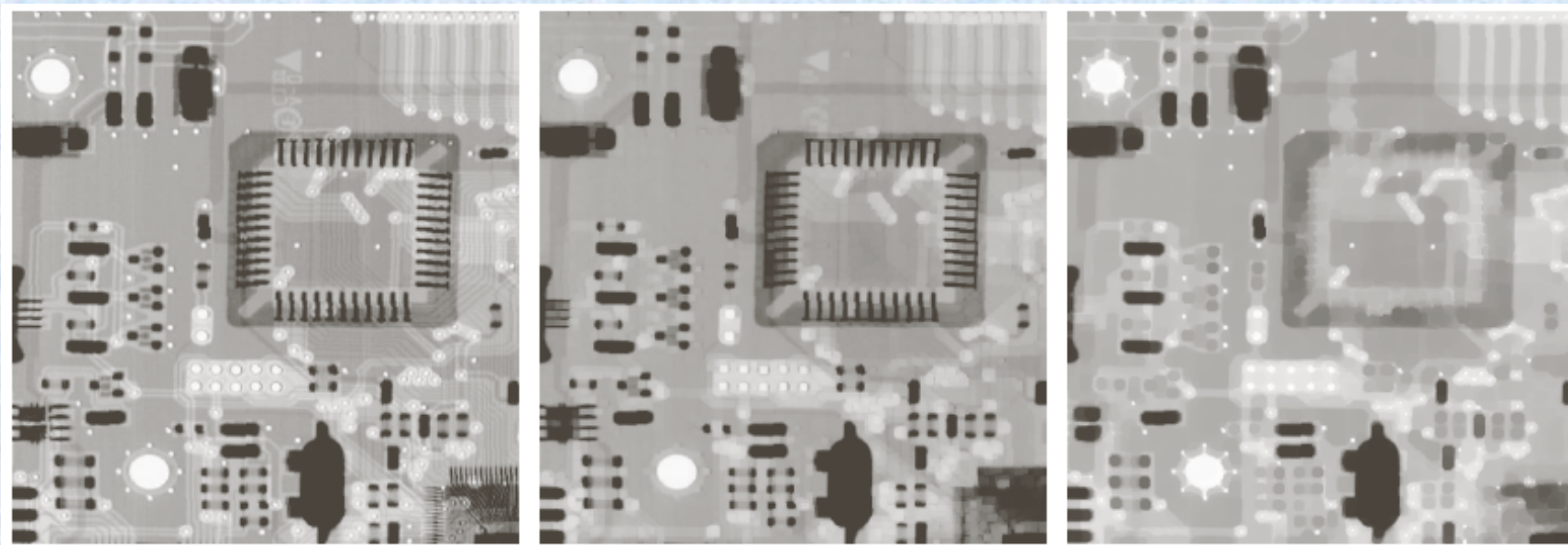
•开操作 $f \circ b = (f \ominus b) \oplus b$

•闭操作 $f \bullet b = (f \oplus b) \ominus b$



开运算通常对图像轮廓进行平滑，使狭窄的“地峡”形状断开，去掉细的突起。

闭运算也是趋向于平滑图像的轮廓，它一般使窄的断开部位和细长的沟融合，填补轮廓上的间隙。



a b c

FIGURE 9.37 (a) A gray-scale X-ray image of size 448×425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.