

Research Article

Virtual Constraints Based Control Design of an Inclined Translational Oscillator with Rotational Actuator System

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Translational oscillator with rotational actuator (TORA) system, whose translational and rotational movements occur in horizontal planes, is a benchmark of underactuated mechanical systems for studying of control techniques. Currently, the research work of the benchmark mainly focuses on stabilizing control of equilibrium points of the dynamical system. The problem of steering TORA to arbitrary points in its state space is rarely studied. In this paper, the movements of the TORA system are extended to inclined planes and dynamics of the inclined TORA system is presented firstly. Following that, a trajectory tracking control method based on virtual constraints is proposed to steer the oscillations of the inclined TORA system. A virtual constraints based method is presented to generate periodic trajectories which pass through desired point; and a Lyapunov based control design is proposed to track the generated trajectories. Finally, the performance and feasibility of the proposed control design methodology are illustrated and analyzed according to numerical simulations.

1. Introduction

The translational oscillator with rotational actuator (TORA) system consists of an unactuated translational oscillation cart and an actuated eccentric rotor attached to the cart, which is a benchmark of underactuated mechanical systems [1]. Underactuated mechanical systems were defined as a class of systems having fewer control inputs than the number of configuration variables [2]. It is interesting and challenging to analyze and control design underactuated mechanical systems. Currently, the control objectives of TORA system are to employ the control input torque acting on the rotor to stabilize both the translational position of the unactuated oscillating cart and the rotational position of the actuated eccentric rotor.

Before several interesting and effective control design methods [3, 4] presented in the special issue [5] entitled *International Journal of Robust and Nonlinear Control* in 2005, Jankovic et al. [6] have already presented several cascade-based effective controllers for the TORA system in 1994. Lee and Chang [7] combined an adaptive backstepping control scheme based on a wavelet-based neural network

and a compensated controller to improve the closed-loop control performance of TORA system. Petres et al. [8] studied approximation and complexity trade-off by tensor product model transformation in control design with TORA system. In [9], an equivalent-input-disturbance method was proposed for TORA system with two steps. Moreover, several control design techniques resulting in controllers with only rotor angle feedback were presented in [10, 11], which could be easier to be implemented in practice.

In the literature, the studied TORA system lies on horizontal planes, and there are infinite equilibriums of the rotor. In [12], dynamics and control design of a TORA system with rotating motion in a vertical plane were studied. Due to the gravity effect in the dynamics, there are only two equilibriums of the rotor, and a simple PD (proportional derivative) controller was designed to stabilize the cart oscillation while bringing the rotor angle to its downward equilibrium. Avis et al. [13] compared energy-based controller and sliding mode based controller designed for TORA system with rotating motion in a vertical plane. Since there does not exist absolute horizontal or vertical plane in practical

