Bio144, 4./5. April 2019

Solution to practical part 7: Exercise on linear algebra

1. The matrices and vectors can be generated in R:

```
A <- matrix(c(4,2,3,1,4,6),byrow=TRUE,nrow=2)
B <- matrix(c(0,3,6,-1,-1,0),byrow=TRUE,nrow=2)
x <- c(-1,2,-3)
y <- c(5,3,-2)
```

a)
```
2*A
```

```
##      [,1] [,2] [,3]
## [1,]    8    4    6
## [2,]    2    8   12
```

b)
```
A + B
```

```
##      [,1] [,2] [,3]
## [1,]    4    5    9
## [2,]    0    3    6
```

c) Calculating the results by hand:

$$\begin{bmatrix} 4 & 2 & 3 \\ 1 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 3 & -1 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 4\cdot 0 + 2\cdot 3 + 3\cdot 6 & 4\cdot(-1) + 2\cdot(-1) + 3\cdot 0 \\ 1\cdot 0 + 4\cdot 3 + 6\cdot 6 & 1\cdot(-1) + 4\cdot(-1) + 6\cdot 0 \end{bmatrix} = \begin{bmatrix} 24 & -6 \\ 48 & -5 \end{bmatrix}$$

Checking with R:

```
A%*%t(B)
```

```
##      [,1] [,2]
## [1,]   24   -6
## [2,]   48   -5
```

d) Again, start by hand:

$$\begin{bmatrix} 4 & 2 & 3 \\ 1 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 4\cdot(-1) + 2\cdot 2 + 3\cdot(-3) \\ 1\cdot(-1) + 4\cdot 2 + 6\cdot(-3) \end{bmatrix} = \begin{bmatrix} -9 \\ -11 \end{bmatrix}$$

And check by R:

```
A %*% x
```

```
##      [,1]
## [1,]   -9
## [2,]  -11
```

e) Not defined, wrong dimensions.

f) Not defined, wrong dimensions.

g)
```
A %*% t(A)

##      [,1] [,2]
## [1,]   29   30
## [2,]   30   53
```

h)
```
t(A) %*% A

##      [,1] [,2] [,3]
## [1,]   17   12   18
## [2,]   12   20   30
## [3,]   18   30   45
```

i)
```
t(x)%*%x

##      [,1]
## [1,]   14
```

j)
```
x%*%t(x)

##      [,1] [,2] [,3]
## [1,]    1   -2    3
## [2,]   -2    4   -6
## [3,]    3   -6    9
```

2.

a) In the lecture we have defined the covariate vectors $\boldsymbol{x}^{(1)}$ and $\boldsymbol{x}^{(2)}$, the data matrix $\tilde{\boldsymbol{X}}$, the $\boldsymbol{\beta}$ vector, and the response vector:

```
x1 <- c(0,1,2,3,4)
x2 <- c(4,1,0,1,4)
Xtilde <- matrix(c(rep(1,5),x1,x2),ncol=3)
t.beta <- c(10,5,-2)
t.y <- Xtilde%*%t.beta

t.e <- rnorm(5,0,1)
t.Y <- t.y  + t.e
r.lm <- lm(t.Y ~ x1 + x2)
summary(r.lm)$coef

##              Estimate Std. Error   t value   Pr(>|t|)
## (Intercept)  9.849794  1.8118749  5.436244 0.03221176
## x1           5.227408  0.6088094  8.586280 0.01329419
## x2          -1.824631  0.5145379 -3.546154 0.07114011
```

```r
solve(t(Xtilde) %*% Xtilde)  %*%  t(Xtilde) %*% t.Y

##            [,1]
## [1,]  9.849794
## [2,]  5.227408
## [3,] -1.824631
```

b) Generate a matrix of true value `t.y` and a matrix of residual errors `t.E` (instead of generating a separate `t.e` each time). The 100 observed vectors are then stored in a 5×100 matrix `t.Y`:

```r
t.y <- matrix(rep(t.y,100),nrow=5,byrow=F)
t.E <- matrix(rnorm(500,0,1),nrow=5)
t.Y <- t.y  + t.E
```

100 iterations of the regression:

```r
r.coef <- matrix(NA,ncol=3, nrow=100)
for (i in 1:100) {
  r.coef[i,] <- lm(t.Y[,i] ~x1 + x2)$coefficients
}
```

c) We procude the required graphs using `ggplot`. Load the libraries:

```r
library(ggplot2)
library(tidyr)
library(dplyr)
```

For the histograms, first convert `r.coef` into a data frame and rename the columns:

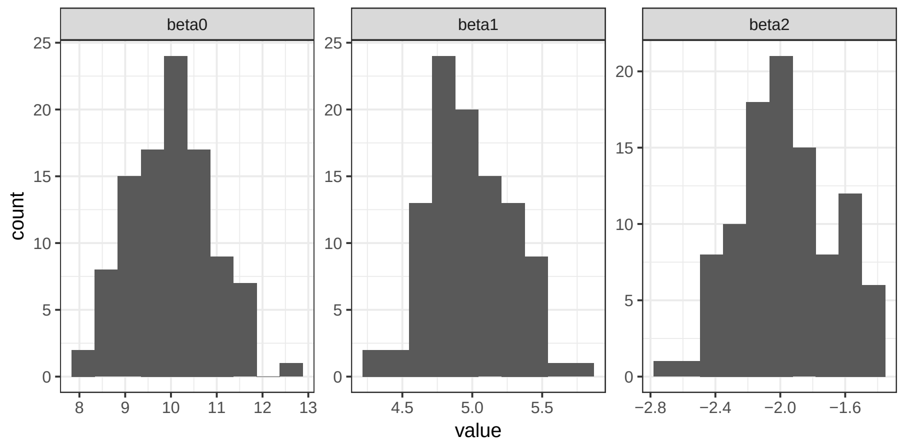
```r
r.coef <- data.frame(r.coef)
names(r.coef) <- c("beta0","beta1","beta2")
```

Then either produce three separate plots using

```r
ggplot(r.coef,aes(x=beta0)) + geom_histogram()
ggplot(r.coef,aes(x=beta1)) + geom_histogram()
ggplot(r.coef,aes(x=beta2)) + geom_histogram()
```

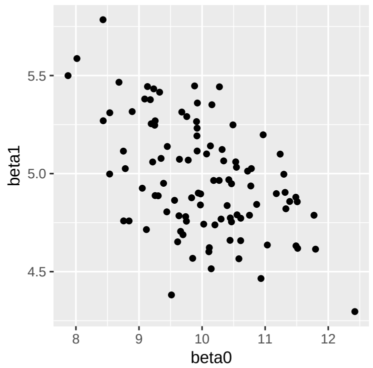
Or use the `gather()` function from the `tidyr` package, which has the advantage that you can then use `facet_wrap()`:

```r
ggplot(gather(r.coef, key=variable, value=value), aes(value)) +
  geom_histogram(bins=10) + facet_wrap(~variable, scales = "free") +
  theme_bw()
```

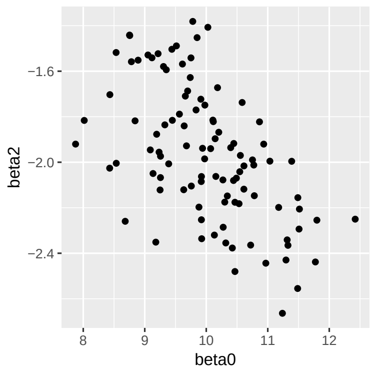


For the scatterplots we can continue to use `r.coef`:

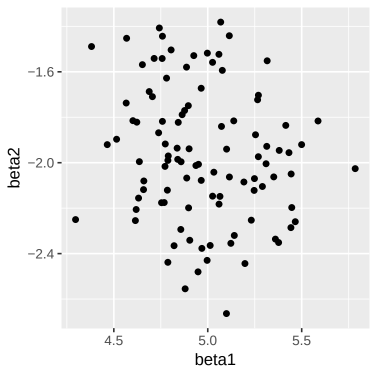
```
ggplot(r.coef,aes(x=beta0,y=beta1)) + geom_point()
```




```
ggplot(r.coef,aes(x=beta0,y=beta2)) + geom_point()
```




```
ggplot(r.coef,aes(x=beta1,y=beta2)) + geom_point()
```



Observation: β_0 seems to be correlated with β_1 and β_2.