Bio144, 6./7. April 2017

Solution to practical part 7: Exercise on linear algebra

1. The matrices and vectors can be generated in R:

```
> A <- matrix(c(4,2,3,1,4,6),byrow=TRUE,nrow=2)
> B <- matrix(c(0,3,6,-1,-1,0),byrow=TRUE,nrow=2)
> x <- c(-1,2,-3)
> y <- c(5,3,-2)
```

a) `> 2*A`

```
     [,1] [,2] [,3]
[1,]    8    4    6
[2,]    2    8   12
```

b) `> A + B`

```
     [,1] [,2] [,3]
[1,]    4    5    9
[2,]    0    3    6
```

c) Calculating the results by hand:

$$\begin{bmatrix} 4 & 2 & 3 \\ 1 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1 \\ 3 & -1 \\ 6 & 0 \end{bmatrix} = \begin{bmatrix} 4\cdot 0 + 2\cdot 3 + 3\cdot 6 & 4\cdot(-1) + 2\cdot(-1) + 3\cdot 0 \\ 1\cdot 0 + 4\cdot 3 + 6\cdot 6 & 1\cdot(-1) + 4\cdot(-1) + 6\cdot 0 \end{bmatrix} = \begin{bmatrix} 24 & -6 \\ 48 & -5 \end{bmatrix}$$

Checking with R:

`> A%*%t(B)`

```
     [,1] [,2]
[1,]   24   -6
[2,]   48   -5
```

d) Again, start by hand:

$$\begin{bmatrix} 4 & 2 & 3 \\ 1 & 4 & 6 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix} = \begin{bmatrix} 4\cdot(-1) + 2\cdot 2 + 3\cdot(-3) \\ 1\cdot(-1) + 4\cdot 2 + 6\cdot(-3) \end{bmatrix} = \begin{bmatrix} -9 \\ -11 \end{bmatrix}$$

And check by R:

`> A %*% x`

```
     [,1]
[1,]   -9
[2,]  -11
```

e) Not defined, wrong dimensions.

f) Not defined, wrong dimensions.

g) `> A %*% t(A)`

```
     [,1] [,2]
[1,]   29   30
[2,]   30   53
```

h) `> t(A) %*% A`

```
     [,1] [,2] [,3]
[1,]   17   12   18
[2,]   12   20   30
[3,]   18   30   45
```

i) `> t(x)%*%x`

```
     [,1]
[1,]   14
```

j) `> x%*%t(x)`

```
     [,1] [,2] [,3]
[1,]    1   -2    3
[2,]   -2    4   -6
[3,]    3   -6    9
```

2.

a) In the lecture we have defined the covariate vectors $\boldsymbol{x}^{(1)}$ and $\boldsymbol{x}^{(2)}$, the data matrix $\tilde{\boldsymbol{X}}$, the $\boldsymbol{\beta}$ vector, and the response vector:

```
> x1 <- c(0,1,2,3,4)
> x2 <- c(4,1,0,1,4)
> Xtilde <- matrix(c(rep(1,5),x1,x2),ncol=3)
> t.beta <- c(10,5,-2)
> t.y <- Xtilde%*%t.beta
> t.e <- rnorm(5,0,1)
> t.Y <- t.y  + t.e
> r.lm <- lm(t.Y ~ x1 + x2)
> summary(r.lm)$coef
```

```
             Estimate Std. Error   t value    Pr(>|t|)
(Intercept) 10.562244  0.7687569 13.739382 0.005255709
x1           4.615732  0.2583106 17.868925 0.003117229
x2          -1.966152  0.2183123 -9.006146 0.012105420
```

```
> solve(t(Xtilde) %*% Xtilde)  %*%  t(Xtilde) %*% t.Y
```

```
          [,1]
[1,] 10.562244
[2,]  4.615732
[3,] -1.966152
```

b) Generate a matrix of true value `t.y` and a matrix of residual errors `t.E` (instead of generating a separate `t.e` each time). The 100 observed vectors are then stored in a 5×100 matrix `t.Y`:

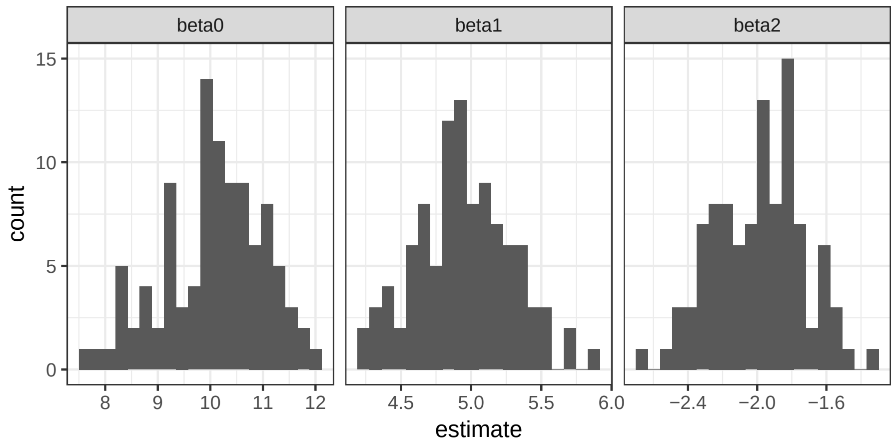
```
> t.y <- matrix(rep(t.y,100),nrow=5,byrow=F)
> t.E <- matrix(rnorm(500,0,1),nrow=5)
> t.Y <- t.y  + t.E
```

The `apply` functions applies the defined function on all columns of `t.Y` (note that if the second argument was `=1` the function would be applied to all rows)

```
> r.coef <- t(apply(t.Y, 2, FUN = function(y) lm(y ~ x1 + x2)$coefficients))
```

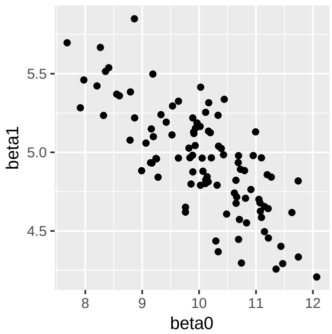
c) Following the course philosophy, we procude the required graphs using `ggplot`. For the histograms, rearrange the data a bit, namely by generating a data frame `r.data` that contains as one column all estimates of the βs, and as a second column an indicator if the estimate is for β_0, β_1 or β_2.

```
> library(ggplot2)
> library(dplyr)
> r.data <- data.frame(estimate=c(r.coef[,1],r.coef[,2],r.coef[,3]))
> r.data <- mutate(r.data,beta=rep(c("beta0","beta1","beta2"),each=100))
> ggplot(r.data,aes(x=estimate)) +
+   facet_wrap(~beta,scales="free_x") +
+   geom_histogram(bins=20) +
+   theme_bw()
```

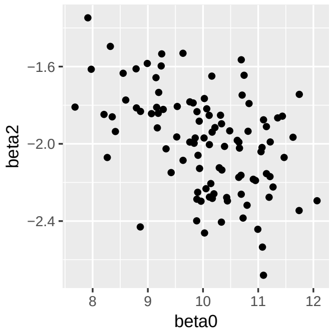


For the scatterplots we can continue to use `r.coef` with the three columns, but we need to convert it from matrix to a data frame.

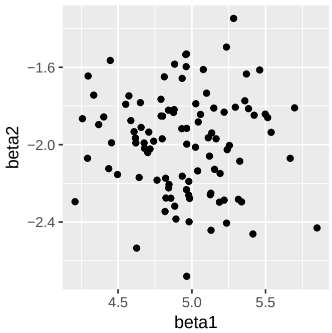
```
> r.coef <- data.frame(r.coef)
> names(r.coef) <- c("beta0","beta1","beta2")
> par(mfrow=c(1,3))
> ggplot(r.coef,aes(x=beta0,y=beta1)) + geom_point()
```




```
> ggplot(r.coef,aes(x=beta0,y=beta2)) + geom_point()
```




```
> ggplot(r.coef,aes(x=beta1,y=beta2)) + geom_point()
```



Observation: β_0 seems to be correlated with β_1 and β_2.