

Practical part 7: Exercise on linear algebra

1. Let us use the following matrices and vectors:

$$\mathbf{A} = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 4 & 6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 3 & 6 \\ -1 & -1 & 0 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} -1 \\ 2 \\ -3 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}$$

Calculate all of the following expressions, if they are defined. Solve at least a)-d) by hand.

- | | | | |
|------------------------------------|------------------------------------|------------------------------------|------------------------------------|
| a) $2 \cdot \mathbf{A}$ | b) $\mathbf{A} + \mathbf{B}$ | c) $\mathbf{A} \cdot \mathbf{B}^T$ | d) $\mathbf{A} \cdot \mathbf{x}$ |
| e) $\mathbf{A} \cdot \mathbf{B}$ | f) $\mathbf{B}^T \cdot \mathbf{y}$ | g) $\mathbf{A} \cdot \mathbf{A}^T$ | h) $\mathbf{A}^T \cdot \mathbf{A}$ |
| i) $\mathbf{x}^T \cdot \mathbf{x}$ | j) $\mathbf{x} \cdot \mathbf{x}^T$ | | |

R-hints:

- A matrix can be created for example as
`A <- matrix(c(4,2,3,0,3,6),byrow=TRUE,nrow=2)`
- `t(A)` corresponds to \mathbf{A}^T .
- Make sure you understand the difference between $\mathbf{A} * \mathbf{B}$ and $\mathbf{A} \%*\% \mathbf{B}$.

2. a) Repeat the example given in slides 35–38 of lecture 6 using R.

- b) Repeat the procedure from a) 100 times. In each iteration, store the estimated coefficient vector $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$. Note that the estimates will be a bit different in each iteration, because another residual vector `t.e` is used.
- c) Plot the histograms of the β s, the coefficients as a function of the simulation number, and scatterplots of $\hat{\beta}_1$ vs. $\hat{\beta}_0$, $\hat{\beta}_2$ vs. $\hat{\beta}_0$ and $\hat{\beta}_2$ vs. $\hat{\beta}_1$.

R-hints: The 100 simulations can be generated elegantly as follows:

- Generate an error matrix of dimension 5×100 . Each column contains the errors of one experiment:
`t.E <- matrix(rnorm(500),ncol=...)`
- Use `t.E` to generate the observations:
`t.y <- matrix(rep(t.y,100),nrow=5,byrow=F)`
`t.Y <- t.y + t.E`
- Define a results matrix of dimension 3×100 that contains in each column the estimates $(\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)$ of an experiment, and fill it with the results of the

100 iterations:

```
r.coef <- matrix(NA,ncol=3, nrow=100)
for (i in 1:100) {
  r.coef[i,] <- lm(t.Y[,i] ~ x1 + x2)$coefficients
}
```

or more elegantly with `apply()`:

```
r.coef <- t(apply(t.Y, 2, FUN = function(y) lm(...)$...))
```