

# Principles of Bayesian Statistics

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```
library(knitr)
opts_knit$set(root.dir='../..')
getwd()
```

```
## [1] "/home/wmmurrah/FOCAL/Methods/BayesianAnalysis/reports/Binomial"
```

## Bayes Theorem:

$$p(B|A) = \frac{p(A|B)p(A)}{p(A)}.$$

### proof:

We know that  $p(A, B) = p(A|B)p(B)$  and  $p(B, A) = p(B|A)p(A)$ . We also know that  $p(A, B) = p(B, A)$ . Therefore we know:

$$p(B|A)p(A) = p(A|B)p(B).$$

If we divide both sides of this equation by  $p(A)$ , we get Bayes Theorem.

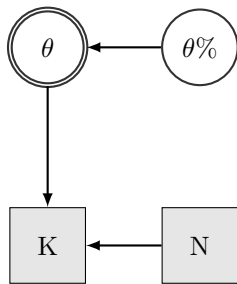
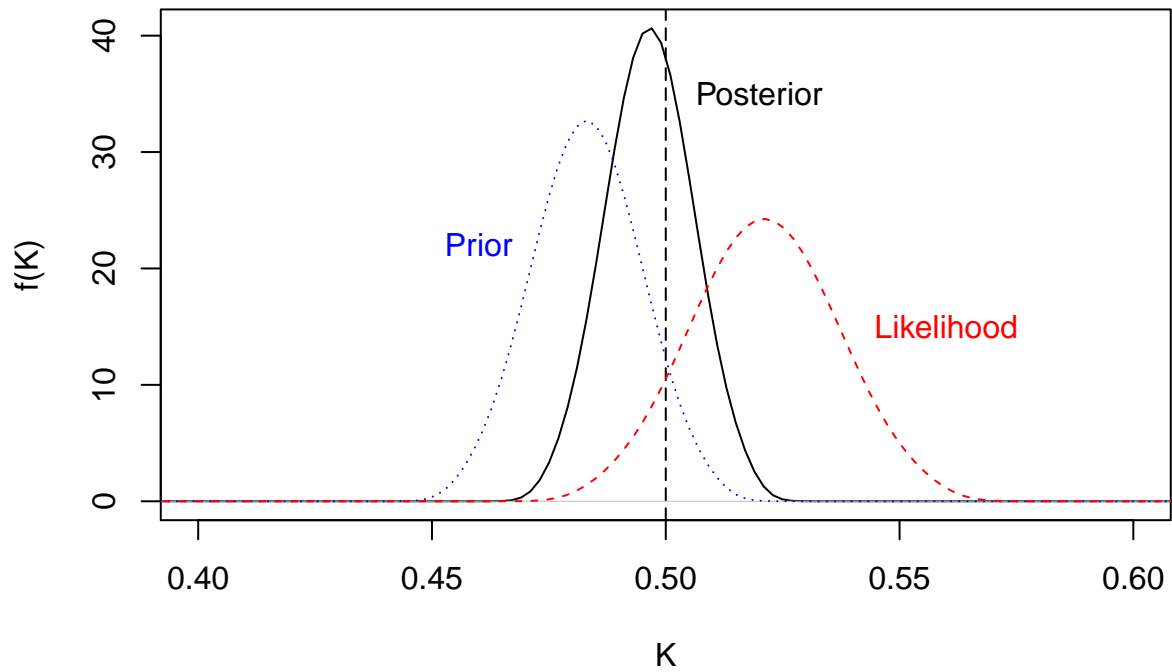
## Polling Example

This example is taken from Chapters 1 and 2 of *Introduction to Applied Bayesian Statistics and Estimation for Social Scientists* by Scott Lynch.

```
K <- seq(0,1, .01)

posterior <- qbeta(K, 1498, 1519)
prior <- qbeta(K, 942, 1008)
likelihood <- qbeta(K, 557, 512)
plot(density(posterior), xlim=c(.4,.6),
     main='Prior, Posterior, and Likelihood Densities for Polling Example',
     ylab='f(K)', xlab='K')
abline(v=.5, lty=17)
lines(density(prior), col='blue', lty=15)
lines(density(likelihood), col='red', lty=20)
text(.56, 15, "Likelihood", col='red')
text(.52, 35, "Posterior", col='black')
text(.46, 22, "Prior", col='blue')
```

## Prior, Posterior, and Likelihood Densities for Polling Example



$$\begin{aligned}
 K &\sim \text{Binomial}(\theta, N) \\
 \theta &= \theta\% / 100 \\
 \theta\% &\sim \text{Uniform}(0, 100)
 \end{aligned}$$