Fixed Income Securities

Finance 529Q, Spring 2024

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Team Assignment 2

Question 1

Consider the following bond portfolio consisting of four US treasury securities, which pay coupon semi-annually. The first column is the maturity date. The second column is the coupon rate. The third column is the clean price, per $100 of face value, as of the settlement date 2023-12-15. The fourth column is the amount of face value of each bond in the portfolio.

Maturity	Coupon	Clean Price	Face Value
2025-05-15	2.75 %	97.447	4,000,000
2028-08-15	2.875 %	95.403	5,000,000
2032-05-15	2.875 %	91.262	6,000,000
2040-08-15	3.875 %	96.626	7,000,000

1a. Find the modified duration and convexity of each bond.

1b. Find the modified duration and convexity of the bond portfolio.
Definition: the modified duration of a bond portfolio is the weighted average of the modified durations of the bonds in the portfolio, using market value weights. The convexity of a bond portfolio is the weighted average of the convexities of the bonds in the portfolio, using market value weights

1c. Suppose all yields increase by 10 bp, what is the first order approximation of the percentage change of the market value of this portfolio.

1d. Suppose all yields increase by 10 bp, what is the second order approximation of the percentage change of the market value of this portfolio.

1e. Suppose all yields increase by 10 bp, what is the actual percentage change of the market value of this portfolio.

1f. Suppose the yields change as follows:

Maturity	yield change
2025-05-15	+15 bp
2028-08-15	+13 bp
2032-05-15	+11 bp

2040-08-15 +9 bp

Approximate the percentage change of the market value of this portfolio using its modified duration and convexity. Compare this approximation to the actual percentage change of the market value of this portfolio.

Question 2

Download the data file: TeamAssignment2_Q2_bond.csv. It contains the following information for US treasury securities:

Column 1: maturity date
Column 2: coupon rate (in decimal format)
Column 3: (clean) price per $100 of face value (in decimal format)

The prices are for settlement on 2023-12-15.

Estimate the parameters of the nonlinear yield regression as discussed in slides 17-21 of the "Term Structure II" presentation in Lecture 3, where the spot rate at maturity t years, Z(t), is given by the following equation:.

$$Z(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \alpha_4 t^4 + \alpha_5 t^5$$

The parameters to be estimated are $\theta = (\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5)$

2a) What are the estimated parameters? [Hint: replace the gfun4() function using the above equation. Then try the starting value of (0.01,0,0,0,0,0) for the six parameters.]

2b) Compare the actual prices and predicted prices of the bonds based on estimated parameters in part 2a.

2c) Use the estimated parameters to "predict" the prices of the coupon strips in the data file: TeamAssignment2_Q2_ci.csv. It contains the following information for US treasury securities:

Column 1: maturity date
Column 2: price per $100 of face value (in decimal format)

The prices are for settlement on 2023-12-15.
Compare the actual prices and predicted prices of the coupon strips using the nonlinear yield regression estimated in 2a.

2d) Use the estimated parameters to "predict" the prices of the principal strips in the data file: TeamAssignment2_Q2_sp.csv. It contains the following information for US treasury securities:

Column 1: maturity date
Column 2: price per $100 of face value (in decimal format)

The prices are for settlement on 2023-12-15.
Compare the actual prices and predicted prices of the coupon strips using the nonlinear yield regression estimated in 2a.

2e) Discuss how the prediction errors differ for coupon strips (in 2c) and principal strips (in 2d).

Question 3:

Here is the definition of "par rates". Given the spot rate curve, Z(t). [Remember, Z(t) is the yield-to-maturity of the zero coupon bond with a maturity of t years.] Then the par rate, C(t), is defined as the coupon rate of a bond with a maturity of t years whose full price is par, that is, \$100 per \$100 of face value.

For the purpose of this question, we will assume that bonds have annual coupon payments. In this case, we will use annual compounding for interest rates (instead of semi-annual compounding, as we have been doing so for treasury securities).

Under annual coupon payments and annual compounding, the par rate C(t) satisfies the equation:
$\frac{100C(t)}{(1+Z(1))} + \frac{100C(t)}{(1+Z(2))^2} + \cdots + \frac{100C(t)+100}{(1+Z(t))^t} = 100$ for any t = 1, 2, …

Download the data file: TeamAssignment2_Q3.csv. It contains the par rates of default-free bonds with annual coupon payments on the settlement date **2021-12-30**:

maturity	par.rate
2022-12-30	0.003524
2023-12-30	0.007097
2024-12-30	0.009603
2025-12-30	0.011592
2026-12-30	0.013155
2027-12-30	0.014330
2028-12-30	0.015139
2029-12-30	0.015675
2030-12-30	0.016172
2031-12-30	0.016712

3a) Find the discount factors corresponding to the above ten dates.

Hint: The discount factors for cash flow in 1 year, D(1), can be found using this equation:

[100 + 100 * C(1)] * D(1) = 100

The left hand side is the present value of future cash flows (which is \$100 of face value, plus \$100 * C(1) of coupon interest) in one year, multiplied by the discount factor for one year, D(1). The right hand side is the (full) price today, which is \$100. Given C(1), you can infer what D(1) is.

The two year par rate, C(2), is the coupon rate on a two year security which is priced at par. This security satisfies the equation:

100 * C(2) * D(1) + [100 + 100 * C(2)] * D(2) = 100

The left hand side is the present value of the future cash flows of the two-year bond, and the right hand side is its (full) price. We found D(1) already. So we can solve for D(2).

This results in a recursive solution for the set of D(t), analogous to the "recursive stripping" calculation in "Term Structure: Part II".

3b) Use the discount factors to find the corresponding spot rates. Now, use this information to approximate the present value of an annuity that pays $100,000 on Jun 30 of each year, starting 6/30/2022 and ending 6/30/2031. For the purpose of this question, assume that Jun 30 and Dec 30 of each year are business days.

Submission:

Your submission consists of two files uploaded to Canvas.

File 1: PDF file of the following powerpoint slides.
Slide 1: Your team number and members.
Slide 2: Your answer to Question 1f, plus a brief explanation.
Slide 3: Your answer to Questions 2a-2c, plus a brief explanation of your estimation method.
Slide 4: Your answer to Questions 3a-3b, plus a brief explanation of your calculations.

Note: An important objective of the team assignments is to help you sharpen your presentation skills. Your slides should be clear, informative, and well organized. Each slide should have a clear message, and justification of that message, using words; include tables or graphs if needed. Don't include R code.

File 2: One R script with all your calculations for the entire assignment. Please provide comments for each part. For example:
Part 1 – R code for Question 1, all parts
Part 2 – R code for Question 2, all parts
Part 3 – R code for Question 3, all parts