Introduction to Nonlinear Programming

Homework #6 – Due Thursday, May 23

1. We let [0, 5] be the initial interval of uncertainty and require the final length of uncertainty must be lower than 0.01. Find the minimum of $6e^{-2\lambda}+2\lambda^2$ by the Dichotomous search method (let $\varepsilon = 0.001$).

2. Consider the function *f* defined by $f(x)=\left(x_1+x_2^3\right)^2+2\left(x_1-x_2-4\right)^4$. Given a point x_1 and a nonzero vector **d**, let $\theta(\lambda)=f\left(x_1+\lambda\mathbf{d}\right)$.
   a. Obtain an explicit expression for $\theta(\lambda)$.
   b. For $x^1=(0,0)^t$ and $\mathbf{d}=(1,1)^t$, using the Fibonacci method, find the value of λ that solves the problem to minimize $\theta(\lambda)$ subject to $\lambda \in \mathbf{R}$. (We also let the final length of uncertainty ε be 0.01 and [0, 5] be the initial interval of uncertainty.)
   c. For $x^1=(5,4)^t$ and $\mathbf{d}=(-2,1)^t$, using the Golden section method, find the value of λ that solves the problem to minimize $\theta(\lambda)$ subject to $\lambda \in \mathbf{R}$. (We let $[a_1, b_1]=[-2, 2]$ and the length of uncertainty l is 0.01.)

3. We let [0, 5] be the initial interval of uncertainty and require the final length of uncertainty must be lower than 0.01. Find the minimum of $6e^{-2\lambda}+2\lambda^2$ by each of the following procedures:
   a. Newton's method ($\lambda_1 = 1$, $\varepsilon = 0.001$).
   b. Bisection search method.

4. Consider the problem to minimize $\left(3-x_1\right)^2+7\left(x_2-x_1^2\right)^2$. Starting from the point (0, 0), solve the problem by the following procedures:
   a. The cyclic coordinate method. ($\varepsilon = 0.2$)
   b. The method of Davidon-Fletcher-Powell. ($\varepsilon = 0.2$)
   c. The method of steepest descent. ($\varepsilon = 2$)