Introduction to Nonlinear Programming

Homework #7 – Due Thursday, May 30

1. Solve the problem given below with the penalty function method and barrier function method.

   Minimize $x_1^2 + 4x_2^2 - 8x_1 - 16x_2$

   Subject to $x_1 + x_2 \le 5$

   $0 \le x_1 \le 3, x_2 \ge 0$

   (a) Penalty function method (The penalty function is set to be $\alpha(x) = [\max\{0, g_i(x)\}]^2$, $\mu_1 = \frac{1}{2}, \beta = 2$, $\varepsilon = 0.03$. The starting point is $(4,2)$.)

   (b) Barrier function method (The barrier function is set to be $B(x) = \sum_{i=1}^{m} \frac{-1}{g_i(x)}$, $\mu_1 = 8, \beta = 0.5$, $\varepsilon = 0.6$. The starting point is $(1,1)$.)

   Hint: You can solve unconstrained problem with Bisection Method ($[a_1, b_1] = [-100, 100]$, $\varepsilon = 0.00001$). In (a), $d_k = -\nabla[f(x_k) + \mu_k \alpha(x_k)]$ and $\theta(\lambda) = f(x_k + \lambda d_k) + \mu_k \alpha(x_k + \lambda d_k)$. In (b), $d_k = -\nabla[f(x_k) + \mu_k B(x_k)]$ and $\theta(\lambda) = f(x_k + \lambda d_k) + \mu_k B(x_k + \lambda d_k)$. After finding λ_k at iteration k, you can get $x_{k+1} (= x_{\mu_k}) = x_k + \lambda_k d_k$.