Notably, the added constraint does not affect the original optimal solution, where the objective function is as (1) and subject to the constraint (2).

$$min\left(loss_o\left(\tilde{X}_{CLM}\right)\right)=min\left(\left(X-\tilde{X}_{CLM}\right)^2\right) \tag{1}$$

$$\text{s.}t.g\left(\tilde{X}_{CLM}\right)=\tilde{X}_{cons}-\sum_{i=1}^{z}\tilde{X}_{CLM\cdot\mathrm{i}}=0 \tag{2}$$

This can be explained as follows. The optimal solution of the $\min(l_{\mathrm{CLM}})$ lies on the common tangent plane of the solution hyperplane of (1) and (2), expressed as (3). It can be further converted into solving partial derivatives of the two variables separately, as (4), which has the common optimal solution to the original question, i.e., $\min\left(l_{\mathrm{o}}(\tilde{X}_{CLM})\right)$ with $\mathrm{s.t.}\, g(\tilde{X}_{CLM})=0$.

$$\nabla_{\left(\tilde{X}_{CLM},\lambda_e\right)}loss_{CLM}\left(\tilde{X}_{CLM},\lambda_e\right)=0 \tag{3}$$

$$\begin{cases}\nabla_{\tilde{X}_{CLM}}loss_{CLM}\left(\tilde{X}_{CLM},\lambda_e\right)\\=\nabla_{\tilde{X}_{CLM}}loss_o\left(\tilde{X}_{CLM}\right)+2\lambda_e\nabla_{\tilde{X}_{CLM}}g\left(\tilde{X}_{CLM}\right)=0\\\nabla_{\lambda_e}loss_{CLM}\left(\tilde{X}_{CLM},\lambda_e\right)\\=g\left(\tilde{X}_{CLM}\right)=0\end{cases} \tag{4}$$