

# Introduction to Residual Neural Network (ResNet)

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# Overview

- ▶ Background
- ▶ Residual neural network
- ▶ Variants of residual blocks
- ▶ Some analysis

# Why Deep Neural Networks?

- ▶ Shallow NNs (single hidden layer)
  - ▶ <sup>1</sup>Universal approximation theorem (uniformly)
  - ▶ <sup>2</sup> $\epsilon^{-d/n}$  neurons can approximate any  $C^n$ -function on a compact set in  $\mathbb{R}^d$  with error  $\epsilon$
- ▶ Deep NNs: better than shallow NNs (of comparable size)
  - ▶ <sup>3</sup>Exists a function expressible by a 3-layer NN, which cannot be approximated by any 2-layer network (unless exponentially large)
  - ▶ <sup>4</sup> $\frac{\text{size}_{\text{deep}}}{\text{size}_{\text{shallow}}} \approx \epsilon^d$

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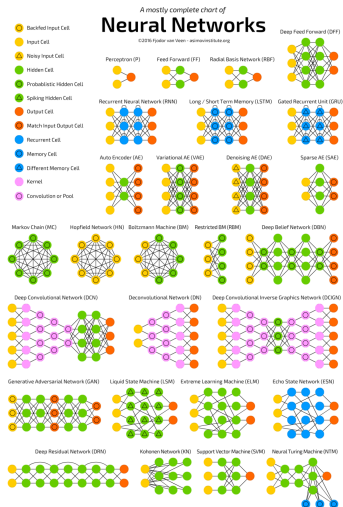
<sup>1</sup>Cybenko, Math. Control Signals Syst., 1989; Hornik et al., Neural Netw., 1989.

<sup>2</sup>Mhaskar, Neural Comput., 1996.

<sup>3</sup>Eldan & Shamir, COLT, 2016.

<sup>4</sup>Mhaskar & Poggio, Anal. Appl., 2016; Mhaskar et al., AAI, 2017; Poggio et al., IJAC, 2017.

“In theory, theory and practice are the same. In practice, they are not.” — Albert Einstein?



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# Residual Neural Network<sup>5</sup>

- ▶ Deep networks are hard to train: vanishing gradients
- ▶ Core idea: “identity shortcut connection” that skips one or more layers
- ▶ Widely used: simple & powerful

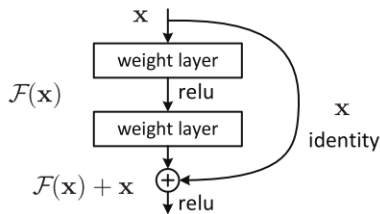
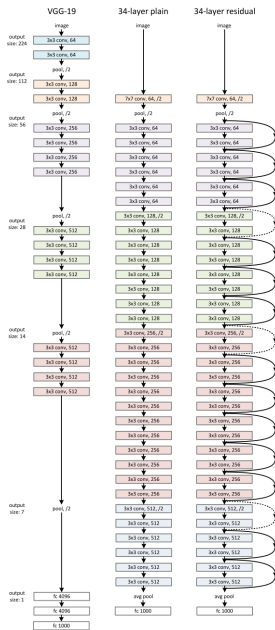


Figure : A residual block

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<sup>5</sup>He et al., CVPR, 2016.

# Residual Neural Network



# Residual Neural Network

- ▶  $\mathcal{H}(\mathbf{x}) = \mathcal{F}(\mathbf{x}) + \mathbf{x} \Rightarrow \mathcal{F}(\mathbf{x}) = \mathcal{H}(\mathbf{x}) - \mathbf{x}$  (residual)
- ▶ *Hypotheses*: the residual may be an easier function to fit

$\mathcal{F}$ ?

- ▶ If  $\mathcal{F}$  has two layers,  $\mathcal{F}(\mathbf{x}) = W_2\sigma(W_1\mathbf{x})$
- ▶ If  $\mathcal{F}$  has one layers,  $\mathcal{F}(\mathbf{x}) = W_1\mathbf{x}$ ,  
 $\mathcal{H}(\mathbf{x}) = W_1\mathbf{x} + \mathbf{x} = (W_1 + 1)\mathbf{x}$ . No advantage!

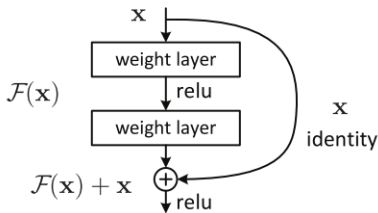


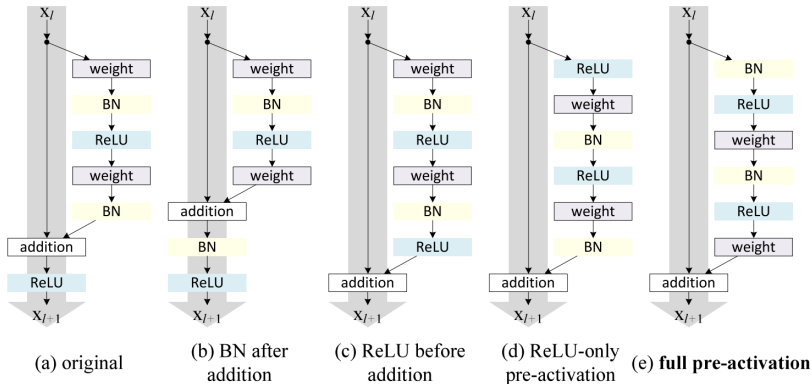
Figure : A residual block



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# Variants of Residual Blocks<sup>6</sup>



<sup>6</sup>He et al., ECCV, 2016.

# Variants of Residual Blocks

Accuracy can be gained more efficiently by increasing the cardinality than by going deeper or wider.

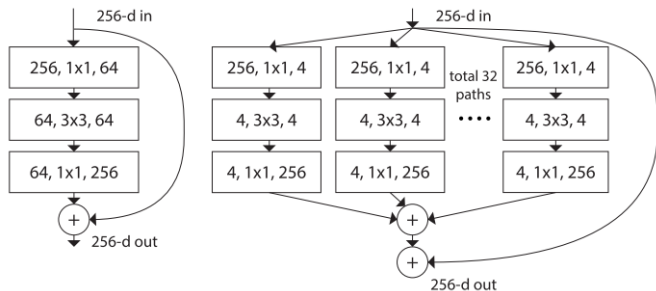


Figure : ResNeXt<sup>7</sup>: split-transform-merge

<sup>7</sup>Xie et al., CVPR, 2017.

# Variants of Residual Blocks

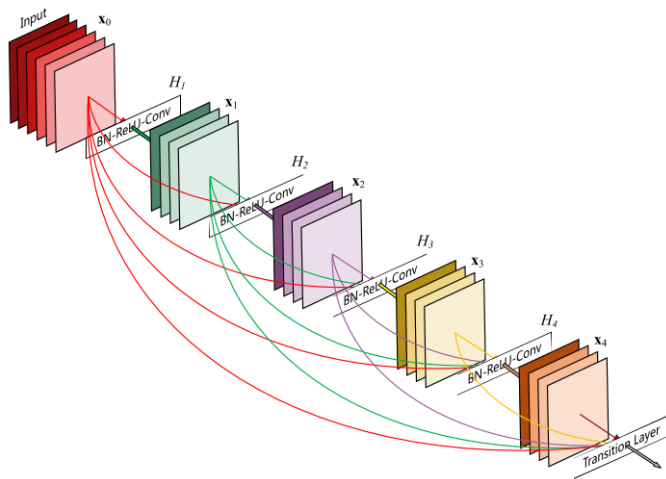


Figure : DenseNet<sup>8</sup>

<sup>8</sup>Huang et al., CVPR, 2017.

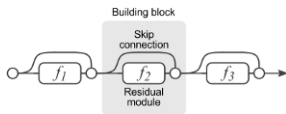
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# Unraveled View<sup>9</sup>

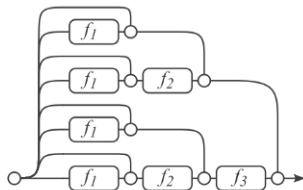
$$\begin{aligned}y_3 &= y_2 + f_3(y_2) \\&= [y_1 + f_2(y_1)] + f_3(y_1 + f_2(y_1)) \\&= [y_0 + f_1(y_0) + f_2(y_0 + f_1(y_0))] + f_3(y_0 + f_1(y_0) + f_2(y_0 + f_1(y_0)))\end{aligned}$$

$2^n$  paths connecting input to output layers



(a) Conventional 3-block residual network

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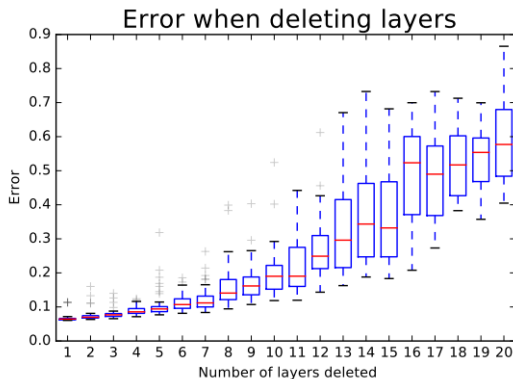


(b) Unraveled view of (a)

<sup>9</sup>Veit et al., NIPS, 2016.

# Ensemble-like Behavior<sup>10</sup>

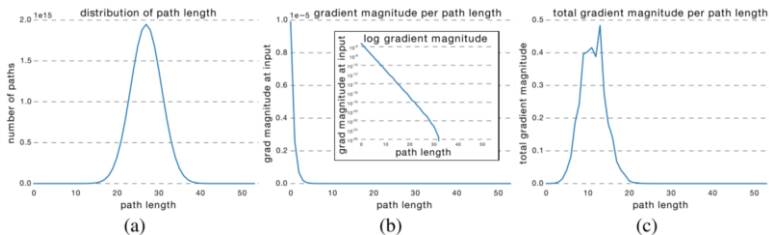
- ▶ Lesion study: randomly deleting several modules
- ▶ Paths do not strongly depend on each other



<sup>10</sup>Veit et al., NIPS, 2016.

# Vanishing Gradients?<sup>11</sup>

- ▶ The effective paths are relatively shallow
- ▶ Only the short paths contribute gradients
- ▶ ResNet does not resolve vanishing gradients by preserving gradient flow throughout the entire network. Rather, they enable very deep networks by *shortening the effective paths*.



<sup>11</sup>Veit et al., NIPS, 2016.



# Universal Approximation

Recall<sup>12</sup>:

- ▶ To approximate any continuous function  $[0, 1]^d \rightarrow \mathbb{R}$  by ReLU NN: minimal width is  $d + 1$

ResNet with one hidden neuron:

$$\mathcal{H}(\mathbf{x}) = \mathbf{W}_2 \text{ReLU}(\mathbf{W}_1 \cdot \mathbf{x} + b) + \mathbf{x}$$

- ▶ Identity map ( $d$  dim) + one hidden neuron =  $d + 1$  units

ResNet with one neuron per hidden layer: universal approximation (in  $L^1$ ) for any Lebesgue-integrable function as the depth  $\rightarrow \infty$ .<sup>13</sup>

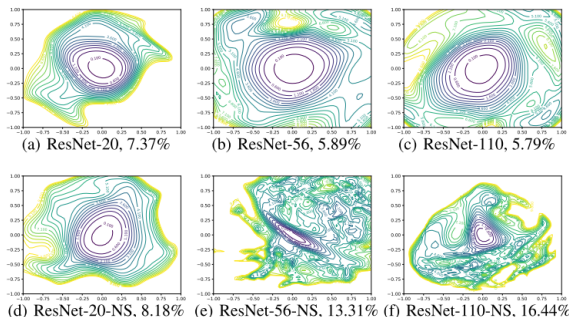
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<sup>12</sup>Hanin et al., arXiv, 2017.

<sup>13</sup>Lin & Jegelka, NIPS, 2018.

# Why Easier to Train?

- ▶ 2D visualization of the loss surface by “filter normalization” method<sup>14</sup>
- ▶ BoostResNet<sup>15</sup>: a training algorithm (non-differentiable), training error decays exponentially with depth



<sup>14</sup>Li et al., NIPS, 2018.

<sup>15</sup>Huang et al., ICML, 2018.

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# Cerebral Cortex

- ▶ Cajal Ramon (the father & the mother of modern neuroscience)
- ▶ Pyramidal cells (1888)

