

# COE 352 Project Z

Derivation of weak form

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = f(x, t)$$

$$(x, t) \in (0, 1) \times (0, 1)$$

$$u(x, 0) = \sin \pi x$$

$$u(0, t) = u(1, t) = 0$$

$$f(x, t) = (\pi^2 - 1) e^{-t} \sin \pi x$$

$$u(x, t) = e^{-t} \sin \pi x$$

$$\int_0^1 \frac{\partial u}{\partial t} v(x) dx - \int_0^1 \frac{\partial^2 u}{\partial x^2} v(x) dx = \int_0^1 f(x, t) v(x) dx$$

$$u = v(x)$$

$$du = v'(x) dx$$

$$v = \frac{\partial u}{\partial x}$$

$$v = \frac{\partial u}{\partial x}$$

$$dv = \frac{d^2 u}{dx^2} dx$$

$$\int_0^1 \frac{\partial u}{\partial t} v(x) dx - \left[ v(x) \frac{du}{dx} \right]_0^1 + \int_0^1 \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} dx = \int_0^1 f(x, t) v(x) dx$$

$$v(x) = \phi_i(x)$$

$$\int_0^1 \frac{\partial u}{\partial t} \phi_i(x) dx + \int_0^1 \frac{\partial u}{\partial x} \frac{\partial \phi_i}{\partial x} dx - \int_0^1 f(x, t) \phi_i(x) dx = 0$$