Task 20-01 Part A

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Problem Statement

Given the following the piecewise function and its integral, we will solve for c

1. $\psi(x) = \begin{cases} 0 & x < 0 \\ ce^{\frac{-x}{L}} & x \geq 0 \end{cases}$
2. $\int_{-\infty}^{+\infty} |\psi(x)|^2 dx = 1$

Work

$$\int_{-\infty}^{+\infty} |\psi(x)|^2 dx = 1$$

Since $\psi(x)$ is piecewise we can split the integral via the regions defined by the piecewise. eg:

$$\int_{-\infty}^{0} |\psi(x)|^2 dx + \int_{0}^{\infty} |\psi(x)|^2 dx = 1$$

Plug in the corresponding definitions of each region.

$$\int_{-\infty}^{0} |0|^2 dx + \int_{0}^{\infty} |ce^{\frac{-x}{L}}|^2 dx = 1$$

The expression $ce^{\frac{-x}{L}}$ will always be positive

$$\int_{0}^{\infty} |ce^{\frac{-x}{L}}|^2 dx = 1$$

$$\int_{0}^{\infty} c^2 e^{\frac{-2x}{L}} dx = 1$$

We want to use u substitution to integrate.

$$u = \frac{-2x}{L}$$

$$du = (\frac{-2}{L})dx$$

$$(\frac{L}{-2})du = dx$$

Substitue in the value for dx

$$\int_0^{\infty} \frac{-Lc^2}{2} e^u du = 1$$

$$\frac{-Lc^2}{2} \int_0^{\infty} e^u du = 1$$

Integrate

$$\frac{-Lc^2}{2} \left[e^u\right]_0^{\infty} = 1$$

Substitute back in the expression for u

$$\frac{-Lc^2}{2} \left[e^{\frac{-2x}{L}}\right]_0^{\infty} = 1$$

$$\frac{-Lc^2}{2} \left[e^{-\infty} - e^0\right] = 1$$

$e^{-\infty}$ goes to zero and e^0 simplifies to one

$$\frac{-Lc^2}{2}[0-1] = 1$$

$$\frac{Lc^2}{2} = 1$$

$$c^2 = \frac{2}{L}$$

Substitute in the given value for L

$$c^2 = \frac{2}{1}$$

$$c^2 = 2$$

$$c = \sqrt{2}$$

Therefore therefore the constant c is $c = \sqrt{2}$.