Single qubit interference – HPH

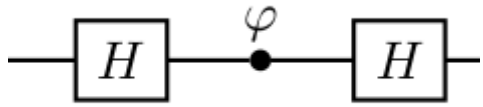

Figure 1: hph circuit

Claim: The above circuit can be represented as the product of three matrices HPH. We claim:

$$HPH = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & e^{i\psi}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} = e^{i\frac{\psi}{2}}\begin{bmatrix}\cos\frac{\psi}{2} & -i\sin\frac{\psi}{2}\\ -i\sin\frac{\psi}{2} & \cos\frac{\psi}{2}\end{bmatrix}$$

Proof:

Multiply

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & e^{i\psi}\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 1\\ e^{i\psi} & -e^{i\psi}\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix}1+e^{i\psi} & 1-e^{i\psi}\\ 1-e^{i\psi} & 1+e^{i\psi}\end{bmatrix}$$

By claim [1], we can then covert to polar form:

$$e^{i\frac{\psi}{2}}\begin{bmatrix}\cos\frac{\psi}{2} & -i\sin\frac{\psi}{2}\\ -i\sin\frac{\psi}{2} & \cos\frac{\psi}{2}\end{bmatrix}$$