Converting 1 + eipsi to polar

Claim: Given the complex number $z = 1 + e^{i\theta}$, we claim:

$$z = 1 + e^{i\theta} = 2\cos\frac{\theta}{2}e^{i\frac{\theta}{2}}$$

Proof:

We will show the claim by showing there exist (r, ϕ) such that $z = re^{i\phi} = 2\cos\frac{\theta}{2}e^{i\frac{\theta}{2}}$. In other words, z in exponential form with $r = 2\cos\frac{\theta}{2}$ and angle $\phi = \frac{\theta}{2}$.

$z = (x, y)$, we want r and θ such that $z = (x, y) = (r\cos\theta, r\sin\theta)$.

Find r:

$$r = |z|$$

The modulus of a complex number is the length in Euclidean space. So take the square root of the product of the complex number and its conjugate:

$$\begin{gathered}
= \sqrt{(1 + e^{i\theta})(1 + e^{-i\theta})} \\
= \sqrt{1 + e^{-i\theta} + e^{i\theta} + e^{i\theta}e^{-i\theta}} \\
= \sqrt{1 + e^{-i\theta} + e^{i\theta} + 1} \\
= \sqrt{2 + e^{-i\theta} + e^{i\theta}}
\end{gathered}$$

Use Euler's formula for e's:

$$\begin{gathered}
= \sqrt{2 + \cos\theta + i\sin\theta + \cos\theta - i\sin\theta} \\
= \sqrt{2 + \cos\theta + \cos\theta + i\sin\theta - i\sin\theta} \\
= \sqrt{2 + 2\cos\theta} \\
= \sqrt{2}\sqrt{1 + \cos\theta}
\end{gathered}$$

Use the trigonometric double angle identity $2\cos^2\frac{\theta}{2} = 1 + \cos\theta$:

$$= \sqrt{2}\sqrt{2\cos^2\frac{\theta}{2}}$$

$$= \sqrt{2}\sqrt{2}\sqrt{\cos^2\frac{\theta}{2}}$$

$$= 2\cos\frac{\theta}{2}$$

So $r = 2\cos\frac{\theta}{2}$.

Find $\phi:$

$$z = r\cos\phi + rsin\phi$$

$$z = r\cos\phi + rsin\phi$$

$$\tan\phi =$$