1.11.4 Distant photon emitters

Part A

Let H_1 and H_2 denote the click outcome of detectors 1 and 2 respectively. That is, H_i means detector i registered a click. Click events are independent of one another, that is, detector 1 outputting H_1 occurs independently of detector 2 outputting H_2. Hence, the probability that both click simultaneously is:

$$pr(H_1 \cap H_2) = pr(H_1)pr(H_2)$$

$$pr(H_1) = |z_1|^2 = |z_{1a} + z_{1b}|^2$$
$$pr(H_2) = |z_1|^2 = |z_{2a} + z_{2b}|^2$$

where z_{1a} and z_{1b} denote the probability amplitudes corresponding to photon A and photon B traveling to detecter 1. The same is true for z_{2a} and z_{2b} but for detector 2:

Now we compute both probabilities, and then compute the joint probability.

Detector 1

$$\begin{gathered} pr(H_1) = |z_1|^2 = |z + ze^{i\phi}|^2 \\ = (z + ze^{i\phi})(z^\dagger + z^\dagger e^{-i\phi}) \\ = zz^\dagger + zz^\dagger e^{-i\phi} + zz^\dagger e^{i\phi} + zz^\dagger e^{i\phi} e^{-i\phi} \\ = p + pe^{-i\phi} + pe^{i\phi} + p \\ = 2p + pe^{-i\phi} + pe^{i\phi} \\ = p(2 + e^{-i\phi} + e^{i\phi}) \\ = p(2 + 2\cos\phi) \\ = 2p(1 + 1\cos\phi) \end{gathered}$$

Detector 2

$$\begin{gathered} pr(H_2) = |z_2|^2 = |z + ze^{i\phi}|^2 \\ = (z + ze^{i\phi})(z^\dagger + z^\dagger e^{-i\phi}) \\ = zz^\dagger + zz^\dagger e^{-i\phi} + zz^\dagger e^{i\phi} + zz^\dagger e^{i\phi} e^{-i\phi} \\ = p + pe^{-i\phi} + pe^{i\phi} + p \\ = 2p + pe^{-i\phi} + pe^{i\phi} \\ = p(2 + e^{-i\phi} + e^{i\phi}) \\ = p(2 + 2\cos\phi) \\ = 2p(1 + 1\cos\phi) \end{gathered}$$

Final joint probability

$$\begin{aligned} pr(H_1 \cap H_2) &= 2p(1 + 1\cos\phi)2p(1 + 1\cos\phi) \\ &= 4p^2(1 + 1\cos\phi)(1 + 1\cos\phi) \\ &= 4p^2(1 + 1\cos\phi)(1 + 1\cos\phi) \\ &= 4p^2(1 + \cos\phi)^2, \text{where } p = |z|^2 \end{aligned}$$

Part B

Assuming $z \approx \frac{1}{r}e^{\frac{i2\pi}{\lambda}}$ where r is the distance between the detectors and the starts and λ is a fixed constant, we can determine r by first running a number of experiments to approximate the probability $Pr(H_1 \cap H_2)$ say as x. That is $x \approx Pr(H_1 \cap H_2)$

We can then solve for r:

$$\begin{aligned} x &= 4p^2(1 + \cos\phi)^2 \\ x &= 4p^2(1 + \cos\phi)^2 \\ \frac{x}{4(1 + cos\phi)^2} &= p^2 \\ p^2 &= \frac{x}{4(1 + cos\phi)^2} \\ p &= \sqrt{\frac{x}{4(1 + cos\phi)^2}} \\ p &= \frac{\sqrt{x}}{2(1 + cos\phi)} \end{aligned}$$

Since $p = |z|^2$

$$\begin{aligned} |\frac{1}{r}e^{\frac{i2\pi}{\lambda}}|^2 &= \frac{\sqrt{x}}{2(1 + cos\phi)} \\ \frac{1}{r}e^{\frac{i2\pi}{\lambda}}\frac{1}{r}e^{\frac{-i2\pi}{\lambda}} &= \frac{\sqrt{x}}{2(1 + cos\phi)} \\ \frac{1}{r^2} &= \frac{\sqrt{x}}{2(1 + cos\phi)} \\ r^2 &= \frac{2(1 + cos\phi)}{\sqrt{x}} \\ r^2 &= \frac{2(1 + cos\phi)}{\sqrt{x}} \end{aligned}$$