2.14.2 Change of basis

Answer:

$$|\psi\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$$

We can rewrite ψ in normal vector notation:

$$|\psi\rangle = \frac{3}{5}\begin{bmatrix}1\\0\end{bmatrix} + \frac{4}{5}\begin{bmatrix}0\\1\end{bmatrix}$$

$$|\psi\rangle = \begin{bmatrix}\frac{3}{5}\\\frac{4}{5}\end{bmatrix}$$

We can think of that vector as the coordate vector for $|\psi\rangle$ in the standard basis. We want to find the coordinate vector $|\psi\rangle$ in the hadamard basis. That is we solve for α and β:

$$|\psi\rangle = \begin{bmatrix}\frac{3}{5}\\\frac{4}{5}\end{bmatrix} = \alpha|+\rangle + \beta|-\rangle$$

$$\begin{bmatrix}\frac{3}{5}\\\frac{4}{5}\end{bmatrix} = \frac{\alpha}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix} + \frac{\beta}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$$

$$\begin{bmatrix}\frac{3\sqrt{2}}{5}\\\frac{4\sqrt{2}}{5}\end{bmatrix} = \alpha\begin{bmatrix}1\\1\end{bmatrix} + \beta\begin{bmatrix}1\\-1\end{bmatrix}$$

$$\begin{bmatrix}\frac{3\sqrt{2}}{5}\\\frac{4\sqrt{2}}{5}\end{bmatrix} = \begin{bmatrix}\alpha+\beta\\\alpha-\beta\end{bmatrix}$$

From the matrix equation:

$$\frac{3\sqrt{2}}{5} = \alpha + \beta$$

$$\frac{4\sqrt{2}}{5} = \alpha - \beta$$

Adding the equations:

$$2\alpha = \frac{7\sqrt{2}}{5}$$

$$\alpha = \frac{7\sqrt{2}}{10}$$

Subtracting the equations:

$$2\beta = -\frac{\sqrt{2}}{5}$$

$$\beta = -\frac{\sqrt{2}}{10}$$

Therefore:

$$\alpha = \frac{7\sqrt{2}}{10} \approx 0.99$$

$$\beta = -\frac{\sqrt{2}}{10} \approx -0.14$$