## Single qubit interference – HPH

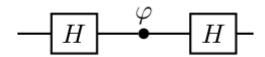


Figure 1: hph circuit

**Claim:** The above circuit can be represented as the product of three matrices HPH. We claim:

$$HPH = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\psi} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = e^{i\frac{\psi}{2}} \begin{bmatrix} \cos\frac{\psi}{2} & -i\sin\frac{\psi}{2} \\ -i\sin\frac{\psi}{2} & \cos\frac{\psi}{2} \end{bmatrix}$$

## **Proof:**

Multiply

$$\begin{split} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\psi} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ e^{i\psi} & -e^{i\psi} \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 1 + e^{i\psi} & 1 - e^{i\psi} \\ 1 - e^{i\psi} & 1 + e^{i\psi} \end{bmatrix} \end{split}$$

By claim [1], we can then covert to polar form:

$$e^{i\frac{\psi}{2}} \begin{bmatrix} \cos\frac{\psi}{2} & -i\sin\frac{\psi}{2} \\ -i\sin\frac{\psi}{2} & \cos\frac{\psi}{2} \end{bmatrix}$$