Converting 1 + eipsi to polar

Claim 1: Given the complex number $z = 1 + e^{i\theta}$, we claim:

$$z = 1 + e^{i\theta} = 2e^{i\frac{\psi}{2}} \cos \frac{\psi}{2}$$

Proof:

$$1 + e^{i\psi}$$
$$\Rightarrow <1 = e^{i \cdot 0}>$$
$$e^{i\cdot 0} + e^{i\psi}$$
$$\Rightarrow <0 \equiv \frac{\psi}{2} - \frac{\psi}{2}, \psi \equiv \frac{\psi}{2} + \frac{\psi}{2}>$$
$$e^{i(\frac{\psi}{2} - \frac{\psi}{2})} + e^{i(\frac{\psi}{2})}$$
$$\Rightarrow$$
$$e^{i\frac{\psi}{2}} e^{-i\frac{\psi}{2}} + e^{i\frac{\psi}{2}} e^{i\frac{\psi}{2}}$$
$$\Rightarrow$$
$$e^{i\frac{\psi}{2}} \left(e^{-i\frac{\psi}{2}} + e^{i\frac{\psi}{2}} \right)$$
$$\Rightarrow < \text{Euler's identity: } e^{i\theta} = \cos\theta + i \sin\theta>$$
$$e^{i\frac{\psi}{2}} \left[(\cos\frac{\psi}{2} + i \sin\frac{\psi}{2}) + (\cos\frac{\psi}{2} - i\sin\frac{\psi}{2}) \right]$$
$$\Rightarrow$$
$$e^{i\frac{\psi}{2}} \left(2\cos\frac{\psi}{2} \right)$$
$$\Rightarrow$$
$$2e^{i\frac{\psi}{2}} \cos\frac{\psi}{2}$$

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Claim 1.5: Given the complex number $z = 1 - e^{i\theta}$, we claim:

$$z = 1 - e^{i\theta} = -2i \cdot e^{i\frac{\psi}{2}} \cdot \sin\frac{\psi}{2}$$

Proof:

$$1 - e^{i\psi}$$
$$\Rightarrow <1 = e^{i \cdot 0}>$$
$$e^{i\cdot 0} + e^{i\psi}$$
$$\Rightarrow <0 \equiv \frac{\psi}{2} - \frac{\psi}{2}, \psi \equiv \frac{\psi}{2} + \frac{\psi}{2}>$$

$$e^{i(\frac{\psi}{2}-\frac{\psi}{2})} - e^{i(\frac{\psi}{2}+\frac{\psi}{2})}$$

$$\Rightarrow$$

$$e^{i\frac{\psi}{2}}e^{-i\frac{\psi}{2}} - e^{i\frac{\psi}{2}}e^{i\frac{\psi}{2}}$$

$$\Rightarrow$$

$$e^{i\frac{\psi}{2}}\left(e^{-i\frac{\psi}{2}} - e^{i\frac{\psi}{2}}\right)$$

$$\Rightarrow < \text{Euler's identity: } e^{i\theta} = \cos\theta + i\sin\theta >$$

$$e^{i\frac{\psi}{2}}\left[\left(\cos\frac{\psi}{2} - i\sin\frac{\psi}{2}\right) - \left(\cos\frac{\psi}{2} - i\sin\frac{\psi}{2}\right)\right]$$

$$\Rightarrow$$

$$e^{i\frac{\psi}{2}}\left[\cos\frac{\psi}{2} - i\sin\frac{\psi}{2} - \cos\frac{\psi}{2} + i\sin\frac{\psi}{2}\right]$$

$$\Rightarrow$$

$$e^{i\frac{\psi}{2}}\left(-2i\sin\frac{\psi}{2}\right)$$

$$\Rightarrow$$

$$-2i \cdot e^{i\frac{\psi}{2}} \cdot \sin\frac{\psi}{2}$$

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