Notes

Suppose we have a one dimensional universe represented by an array whose entries contain the state of each cells (i.e. one or zero). We can define a high order operator which describes the evolution of the universe also refered to as the update function $uf(|bbb\rangle)$ where $|bbb\rangle$ is the current state.

High Order Definition of update function

Suppose we define the uf in the following high order definition

$$
\begin{aligned}
|0b0\rangle &\mapsto |0b0\rangle \\
|1b0\rangle &\mapsto |1b0\rangle \\
|0b1\rangle &\mapsto |0b1\rangle \\
|1b1\rangle &\mapsto |1b1\rangle
\end{aligned}
$$

$$
\begin{aligned}
|0b0\rangle &\mapsto |0b0\rangle \\
|0b1\rangle &\mapsto |0b1\rangle \\
|1b0\rangle &\mapsto |0b1\rangle \\
|1b1\rangle &\mapsto |1b1\rangle
\end{aligned}
$$

$$
\begin{aligned}
|1b0\rangle &\mapsto |1b0\rangle \\
|1b0\rangle &\mapsto |1b0\rangle \\
|1b1\rangle &\mapsto |1b1\rangle
\end{aligned}
$$

Low Order

$$
\begin{gathered}
|0\rangle\langle 0| I |0\rangle\langle 0| \\
|1\rangle\langle 1| I |0\rangle\langle 0| \\
|0\rangle\langle 0| I |1\rangle\langle 1| \\
|1\rangle\langle 1| A |1\rangle\langle 1|
\end{gathered}
$$

$$
\begin{gathered}
|0\rangle\langle 0| \cdot |0\rangle\langle 0| \cdot I \\
|0\rangle\langle 0| |1\rangle\langle 1| A \\
|1\rangle\langle 1| |0\rangle\langle 0| I \\
L |1\rangle\langle 1| |1\rangle\langle 1| A
\end{gathered}
$$

$$
\begin{gathered}
I \cdot |0\rangle\langle 0| \cdot |0\rangle\langle 0| \\
I |0\rangle\langle 0| |1\rangle\langle 1| \\
I |1\rangle\langle 1| |0\rangle\langle 0| \\
A |1\rangle\langle 1| |1\rangle\langle 1|
\end{gathered}
$$