

Single qubit interference – HPH



Figure 1: hph circuit

Claim: The above circuit can be represented as the product of three matrices HPH . We claim:

$$HPH = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\psi} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = e^{i\frac{\psi}{2}} \begin{bmatrix} \cos \frac{\psi}{2} & -i \sin \frac{\psi}{2} \\ -i \sin \frac{\psi}{2} & \cos \frac{\psi}{2} \end{bmatrix}$$

Proof:

Multiply

$$\begin{aligned} & \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\psi} \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ e^{i\psi} & -e^{i\psi} \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 1 + e^{i\psi} & 1 - e^{i\psi} \\ 1 - e^{i\psi} & 1 + e^{i\psi} \end{bmatrix} \end{aligned}$$

By claim [1] and [1.5], we can then covert to polar form:

$$e^{i\frac{\psi}{2}} \begin{bmatrix} \cos \frac{\psi}{2} & -i \sin \frac{\psi}{2} \\ -i \sin \frac{\psi}{2} & \cos \frac{\psi}{2} \end{bmatrix}$$