Notes

Suppose we have a one dimensional universe represented by an array whose entries contain the state of each cells (i.e. one or zero). We can define a high order operator which describes the evolution of the universe also refered to as the update function $uf(|bbb\rangle)$ where $|bbb\rangle$ is the current state.

High Order Definition of update function - No unitrary (no dense linear algebra)

Suppose we define the uf in the following high order definition

Bit 0 Rule

$$\begin{aligned} |00b\rangle &\mapsto |00b\rangle \\ *\,|01b\rangle &\mapsto |01\bar{b}\rangle \\ |10b\rangle &\mapsto |10b\rangle \\ *\,|11b\rangle &\mapsto |11\bar{b}\rangle \end{aligned}$$

Bit 1 Rule

$$\begin{aligned} |0b0\rangle &\mapsto |0b0\rangle \\ *\,|0b1\rangle &\mapsto |0\bar{b}1\rangle \\ *\,|1b0\rangle &\mapsto |1\bar{b}0\rangle \\ |1b1\rangle &\mapsto |1b1\rangle \end{aligned}$$

Bit 2 Rule

$$\begin{aligned} |b00\rangle &\mapsto |b00\rangle \\ |b01\rangle &\mapsto |b01\rangle \\ *\,|b10\rangle &\mapsto |\bar{b}10\rangle \\ *\,|b11\rangle &\mapsto |\bar{b}11\rangle \end{aligned}$$

Low Order - Dense linear algebra based definition with unitrary

Bit 0 Rule

$$\begin{gathered} |0\rangle\langle 0| \cdot |0\rangle\langle 0| \cdot I \\ |0\rangle\langle 0| \cdot |1\rangle\langle 1| \cdot A \\ |1\rangle\langle 1| \cdot |0\rangle\langle 0| \cdot I \\ |1\rangle\langle 1| \cdot |1\rangle\langle 1| \cdot A \end{gathered}$$

Bit 0 Rule

$$\begin{gathered}|0\rangle\langle 0|\cdot I\cdot|0\rangle\langle 0| \\ |0\rangle\langle 0|\cdot A\cdot|1\rangle\langle 1| \\ |1\rangle\langle 1|\cdot A\cdot|0\rangle\langle 0| \\ |1\rangle\langle 1|\cdot I\cdot|1\rangle\langle 1|\end{gathered}$$

Bit 2 Rule

$$\begin{gathered}I\cdot|0\rangle\langle 0|\cdot|0\rangle\langle 0| \\ I\cdot|0\rangle\langle 0|\cdot|1\rangle\langle 1| \\ A\cdot|1\rangle\langle 1|\cdot|0\rangle\langle 0| \\ A\cdot|1\rangle\langle 1|\cdot|1\rangle\langle 1|\end{gathered}$$

Generalization

In general given a bit vector $|a_0 a_1 b\rangle$ where b is to be updated, we can describe the update with a boolean function $f : B^3 \mapsto B^1$. From the above example we could define the boolean function as $x_3 = f(\vec{s}) = (x_0 \wedge x_1) \vee x_3$ where 3 is the bit we are updating (b). So in general we need a boolean function for each bit.

bit 0: $x_0 = f(\vec{s}) = x_1 \wedge \neg x_0$

bit 1: $x_1 = f(\vec{s}) = \neg x_1 \vee x_0$

bit 2: $x_2 = f(\vec{s}) = (x_0 \wedge x_1) \vee x_3$

Crux: I think the crux of this is that given a high level definition of a CA we need a systematic way to derive a unitary matrix which carries out the computation elegantly.

Example: CCNOT. High vs Low order definitions

$$\begin{array}{rl} |00b\rangle & \mapsto |00b\rangle \\ |00b\rangle & \mapsto |01b\rangle \\ |10b\rangle & \mapsto |10b\rangle \\ *\,|11b\rangle & \mapsto |11\bar{b}\rangle \end{array} \longrightarrow \begin{array}{c} |0\rangle\langle 0|\cdot|0\rangle\langle 0|\cdot I \\ |0\rangle\langle 0|\cdot|1\rangle\langle 1|\cdot I \\ |1\rangle\langle 1|\cdot|0\rangle\langle 0|\cdot I \\ |1\rangle\langle 1|\cdot|1\rangle\langle 1|\cdot X \end{array}$$

Therefore:

So rule f_0 is a linear operator, defined as:

$f_0 = |0\rangle\langle 0|\cdot|0\rangle\langle 0|\cdot I + |0\rangle\langle 0|\cdot|1\rangle\langle 1|\cdot I + |1\rangle\langle 1|\cdot|0\rangle\langle 0|\cdot I + |1\rangle\langle 1|\cdot|1\rangle\langle 1|\cdot X,$

Hence f_0 implements ccnot

We could then have a rule for the other 2 bits f_1, f_2

The evolution of the entire system is therefore the set of operators associated with each qubit F

Therefore:

$$F = \{f_0, f_1, f_2\}$$

The matrix representation of the entire eovlution is therefore given as:

$$F = f_0 + f_1 + f_2$$

Crux: My formalism

CA: **Input:** 3 element tuple $(F,)$

Laws of Quantum based logic

Laws of Chuckian Algebra