

Lecture 2 Summary



BNF is used to specify the syntax of numerals.

$$\begin{aligned} \langle \text{Digit} \rangle &::= 0 \mid 1 \\ \langle \text{Numeral} \rangle &::= \langle \text{Digit} \rangle \mid \langle \text{Numeral} \rangle \langle \text{Digit} \rangle \end{aligned}$$

BNF rules define a **language**: a set of strings. Every string in the language is generated by the rules. The only two strings in the language of $\langle \text{Digit} \rangle$ s are 0 and 1. Every string in the language of $\langle \text{Numeral} \rangle$ s is either a $\langle \text{Digit} \rangle$ or of the **form** $N D$ where N is a $\langle \text{Numeral} \rangle$ and D is a $\langle \text{Digit} \rangle$.

This allows us to define functions on a language **inductively**: by considering the forms that the strings of the language can take, following the BNF options. For example, we define the integer $[[D]]digit$ as a function of $\langle \text{Digit} \rangle D$ by considering the two options defining $\langle \text{Digit} \rangle$: D is either 0 or 1. Thus we define the function:

$$\begin{aligned} [[0]]digit &= 0 \\ [[1]]digit &= 1 \end{aligned}$$

Similarly, we define the integer $[[N]]numeral$ as a function of $\langle \text{Numeral} \rangle N$ by noting that every $\langle \text{Numeral} \rangle$ is either a $\langle \text{Digit} \rangle$ or of the form $N D$:

$$\begin{aligned} [[D]]numeral &= [[D]]digit && \text{for any } \langle \text{Digit} \rangle D \\ [[N D]]numeral &= 2 * [[N]]numeral + [[D]]digit && \text{for any } \langle \text{Numeral} \rangle N \text{ and } \langle \text{Digit} \rangle D \end{aligned}$$

Key Points

The key points that will recur throughout the module are:

- **Syntax** is strings: things that are written down. Syntactic **languages** are sets of strings, and these can be defined by BNF.
- **Denotation functions** map syntax to semantics, and can be defined **inductively** on the form of their inputs: each BNF option defines one form that the input can take.

You should be able to:

- use BNF to define languages
see the [tutorial on BNF](#) by Lars Marius Garshol, and the [Problem Sheet for Week 2](#);
- define functions inductively
see the [Problem Sheet for Week 2](#).

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