Comp 317: Semantics of Programming Languages

Problem Sheet 4

1. Consider the following Maude specification.

   ```
   fmod NUMERAL-EXPRESSION is

     sorts Digit NumeralExp .
     subsort Digit < NumeralExp .

     ops 0 1 : -> Digit .

     op _ _ : NumeralExp Digit -> NumeralExp .

     op _+_ : NumeralExp NumeralExp -> NumeralExp .

   endfm
   ```

   (a) What is the meaning of this module?

   (b) Describe its signature (i.e., say what S and Σ are).

   (c) Describe the term algebra for this signature.

   (d) One model of this signature is A, where we define
   - $A_{\texttt{Digit}} = \{0, 1\}$
   - $A_{\texttt{NumeralExp}} = \{0, 1, 2, \ldots\}$
   - $A_0 = 0$
   - $A_1 = 1$
   - $A_{\texttt{\_\_}}(x, y) = 2x + y$
   - $A_+(x, y) = x + y$

   Describe the functions $h_{\texttt{Digit}}, h_{\texttt{NumeralExp}} : T_\Sigma \to A$.

   (e) Evaluate the following terms in A (i.e., what does $h_{\texttt{NumeralExp}}$ send these terms to?):
   i. 001
   ii. 11001
   iii. 001 + 101
   iv. 11 + 111
   v. (10 + 1011)1

   (f) Define another model of `NUMERAL-EXPRESSION`, and repeat parts (d) and (e) for that model.

(g) (Harder) Write a Maude module `NUMERAL-EXPRESSION-SEMANTICS` that imports `NUMERAL-EXPRESSION` and `INT` (Maude's built-in integers), and declares two 'semantics' operations

```
op [[_]] : Digit -> Int .
op [[_]] : NumeralExp -> Int .
```

and gives equations describing the semantics of binary numerals (test the equations on the terms in part (e)).