Supplementary Materials: GORICA-Based Selection of RIs in RI-CLPM

The four possible models: CLPM (zero RIs), RI-CLPM(κ) and RI-CLPM(ω) (one RI), and RI-CLPM(full) (two RIs). These four models can be expressed in different manners, as depicted in Equation 1.

$$
\begin{aligned}
&H_0 : \text{Var}_{[\kappa]} = 0, \text{Var}_{[\omega]} = 0 && \text{(CLPM)} \\
&H_2 : \text{Var}_{[\kappa]} < 0, \text{Var}_{[\omega]} < 0 && \text{(CLPM)} \\
\\
&H_{1b} : \text{Var}_{[\kappa]} > \text{bound}, \text{Var}_{[\omega]} > \text{bound} && \text{(RI-CLPM (full))} \\
\\
&H_{3b} : \text{Var}_{[\kappa]} > \text{bound}, \text{Var}_{[\omega]} < 0 && \text{(RI-CLPM(}\kappa\text{))} \\
&H_{3ab} : \text{Var}_{[\kappa]} > \text{bound}, \text{Var}_{[\omega]} = 0 && \text{(RI-CLPM(}\kappa\text{))} \\
&H_{3bb} : \text{Var}_{[\kappa]} > \text{bound}, \text{Var}_{[\omega]} < \text{bound} && \text{(RI-CLPM(}\kappa\text{))} \\
\\
&H_{4b} : \text{Var}_{[\kappa]} < 0, \text{Var}_{[\omega]} > \text{bound} && \text{(RI-CLPM(}\omega\text{))} \\
&H_{4ab} : \text{Var}_{[\kappa]} = 0, \text{Var}_{[\omega]} > \text{bound} && \text{(RI-CLPM(}\omega\text{))} \\
&H_{4bb} : \text{Var}_{[\kappa]} < \text{bound}, \text{Var}_{[\omega]} > \text{bound} && \text{(RI-CLPM(}\omega\text{))}
\end{aligned} \tag{1}
$$

Table 1 provides a classification of the true and correct hypotheses for GORICA in the context of the bivariate model. The various sets of hypotheses are repeated in the second column of Table 1 where the hypotheses themselves can be found in Equation 1. In some cases, there may be multiple hypotheses that are correct, but typically, one is the best hypothesis due to its smaller complexity. This hypothesis is referred to the true one. However, when the truth lies on the border of some or all of the correct hypotheses, the border is actually true and the THR will not reach 1. Notably, this can happen when at least one population RI variance is 0 and this is part of the hypothesis. For instance, in Set 1, the set of hypothese consists of H_0, H_{1b}, and H_u. H_0 is the true hypothesis, but equalities are never exactly true. Since H_u is also correct, H_u will receive some support as well. Hence, the THR of H_0 will not reach 1. Another scenario when comparing H_{1b} and H_u, despite H_{1b} being the true hypothesis, its THR would unlikely reach 1 because H_{1b} is a subset of H_u, implying that both H_{1b} and H_u are true.

Table 1: Classification of correct and true hypotheses for the GORICA in a bivariate model.

$\mathbf{Cov}(\overrightarrow{RI})$	**GORICA**		
	Set of Hypotheses	**Correct Hypothesis/-es**	**True Hypothesis/-es**
	Set 1: H_0, H_{1b}, H_u	H_0, H_u	H_0
	Set 2: H_{1b}, complement H_{1b}	complement H_{1b}	complement H_{1b}
$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$	Set 3: H_{1b}, H_2, H_u	H_2, H_u	H_2
	Set 4: H_0, H_{1b}, H_2, H_{3b}, H_{4b}	H_0, H_2	H_0
	Set 5: H_0, H_{1b}, H_{3ab}, H_{4ab}, H_u	H_0, H_u	H_0
	Set 6: H_{1b}, H_2, H_{3bb}, H_{4bb}	H_2	H_2
	Set 1: H_0, H_{1b}, H_u	H_u	H_u
	Set 2: H_{1b}, complement H_{1b}	complement H_{1b}	complement H_{1b}
$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$	Set 3: H_{1b}, H_2, H_u	H_u	H_u
	Set 4: H_0, H_{1b}, H_2, H_{3b}, H_{4b}	H_{4b}	H_{4b}
	Set 5: H_0, H_{1b}, H_{3ab}, H_{4ab}, H_u	H_{4ab}, H_u	H_{4ab}
	Set 6: H_{1b}, H_2, H_{3bb}, H_{4bb}	H_{4bb}	H_{4bb}
	Set 1: H_0, H_{1b}, H_u	H_{1b}, H_u	H_{1b}
	Set 2: H_{1b}, complement H_{1b}	H_{1b}	H_{1b}
$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$,	Set 3: H_{1b}, H_2, H_u	H_{1b}, H_u	H_{1b}
$\begin{bmatrix} 1 & -0.62 \\ -0.62 & 1 \end{bmatrix}$,	Set 4: H_0, H_{1b}, H_2, H_{3b}, H_{4b}	H_{1b}	H_{1b}
$\begin{bmatrix} 1 & 0.5 \\ 0.5 & 2 \end{bmatrix}$,	Set 5: H_0, H_{1b}, H_{3ab}, H_{4ab}, H_u	H_{1b}, H_u	H_{1b}
$\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$	Set 6: H_{1b}, H_2, H_{3bb}, H_{4bb}	H_{1b}	H_{1b}