XII

Homework Review

Chapter 9

1.1 Let $L_1 = L(ab^* a) = \{aa, aba, abba, abbba, \ldots\}$
$L_2 = ((ab)^* a) = \{a, aba, ababa, abababa, \ldots\}$
$L_3 = ((ab)^+ a) = \{aba, ababa, abababa, \ldots\}$

a) List the members of $L_1 \cap L_2$.

aba

b) Present one word of length 10 in $L_1 \circ L_2$

abbbaababa

c) List, in order of increasing length, the first five words of L_2^*.

$L_2^* = ((ab)^* a)^* = \{\epsilon, a, aa, aaa, aba, \ldots\}$

d) List the word(s) in $L_2 \setminus L_3$.

a

Every other word in L_2 also appears in L_3.

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1.3 $\Sigma = \{0, 1\}$. Describe in English the languages over Σ denoted by the following regular expressions:

a) $0^* 1 0^* 1 0^* 1 0^*$

b) $0 (0 + 1)^* 0$

1.5 a) Present a regular expression that denotes the set of even binary strings.

b) Odd binary strings:

c) for those binary numerals that name multiples of 8:

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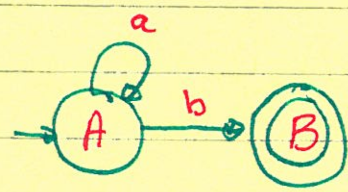
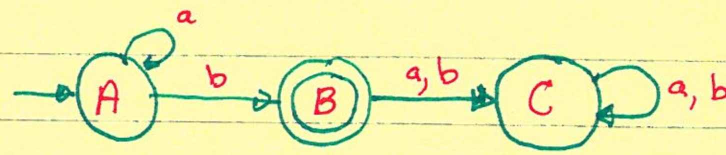
2.3 Create a dfa $\cong$ L(E)

a) $a^* b$



// this machine is non-deterministic



// a dfa

b) $b^*(ab)^+$

d) $(a+b)^* b$

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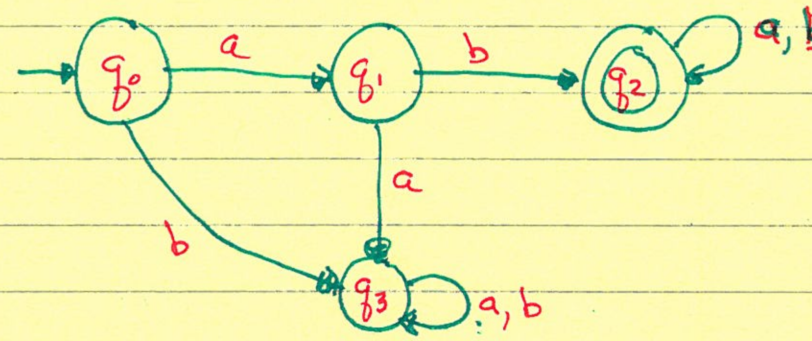
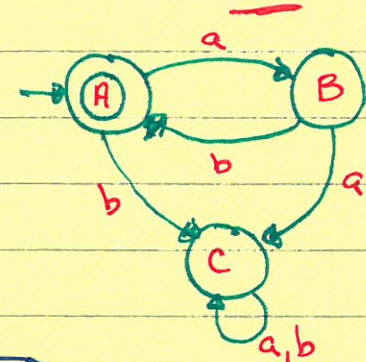
2.8 a) $L(M) = ?$



3.3 a) dfa $\cong E(ab)^* = L$

this machine is non-deterministic

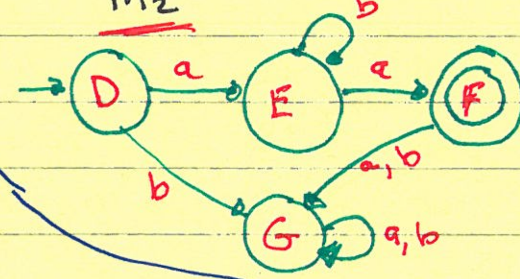
M₁



A dfa

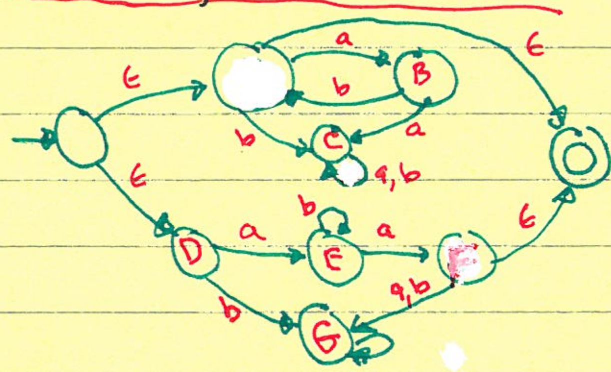
b) dfa $\cong E(ab^*a) = L'$

M₂



c) Combining M_1 and M_2

M_3



Is $L(M_3) = L(M_1) \cup L(M_2)$?

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8.1

$$G: \begin{array}{ll} (1) & S \to aaA \\ (2)(3) & A \to aaA \mid B \\ (4)(5) & B \to b \mid bB \end{array}$$

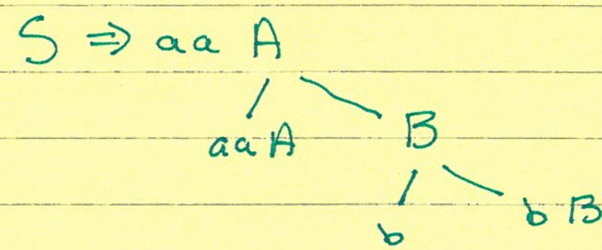
// right linear - note this is a shortcut for regular rules.

a) non terminals of G: S, A, B

terminals of G: a, b

Number of productions: 5

b) $L(G) = ?$



$L(G) = \{(aa)^n b^m \mid n \geq 1, m \geq 1\}$ // clearly ϵ not in L.

c) regular expression E s.t. $L(E) = L(G)$

$(aa)^+ b^+$ or $(aa)(aa)^* bb^*$

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8.2 G: (1)(2) $S \to aaaA \mid A$

(3)..(5) $A \to aaa \mid aaaA \mid B$

(6)..(8) $B \to bb \mid bbB \mid C$

(9)..(11) $C \to c \mid cC \mid$

never write a rule this way!!!

Rewrite rule 11 as $C \to \epsilon$

a) nonterminals of G:

terminals of G:

number of production rules:

b) $L(G) = ?$

c) Regular expression E s.t. $L(E) = L(G)$.

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8.3 G: (1)(2) $S \to aS'bbccc \mid \epsilon$

Rule 1 is not a regular rule! $N \to (N+T)^*$ is context free

(3)(4) $S' \to aS'bbCCC \mid \epsilon$

(5) $Cb \to bC$

(6) $Cc \to cc$

Rule 5 is not even context free! It is context sensitive!

a) nonterminals:

terminals:

number of production rules:

b) $L(G) = ?$

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8.8 Grammar of example 9.8.3

$$\begin{aligned} &(1)(2) \quad S \to aS'bc \mid \epsilon \\ &(3)(4) \quad S' \to aS'bC \mid \epsilon \\ &(5) \quad Cb \to bC \\ &(6) \quad Cc \to cc \end{aligned}$$

rules 1, 3 are context free rules

rules 5, 6 are context sensitive rules

Give two distinct derivations for the word $a^4b^4c^4$.

$$S \underset{(1)}{\Rightarrow} aS'bc \underset{(3)}{\Rightarrow} aaS'bCbc \underset{(3)}{\Rightarrow} aaaS'bCbCbc$$

$$\underset{(3)}{\Rightarrow} aaaaS'bCbCbCbc \underset{(4)}{\Rightarrow} aaaabCbCbCbc$$

$$\overset{*}{\underset{(5)}{\Rightarrow}} aaaabbbbCCCc \overset{*}{\underset{(6)}{\Rightarrow}} aaaabbbbcccc$$

$$S \underset{(1)}{\Rightarrow} aS'bc \overset{*}{\underset{3}{\Rightarrow}} aaaaS'bCbCbCbc$$

then what ?...

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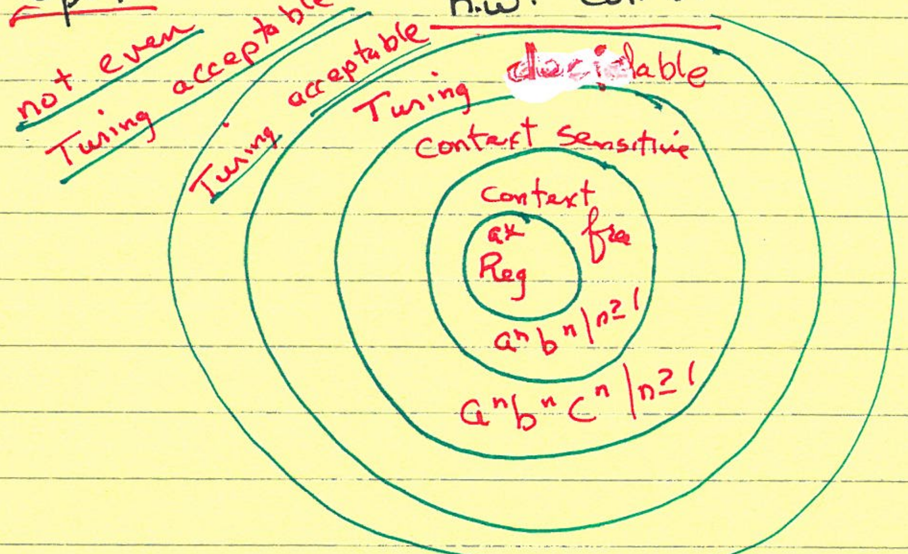


9.9.1 Apply Thm 9.15 to grammar G to obtain an fa M s.t. $L(M) = L(G)$

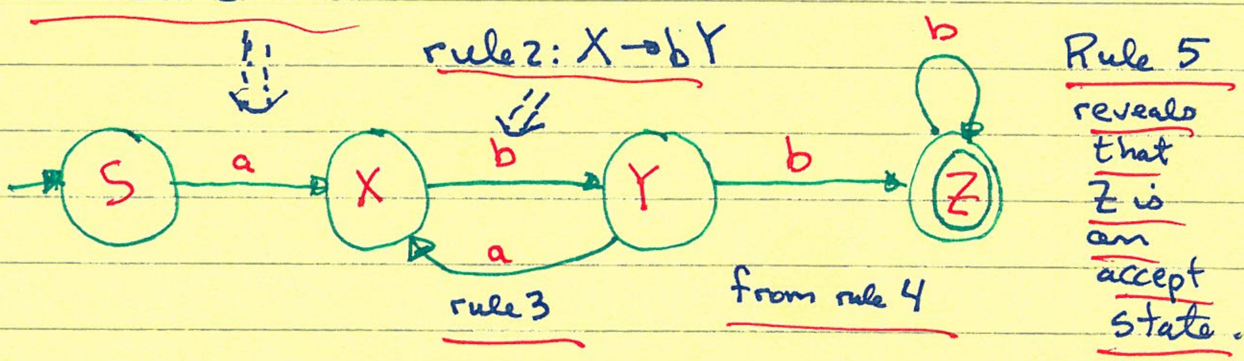
G: (1) $S \to aX$
(2) $X \to bY$
(3)-(5) $Y \to aX \mid bZ \mid b$
(6)(7) $Z \to bZ \mid b$

$S \underset{(1)}{\Rightarrow} aX \underset{(2)}{\Rightarrow} abY$

rules 3 & 4 yield a loop of (ab)'s
rule 5 a 'b'
rules 6, 7 a loop of b's.

$L(G) = ab(ab)^*bb^*$ rules 6, 7

rule 1: $S \to aX$



$L(M) = ab(ab)^*bb^* = L(G)$

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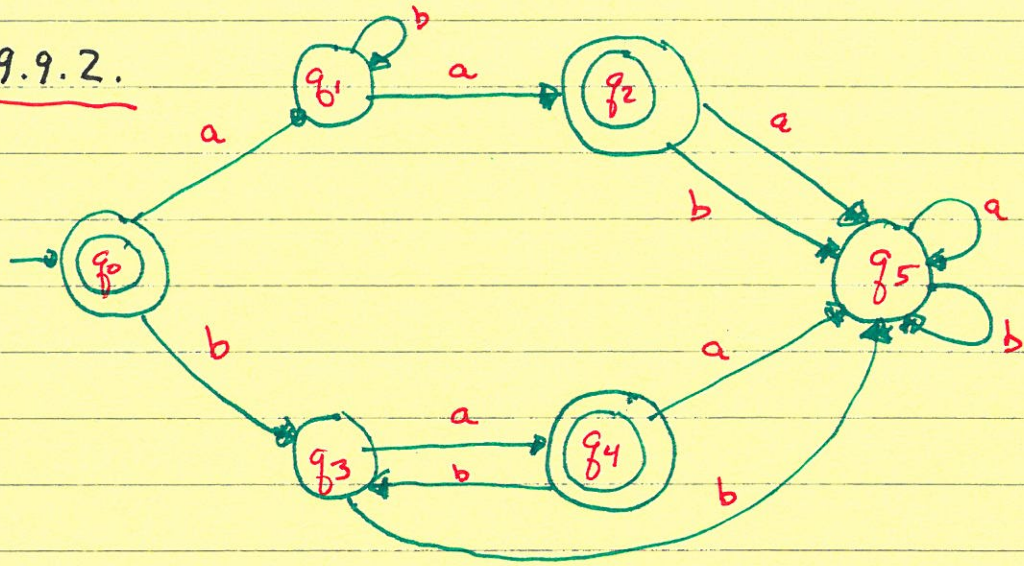
9.9.2.

a)



(1)(2)(3) $Q_0 \to aQ_1 \mid bQ_3 \mid \epsilon$

(4)(5)(6) $Q_1 \to bQ_1 \mid aQ_2 \mid a$ — or $Q_1 \to bQ_1 \mid aQ_2$ and $Q_2 \to \epsilon$

(7)(8) $Q_2 \to aQ_5 \mid bQ_5$

(9)(10)(11) $Q_3 \to aQ_4 \mid a \mid bQ_5$ — similar to above

(12)(13) $Q_4 \to bQ_3 \mid aQ_5$

(14)(15) $Q_5 \to aQ_5 \mid bQ_5$

b) $w = bababa$

$(q_0, bababa) \vdash q_3, ababa \vdash q_4, baba \vdash q_3, aba$
$\vdash q_4, ba \vdash q_3, a \vdash q_4, \epsilon$

$w = bababa \in L(M)$.

$Q_0 \underset{(2)}{\Rightarrow} bQ_3 \underset{(9)}{\Rightarrow} baQ_4 \underset{(12)}{\Rightarrow} babQ_3$
$\underset{(9)}{\Rightarrow} babaQ_4 \underset{(12)}{\Rightarrow} bababQ_3 \underset{(10)}{\Rightarrow} bababa$

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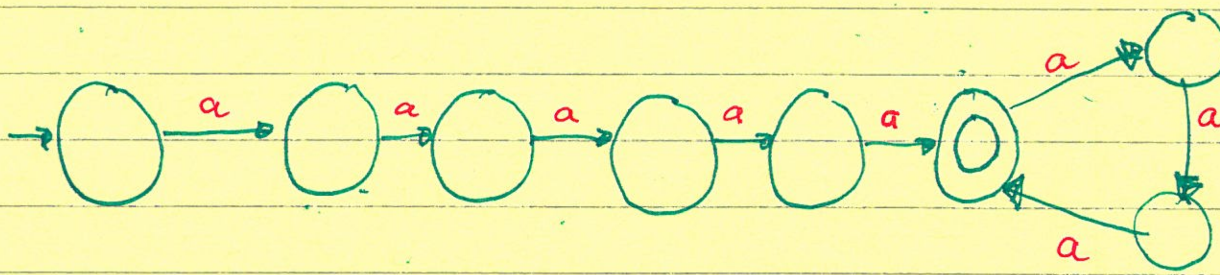
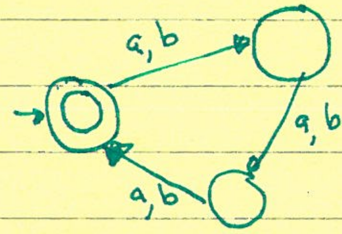
10.3 a) Show that the language
$L = \{a^n \mid n = 5 + 3K \text{ for } K \geq 0\}$ is regular.



b) fixed $i, j \geq 0$
$L = \{a^n \mid n = i + jK \text{ for } K \geq 0\}$ is regular.

10.8 $L = \{a, b\}$, dfa for $L =$

a) $\{w \mid |w| \bmod 3 = 0\}$



b) $\{w \mid |w| \bmod 4 \neq 0\}$