XV

cp.10

Chapter 10 homework

1.1) context free grammars

a) $S \rightarrow_{(1)(2)} aSbb \mid \epsilon$

$S \underset{(1)}{\Rightarrow} aSbb \underset{(1)}{\Rightarrow} aaSbbbb \underset{2}{\Rightarrow} aabbbb$

Each application of rule 1 yields one a and two b's. Finally an application of rule 2 removes the sole nonterminal 'S'.

$L = \{ a^n b^{2n} \mid n \geq 0 \}$ yes, ϵ is in L.

b)
$S \rightarrow X \mid Y \mid \epsilon$
$X \rightarrow aXb \mid \epsilon$
$Y \rightarrow aYbb \mid \epsilon$

$X \overset{*}{\Rightarrow} a^n b^n$

$y \overset{*}{\Rightarrow} a^n b^{2n}$

$\therefore L(G) = \{ a^n b^n \cup a^n b^{2n} \mid n \geq 0 \}$ yes, ϵ is in L.

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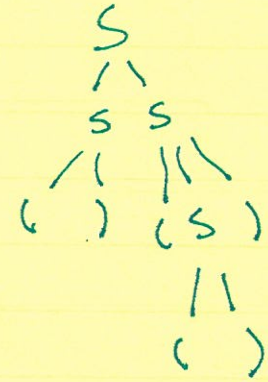
o 1.2 a) Let L be the language of well-balanced parenthesized. We conside ϵ as in L.

If $w \in L$, then so is (w)
and so is ww
and of course w too

So ...

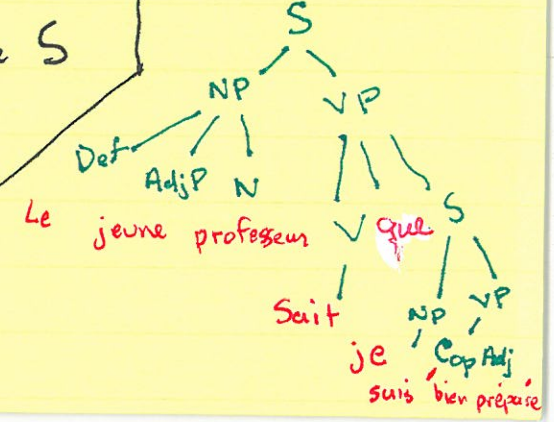
$S \to (S) \mid SS \mid \epsilon$

then for $()(())$ we have



1.3) S → NP VP
NP → Det Adj P N | Det N | ϵ |
Adj P N | N
N → Pron | professeur
VP → Cop Adj P | V que S
Adj P → Adv Adj | Adj
Det → le
Adj → jeune | préparée
V → sait
Pron → je

Cop → suis
Adv. → bien



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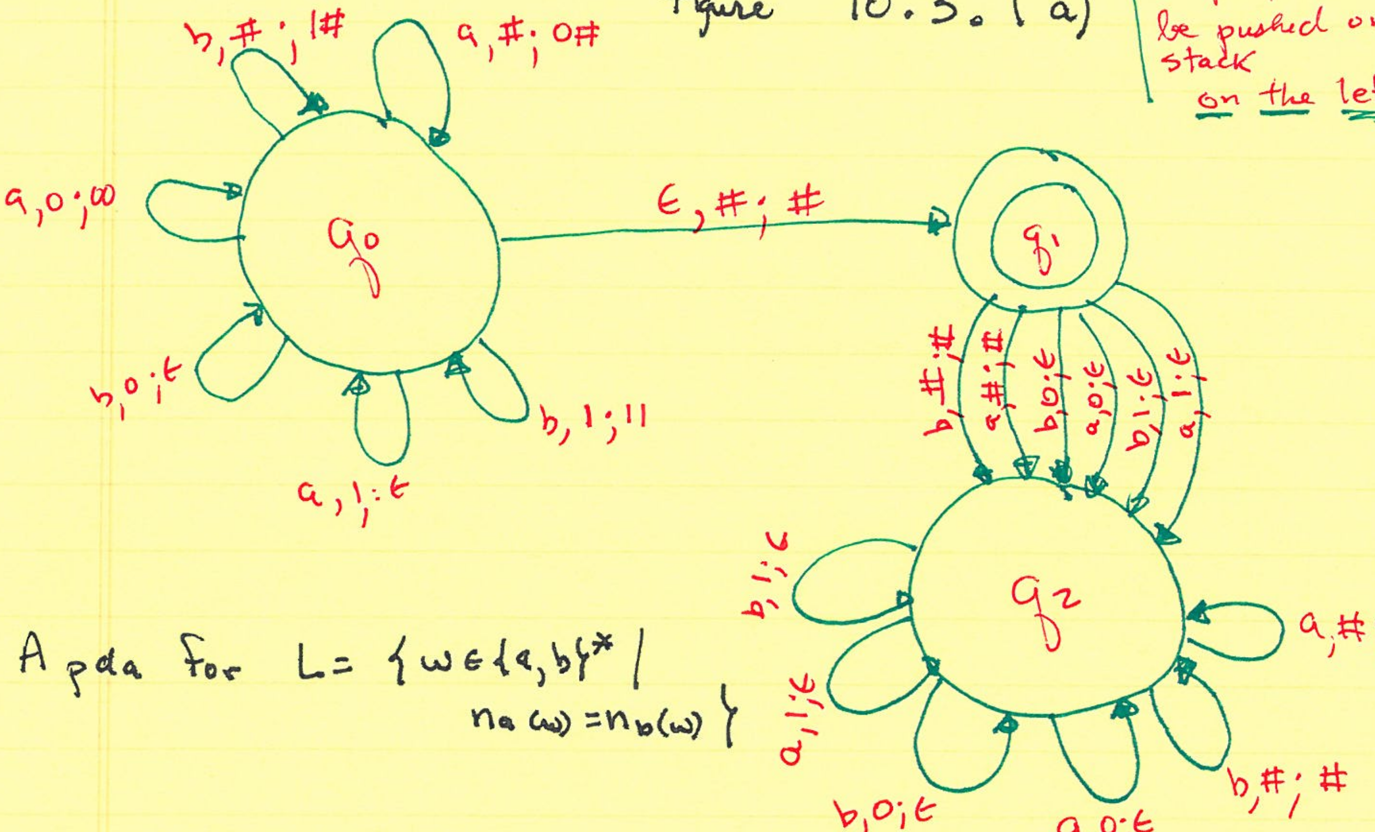
3.1) Consult pda in ex. 10.3.1 figure 10.3.1 a)

// Note, top symbol to be pushed on stack on the left.



A pda for $L = \{ w \in \{a,b\}^* \mid n_a(w) = n_b(w) \}$

a) Show that $w = abba \in L(M)$

$(q_0, abba; \#)$
$\vdash (q_0, bba; 0\#)$
$\vdash (q_0, ba; \#)$
$\vdash (q_0, a; 1\#)$
$\vdash (q_0, \epsilon; \#)$
$\vdash (q_1, \#; \#)$

// note, in this example we assume M comes "pre-loaded" with #

// notice, M "travels" to q_1 but stack is not emptied.

M accepts by accept state.

Notice $aba \notin L(M)$. We would be in q_0 with '0' on stack. Stuck!

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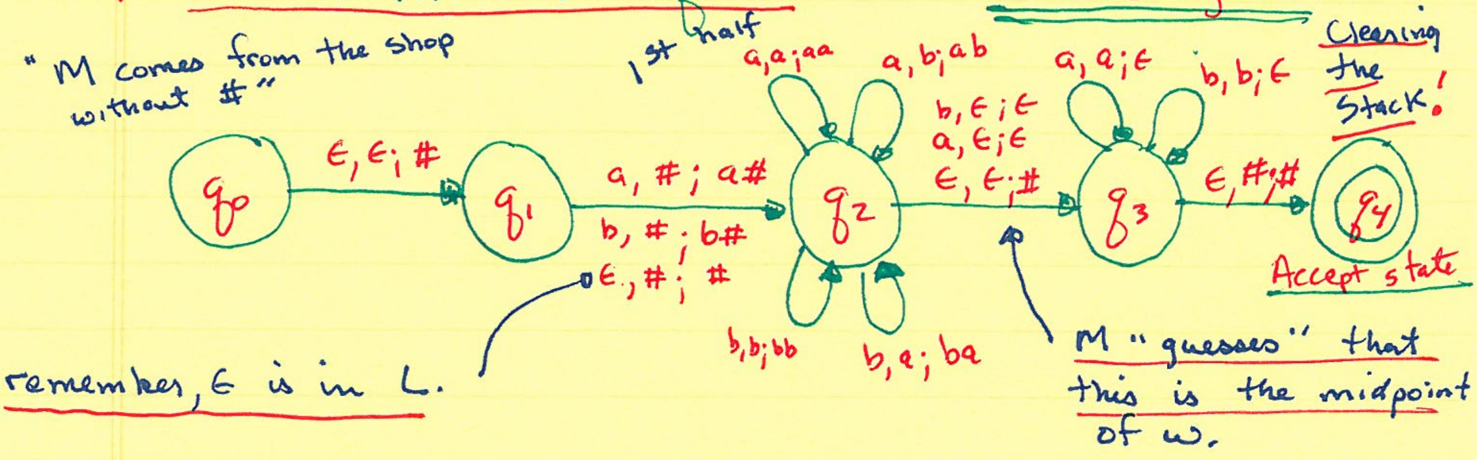
- 3.5) Design a (nondeterministic) pda that accepts the language of palindromes over $\Sigma = \{a, b\}$.

ie. $L = \{ w \in \{a,b\}^* \mid w = w^R \}$

hence, strings in L are $\epsilon, a, b, aa, bb, aba, bab, \ldots \}$

Well, our machine will push symbols in 1st half, and then pop from 2nd half.
Basic Question: How will M know when it is in the middle of w ... It will guess!!!



- Consider w = abba

$(q_0, abbba; \epsilon)$

$\vdash (q_1, abbba; \#)$ Place '#' on stack

$\vdash (q_2, bbba; a\#)$ During 1st half of string – place appropriate symbols on stack.

$\vdash (q_2, bba; ba\#)$ M guesses this 'b' is middle

$\vdash (q_3, ba; ba\#)$ Pop these stack symbols if input matches

$\vdash (q_3, a; a\#)$

$\vdash (q_3, \epsilon; \#) \vdash (q_4, \epsilon; \#)$ String entirely read; q_4 is Accept state cp.10

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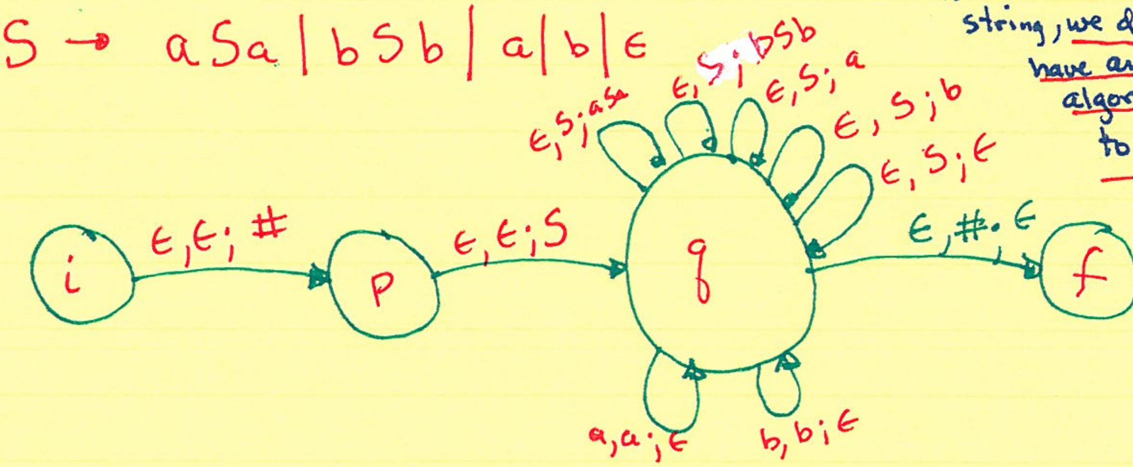
- $L = \{w \in \{a,b\}^* \mid w = w^R\}$ revisited

It is impossible for M to accept this language unless it can guess (and of course, guess correctly!) where the midpoint of a string occurs. Hence, any pda to accept this language must be nondeterministic

As a 2nd approach, we construct a pda directly from a cfg for this language.

- $S \to aSa \mid bSb \mid a \mid b \mid \epsilon$

- Notice, that when presented with a string, we do not have an algorithm yet to process w!



- We consider w= abbba once again.

$(i, abbba; \epsilon) \vdash (p, abbba; \#) \vdash (q, abbba; S\#)$

$\vdash (q, abbba; aSa\#) \vdash (q, bbba; Sa\#) \vdash (q, bbba; bSba\#) \vdash (q, bba; Sba\#)$
$\vdash (q, bba; bba\#) \vdash (q, ba; ba\#) \vdash (q, a; a\#) \vdash (q, \epsilon; \#) \vdash (q, \epsilon, \epsilon)$

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3.6) Which of these languages do you suspect can be recognized by a pda?

a) $\{a^n b c^n \mid n \geq 0\}$

yes. Stack an 'a' for each 'a' in prefix.
Scan past b.
Pop an a for each 'c' in suffix.
If the stack is "empty" then accept.

c) $\{a^n b c^{2n} \mid n \geq 0\}$

yes. Stack an 'a' for each 'a' in prefix.
Scan past 'b'.
Pop an 'a' for each two c's in suffix.
If the stack is "empty" then accept.

d) $\{a^n b^n c^n \mid n \geq 0\}$??? NO ... why not?

e) $\{a^n b^n c^m d^m \mid n, m \geq 0\}$??? YES ... why?

f) $\{a^n b^n c^n d^n \mid n \geq 0\}$???

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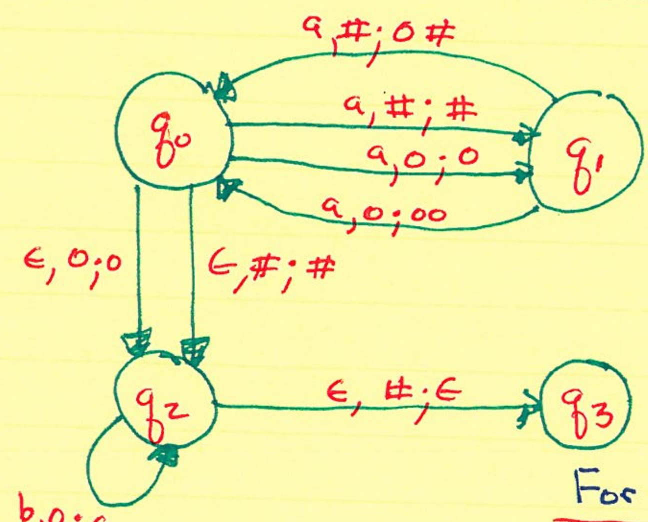
3.6 con't.)

g) $\{a^n b^m c^n d^m \mid n, m \geq 0\}$???

h) $\{a^n b^m c^n d^m \mid 0 \leq n \leq m\}$???

4.1) What language is accepted by empty stack?



- M comes preloaded with #.
- Notice that for each two a's scanned, one '0' is placed on stack.

For each 'b' that is scanned, a single '0' is popped.

The empty string drives M to q_3.

Therefore L(M) = ???

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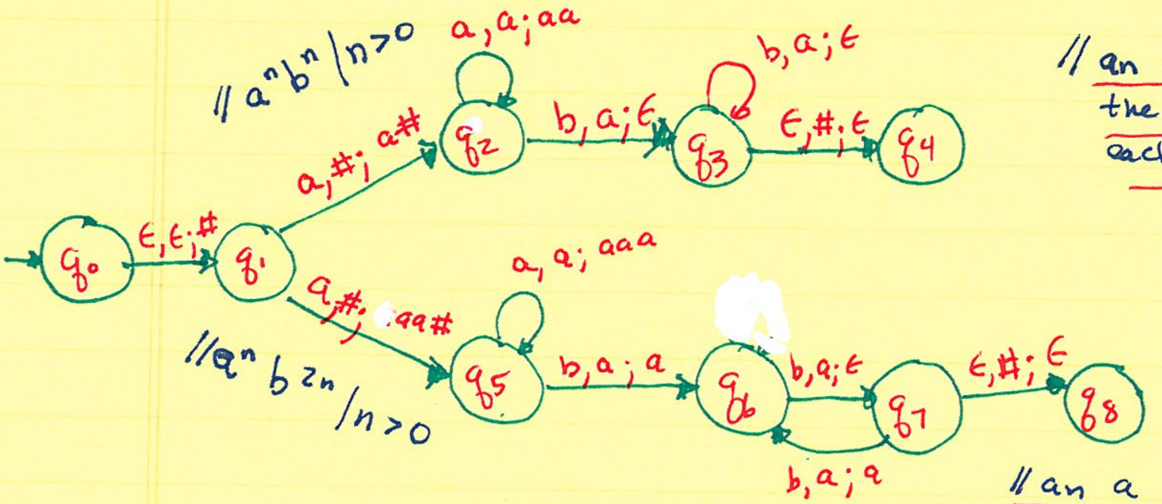
8.2.) Show that the language $L = \{a^n b^n \mid n > 0\} \cup \{a^n b^{2n} \mid n > 0\}$ is accepted by some npda but not by any dpda, i.e. L is a cfl,

but not a dcfl.



// an a is erased from the stack for each b scanned.

// an a is erased from the stack for each two b's scanned.

- There is a bifurcation at q_1
  M heads north if $n_a(w) = n_b(w)$
  and it
  heads south if there are twice as many b's.

It must guess about a portion of w it cannot see.

If M can guess 'n' then it could recognize $\{a^n b^n c^n \mid n > 0\}$ which of course, it cannot.