Homework

5.6.1.1

1 $P(p)A = LU$ $\quad \langle A(b_0 | b_1 \cdots b_{m-1}) = (e_0 | e_1 | \cdots e_{m-1}) \rangle$

$P(p)Ab_j = LUb_j$

$P(p)e_j = LUb_j$

$P(p)e_j = Lz$

$Ub_j = z$.

Suppose we have $B = (b_0 | b_1 \cdots b_{m-1})$

$LUB = P(p)I$

$P(p)LUB = I$

$P(p)LU = A$

$AB = I$

2. for $j = 0, \cdots m-1$ .... m

Compute LU .... m^3

Solve $Lz = P(p)e_j$ .... m^2

Solve $Ub_j = z$ .... m^2

end for

$\text{Cost} = m(m^3 + m^2 + m^2) \approx O(m^4)$.

3. We can compute LU individully outside the Loop. and the cost would be:

$\text{cost} = \cancel{m(m^2+m^2) + m^3}$

$m^3 + m(m^2 + m^2) = m^3 + m(2m^2)$

$\approx O(m^3)$

4. $x = By$. $\quad$ Cost $= m^2$, remain two triangle compute $\sim 2m^2$.

$Lz \sim m^2$. $\quad Ub_j = z \sim m^2$

This is faster than Loop that compute the inverse. LU decomposition for solving a system are more efficient.

5.6.1.2.

1.

$$L \cdot L^{-1} = \left(\begin{array}{c|c} 1 & 0 \\ \hline l_{21} & L_{22} \end{array}\right)\left(\begin{array}{c|c} 1 & 0 \\ \hline -L_{22}^{-1} l_{21} & L_{22}^{-1} \end{array}\right)$$

$$= \left(\begin{array}{c|c} 1 & 0 \\ \hline l_{21} - L_{22} L_{22}^{-1} l_{21} & L_{22} L_{22}^{-1} \end{array}\right)$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & I \end{pmatrix}$$

$$= I.$$

So, $L^{-1} = \left(\begin{array}{c|c} 1 & 0 \\ \hline -L_{22}^{-1} l_{21} & L_{22}^{-1} \end{array}\right)$.

2. $z = -L_{22}^{-1} l_{21}$

$l_{21} = -L_{22} z$

compute the lower triangular for z.

$z = L(-L_{22}, l_{21})$

3. ~~while $n(L_{TL}) < n(L)$~~

$$\cancel{\left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \rightarrow \left(\quad \right)}$$

$\cancel{l_{21} :=}$

$$\cancel{\left(\begin{array}{c|c} L_{TL} & L_{TR} \\ \hline L_{BL} & L_{BR} \end{array}\right) \leftarrow \left(\quad \right)}$$

~~endwhile~~.

Suppose in ith iteration.

$\text{Cost}(z = L(-L_{22}, l_{21})) = (m-i)^2$

$$\sum_{i=0}^{m-1} (m-i)^2$$

$$= \sum_{i=0}^{m-1} (m^2 - 2i + i^2)$$

$$= m^3 - 2m \sum_{i=0}^{m-1} i + \sum_{i=0}^{m-1} i^2$$

$$= m^3 - 2m\left(\tfrac{1}{2}m^2 - \tfrac{1}{2}m\right) + \tfrac{1}{6}(2m^3 + 3m^2 + 1)$$

$$= m^3 - m^3 + m^2 + \tfrac{1}{3}m^3 + \tfrac{1}{2}m^2 + \tfrac{1}{6}$$

$$= \tfrac{1}{3}m^3 + \tfrac{3}{2}m^2 + \tfrac{1}{6}.$$

Homework 5.6.1.3.

1. $P(p) A = LU$

Compute the inversion of $L \Rightarrow L^{-1}$

Solve $UX = L^{-1}$ for X. $\Rightarrow X = U^{-1}L^{-1}$

$$= \cancel{(UL)}^{-1} (LU)^{-1}$$

$$= (P(p)A)^{-1}$$

$$= A^{-1}P(p)$$

$$A^{-1} := XP(p)$$

$$= A^{-1}P(p)P(p) = A^{-1}.$$

2. $UX = L$ for X.

$\Rightarrow Ux_i = e_i$ m times

3. U ~~slove~~ solve $\sim m^2$

m times $\sim m$.

Total cost $= m \cdot m^2 = m^3$.

4. Compute LU $\sim m^2$

Invert L $\sim m^3$

Solve $UX = L$ $\sim m^3$

Compute $A^{-1} := XP(p) \sim k$

Total Cost $\sim O(m^3)$.