Washington State University
EE 521 Analysis of Power System
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Project 5
Optimal Power Flow

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Overview

Many power system applications, such as the power flow, offer only a snapshot of the system operation. Usually, the system planner or operator is interested in the effect that making adjustments to the system parameters will have on the power flow through lines or system losses. Rather than making the adjustments in a random fashion, the system planner will attempt to optimize the adjustments according to some objective costs, reservoir water levels, or system losses, among others. The optimal power flow problem is to formulate the power flow problem to find system voltages and generated powers within the framework of the objective function. In this project, OPF is applied in the IEEE 14-bus system to minimize the cost.

Economic Dispatch

For 14 buses system, the generator costs are:

$$F_1(P_1) = 0.004P_1^2 + 8P_1$$
 (1)

$$F_2(P_2) = 0.0048P_2^2 + 6.4P_1$$
 (2)

Object function:

$$f1 = \frac{\partial \zeta}{\partial \lambda} = g(x, u) = 0 \tag{3}$$

Also,

$$L = f - \lambda(P1 + P2 - 259) \tag{4}$$

For the optimal power flow

$$g = \begin{bmatrix} \Delta P_i \\ \Delta Q_i \end{bmatrix}$$
 (5)

$$x = \begin{bmatrix} \delta_i \\ V_i \end{bmatrix} \tag{6}$$

$$\frac{\partial \zeta}{\partial u} = \frac{\partial f}{\partial u} - \left[\frac{\partial g}{\partial u}\right]^{\mathsf{T}} \lambda = \frac{\partial f}{\partial u} - \left[\frac{\partial g}{\partial u}\right]^{\mathsf{T}} \left[\frac{\partial g}{\partial x}\right]^{\mathsf{T}^{-1}} \left[\frac{\partial f}{\partial x}\right] \tag{7}$$

Result

Solve the economic dispatch for P(total)=259 MW

P1_1 is 50.364 MW P2_1 is 208.636 MW F_1 is 1957.268 f

From base PF, use P(total)=P1+P2 to solve economic dispatch

P1_2 is 57.633 MW P2_2 is 214.694 MW F_2 is 2069.635 f

Solve OPF with u=p2

P1_3 is 55.805 MW P2_3 is 212.036 MW P12_3 is 9.299 MW F_3 is 2031.731 f

Solve OPF with line constraint P12 ≤ 5MW

P1_4 is 55.815 MW P2_4 is 212.026 MW P12_4 is 9.307 MW F_4 is 2031.732 f

Code

clc											
clear all											
% Brah	en Bus	To bus	R	X	Line	Rtio					
%		Charging B									
linedata=[1	2 (0.01938	0.05917	0.0528	1					
	1	5	0.05403	0.22304	0.0492	1					
	2	3	0.04699	0.19797	0.0438	1					
	2	4	0.05811	0.17632	0.0340	1					
	2	5	0.05695	0.17388	0.0346	1					
	3	4	0.06701	0.17103	0.0128	1					
	4	5	0.01335	0.04211	0.0	1					
	4	7	0.0	0.20912	0.0	0.978					
	4	9	0.0	0.55618	0.0	0.969					
	5	6	0.0	0.25202	0.0	0.932					
	6	11	0.09498	0.19890	0.0	1					
	6	12	0.12291	0.25581	0.0	1					
	6	13	0.06615	0.13027	0.0	1					
	7	8	0.0	0.17615	0.0	1					
	7	9	0.0	0.11001	0.0	1					
	9	10	0.03181	0.08450	0.0	1					
	9	14	0.12711	0.27038	0.0	1					
	10	11	0.08205	0.19207	0.0	1					
	12	13	0.22092	0.19988	0.0	1					
	13	14	0.17093	0.34802	0.0	1];					
		=length(line									
	a=linedata(mber of Bus							
	b=linedata((:,2);	% Nu	mber of to B	Buses						
	R=linedata	(:,3);	% Ge	t the resistar	nce						
	X=linedata	(:,4);	% Ge	et the Reacta	nce						
	B_Chargin	g=i*linedata	(:,5)/2;	% Ge	t B/2						
	T=linedata	(:,6);	% G	et the ratio of	of transformer						
	Ti=T*i;										
%						6 '	7	8 9)		
10 11 12 13 14 15											
% Bi Type Fin-V FinAng-Deg PL-MW QL-MVAR PGen QGen-MVAR											
BaseKV DesiredVolts MaxMVAR MinMVAR ShuntG ShuntB Remote											
node =	· [1	3 1	.060	0.0	0.0	0.0	232.4	-16.9	0.0		

1.060	0.0		0.0	0.0	0.0	0;				
	2	2	1.045	-4.98		21.7	12.7	40.0	42.4	0.0
1.045	50.0		-40.0	0.0	0.0	0;				
	3	2	1.010	-12	.72	94.2	19.0	0.0	23.4	0.0
1.010	40.0		0.0	0.0	0.0	0;				
	4	0	1.019	-10	.33	47.8	-3.9	0.0	0.0	0.0
0.0	0.0		0.0	0.0	0.0	0;				
	5	0	1.020	-8	.78	7.6	1.6	0.0	0.0	0.0
0.0	0.0		0.0	0.0	0.0	0;				
	6	2	1.070	-14	.22	11.2	7.5	0.0	12.2	0.0
1.070	24.0		-6.0	0.0	0.0	0;				
	7	0	1.062	-13		0.0	0.0	0.0	0.0	0.0
0.0	0.0		0.0	0.0	0.0	0;				
	8	2	1.090	-13	.36	0.0	0.0	0.0	17.4	0.0
1.090	24.0		-6.0			0;				
	9	0	1.056	-14	.94		16.6	0.0	0.0	0.0
0.0	0.0		0.0	0.0	0.19	0;				
	10	0			.10		5.8	0.0	0.0	0.0
0.0	0.0		0.0			0;				
	11	0		-14.79		3.5	1.8	0.0	0.0	0.0
0.0	0.0		0.0			0;				
	12	0	1.055	-15		6.1	1.6	0.0	0.0	0.0
0.0	0.0		0.0			0;				
	13	0				13.5	5.8	0.0	0.0	0.0
0.0	0.0		0.0			0;				
	14	0				14.9	5.0	0.0	0.0	0.0
0.0	0.0		0.0	0.0	0.0	0];				

numberbus=max(node(:,1));

% Get the number of the node

V=node(:,10);

V(~V)=1;

% The initial voltage from 1

Vangle=zeros(numberbus,1); % The initial angle of voltage from 0

PG0=node(:,7)/100;

P0=(node(:,7)-node(:,5))/100; Q0=(node(:,8)-node(:,6))/100; C=node(:,13)+i*node(:,14); % Get P of each bus

% Get Q of each bus

% Get the Shunt

PVbus=node(:,2)==2; % Define 2 is PV bus PQbus=node(:,2)==0; % Define 0 is PQ bus

numberPVbus=sum(PVbus); % Define The number of PV bus numberPQbus=sum(PQbus); % Define the number of PQ bus PQ=find(node(:,2)==0|node(:,2)==1); % Find the order of PQ from the form

```
Y=(1./Z);
                           % Define the resistance
                               =====off Diagonal for Y bus==
         Ybus=zeros(14,14);
                                  % Create a new empty Y matrix bus
         for k=1:numberline
          Ybus(linedata(k,1), linedata(k,2)) = Ybus(linedata(k,1), linedata(k,2)) - Y(k)/T(k);\\
          Ybus(linedata(k,2),linedata(k,1))=Ybus(linedata(k,1),linedata(k,2));
         end
                                   =Diagonal for Y bus=
        for m=1:numberbus
             for n=1:numberline
                 if m == linedata(n, 1);
                      Ybus(m,m) = Ybus(m,m) + Y(n)/(T(n)^2) + B\_Charging(n);
                 elseif m == linedata(n,2);
                      Ybus(m,m)=Ybus(m,m)+Y(n)+B_Charging(n);
                 end
            end
             Ybus(m, m) = Ybus(m, m) + C(m);
        end
    Yabs=abs(Ybus);
                          % Get the real of Y bus
    Yangle=angle(Ybus);
                              % Get the angle of Y bus
    Gi=real(Ybus);
                                 % Get the real of Y bus
    Bi=imag(Ybus);
                               % Get the imag of Y bus
%% 1st part
    PTotal=259;
    PP=[0.008,0,-1;0 0.0096 -1;1 1 0];
    L=[-8;-6.4;PTotal];
    c=inv(PP)*L;
```

% Define the impedance

Z=R+i*X;

```
P1_1=c(1);
  P2 1=c(2);
   F_1 = 0.004 * P1_1^2 + 8 * P1_1 + 0.0048 * P2_1^2 + 6.4 * P2_1;
   %% 2nd part . - Solve economic dispatch
  Pi=zeros(numberbus,1);
    Qi=zeros(numberbus,1);
    for m=1:numberbus
        for n=1:numberbus
            Pi(m) = Pi(m) + Yabs(m,n) *V(m) *V(n) * cos(Vangle(m) - Vangle(n) - Yangle(m,n));
            Qi(m)=Qi(m)+Yabs(m,n)*V(m)*V(n)*sin(Vangle(m)-Vangle(n)-Yangle(m,n));
        end
    end
    dP=P0-Pi;
    dQ=Q0-Qi;
    dm=[dP(2:end);dQ(PQ)]; % Get the first time mismatches
    Tol=1
    Iter=0;
while max(abs(Tol))>1e-2 && (Iter<10)
    %%%%%%%%% Jacobian %%%%%%%%%%%
    for m=1:(numberPQbus+numberPVbus)
        for n=1:(numberPQbus+numberPVbus)
    if m==n
        J11(m,n)=-Qi(m+1)-Bi(m+1,n+1)*V(m+1)^2;
    else
        J11(m,n) = V(m+1) * V(n+1) * Yabs(m+1,n+1) * sin(Vangle(m+1) - Vangle(n+1) - Yangle(m+1,n+1));
    end
        end
    end
```

```
%%%%%%%%%% J12 %%%%%%%%%%%%%%%%%
 for m=2:(numberPQbus+numberPVbus+1)
                    for n=2:numberPQbus+1
                                      k=PQ(n-1);
                                       if k==m
                   J12(m-1,n-1)=Pi(k)/V(k)+Gi(m,k)*V(k);
else
                    J12(m-1,n-1)=V(m)*Yabs(m,k)*cos(Vangle(m)-Vangle(k)-Yangle(m,k));
 end
                    end
 end
 for m=2:numberPQbus+1
                   k=PQ(m-1)
                   for n=2:(numberPQbus+numberPVbus+1)
                   J21(m-1,n-1)=Pi(k)-Gi(k,n)*V(k)^2;
else
                   J21(m-1,n-1) = -1*V(k)*V(n)*Yabs(k,n)*cos(Vangle(k)-Vangle(n)-Yangle(k,n));
end
                    end
 end
 {}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}
 for m=2:numberPQbus+1
                   k1=PQ(m-1)
                    for n=2:numberPQbus+1
                                      k2=PQ(n-1)
                                      if k1 == k2
                   J22(m-1,n-1)=Qi(k1)/V(k1)-Bi(k1,k2)*V(k1);
else
                    J22(m-1,n-1)=V(k1)*Yabs(k1,k2)*sin(Vangle(k1)-Vangle(k2)-Yangle(k1,k2));
 end
                    end
 end
J=[J11 J12; J21 J22];
```

```
X0 \hspace{-0.05cm}=\hspace{-0.05cm} [Vangle(2:\hspace{-0.05cm}end);\hspace{-0.05cm} V(PQ)]; \hspace{0.3cm} \% \hspace{0.3cm} Create \hspace{0.3cm} a \hspace{0.3cm} vector \hspace{0.3cm} that \hspace{0.3cm} made \hspace{0.3cm} of \hspace{0.3cm} Vangle \hspace{0.3cm} and \hspace{0.3cm} V
   X=X0+inv(J)*(dm);
   Vangle0=X(1:numberbus-1); % Get the Vangle after the calculation
   V0=X(numberbus:end);
                                      % Get the V after the calculation
   Vangle=[0;Vangle0];
                                  % Get the Vagnle excepting slack bus
   V=node(:,10);
                                   % Get the V from the node data
   V(~V)=V0;
                                      % Use the V0 to instead of 0
   \%\%\%\%\% Get a new P and Q after the calculation \%\%\%\%\%\%\%\%\%\%\%\%\%
    Pi=zeros(numberbus,1);
    Qi=zeros(numberbus,1);
   for m=1:numberbus
        for n=1:numberbus
              Pi(m) = Pi(m) + Yabs(m,n) *V(m) *V(n) * cos(Vangle(m) - Vangle(n) - Yangle(m,n));
              Qi(m)=Qi(m)+Yabs(m,n)*V(m)*V(n)*sin(Vangle(m)-Vangle(n)-Yangle(m,n));
        end
   end
  dP=P0-Pi;
  dQ=Q0-Qi;
  dm=[dP(2:end);dQ(PQ)]; % Mismatches
  Tol=dm;
   Iter=Iter+1;
PG=Pi+node(:,5)/100;
PTotal2=sum(PG(1:2)) * 100;
PP2 = [0.008, 0, -1; 0 \ 0.0096 \ -1; 1 \ 1 \ 0];
L2 = [-8; -6.4; PTotal2];
P = inv(PP2) * L2;
P1_2 = P(1);
```

end

```
F 2 = 0.004 * (P1 2)^2 + 8 * P1 2 + 0.0048 * (P2 2)^2 + 6.4 * P2 2;
    %% Solve OPF with u=P2
    P1=P1_2;
    P2=P2_2/100;
    P12=40;
    lu= 1e3;
      V=node(:,10);
      V(\sim V)=1;
                                           % The initial voltage from 1
                                        % The initial angle of voltage from 0
      Vangle=zeros(numberbus,1);
     while abs(lu) > 1e-4
              if P2 > 0
               PG0(2) = P2;
              end
              P0=PG0-node(:,5)/100;
              %%%%%%%%%%%%%%%%%
                                        Get
                                                    the
                                                               initial
                                                                             P
                                                                                       and
                                                                                                  Q
Pi=zeros(numberbus,1);
      Qi=zeros(numberbus,1);
      for m=1:numberbus
           for n=1:numberbus
               Pi(m) = Pi(m) + Yabs(m,n) *V(m) *V(n) * cos(Vangle(m) - Vangle(n) - Yangle(m,n));
               Qi(m) = Qi(m) + Yabs(m,n) *V(m) *V(n) *sin(Vangle(m)-Vangle(n)-Yangle(m,n));
          end
      end
      dP=P0-Pi;
      dQ=Q0-Qi;
      dm=[dP(2:end);dQ(PQ)]; % Get the first time mismatches
```

 $P2_2 = P(2);$

```
Tol=1;
         Iter=0;
while max(abs(Tol))>1e-2 && (Iter<10)
   %%%%%%%%% Jacobian %%%%%%%%%%%
   for m=1:(numberPQbus+numberPVbus)
       for n=1:(numberPQbus+numberPVbus)
   if m==n
       J11(m,n)=-Qi(m+1)-Bi(m+1,n+1)*V(m+1)^2;
   else
       J11(m,n)=V(m+1)*V(n+1)*Yabs(m+1,n+1)*sin(Vangle(m+1)-Vangle(n+1)-Yangle(m+1,n+1));
   end
       end
   end
   \frac{0}{0}
   for m=2:(numberPQbus+numberPVbus+1)
       for n=2:numberPQbus+1
           k=PQ(n-1);
           if k==m
       J12(m-1,n-1)=Pi(k)/V(k)+Gi(m,k)*V(k);
   else
       \label{eq:costangle} J12(m-1,n-1) = V(m)*Yabs(m,k)*cos(Vangle(m)-Vangle(k)-Yangle(m,k));
   end
       end
   end
   for m=2:numberPQbus+1
       k=PQ(m-1);
       for n=2:(numberPQbus+numberPVbus+1)
       J21(m-1,n-1)=Pi(k)-Gi(k,n)*V(k)^2;
   else
       J21(m-1,n-1)=-1*V(k)*V(n)*Yabs(k,n)*cos(Vangle(k)-Vangle(n)-Yangle(k,n));
   end
       end
   end
   %%%%%%%%% J22 %%%%%%%%%%%%%%%%
   for m=2:numberPQbus+1
```

```
k1=PQ(m-1);
    for n=2:numberPQbus+1
        k2=PQ(n-1);
        if k1 == k2
    J22(m-1,n-1)=Qi(k1)/V(k1)-Bi(k1,k2)*V(k1);
else
    J22(m-1,n-1)=V(k1)*Yabs(k1,k2)*sin(Vangle(k1)-Vangle(k2)-Yangle(k1,k2));
end
    end
end
J=[J11 J12; J21 J22];
X0=[Vangle(2:end);V(PQ)]; % Create a vector that made of Vangle and V
X=X0+inv(J)*(dm);
Vangle0=X(1:numberbus-1); % Get the Vangle after the calculation
V0=X(numberbus:end);
                         % Get the V after the calculation
Vangle=[0;Vangle0];
                       % Get the Vagnle excepting slack bus
V=node(:,10);
                       % Get the V from the node data
V(\sim V)=V0;
                          % Use the V0 to instead of 0
Pi=zeros(numberbus,1);
 Qi=zeros(numberbus,1);
for m=1:numberbus
    for n=1:numberbus
        Pi(m) = Pi(m) + Yabs(m,n) *V(m) *V(n) * cos(Vangle(m) - Vangle(n) - Yangle(m,n));
        Qi(m) = Qi(m) + Yabs(m,n) *V(m) *V(n) *sin(Vangle(m) - Vangle(n) - Yangle(m,n));
    end
end
dP=P0-Pi;
dQ=Q0-Qi;
```

```
dm=[dP(2:end);dQ(PQ)]; % Mismatches
                         Tol=dm;
                         Iter=Iter+1;
        end
                PG=Pi+node(:,5)/100;
                if P12 == 0
                P12_{max} = 0;
                else
                P12_{max} = P12;
                 end
                dgx = -J;
                dgu = [1, zeros(1, 21)];
                dfu = 0.0096 * P2 * 100 + 6.4;
                dP1x = [];
                for i = 2:numberbus
                 dP1x = [dP1x; Yabs(1,i) * V(1) * V(i) * sin(Vangle(1) - Vangle(i) - Yangle(1,i))];
                end
                for j = PQ'
                dP1x = [dP1x; Yabs(1,j) * V(1) * cos(Vangle(1) - Vangle(j) - Yangle(1,j))];
                end
                 P12\_tem = (Yabs(1,2) * V(1) * V(2) * cos(Vangle(1) - Vangle(2) - Yangle(1,2)) - Yabs(1,2) * V(1)^2 *
cos(Yangle(1,2))) * 100;
                if abs(P12) > P12\_max
                dP12_x = [Yabs(1,2) * V(1) * V(2) * sin(Vangle(1) - Vangle(2) - Yangle(1,2)) zeros(1,21)]';
                else
                dP12_x = 0;
                 end
                 dfx = (0.008 * P1 + 8) * dP1x + 2 * (P12_tem - P12_max) * dP12_x;
                dlu = (dfu - dgu * inv(dgx') * dfx);
```

```
P2 = P2 - dlu/100;
                  lu=dlu;
                      end
                      P1_3=P1;
                      P2 3 = P2 * 100;
                      P12_3 = (Yabs(1,2) * V(1) * V(2) * cos(Vangle(1) - Vangle(2) - Yangle(1,2)) - Yabs(1,2) * V(1)^2 * V(1)^2 * V(1)^3 + V(1)^4 + V
cos(Yangle(1,2))) * 100;
                      F_3 = 0.004 * P1^2 + 8 * P1 + 0.0048 * P2_3^2 + 6.4 * P2_3;
                      %% Solve OPF with line constraint %%
                      P2 = P2/100;
                      P12 = 5;
                      lu= 1e3;
                            V=node(:,10);
                            V(\sim V)=1;
                                                                                                                                                                                       % The initial voltage from 1
                            Vangle=zeros(numberbus,1);
                                                                                                                                                                           % The initial angle of voltage from 0
                            while abs(lu) > 1e-4
                                                           if P2 > 0
                                                                PG0(2) = P2;
                                                           end
                                                           P0=PG0-node(:,5)/100;
                                                           \% \% \% \% \% \% \% \% \% \% \% \% \% \%
                                                                                                                                                                            Get
                                                                                                                                                                                                                              the
                                                                                                                                                                                                                                                                             initial
                                                                                                                                                                                                                                                                                                                                      P
                                                                                                                                                                                                                                                                                                                                                                                 and
                                                                                                                                                                                                                                                                                                                                                                                                                                  Q
Pi=zeros(numberbus,1);
                            Qi=zeros(numberbus,1);
```

P1 = PG(1) * 100;

```
for m=1:numberbus
        for n=1:numberbus
            Pi(m)=Pi(m)+Yabs(m,n)*V(m)*V(n)*cos(Vangle(m)-Vangle(n)-Yangle(m,n));
            Qi(m)=Qi(m)+Yabs(m,n)*V(m)*V(n)*sin(Vangle(m)-Vangle(n)-Yangle(m,n));
        end
    end
    dP=P0-Pi;
   dQ=Q0-Qi;
    dm=[dP(2:end);dQ(PQ)]; % Get the first time mismatches
          Tol=1;
          Iter=0;
while max(abs(Tol))>1e-2 && (Iter<10)
   %%%%%%%%% Jacobian %%%%%%%%%%%
   0\%0\%0\%0\%0\%0\%0\%0\%0\%0 J11 0\%0\%0\%0\%0\%0\%0\%0\%0\%0\%0
    for m=1:(numberPQbus+numberPVbus)
        for n=1:(numberPQbus+numberPVbus)
    if m==n
        J11(m,n)=-Qi(m+1)-Bi(m+1,n+1)*V(m+1)^2;
    else
        J11(m,n)=V(m+1)*V(n+1)*Yabs(m+1,n+1)*sin(Vangle(m+1)-Vangle(n+1)-Yangle(m+1,n+1));
    end
        end
    end
    for m=2:(numberPQbus+numberPVbus+1)
        for n=2:numberPQbus+1
            k=PQ(n-1);
            if k==m
        J12(m-1,n-1)=Pi(k)/V(k)+Gi(m,k)*V(k);
   else
        J12(m-1,n-1)=V(m)*Yabs(m,k)*cos(Vangle(m)-Vangle(k)-Yangle(m,k));
    end
        end
    end
```

 ${}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}$

```
for m=2:numberPQbus+1
                 k=PQ(m-1);
                 for n=2:(numberPQbus+numberPVbus+1)
                                 if n==k
                J21(m-1,n-1)=Pi(k)-Gi(k,n)*V(k)^2;
else
                 J21(m-1,n-1) = -1*V(k)*V(n)*Yabs(k,n)*cos(Vangle(k)-Vangle(n)-Yangle(k,n));
 end
                 end
 end
 {}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}{}^{9}\!\!/_{\!0}
 for m=2:numberPQbus+1
                k1=PQ(m-1);
                for n=2:numberPQbus+1
                                 k2=PQ(n-1);
                                 if k1 == k2
                J22(m-1,n-1)=Qi(k1)/V(k1)-Bi(k1,k2)*V(k1);
else
                J22(m-1,n-1)=V(k1)*Yabs(k1,k2)*sin(Vangle(k1)-Vangle(k2)-Yangle(k1,k2));
 end
                 end
end
J=[J11 J12; J21 J22];
 X0=[Vangle(2:end);V(PQ)]; % Create a vector that made of Vangle and V
 X=X0+inv(J)*(dm);
 Vangle0=X(1:numberbus-1); % Get the Vangle after the calculation
 V0=X(numberbus:end);
                                                                                                  % Get the V after the calculation
 Vangle=[0;Vangle0];
                                                                                        % Get the Vagnle excepting slack bus
 V=node(:,10);
                                                                                           % Get the V from the node data
 V(\sim V)=V0;
                                                                                                   % Use the V0 to instead of 0
 Pi=zeros(numberbus,1);
```

```
Qi=zeros(numberbus,1);
  for m=1:numberbus
       for n=1:numberbus
            Pi(m) = Pi(m) + Yabs(m,n) *V(m) *V(n) * cos(Vangle(m) - Vangle(n) - Yangle(m,n));
            Qi(m) = Qi(m) + Yabs(m,n) *V(m) *V(n) *sin(Vangle(m) - Vangle(n) - Yangle(m,n));
       end
  end
  dP=P0-Pi;
  dQ=Q0-Qi;
  dm=[dP(2:end);dQ(PQ)]; % Mismatches
  Tol=dm;
  Iter=Iter+1;
PG=Pi+node(:,5)/100;
if P12 == 0
P12_{max} = 0;
else
P12_{max} = P12;
end
dgx = -J;
dgu = [1, zeros(1, 21)];
dfu = 0.0096 * P2 * 100 + 6.4;
dP1x = [];
for i = 2:numberbus
dP1x = [dP1x; Yabs(1,i) * V(1) * V(i) * sin(Vangle(1) - Vangle(i) - Yangle(1,i))];
end
for j = PQ'
dP1x = [dP1x; Yabs(1,j) * V(1) * cos(Vangle(1) - Vangle(j) - Yangle(1,j))];
end
```

end

```
P12\_tem = (Yabs(1,2) * V(1) * V(2) * cos(Vangle(1) - Vangle(2) - Yangle(1,2)) - Yabs(1,2) * V(1)^2 *
cos(Yangle(1,2))) * 100;
                       if abs(P12) > P12\_max
                       dP12_x = [Yabs(1,2) * V(1) * V(2) * sin(Vangle(1) - Vangle(2) - Yangle(1,2)) zeros(1,21)]';
                        else
                       dP12_x = 0;
                        end
                       dfx = (0.008 * P1 + 8) * dP1x + 2 * (P12_tem - P12_max) * dP12_x;
                       dlu = (dfu - dgu * inv(dgx') * dfx);
                        P1 = PG(1) * 100;
                       P2 = P2 - dlu/100;
                        lu=dlu;
                             end
                             P1_4=P1;
                             P2 4 = P2 * 100;
                            P12_4 = (Yabs(1,2) * V(1) * V(2) * cos(Vangle(1) - Vangle(2) - Yangle(1,2)) - Yabs(1,2) * V(1)^2 * V
cos(Yangle(1,2))) * 100;
                             F_4 = 0.004 * P1^2 + 8 * P1 + 0.0048 * P2_4^2 + 6.4 * P2_4;
                              fprintf('P1_1 is %.3f MW\n', P1_1);
                              fprintf('P2_1 is %.3f MW\n', P2_1);
                              fprintf('F_1 is %.3f f\n', F_1);
                              fprintf('P1_2 is %.3f MW\n', P1_2);
                              fprintf('P2 2 is %.3f MW\n', P2 2);
                              fprintf('F_2 is \%.3f f\n', F_2);
                              fprintf('P1_3 is %.3f MW\n', P1_3);
                              fprintf('P2 3 is %.3f MW\n', P2 3);
                              fprintf('P12 3 is %.3f MW\n', P12 3);
                              fprintf('F_3 is %.3f f\n', F_3);
```

```
fprintf('P1_4 is %.3f MW\n', P1_4);
fprintf('P2_4 is %.3f MW\n', P2_4);
fprintf('P12_4 is %.3f MW\n', P12_4);
fprintf('F_4 is %.3f f\n', F_4);
```