Mathematical Notes

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Summary

This document provides a structured template for mathematical notes, including definitions, theorems, lemmas, properties, examples, pseudocode, tables, and images. The content is organized to facilitate clear understanding and reference, with a focus on mathematical rigor and aesthetic presentation.

1 Introduction

This section introduces the main topics covered in the notes. You can provide an overview of the subject matter here.

2 Definitions and Properties

Definition 2.1. A group is a set G equipped with a binary operation $\cdot$ that satisfies the following axioms:

i. Closure: For all $a, b \in G$, $a \cdot b \in G$.

ii. Associativity: For all $a, b, c \in G$, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

iii. Identity: There exists an element $e \in G$ such that for all $a \in G$, $e \cdot a = a \cdot e = a$.

iv. Inverse: For each $a \in G$, there exists an element $b \in G$ such that $a \cdot b = b \cdot a = e$.

Property 2.2. Every group has a unique identity element.

3 Theorems and Proofs

Theorem 3.1. Let G be a group. Then the identity element e in G is unique.

Proof. Suppose e and e' are both identity elements in G. Then, for all $a \in G$, we have $e \cdot a = a$ and $a \cdot e' = a$. Consider $e \cdot e'$. Since e is an identity, $e \cdot e' = e'$. Since e' is an identity, $e \cdot e' = e$. Thus, $e = e'$, proving the identity is unique. □

Lemma 3.2. In any group G, the inverse of each element is unique.

Proof. Let $a \in G$ have two inverses b and c. Then $a \cdot b = e$ and $a \cdot c = e$. Multiply both sides of $a \cdot b = e$ by c on the right: $(a \cdot b) \cdot c = e \cdot c = c$. By associativity, $a \cdot (b \cdot c) = c$. Since $a \cdot c = e$, we have $a \cdot (b \cdot c) = e$. Thus, $b \cdot c$ is an inverse of a. Since b is an inverse, $b \cdot c = b$. Multiply both sides by the inverse of b: $c = b$, so the inverse is unique. □

Example 3.3. Consider the group $(\mathbb{Z}, +)$. The identity element is 0, and the inverse of any integer a is $-a$, since $a + (-a) = 0$.

Remark 3.4. The concepts introduced here can be extended to other algebraic structures, such as rings and fields.

4 Pseudocode Example

5 Table Example

6 Image Inclusion Examples

Algorithm 1 Group Element Inverse Check

```
procedure FindInverse(a, G, ·, e)
    for each b ∈ G do
        if a · b = e and b · a = e then
            return b
        end if
    end for
    return None
end procedure
```

Table 1: Properties of Common Groups

Group	Identity Element	Inverse of a
$(\mathbb{Z}, +)$	0	$-a$
$(\mathbb{R}^*, \cdot)$	1	$1/a$
$(S_n, \circ)$	Identity permutation	Inverse permutation

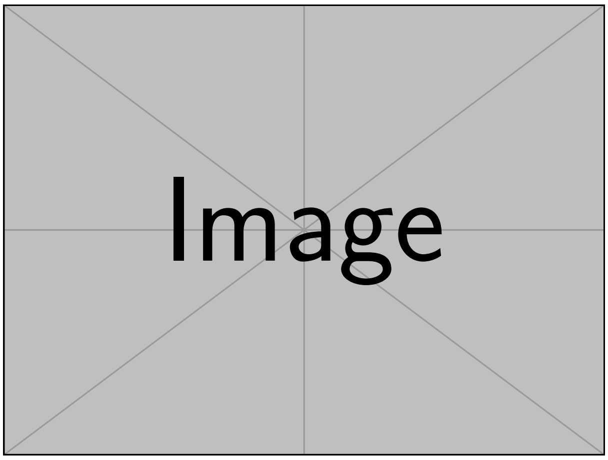

Figure 1: A diagram illustrating a group structure.

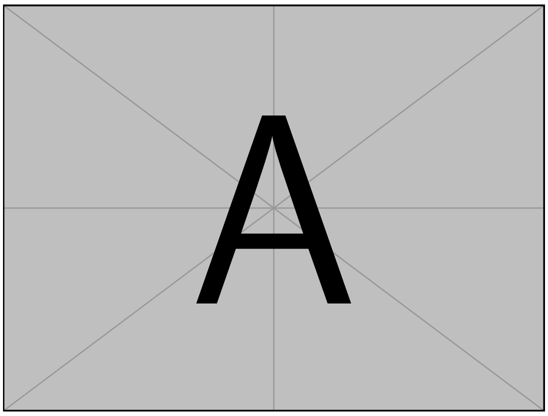

Figure 2: First group diagram.

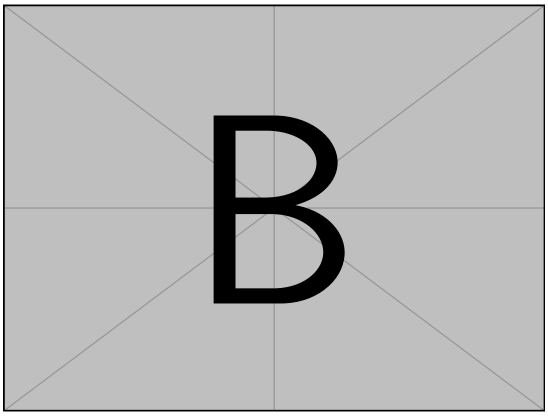

Figure 3: Second group diagram.