A review of methods to measure and calculate train resistances

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Abstract: This paper discusses a well-known tool for calculating train resistance to motion and its suitability for describing operations at high speed. The tool, originally developed by Armstrong and Swift [**1**], also permits the estimation of the contribution to aerodynamic resistance of various features of the architecture of a train. They compare this approach with the results of other formulae for calculating train resistance, as well as published measurements taken during experimental work. It is concluded that Armstrong and Swift's expressions can be considered to provide good estimates for the coefficients to the Davis equation for both high-speed and suburban trains that fit the British loading gauge and have a power car–trailer ratio of 1:3 or less without the need for run-down testing. However, the expressions are not suitable for trains with a predominance of powered axles.

Keywords: aerodynamic drag, ballasted track, Davis equation, run-down testing, Shinkansen, slab track, tractive effort, train à grande vitesse (TGV), train resistance

NOTATION

a	distance between the axles of a bogie (m)
a_1	mass-related empirical constant (N/t)
a_2	mass-related empirical constant (N/t)
A	mass-related coefficient of mechanical resistance (N), equivalent area in the Sauthoff formula (m^2)
b	coefficient in the Sauthoff formula
b_1	mass-related empirical constant for viscous effects (N s/m t, or kg/s t)
b_2	mass-related empirical constant for viscous effects (N s/m, or kg/s)
b_3	Mass-related empirical constant for viscous effects (N s/m kW)
B	viscous mass-related coefficient of mechanical resistance (N s/m, or kg/s)
c_1	aerodynamic empirical constant (N s^2/m^3, or kg/m^2)
c_2	aerodynamic empirical constant (N s^2/m^3, or kg/m^2)
c_3	aerodynamic empirical constant (N s^2/m^3, or kg/m^2)
c_4	aerodynamic empirical constant (N s^2/m^2, or kg/m)
c_5	aerodynamic empirical constant (N s^2/m^2, or kg/m)
C	coefficient of aerodynamic resistance (N s^2/m^2, or kg/m)
g	acceleration due to gravity, taken as 10 m/s^2 (m/s^2)
k	dimensionless parameter
k_1	shape parameter (N/m^2)
k_2	roughness parameter (N/m^2)
L	train length (m) and locomotive weight (t)
m	train mass (kg), mass per axle (t) and numerical constant in the Strahl formula
M	train mass (t)
n	number of axles, number of coaches
N	number of raised pantographs
p	partial perimeter of the rolling stock (m)
P	total weight of the electric multiple unit (t)
Q	train mass in the Sauthoff formula (t)
r	specific resistance force (m/s^2)
r_c	specific curving resistance (kN/t)
R	resistance force (N)
R_c	curve radius in the horizontal plane (m)
S	front surface cross-sectional area (m^2)
V	velocity (m/s)
w_k	resistance force (kN/t)
X	gradient in form 1 in X
α	angle measured from the horizontal (rad)
λ	dimensionless parameter
μ_r	radial coefficient of friction

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1 INTRODUCTION

It is possible to calculate the resistance to motion of trains on the basis of the fundamental laws of physics as applicable to rolling friction, sliding friction and aerodynamics. Sachs [**2**, pp. 9–34] offers a highly detailed description of this scientific approach, much of which is still valid today. The theory, together with up-to-date experimental results and the data obtained from computational fluid dynamics (CFD), can be used in the optimization of the design of new trains and their components. However, this methodology is not very suitable for the purposes of train performance calculations and operational planning. The approaches are complex, require knowledge of very many parameters and do not necessarily lead to useable train resistance data.

Throughout the history of the rail mode of transport, train resistance has therefore been measured by undertaking run-down tests. Dupuit [**3**] was among the first to undertake experiments and concluded that the rolling friction of a wheel was inversely proportional to the radius of the wheel. In run-down tests, the resistance to motion is estimated either by the measurement of the deceleration versus time while coasting in the open or by the measurement of the tractive effort (force) necessary to maintain a constant velocity at various speeds to cover the working speed range. Calm weather conditions are required to do this and experiments of this nature are generally costly. In consequence, national railways developed empirical equations that could be used to estimate the resistance of a generic type of train, e.g. intercity, suburban, freight, etc.

In Britain and elsewhere, run-down testing on an operational railway is probably no longer an option for a rolling stock manufacturer, a leasing company or a train operating company since it takes up valuable train paths and because of the need to allocate rolling stock and personnel to non-revenue-earning duties. Recently, rolling stock manufacturers have constructed test tracks, such as the Siemens facility on the former Wildenrath airbase in Germany, with the objective of testing and improving the reliability of their products. Train resistance can be measured during such testing but, owing to the geometric constraints of the test tracks, the full operational speed range of high-speed trains, $V \geqslant 200\,\text{km/h}$, can seldom be explored. Opportunities for the experimental identification of the resistance to motion are occasionally available during the commissioning phase of a newly constructed high-speed line. Alternatively, the resistance of a new train can be estimated by the application of a sufficiently accurate empirical calculation tool, based on tests undertaken over the past 100 years.

In the present paper, which concentrates mainly on the application of resistance formulae to high-speed trains, a discussion is presented of an established tool for calculating train resistance that also permits the estimation of the contributions to aerodynamic resistance of various features of the architecture of a train. This approach is compared with the results of other equations for calculating train resistance. The Japanese method for calculating train resistance and the effects of different track forms are also described and discussed.

It must be appreciated that the equations used by the different railway undertakings to calculate the running resistances of trains are generally in the form of a quadratic function, of the structure given in equation (1). However, they are inconsistent with one another in terms of both the units used for the input parameters and the resultant resistance force calculated from these equations. Some results are metric, but not according to the SI system. For example, speeds may be input in km/h or m/s and the resistance force may be expressed in N, daN, daN/t, kN and, inaccurately, kg/t or even t. For consistency, all of the resistance formulae presented are adjusted to SI units. However, the units used in the formulae are always stated in their original form. In all graphical presentations, units are standardized to present the resistance force in kN and the speed in m/s. The latter is an SI unit, and m/s = (km/h)/3.6.

2 CALCULATION OF TRAIN RESISTANCES

2.1 Davis equation

The train resistance force, R, measured by run-down tests, is approximated by a quadratic function that is variously known as the von Borries Formel, the Leitzmann Formel, the fonction de Barbier and, in the Anglo-Saxon world, the Davis equation:

$$R = A + BV + CV^2 \qquad (1)$$

where V is the velocity (m/s) and the coefficients A (N), B (N s/m) and C (N s^2/m^2) are obtained by fitting the coefficients of the Davis equation to the curve obtained by run-down tests. Frank [**4**] and later the UIC simplified the formula by combining the elements $A + BV$ into a single constant that depends on the type of train but is independent of its speed. However, this approach is no longer used.

In addition to the three frictional forces occasioned by the mass, viscous component of mass and aerodynamic characteristics, the traction equipment of trains also has to overcome the resistance to acceleration, gradient force and curving resistance. Acceleration and curving are not described in detail in this paper since the former is simply a function of the masses with the necessary allowance for the rotating components (also known as rotary allowance), while the latter is highly dependent on wheel and rail profiles, track cant and the geometry of the vehicle concerned.

Coefficients A and B of the Davis equation include the mechanical resistances and are thus mass related. Therefore, at lower speeds, say $\leqslant 30$ m/s (≈ 100 km/h), the resistance force is principally dependent on the train mass, one of the reasons for reducing the weight of metro rolling stock and suburban trains. At higher speeds, the CV^2 term becomes dominant since it can be said to relate to the aerodynamic resistance and is the reason for the high installed power of high-speed trains, e.g. 13.2 MW for a Eurostar set. The values of the Davis equation coefficients in equation (1) are usually stated for open air conditions and require modification for tunnel environments in which the resistance force coefficient, (C), is greater than in the open. Sachs [**2**, p. 33, Fig. 1.30] presents the results of a series of measurements of train resistance in different Swiss tunnels. Other methods for calculating train resistance will be compared and contrasted in more detail later on in this paper.

2.2 Armstrong and Swift coefficients

Armstrong and Swift [**1**] created empirical relationships to calculate the coefficients A, B and C of the Davis equation for the electric multiple unit (EMU) rolling stock in service at the time with the former British Rail. All of the units in service at the time had two bogies per vehicle and were close coupled with enclosed gangways. They suggest that the coefficients of the equation may be found using the following constants:

$$A = a_1 \times (\text{total mass of trailer cars}) + a_2 \times (\text{total mass of power cars}) \tag{2a}$$

$$B = b_1 \times (\text{train mass}) + b_2 \times \text{NT} + b_3\text{NP} \times (\text{total power}) \tag{3a}$$

$$\begin{aligned} C = \{&[c_1 \times (\text{head/tail drag coefficient}) \times (\text{cross-sectional area})] \\ &+ [c_2 \times (\text{perimeter})\,(\text{length})] \\ &+ [c_3 \times (\text{perimeter})\,(\text{intervehicle gap}) \times (\text{NT} + \text{NP} - 1)] \\ &+ [c_4 \times (\text{bogie drag coefficient}) \times (\text{number of bogies})] \\ &+ [c_5 \times (\text{number of pantographs})]\} \end{aligned} \tag{4a}$$

where NT is the number of trailer cars and NP is the number of power cars.

The existing literature does not state clearly the dimensions of the different parameters of the formula, but D. S. Armstrong (1998, personal communication) has confirmed that, in equations (2a), (3a) and (4a), the masses are expressed in t, the power in kW, lengths in m, areas in m^2 and the coefficients A in N, B in N s/m and C in N s^2/m^2. The units of the empirical constants a_1 to c_5 are chosen to result in the correct units for A, B and C, e.g. c_3 is in N s^2/m^3. Armstrong and Swift provide values for a_1 to c_5 that lead to the following expressions:

$$A = 6.4 \times (\text{total mass of trailer cars}) + 8.0 \times (\text{total mass of power cars}) \tag{2b}$$

$$B = 0.18 \times (\text{train mass}) + 1\text{NT} + 0.005\text{NP} \times (\text{total power}) \tag{3b}$$

$$\begin{aligned} C = \{&[0.6125 \times (\text{head/tail drag coefficient}) \times (\text{cross-sectional area})] \\ &+ [0.00197 \times (\text{perimeter})\,(\text{length})] \\ &+ [0.0021 \times (\text{perimeter})\,(\text{intervehicle gap}) \times (\text{NT} + \text{NP} - 1)] \\ &+ [0.2061 \times (\text{bogie drag coefficient}) \times (\text{number of bogies})] \\ &+ [0.2566 \times (\text{number of pantographs})]\} \end{aligned} \tag{4b}$$

Validation of equations (2b), (3b) and (4b) will be attempted in Section 4, using data obtained by SNCF as a result of run-down tests and reported by M. de la Broise (1997, personal communication), for the Class 373 Eurostar train.

2.3 Other terms

The gradient force is also mass-related and can be added as an equivalent linear force $F_g(\text{N}) = mg \sin \alpha$ or, simplified for small α:

$$\text{Linear force (kN)} = \frac{Mg}{X} \tag{5}$$

where

m = mass of the train (kg)
M = mass of the train (t)
g = acceleration due to gravity (m/s^2)
α = angle of the gradient, measured from the horizontal (rad)
X = gradient in the form 1 in X

As mentioned before, the effects of resistance due to track curvature are ignored in the present paper since, except for very tight curves of less than about 250 m radius, the effects are small. However, for the interested reader, two formulae for calculating the resistance due to curvature are presented below in equations (6a) and (6b), the first provided by Profillidis [**5**]:

$$r_c = 0.01\left(\frac{k}{R_c}\right) \tag{6a}$$

where

r_c = specific resistance force (kN/t), assuming that the acceleration due to gravity, g, is $10\,\mathrm{m/s^2}$
k = dimensionless parameter, depending on the design of the train, and varies from 500 to 1200, with 800 as the average
R_c = curve radius in a horizontal plane (m)

Sachs [**2**, pp. 44–52] offers a very extensive and detailed analysis of a range of formulae to calculate the resistance to motion in curves. The expression given in (6b) is valid for standard gauge with a gauge play of 0.01 m and $\mu_r = 0.25$ and is accurate to 7 per cent when compared with tabulated values:

$$w_k = (1.6a + 1.62)/R_c \tag{6b}$$

where

w_k = resistance force (kN/t), assuming that the acceleration due to gravity, g, is $10\,\mathrm{m/s^2}$
a = distance between the axles of a bogie or of a vehicle with fixed wheelsets
R_c = curve radius in a horizontal plane (m)

2.4 Summary

For the resistance to motion of a high-speed train, aerodynamic drag plays a dominant role. The effect of vehicle mass is relatively small and designers should pay great attention to reducing the drag contributions from the roof, sides and the nose of trains as well as the equipment mounted below the floor. However, when including acceleration and deceleration, the tractive effort is determined by the running resistance and the product of the effective train mass and acceleration (Newton's first law). The latter become dominant for commuter trains and less energy is consumed with lighter vehicles, even if they use regenerative braking. Regenerative braking using the traction motors will only ever allow recovery of a limited proportion of the energy expended in accelerating the train.

3 OTHER APPROACHES TO THE CALCULATION OF TRAIN RESISTANCE

Profillidis [**5**] and, much earlier, Sachs [**2**] have described the approaches of various national railway undertakings to the calculation of train resistance. Many of the equations given below are empirically or semi-empirically modified versions of the Davis equation and include coefficients with values for a particular type of rolling stock.

The first term, A, of the Davis equation (1) is purely mass dependent, the second term, BV, can be viewed as a viscous component partially dependent on the mass-related rolling resistances of the vehicles under consideration and CV^2 represents the aerodynamic resistances.

3.1 France

French National Railways, SNCF, evaluate the parameters A, BV and CV^2 (kN) as functions of the rolling stock characteristics, using the following expressions:

$$A = 0.00001\left[\lambda M\left(\sqrt{\frac{10\,000}{m}}\right)\right] \tag{7}$$

where M is the total train mass (kg), m is the mass per axle (kg) and λ is a dimensionless parameter with values depending on the rolling stock type, e.g. for SNCF vehicles $0.9 < \lambda < 1.5$, the lower value being applicable to modern rolling stock, the higher value to non-homogeneous freight trains;

$$BV = (3.6 \times 10^{-7})MV \tag{8}$$

for good quality track and modern rolling stock on bogies with roller bearings and where velocity V is measured in m/s;

$$CV^2 = 0.1296[k_1 S(V)^2 + k_2 pL(V)^2] \tag{9}$$

where the first term represents the aerodynamic resistance existing at the front and rear of the train and the second term relates to the aerodynamic resistance generated along the surface, pL ($\mathrm{m^2}$), p is the partial perimeter (m) of the rolling stock down to rail level, with common values around 10 m, L is the train length (m), k_1 ($\mathrm{N/m^2}$) is a parameter depending on the shape of the train, front and rear, and can vary from 20×10^{-4} for conventional rolling stock to 9×10^{-4} for TGV stock, S is the front surface cross-sectional area ($\mathrm{m^2}$), commonly around $10\,\mathrm{m^2}$ and k_2 ($\mathrm{N/m^2}$) is a parameter depending on the condition of the surface, pL, and can vary from 30×10^{-6} for conventional rolling stock to 20×10^{-6} for TGV stock.

Profillidis noted that, for loco-hauled rolling stock, passenger or freight, the various resistance formulae give a large spread, but can be simplified by merging the BV term of equation (1) with the CV^2 term. This is an acceptable simplification for modern passenger rolling stock and a speed range of 0–55 m/s, although it deviates from the approaches used by Frank [4] (see the beginning of Section 2 of this paper). The common practice for hauled stock is to give the resistance per unit weight of rolling stock, also called the specific resistance, r. SNCF have derived the following formulae (where r is in N/kg and V is in m/s):

For passenger rail vehicles on bogies:

$$r = 0.015 + 0.000\,036V^2 \tag{10}$$

For UIC-type vehicles:

$$r = 0.0125 + \left(\frac{0.1296V^2}{6300}\right) \quad \text{or} \quad 0.0125 + 0.000\,021V^2 \tag{11}$$

For two- and three-axle passenger vehicles and express freight train vehicles:

$$r = 0.015 + \left(\frac{0.1296V^2}{2000}\right) \quad \text{or} \quad 0.015 + 0.000\,054V^2 \tag{12}$$

For freight vehicles with a load of 10 t per axle:

$$r = 0.015 + 0.000\,081V^2 \tag{13}$$

For the 18 t per axle case:

$$r = 0.012 + 0.000\,0324V^2 \tag{14}$$

The running resistance, R (kN), of diesel or electric locomotives is again given by an empirical relationship:

$$R = 0.0065L + 0.13n + 0.0036V + 0.003\,888V^2 \tag{15}$$

where

L = locomotive weight (t)
n = number of axles
V = speed (m/s)

Electric multiple units with traction motors on several vehicles are commonly used in high-speed trains and suburban commuter services. The total running resistance, R (kN), for an SNCF suburban EMU is given by

$$R = \left[\left(0.013\sqrt{\frac{10}{m}} + 0.000\,36V\right)P + 0.1296CV^2\right] \tag{16}$$

where

$$C = 0.0035S + 0.0041\,\frac{pL}{100} + 0.002N \tag{17}$$

and

P = total weight of the EMU (t)
m = weight per axle (t)
V = speed (m/s)
S, p, L = same as in the explanation of equation (9)
N = number of raised pantographs

For the French TGV, the total running resistance, R (kN), at speeds V (m/s) is given by

$$R = 25 + 1.188V + 0.070\,372\,8V^2 \tag{18}$$

3.2 Germany

Sachs [2, pp. 15–41] has described German railway practice in which the Strahl formula is used for freight trains and mixed passenger trains, and the Sauthoff formula is used principally for intercity passenger trains but may also be used for freight wagons.

The Strahl formula gives the specific resistance, r (N/kg) for speeds V (m/s) as follows:

$$r = 0.01\left[(1.5\text{–}2.5) + \left(0.007 + \frac{1}{m}\right)\left(\frac{3.6V}{10}\right)^2\right] \tag{19}$$

where

m = 40 for four-axle passenger coaches and similar freight wagons
m = 30 for two- and three-axle passenger carriages
m = 25 for loaded wagons in express parcel trains
m = 10 for empty freight wagons

The Sauthoff formula gives the specific resistance, r (N/kg), for speeds V (m/s) for a train of n coaches and a mass Q (t) as follows:

$$r = \left[1.9 + 3.6bV + 0.0048\,\frac{1}{Q}\,(n + 2.7)A(3.6V + 15)^2\right] \tag{20}$$

where the value of the equivalent area, A, should be taken as 1.45 m^2 for fast trains on the new high-speed lines, 1.55 m^2 for fast trains on other lines and 1.15 m^2 for two- and three-axle passenger coaches, and coefficient b takes the following values: 0.0025 for four-axle vehicles, 0.004 for three-axle vehicles and 0.007 for two-axle vehicles. It should be noted that the integer 15 appearing in equation (20) is the assumed head wind speed (km/h). Sachs used the term w for the resistance force in equations (19) and (20). For consistency, the authors have substituted the term r.

3.3 Japan

T. Maeda (1998, personal communication) observed that, in Japan, the resistance, R (kN), of a train is commonly expressed as

$$R = (A' + B'V)W + C'V^2 \tag{21}$$

where W is the train mass (t). The resistance of the train is estimated by undertaking a run-down test, and the data obtained is arranged in the classic Davis equation formula [equation (1)]. The coefficients A', B' and C' are determined by the least-squares method. However, T. Maeda (1998, personal communication) also pointed out that, because of the use of the least-squares method, the term CV^2 does not necessarily mean an aerodynamic drag, and nor does $A + BV$ necessarily mean a mechanical resistance. The formula should therefore be rearranged physically. A detailed description of a method for evaluating the accuracy of the estimate of aerodynamic resistance was published by Maeda *et al.* [**6**]. Their methodology, reproduced from the paper mentioned, is given in Fig. 1.

It was reported that estimation of aerodynamic resistance according to the procedure shown in Fig. 1 had shown that identical results were obtained for measurement of aerodynamic resistance due to the pressure rise in tunnels and by run-down testing in the open. T. Maeda (1998, personal communication) gave the following formulae for calculating the resistance, R

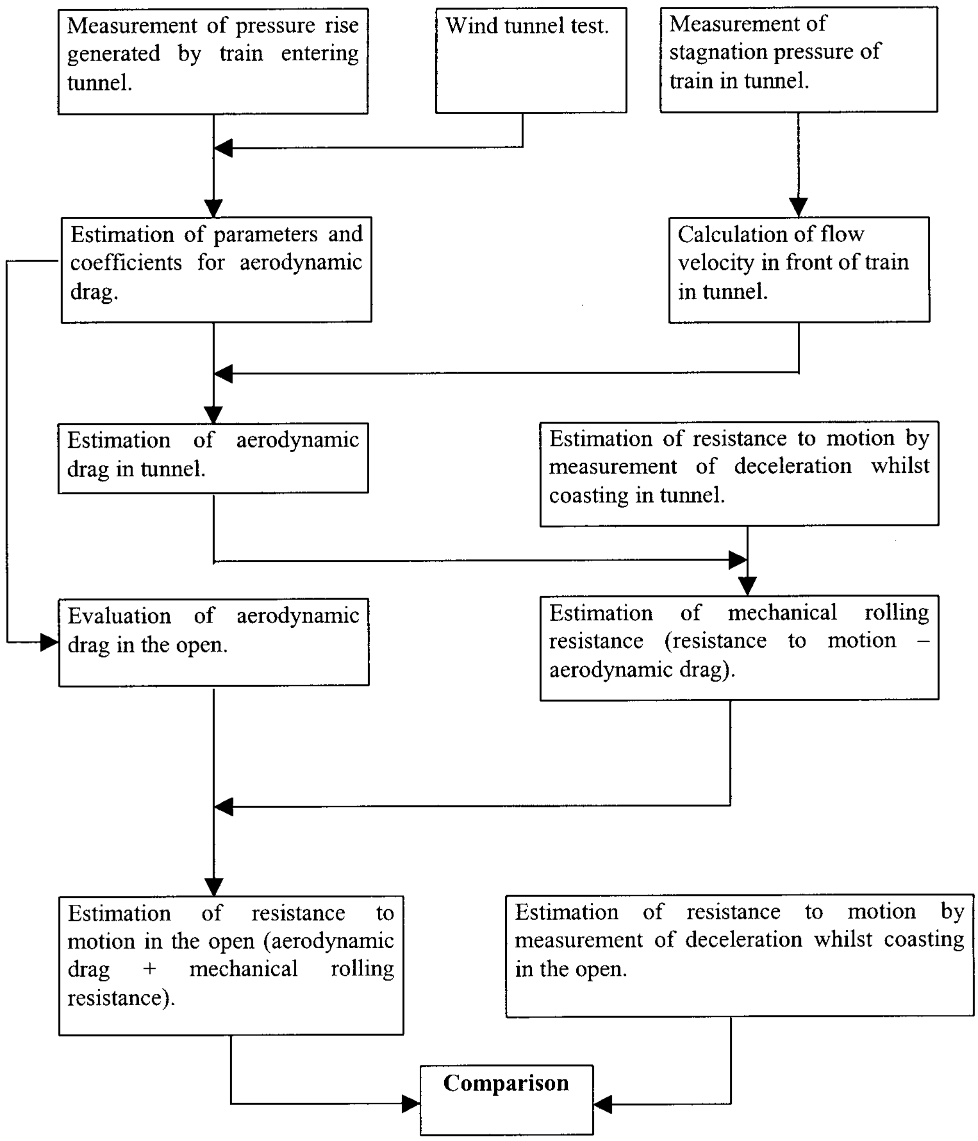

Fig. 1 Method for evaluating the accuracy of the estimate of aerodynamic drag

(kN), of various Shinkansen trains of weight W (t) and at speed V (m/s) as follows:

Series 0, length = 400 m, weight = 869 t

$$R = 10.23 + 0.486V + 0.015\,940\,8V^2 \quad (22)$$

Series 100, length = 400 m, weight = 886 t, on slab track

$$R = 11.06 + 0.109\,44V + 0.015\,616\,8V^2 \quad (23)$$

on ballasted track

$$R = 11.06 + 0.2232V + 0.015\,616\,8V^2 \quad (24)$$

Series 200, length = 300 m, weight = 712 t

$$R = 8.202 + 0.106\,56V + 0.011\,923\,2\,V^2 \quad (25)$$

By now, the reader will have gathered that all railway undertakings base their calculations of train resistance to motion on run-down tests for given types of rolling stock. The Davis equation [equation (1)] is the basis for all such estimations.

4 ANALYSIS OF RESISTANCE EQUATIONS AND THE IMPORTANCE OF FACTORS INFLUENCING AERODYNAMIC RESISTANCE

4.1 Comparison of Armstrong and Swift with measured data

For the aerodynamic resistance of high-speed trains, the CV^2 term swamps the mechanical components of resistance, as shown in Fig. 2 for a Class 373 Eurostar. The figure is derived using coefficients derived from a regression analysis of run-down test results at speeds of up to 300 km/h, undertaken by SNCF, that were supplied by M. de la Broise (1997, personal communication).

The resistance calculated from the Davis equation (1), using de la Broise's coefficients for a Class 373, can be compared with that computed using the same equation but with coefficients A, B and C calculated from Armstrong and Swift's equations (2b), (3b) and (4b) in Fig. 3.

Figure 3 shows that, for the Eurostar train, the mechanical and aerodynamic resistance coefficients of de la Broise produce a value of resistance some 20 per cent smaller than that calculated using formula (1) and equations (2b), (3b) and (4b). Why should this be? Equation (4b) assumes a bogie drag coefficient input value of 0.6 which is typical of British suburban EMUs. If the actual bogie drag coefficient of a Class 373 was less than 0.6, this could account for the difference. Unfortunately, in response to technical queries, Eurostar (UK) Limited (1997, personal communication) stated that the bogie drag coefficient had never been measured for the Class 373. Setting the bogie drag coefficient to zero produced the results shown in Fig. 4.

Figure 4 demonstrates that Armstrong and Swift's equations for calculating the coefficients A, B and C to input into equation (1), with the bogie drag coefficient set to zero, produce a relatively small overestimate of the resistance compared with that calculated from de la Broise's coefficients across the entire speed range. The overestimate of the calculated train resistance is less than 10 per cent greater than that calculated using de la Broise's coefficients at a speed of 300 km/h. Since the contribution of the bogies to aerodynamic drag is

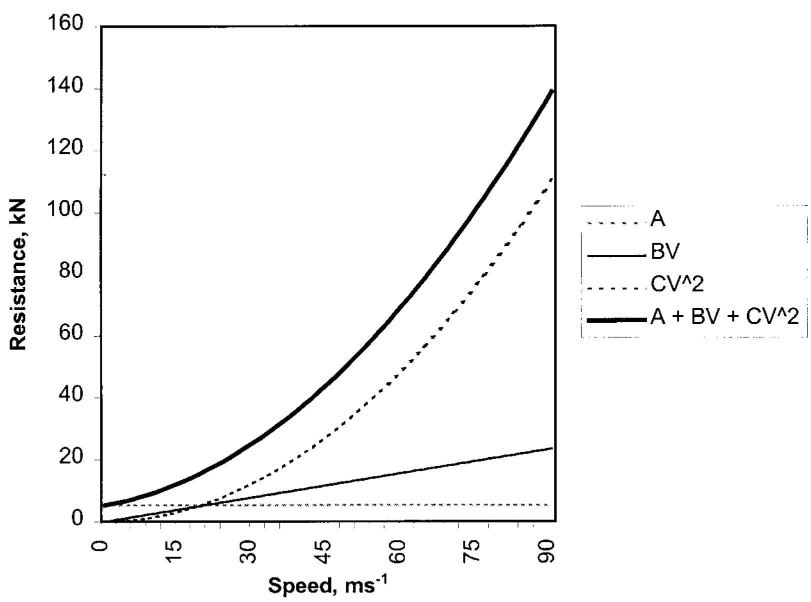

Fig. 2 Variation in the aerodynamic and mechanical components of resistance with speed

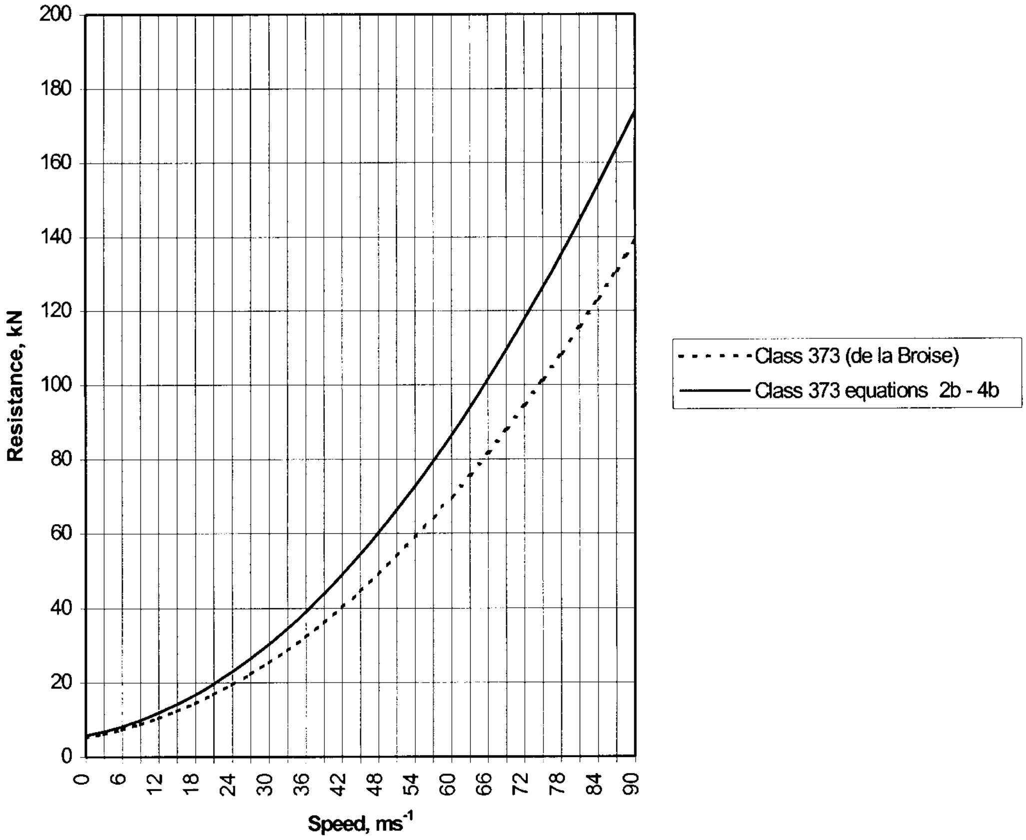

Fig. 3 Comparison of the train resistance calculated by de la Broise's coefficients and that calculated by the unmodified Armstrong and Swift formulae

included in de la Broise's value for coefficient C, Armstrong and Swift's equation (4b) produces a conservative result, and it is suggested that, when using the latter to estimate the resistance of a high-speed train, in the absence of run-down test data, the bogie drag coefficient be omitted from the calculation unless its value is known.

4.2 Parameter sensitivity

Using Armstrong and Swift's equations (2b), (3b) and (4b) to calculate the coefficients A, B and C of the Davis equation, the effects of varying the bogie drag coefficient, the intervehicle gap and the cross-sectional area and perimeter of a train are now investigated. A Class 373 is used as the example, over a speed range from 0 to 300 km/h.

Comparing the curves in Figs 3 and 4, the contribution of bogie drag to the aerodynamic component of resistance, the C coefficient, is significant. Figure 5 shows the effect of varying the bogie drag coefficient between the values of 0.1 and 0.6.

D. S. Armstrong (1998, personal communication) commented that the value of the bogie drag coefficient is strongly affected by the bogie shape, whether it is a motor or trailer bogie, the frame type, the headstock and the type of braking system used. A high-speed train utilizing a large number of ventilated discs per axle, such as the 16 Eurostar trailer vehicles, would be expected to have a relatively large bogie drag coefficient. Reducing the bogie drag coefficient from 0.6 to 0.1 reduces train resistance by some 10 per cent at 300 km/h. This is a worthwhile theoretical reduction which might in practice be achieved by shrouding the bogies and by aerodynamic design of the vehicle underside. However, it might not be attainable because some of the airflow around the train would have to be directed to the brake discs to cool these critical components. Alternatively, if technological advances led to conventional discs being replaced by a system that did not rely on friction and instantaneous heat dissipation (it is possible to use braking energy elsewhere on an electrified network or to store it for distribution over time) to absorb kinetic energy and slow the train, while still satisfying safety regulations, a very low value of bogie drag coefficient could be attainable.

Maeda *et al.* [**6**] have commented that, for Shinkansen trains, shrouding the underfloor of the vehicles will lead to an increase in their cross-sectional area. This will

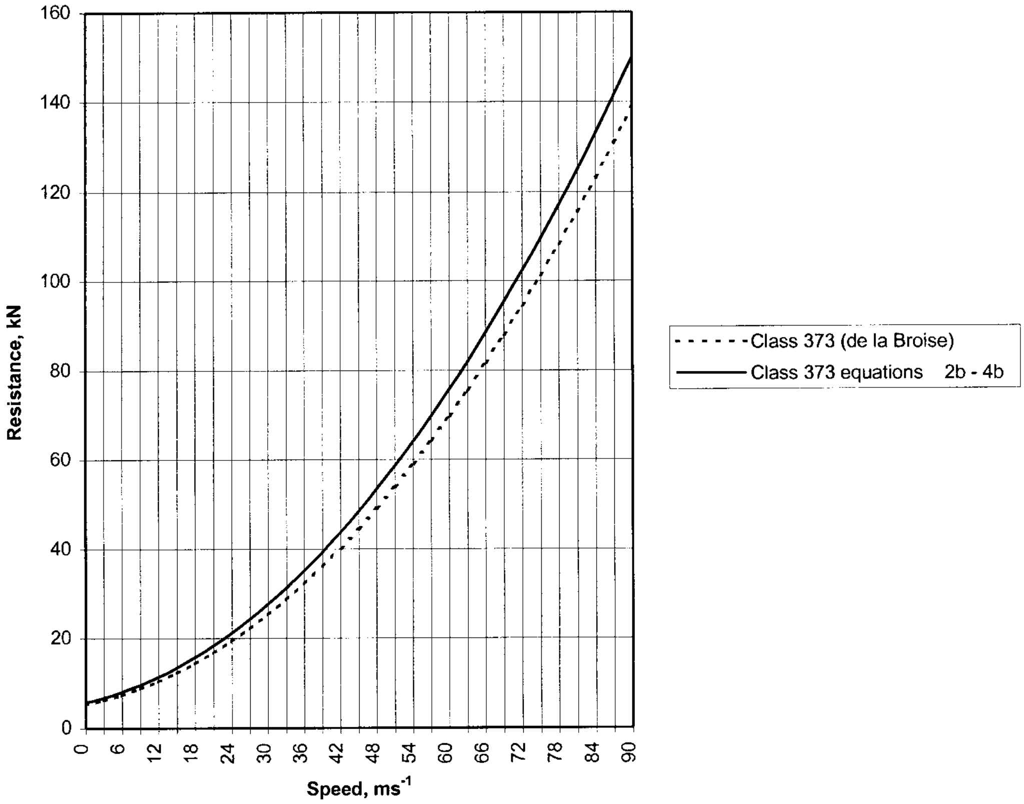

Fig. 4 Comparison of the train resistance calculated by de la Broise's coefficients and that calculated by the Armstrong and Swift formulae with a bogie drag coefficient of 0

increase the aerodynamic drag, possibly to the extent of outweighing the reductions gained by shrouding the undersides. Unless care is taken in design, maintenance may also be made more difficult and time consuming by fitting shrouds to the vehicle undersides. In Japan, the principal reason for shrouding bogies is environmental, to reduce the impact of wheel–rail noise on lineside residents, rather than purely for aerodynamic benefit.

The intervehicle gaps were varied, in steps of 0.2 m, from 0.3 m, the value measured for the Class 373 by the authors, to 0.7, a value typical of a British suburban train. Surprisingly, the effect of more than doubling the intervehicle gap is very small, as Armstrong had observed. For a speed of 83 m/s, the resistance force calculated was 151.8 kN with an intervehicle gap of 0.3 m and 153.0 kN with an intervehicle gap of 0.7 m. The extra power required to overcome this additional effort is 100 kW.

Finally, the height of a Class 373 was reduced by 0.75 m, from 3.334 m, in steps of 0.25 m, which reduced the cross-sectional area of the train from about 9 m^2 to about 7 m^2 and simultaneously had the effect of reducing its perimeter in steps of 0.5 m from about 11.2 m to around 9.7 m. As expected, reducing the height of the train reduced the aerodynamic resistance by some 10 per cent as shown in Fig. 6.

4.3 Discussion of possible measures to reduce drag

Returning to equation (1), it may be observed that the frontal area of a train, its perimeter and its length contribute most significantly to the overall value of the coefficient C. Thus, reducing the height and cross-sectional area of high-speed trains, as in successive generations of Japanese Shinkansen, would yield substantial dividends by reducing the value of C. However, adoption of such measures could give rise to cramped vehicle interiors which might not readily find passenger acceptance, particularly in regions where the average height of the population is approaching 1.8 m. Reducing train length would also reduce C but, if trains are already crowded, reducing their length would lead to more overcrowding unless more trains could be run. The track, infrastructure and signalling system, though, places a finite limit on line capacity.

The special safety regulations of the Channel Tunnel require that, in specific circumstances, a Class 373 can be split into two half-length units. This precluded the fitting of a high-voltage bus to link the power cars at each end of the train as in other TGV variants. Thus, it always has to run with two pantographs raised when working from an overhead line supply. The contribution that the pantographs can make to the C term is referred to in

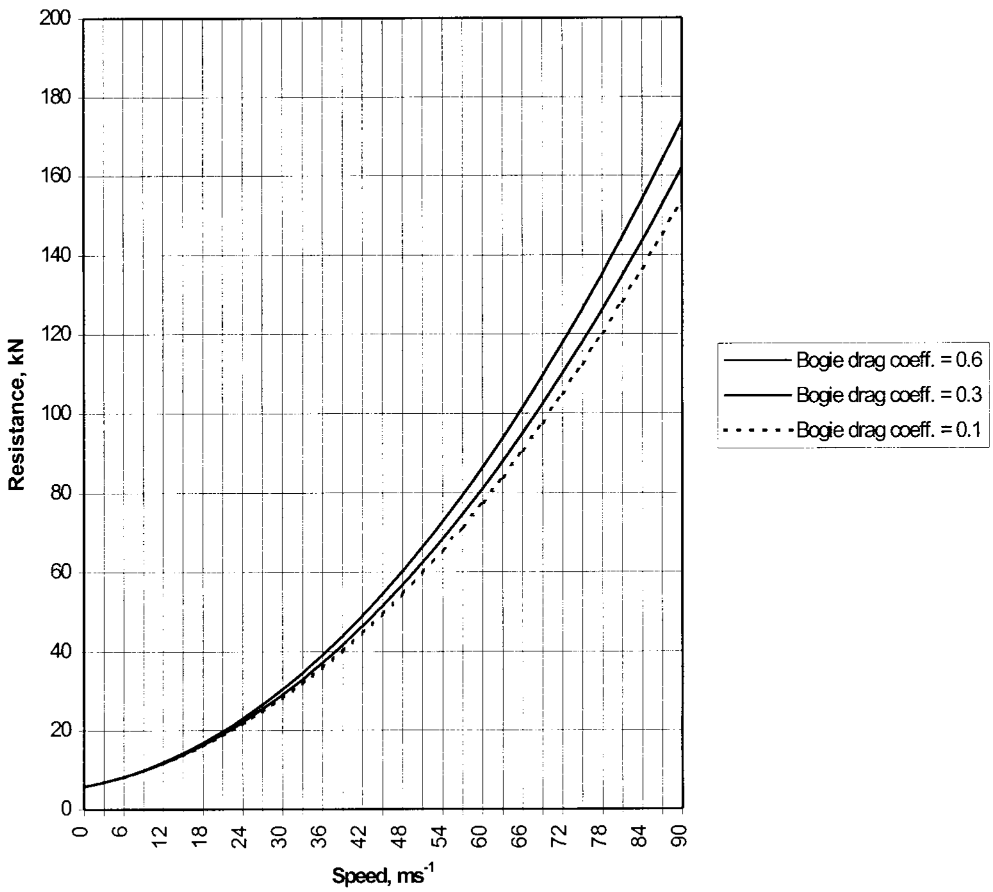

Fig. 5 Effect of varying the bogie drag coefficient

equations (4b) and (17). If the Class 373 could run at 300 km/h with only one pantograph raised, aerodynamic resistance could be reduced, using equation (4), by 1.8 kN or about 1 per cent of the total aerodynamic resistance. The extra power required to overcome this additional effort is 150 kW.

Whatever the relative magnitudes of the individual components of coefficient C, the influence of V^2 is very large, especially at higher speeds, and thus even a small reduction in C will bring great benefits. It is therefore important to reduce aerodynamic resistance by good design of the front and rear of the train, reducing intervehicle gaps to a minimum, shrouding bogies and reducing the number of pantographs. A high-speed train requires streamlined front and rear ends, and the importance of nose shape in terms of its contribution to aerodynamic resistance has been described by Andrews [**7**], to which the interested reader is referred.

An example of good practice in this area is provided by the double-deck TGV Duplex, when compared with a single-deck TGV. The TGV web pages [**8**] reveal that the TGV Duplex, despite being some 0.6 m higher than a single-deck TGV, produces only 4 per cent more drag at 300 km/h than the latter. This is because the nose of the TGV Duplex, designed by Roger Tallon, was improved aerodynamically compared with Cooper's original design. Also, the intervehicle gap was reportedly improved. However, as the authors have demonstrated above, the contribution of the intervehicle gap to aerodynamic drag is not significant. The relatively small increase in drag for the TGV Duplex, at the maximum service speed of 300 km/h, can thus be almost entirely attributed to the improved aerodynamic design of the ends of the TGV Duplex power cars. Moreover, the TGV Duplex can carry up to 45 per cent more passengers than a single-deck TGV for the same length.

4.4 Comparison of other formulae

In Fig. 7, the resistance equations used by a number of different railway undertakings for high-speed trains are plotted, in order to check whether the results they produce bear much resemblance to one another. All the curves in Fig. 7 exhibit the same quadratic form and, with the exception of the Sauthoff formula [equation (20)], they produce results in a relatively narrow range. The latter formula produces a gross overestimate of the resistance of Class 373 resistance which is assumed to be because it is based on trains with different characteristics

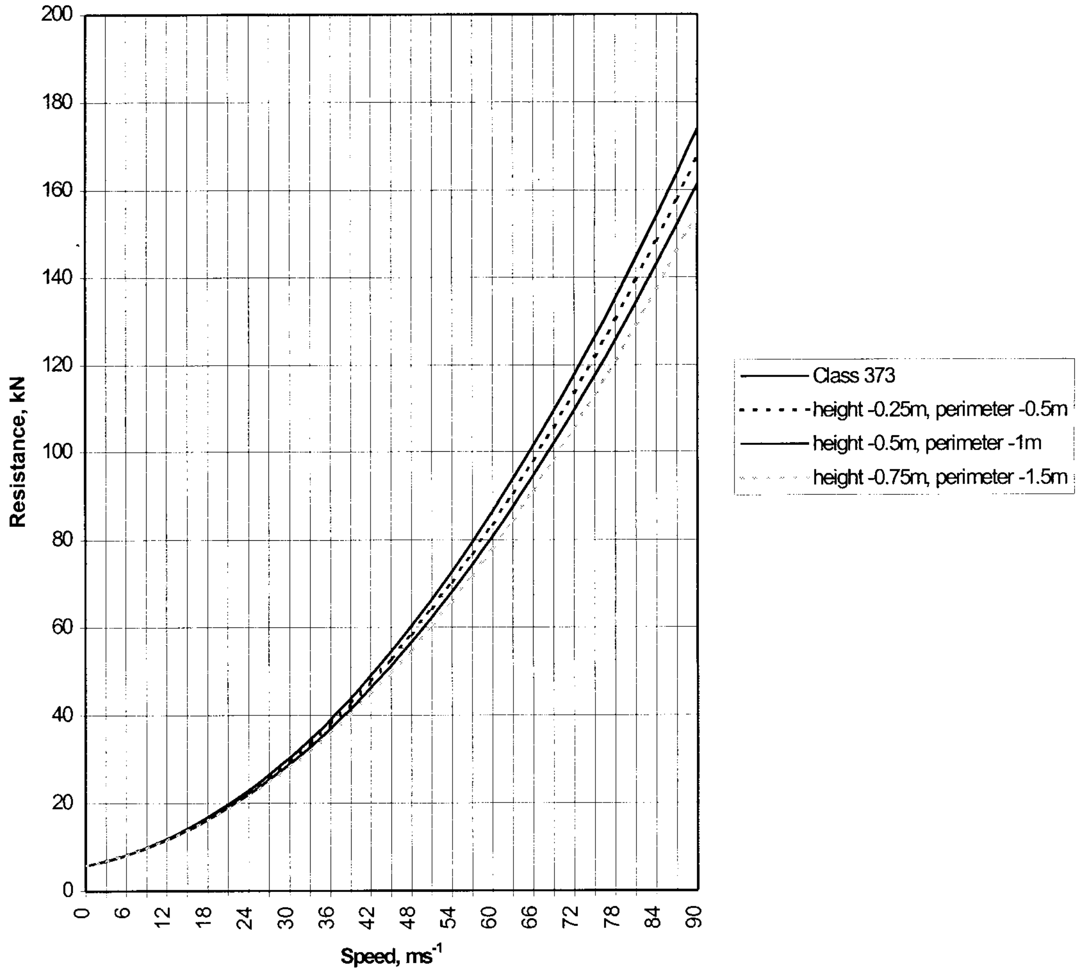

Fig. 6 Effect of reducing the cross-sectional area and perimeter

to the others considered in the figure. The differences within this range are due to variations in the mass, perimeter, cross-sectional area, bogie drag coefficient and track form. As expected, all approaches have validity but, in the authors' opinion, inputting coefficients calculated using Armstrong and Swift's equations (2b), (3b) and (4b) into equation (1) appears to offer the most accurate approach to calculating, to a first approximation, the resistance of a train for which run-down tests have yet to be calculated. The results are acceptable, at least for British trains, or those of similar size including the Class 373, which have a ratio of power to trailer cars of 1:3 or less.

To test the global applicability of Armstrong and Swift's equations, the resistance of a Shinkansen Series 100 train was calculated, with maximum service speed theoretically increased from 230 to 300 km/h, using equations (2b), (3b) and (4b) in (1), equation (20) and equation (24). Input data for the Series 100 Shinkansen was obtained from reference **[6]** and from the Byun Byun Shinkansen web page **[9]**. Values for intercar gap (0.3 m) and bogie drag coefficient were chosen intuitively. The results are reproduced in Fig. 8.

From Fig. 8 and Table 2 it can be seen that Armstrong and Swift's method, with the coefficients of formula (1) calculated using equations (2b), (3b) and (4b), produces a gross overestimate of the resistance of the Shinkansen Series 100 train across virtually the entire speed range, compared with the resistance derived from run-down testing [equation (24)]. The reasons for this are considered below.

Although the Series 100 Shinkansen is of a similar length, mass and power to a Class 373, there the resemblance ends, as Table 1 illustrates clearly. Because equations (2b) and (3b) are dominated by the numbers of power cars and trailers, the much larger ratio of power cars to trailers of the Series 100 Shinkansen compared with contemporary British rolling stock take these formulae outside the range for which Armstrong and Swift derived them. The cross-sectional area and perimeter of the Series 100 train are also much larger than those of a Class 373, which also inflates coefficient C calculated from equation (4b). A comparison of the coefficients A, B and C, calculated from equations (2b), (3b) and (4b) and derived from run-down testing based on equation (14), is given in Table 2.

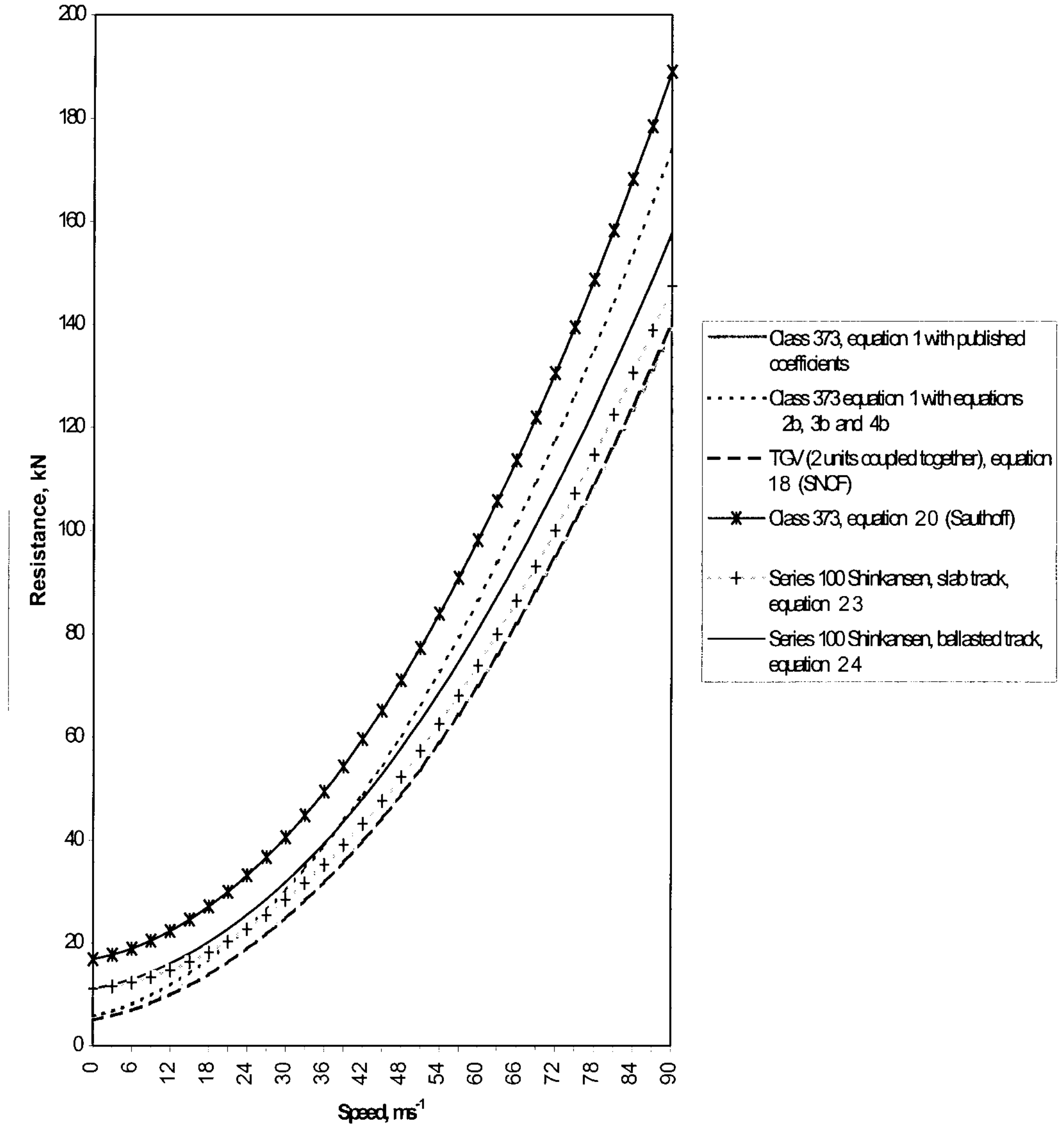

Fig. 7 Comparison of the resistance equations for high-speed trains

Interestingly, inputting data for the Series 100 Shinkansen in Sauthoff's equation (20) produces only a slight overestimate of the resistance compared with that derived from run-down testing using equation (24). At 84 m/s, the respective resistances are 154 and 140 kN.

It is concluded that Armstrong and Swift's method for calculating the coefficients of the Davis equation (1) cannot be applied universally. Equations (2b), (3b) and (4b) are thus limited in application to rolling stock of comparable size to that used in Britain with a ratio of power to trailer cars of 1:3 or less. Sauthoff's equation (20), while producing a consistent overestimate of approximately 10 per cent of train resistance at 300 km/h, appears to have a more general applicability.

Figure 9 shows that the resistance curves for the Series 100 Shinkansen, of a similar length and mass to a Class 373 and with the maximum service speed again theoretically increased from 230 to 300 km/h, but with conventional rather than articulated bogies, are also interesting. Running on slab track [equation (23)] reduces the aerodynamic resistance of the train by some 10 kN, or 7 per cent of the total, compared with running on ballasted track [equation (24)]. This demonstrates that, when optimizing the design of trains to minimize aerodynamic resistance, the trains themselves cannot be considered in isolation from the track on which they run.

As noted in Section 2, for low-speed commuter trains the principal contribution to resistance to motion is the mass of the train. Therefore, the reduction of the mass of commuter trains is a worthwhile exercise. However, since aerodynamic resistance plays a relatively minor role at lower speeds, aerodynamic design is not usually an important consideration. An exception to this could

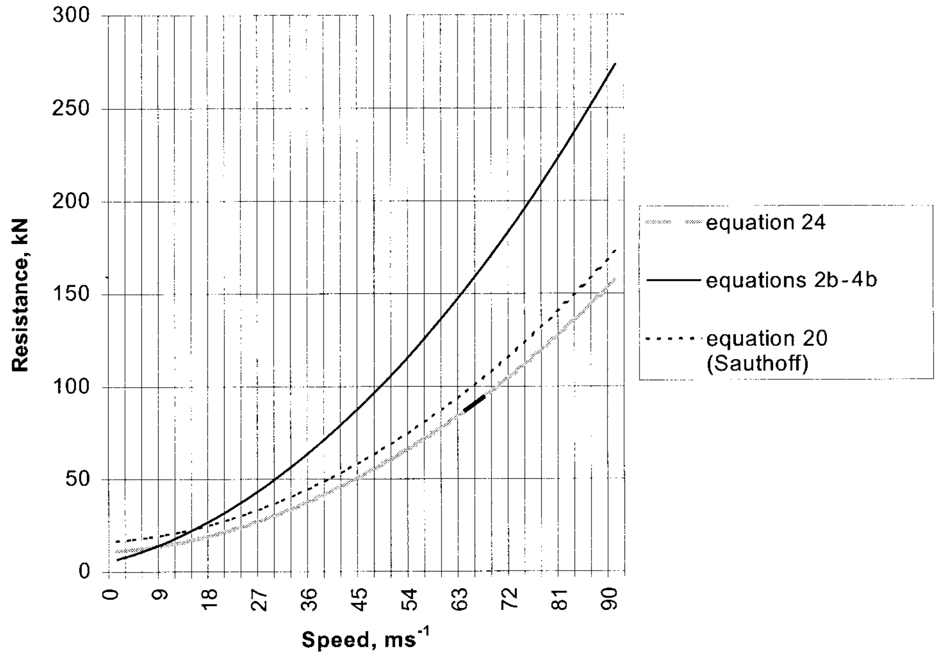

Fig. 8 Resistance of a Shinkansen Series 100 train, calculated by the methods indicated

be for trains operating on a route with a large proportion of its length in tunnels since, depending on the cross-sectional area of the tunnels relative to the train, the amount of aerodynamic resistance may be double that experienced in the open.

Table 1 Comparison of input data for Series 100 Shinkansen and Class 373

Parameter	Series 100	Class 373
Length (m)	402	394
Mass of power cars (t)	672	137
Mass of trailers (t)	184	730
Total mass (t)	856*	867
Power (MW)	11.04	12
Number of power cars	12	2
Number of trailers	4	18
Cross-sectional area (m^2)	12.6	8.9
Perimeter (m)	14.24	11.24
Number of pantographs	3	2
Head drag coefficient	0.075	0.0702
Tail drag coefficient	0.075	0.0743

* For equation (24), derived from run-down testing, the mass quoted by Maeda *et al.* [**6**] was 886 t. The 30 t discrepancy when used in equations (2b), (3b) and (4b) therefore produces a slightly less pessimistic result.

Using the Class 319 electric multiple unit as the example train, the resistance was calculated by various equations appropriate to suburban stock over the speed range 0–160 km/h. The results are plotted in Fig. 10. Figure 10 shows that there is less consistency in the results obtained from the equations shown. The range of peak resistance values was found to extend from 15.5 to 24.4 kN. All the equations chosen were derived for various national railway rolling stock which have different characteristics. This is thought to be the reason for the variability. Again, inputting coefficients calculated using Armstrong and Swift's formulae (2b), (3b) and (4b) into equation (1) appears to offer the most accurate approach to calculating, to a first approximation, in the absence of run-down testing, the resistance of trains that are similarly sized to those used in Britain and that do not have traction power distributed over a majority of axles.

5 CONCLUSIONS

As demonstrated by Figs 3 to 9, good aerodynamic design of vehicles and track is most important for high-

Table 2 Comparison of coefficients according to equations (2b) to (4b) and (24)

Coefficient	Equations (2b) to (4b)	Equation (24)	Difference with respect to equation (24) (%)
A (N)	6554	11 060	−41
B (N s/m)	820.48	223.2	+367
C (N s^2/m^2)	23.8566	15.6168	+153

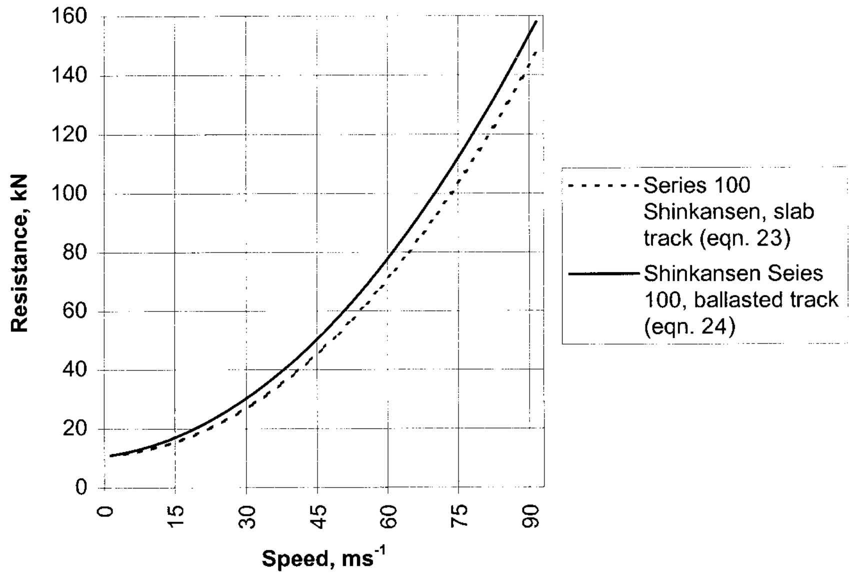

Fig. 9 Demonstration of the reduction in resistance with slab track

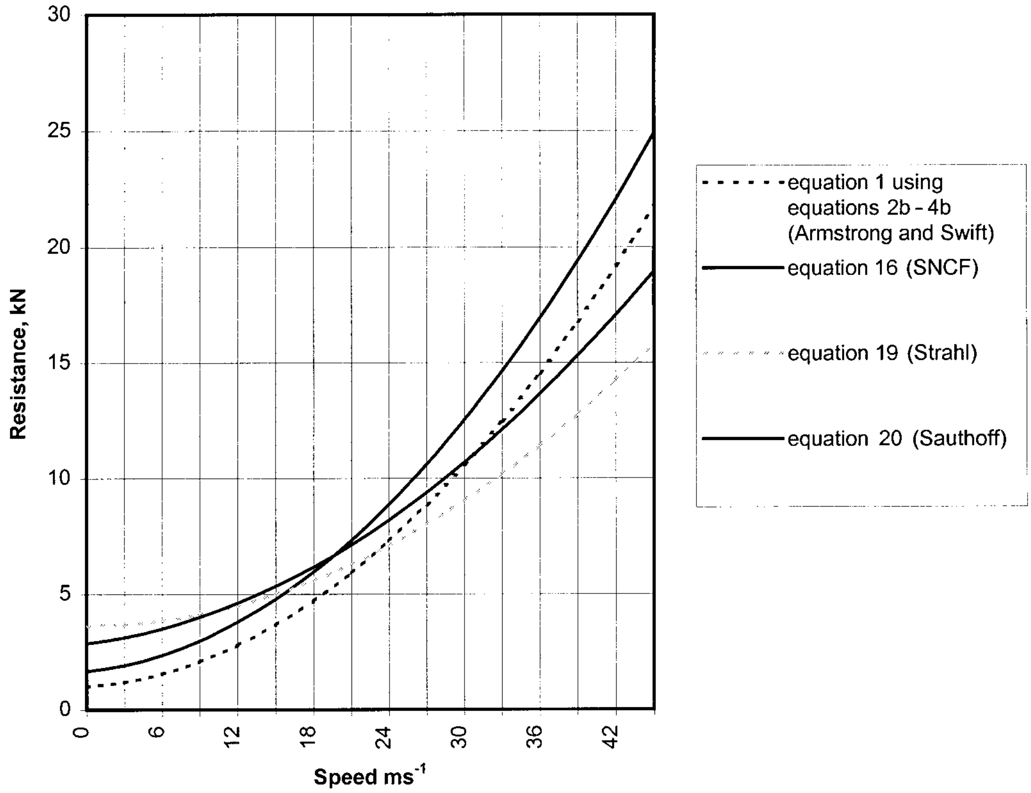

Fig. 10 Comparison of the resistance equations for a suburban train

speed trains, whatever the relative importance of the individual parameters that make up the C component. This is because the influence of coefficient C of the Davis equation increases with the square of the train velocity. Care should be taken to:

(a) provide a streamlined nose and tail (significant),
(b) reduce the cross-sectional area and perimeter (significant),
(c) shroud bogies (significant),
(d) reduce intervehicle gaps (minor effect),
(e) keep the number of pantographs to a minimum (minor effect) and
(f) smooth the underside of the train and surface of the track (significant).

In the absence of run-down testing, to calculate the resistance of trains of similar size to those in use in Britain and with a ratio of power to trailer cars of 1:3 or less, the use of Armstrong and Swift's expressions (2b), (3b) and (4b) can be considered to provide an accurate estimate of the coefficients to input into the Davis equation [equation (1)]. Although the Sauthoff equation (20) consistently overestimates the resistance of trains by some 10 per cent, if this is borne in mind, it does have universal applicability.

Equations to calculate train resistance were compared for high-speed trains (Fig. 7) and a suburban train (Fig. 10). It was found that the best correlation between equations of different undertakings was for high-speed trains. Those for a suburban train, a Class 319, showed a wide spread, probably owing to the various national railway equations reflecting the individual characteristics of their particular types of rolling stock.

The approaches of various national railways to the calculation of resistance and the relative importance of the various individual parameters indicate that the use of run-down tests should be the preferred method. In Japan, however, the comparison of the estimate of aerodynamic resistance by run-down testing and tunnel pressure rise techniques resulted in the same value.

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