

# Qbasis Manual

## I. MATRIX REPRESENTATION OF LOCAL HILBERT SPACE

### A. Fermi-Hubbard Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \sum_{\sigma} (c_{i\sigma}^{\dagger} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}. \quad (1)$$

Local Hilbert space is 4-dimensional:

$$\{|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle\}. \quad (2)$$

In this basis:

$$c_{\uparrow} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad (3a)$$

$$c_{\downarrow} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \quad (3b)$$

From these two operators as input, the code is able to automatically derive the following operators:

$$c_{\uparrow}^{\dagger} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad (4a)$$

$$c_{\downarrow}^{\dagger} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \quad (4b)$$

$$n_{\uparrow} = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 0 & \\ & & & 1 \end{pmatrix}, \quad (4c)$$

$$n_{\downarrow} = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \quad (4d)$$

### B. t-J Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \sum_{\sigma} (c_{i\sigma}^{\dagger} c_{j\sigma} + h.c.) + J \sum_{\langle ij \rangle} \left[ \frac{S_i^+ S_j^- + S_i^- S_j^+}{2} + S_i^z S_j^z - \frac{1}{4} n_i n_j \right], \quad (5)$$

where

$$S_i^+ = c_{i\uparrow}^{\dagger} c_{i\downarrow}, \quad (6a)$$

$$S_i^- = c_{i\downarrow}^{\dagger} c_{i\uparrow}, \quad (6b)$$

$$S_i^z = \frac{1}{2} (c_{i\uparrow}^{\dagger} c_{i\uparrow} - c_{i\downarrow}^{\dagger} c_{i\downarrow}). \quad (6c)$$

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Local Hilbert space is 3-dimensional:

$$\{|0\rangle, |\uparrow\rangle, |\downarrow\rangle\}. \quad (7)$$

In this basis:

$$c_{\uparrow} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (8a)$$

$$c_{\downarrow} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (8b)$$

and all other derived operators can be derived automatically by the code.

### C. Bose-Hubbard Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + h.c.) + \frac{U}{2} \sum_i n_i (n_i - 1) \quad (9)$$

Local Hilbert space restricted to at most  $N_{max}$  bosons:

$$\{|0\rangle, |1\rangle, \dots, |N_{max}\rangle\}. \quad (10)$$

In this basis:

$$b = \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \sqrt{N_{max}} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (11)$$

and other operators should be automatically derived.