# Qbasis Manual

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# Introduction

# Chapter 2

# Model representation

## 2.1 Matrix represenataion of local Hilbert Space

### Fermi-Hubbard Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \sum_{\sigma} (c^\dagger_{i\sigma} c_{j\sigma} + h.c.) + U \sum_i n_{i\uparrow} n_{i\downarrow}. \tag{2.1.1}$$

Local Hilbert space is 4-dimensional:

$$\{|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle\}. \tag{2.1.2}$$

In this basis:

$$c_\uparrow = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{2.1.3a}$$

$$c_\downarrow = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{2.1.3b}$$

From these two opeators as input, the code is able to automatically derive the following operators:

$$c^\dagger_\uparrow = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{2.1.4a}$$

$$c^\dagger_\downarrow = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \tag{2.1.4b}$$

$$n_\uparrow = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 0 & \\ & & & 1 \end{pmatrix}, \tag{2.1.4c}$$

$$n_\downarrow = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \tag{2.1.4d}$$

## t-J Model

$$\mathcal{H} = -t\sum_{\langle ij\rangle}\sum_{\sigma}(c^\dagger_{i\sigma}c_{j\sigma} + h.c.) + J\sum_{\langle ij\rangle}\left[\frac{S^+_i S^-_j + S^-_i S^+_j}{2} + S^z_i S^z_j - \frac{1}{4}n_i n_j\right], \tag{2.1.5}$$

where

$$S^+_i = c^\dagger_{i\uparrow}c_{i\downarrow}, \tag{2.1.6a}$$

$$S^-_i = c^\dagger_{i\downarrow}c_{i\uparrow}, \tag{2.1.6b}$$

$$S^z_i = \frac{1}{2}\left(c^\dagger_{i\uparrow}c_{i\uparrow} - c^\dagger_{i\downarrow}c_{i\downarrow}\right). \tag{2.1.6c}$$

Local Hilbert space is 3-dimensional:

$$\{|0\rangle, |\uparrow\rangle, |\downarrow\rangle\}. \tag{2.1.7}$$

In this basis:

$$c_\uparrow = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2.1.8a}$$

$$c_\downarrow = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2.1.8b}$$

and all other derived operators can be derived automatically by the code.

## Spinless Fermion

$$\mathcal{H} = -t\sum_{\langle ij\rangle}(c^\dagger_i c_j + h.c.) + V_1\sum_{\langle ij\rangle} n_i n_j. \tag{2.1.9}$$

Local Hilbert space is 2-dimensional:

$$\{|0\rangle, |1\rangle\}. \tag{2.1.10}$$

In this basis:

$$c = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \tag{2.1.11}$$

### Bose-Hubbard Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} (b_i^\dagger b_j + h.c.) + \frac{U}{2} \sum_i n_i(n_i - 1) \tag{2.1.12}$$

Local Hilbert space restricted to at most $N_{max}$ bosons:

$$\{|0\rangle, |1\rangle, \dots, |N_{max}\rangle\}. \tag{2.1.13}$$

In this basis:

$$b = \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \sqrt{N_{max}} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{2.1.14}$$

and other operators should be automatically derived.

## 2.2 Lin table

# Chapter 3

# Finite size cluster

# Chapter 4

# Symmetry

## 4.1 Translation symmetry

## 4.2 Weisse table

# Chapter 5

# Lanczos method

# Chapter 6

# Code examples

## 6.1 Exact diagonalization