Qbasis Manual

I. MATRIX REPRESENATAION OF LOCAL HILBERT SPACE

A. Fermi-Hubbard Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \sum_{\sigma} (c_{i\sigma}^{\dagger} c_{j\sigma} + h.c.) + U \sum_{i} n_{i\uparrow} n_{i\downarrow}. \tag{1}$$

Local Hilbert space is 4-dimensional:

$$\{|0\rangle, |\uparrow\rangle, |\downarrow\rangle, |\uparrow\downarrow\rangle\}. \tag{2}$$

In this basis:

$$c_{\uparrow} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{3a}$$

$$c_{\downarrow} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3b}$$

From these two opeators as input, the code is able to automatically derive the following operators:

$$c_{\uparrow}^{\dagger} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \tag{4a}$$

$$c_{\downarrow}^{\dagger} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, \tag{4b}$$

$$n_{\uparrow} = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 0 & \\ & & & 1 \end{pmatrix}, \tag{4c}$$

$$n_{\downarrow} = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}. \tag{4d}$$

B. t-J Model

$$\mathcal{H} = -t \sum_{\langle ij \rangle} \sum_{\sigma} (c_{i\sigma}^{\dagger} c_{j\sigma} + h.c.) + J \sum_{\langle ij \rangle} \left[\frac{S_i^+ S_j^- + S_i^- S_j^+}{2} + S_i^z S_j^z - \frac{1}{4} n_i n_j \right], \tag{5}$$

where

$$S_i^+ = c_{i\uparrow}^\dagger c_{i\downarrow}, \tag{6a}$$

$$S_i^- = c_{i\downarrow}^\dagger c_{i\uparrow}, \tag{6b}$$

$$S_i^z = \frac{1}{2}\left(c_{i\uparrow}^\dagger c_{i\uparrow} - c_{i\downarrow}^\dagger c_{i\downarrow}\right). \tag{6c}$$

Local Hilbert space is 3-dimensional:

$$\{|0\rangle, |\uparrow\rangle, |\downarrow\rangle\}. \tag{7}$$

In this basis:

$$c_\uparrow = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{8a}$$

$$c_\downarrow = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{8b}$$

and all other derived operators can be derived automatically by the code.

C. Spinless Fermion

$$\mathcal{H} = -t\sum_{\langle ij \rangle}(c_i^\dagger c_j + h.c.) + V_1 \sum_{\langle ij \rangle} n_i n_j. \tag{9}$$

Local Hilbert space is 2-dimensional:

$$\{|0\rangle, |1\rangle\}. \tag{10}$$

In this basis:

$$c = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \tag{11}$$

D. Bose-Hubbard Model

$$\mathcal{H} = -t\sum_{\langle ij \rangle}(b_i^\dagger b_j + h.c.) + \frac{U}{2} \sum_i n_i(n_i - 1) \tag{12}$$

Local Hilbert space restricted to at most N_{max} bosons:

$$\{|0\rangle, |1\rangle, \ldots, |N_{max}\rangle\}. \tag{13}$$

In this basis:

$$b = \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \sqrt{N_{max}} \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{14}$$

and other operators should be automatically derived.