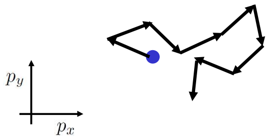
Cooling limit

Damping force $F(v) = -m\gamma v$ leads to velocity reduction $v(t) = v_0 e^{-\gamma t}$.

What limits this reduction?

Momentum diffusion by discreteness of momentum transfer.



For simplicity: discussion in 1D : photons have either momentum $+\hbar k_L$ or $-\hbar k_L$.

Absorption from one beam leads to momentum change $\pm\hbar k_L$, giving rise to momentum diffusion. (We neglect the momentum kick from spontaneous emission.)

This diffusion is counteracted by laser cooling.

In equilibrium, a distribution with finite momentum spread is attained, corresponding to a certain temperature.

Cooling limit

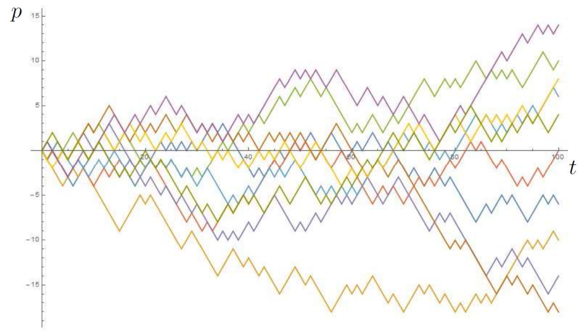
For simplicity:

only one cooling beam pair:

absorbtion changes momentum by either $+\hbar k_L$ or $-\hbar k_L$.

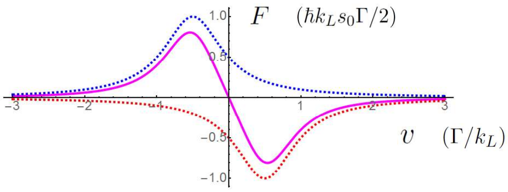
neglect recoil from emitted photon.

The result is a random walk, which leads to momentum diffusion.

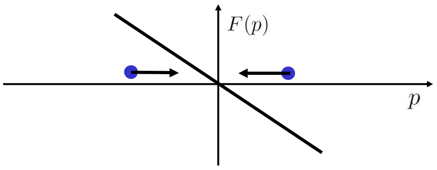


Cooling limit

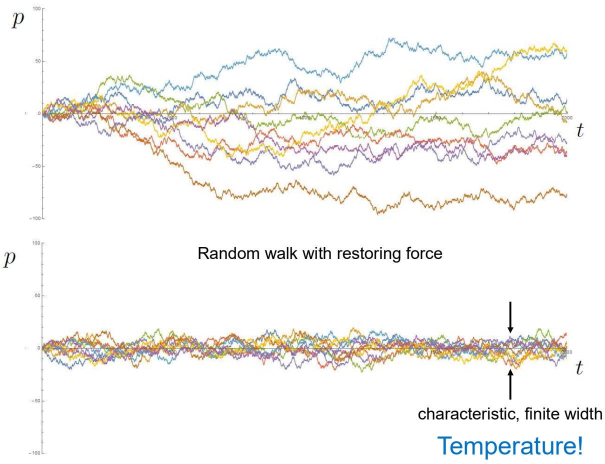
This momentum diffusion is counteracted by laser cooling.



Close to zero momentum the cooling force is approximately linear with momentum:



Laser cooling



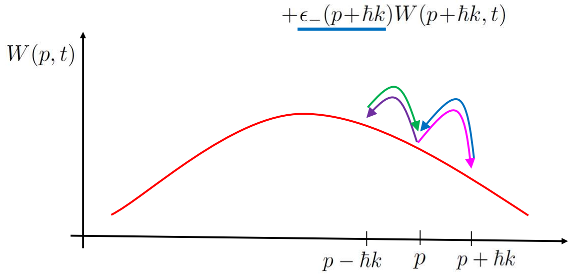
Mathematical description

Momentum distribution of atoms at time t described by $W(p,t)$.

Be $\epsilon_\pm(p)$ the probability to jump by $\pm\hbar k$ in timestep Δt if atom initially has momentum p .

Change of momentum distribution during timestep

$$\begin{aligned} W(p,t+\Delta t) - W(p,t) = &-[\underline{\epsilon_+(p)} + \underline{\epsilon_-(p)}]W(p,t) \\ &+ \underline{\epsilon_+(p-\hbar k)}W(p-\hbar k,t) \\ &+ \underline{\epsilon_-(p+\hbar k)}W(p+\hbar k,t) \end{aligned}$$



Differential equation

$$\begin{aligned} W(p,t+\Delta t) - W(p,t) = &-[\underline{\epsilon_+(p)} + \underline{\epsilon_-(p)}]W(p,t) \\ &+ \underline{\epsilon_+(p-\hbar k)}W(p-\hbar k,t) \qquad (*) \\ &+ \underline{\epsilon_-(p+\hbar k)}W(p+\hbar k,t) \end{aligned}$$

Assume $\hbar k \ll p$. Taylor expansion of $f(p \mp \hbar k, t) = \epsilon_\pm(p \mp \hbar k) W(p \mp \hbar k, t)$

$$\begin{aligned} \epsilon_\pm(p \mp \hbar k) W(p \mp \hbar k, t) = &\ \epsilon(p) W(p,t) \\ &\mp \hbar k \frac{\partial}{\partial p}[\epsilon(p) W(p,t)] \\ &+ \frac{(\hbar k)^2}{2} \frac{\partial^2}{\partial p^2}[\epsilon(p) W(p,t)] + \ldots \end{aligned}$$

Insert into (*), reorder, divide by Δt:

$$\frac{\partial}{\partial t} W(p,t) = -\frac{\partial}{\partial p}[M_1(p) W(p,t)] + \frac{1}{2}\frac{\partial^2}{\partial p^2}[M_2(p) W(p,t)]$$

$$M_1 = [\epsilon_+(p) - \epsilon_-(p)]\frac{\hbar k}{\Delta t} = F(p) = -\gamma p$$

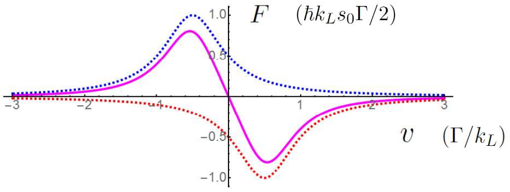
$$M_2 = [\epsilon_+(p) + \epsilon_-(p)]\frac{(\hbar k)^2}{\Delta t}$$

Determination of M_2

$$M_2 = [\epsilon_+(p) + \epsilon_-(p)]\frac{(\hbar k)^2}{\Delta t}$$

$[\epsilon_+(p) + \epsilon_-(p)]$ is the probability of jumping anywhere during time Δt.

We are only interested in region close to zero momentum.



We can approximate the number of jumps by the scattering rate at zero momentum times Δt.

$$M_2 = [\epsilon_+(p) + \epsilon_-(p)]\frac{(\hbar k)^2}{\Delta t} = [2\Gamma_p(\delta)\Delta t]\frac{(\hbar k)^2}{\Delta t} = \Gamma s(\hbar k)^2 \equiv 2D$$

Differential equation

$$\begin{aligned} W(p,t+\Delta t) - W(p,t) = &-[\epsilon_+(p) + \epsilon_-(p)]W(p,t) \\ &+\epsilon_+(p-\hbar k)W(p-\hbar k,t) \\ &+\epsilon_-(p+\hbar k)W(p+\hbar k,t) \end{aligned} \qquad (*)$$

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Stationary solution

$$\frac{\partial}{\partial t}W(p,t) = -\frac{\partial}{\partial p}[M_1(p)W(p,t)] + \frac{1}{2}\frac{\partial^2}{\partial p^2}[M_2(p)W(p,t)]$$

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Stationary solution $\frac{\partial}{\partial t}W(p,t) = 0$:

$$-\gamma p W(p,t) = \frac{1}{2}\frac{\partial}{\partial p}[2DW(p,t)]$$

Ansatz $\quad W(p) = Ce^{-\gamma p^2/2D} \quad$ solves differential equation.

Doppler temperature

Stationary solution

$$W(p) = Ce^{-\gamma p^2/2D}$$

corresponds to a Boltzman distribution

$$W(p) = Ce^{-p^2/2mk_BT}$$

Therefore the temperature of the gas is well defined and

$$k_BT = D/\gamma m = \frac{\hbar\Gamma}{8}\left(\frac{2|\delta|}{\Gamma} + \frac{\Gamma}{2|\delta|}\right)$$

3D calculation gives:

$$k_BT = \frac{\hbar\Gamma}{4}\left(\frac{2|\delta|}{\Gamma} + \frac{\Gamma}{2|\delta|}\right)$$

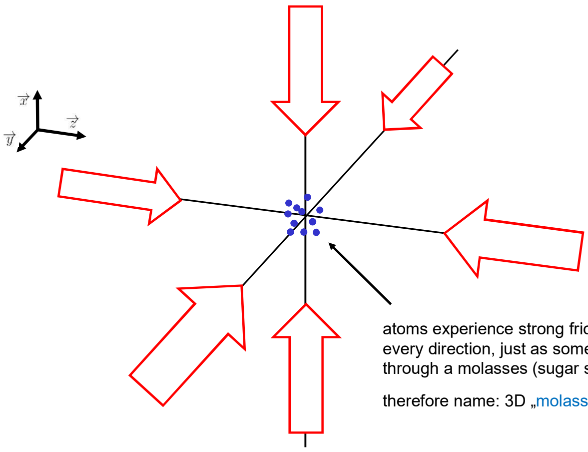
Temperature minimum reached for $\delta = -\frac{\Gamma}{2}$:

$$\boxed{k_BT_{\min} = \hbar\frac{\Gamma}{2}}$$

This temperature is called the Doppler temperature.

Molasses cooling

Use pairs of laser beams in three ~orthogonal directions:



atoms experience strong friction forces in every direction, just as something moving through a molasses (sugar syrup).

therefore name: 3D „molasses“