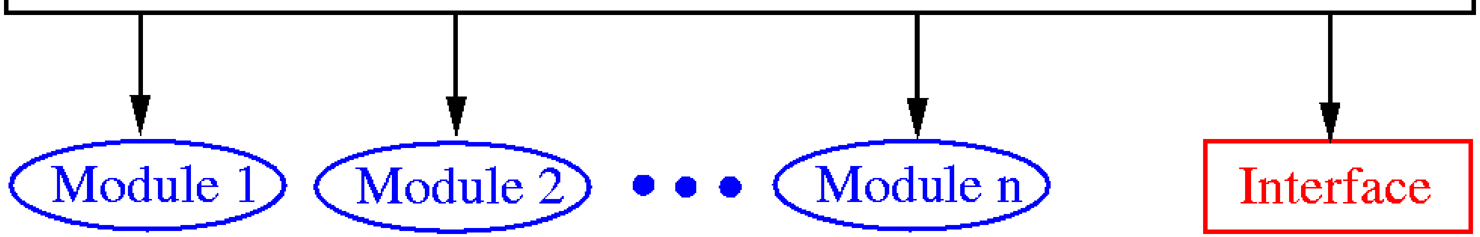
# Partitioning

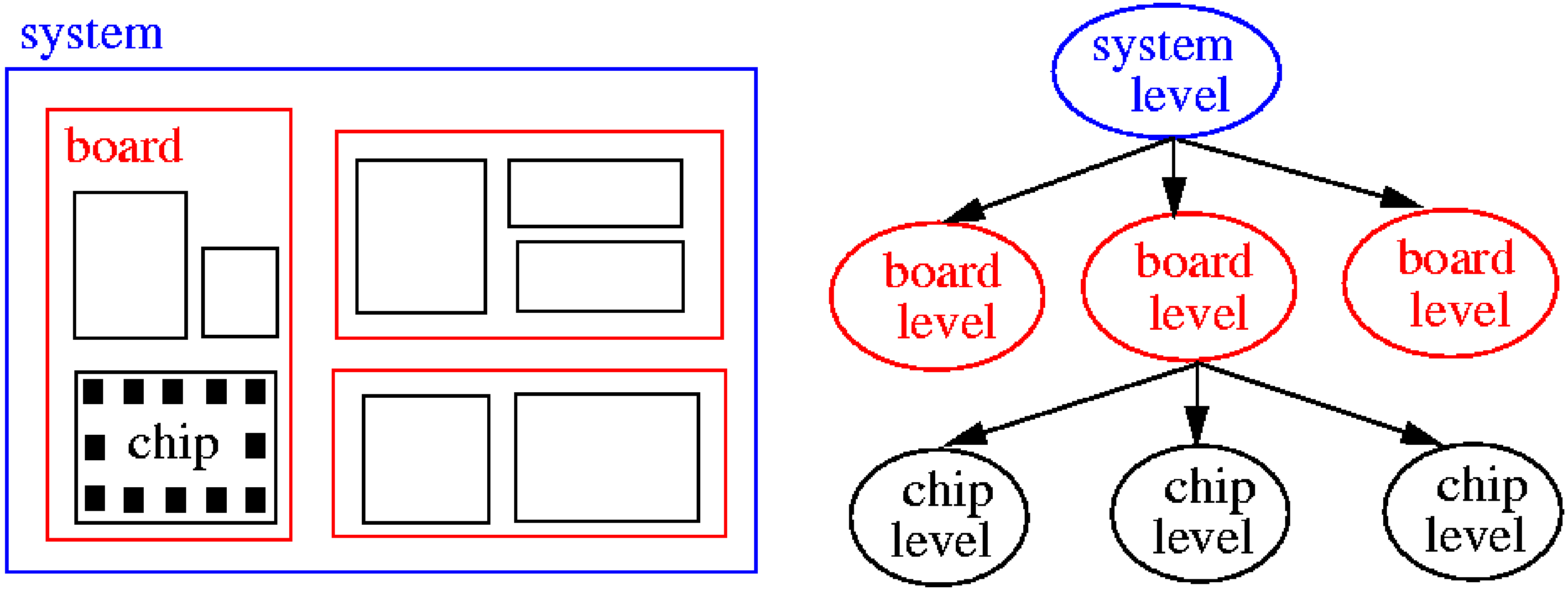
system design

- Decomposition of a complex system into smaller subsystems.
- Each subsystem can be designed independently speeding up the design process.
- Decomposition scheme has to minimize the interconnections among the subsystems.
- Decomposition is carried out hierarchically until each subsystem is of managable size.



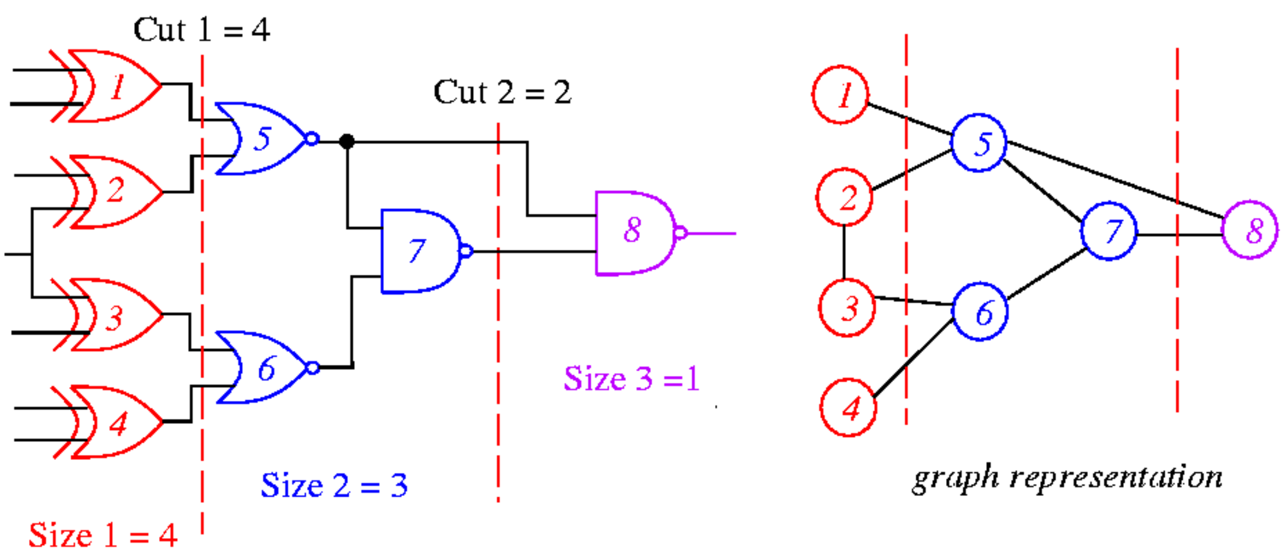
# Levels of Partitioning

- The levels of partitioning: System, Board, Chip.



# Partitioning of a Circuit

- The task of cutting a circuit into smaller parts.
- **Objective:** Partition the circuit into sub-circuits such that every sub-circuit is within a pre-specified size and the # of connections among sub-circuits is minimized.
  - Other possible constraints (e.g., # of pins in a sub-circuit)
  - Other possible objectives (e.g., critical path delay)
- Cutset? Cut size? Size of a sub-circuit?



# Problem Definition

- ***k*-way partitioning:** Given a graph $G(V,E)$, where each vertex $v \in V$ has a **size** $s(v)$ and each edge $e \in E$ has a **weight** $w(e)$, the problem is to divide the set $V$ into $k$ disjoint subsets $V_1, V_2, \ldots, V_k$, such that an objective function is optimized, subject to certain constraints.
- **Bounded size constraint:** The size of the $i$-th subset is bounded by $L_i$ and $U_i$ $\left(\text{i.e., } L_i \le \sum_{v \in V_i} s(v) \le U_i\right)$
- **Min-cut cost between two subsets:** Minimize $\sum_{\forall e=(u,v) \wedge p(u) \neq p(v)} w(e)$, where $p(u)$ is the subset where $u$ is.
- The 2-way, size-constrained partitioning problem is NP-hard, even in its simple form with identical vertex sizes and unit edge weights.

# Kernighan-Lin (KL) Algorithm

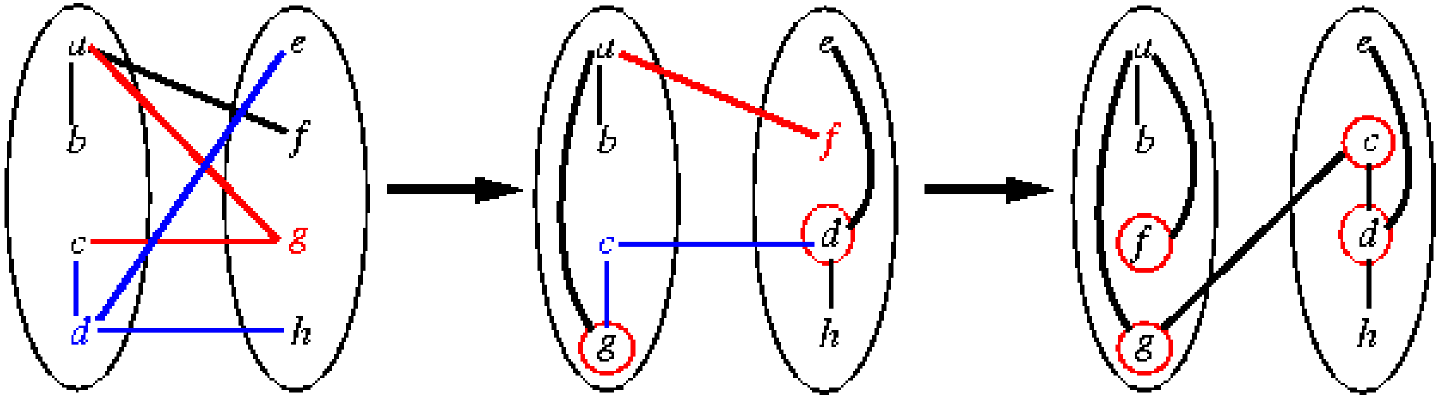
- Kernighan and Lin, “An efficient heuristic procedure for partitioning graphs,” *The Bell System Technical Journal*, vol. 49, no. 2, Feb. 1970.
- An **iterative**, **2-way**, **balanced** partitioning (bi-sectioning) heuristic.
- Restrictions:
  – Assume all vertices are of the same size.
  – Work only for 2-terminal nets.

# Key Idea of KL Algorithm

- Start with any initial partitions *A* and *B*.
- A **pass** (exchanging each vertex exactly once) consists of:
  - Exchange a vertex pair which gives the **maximum gain** $g_i$ **(**i.e., largest decrease or smallest increase in cut size), and **lock** them (which thus are prohibited from participating in any further exchanges).
  - This process continues until all vertices are locked.
  - Find the largest partial sum *G* (i.e., find *k* such that $g_1+...+g_k$ is maximized).
  - Exchange the first *k* pairs.
- Repeat the pass until there is no improvement (i.e., $G=0$).

# KL Algorithm: A Simple Example

- Each edge has a unit weight.



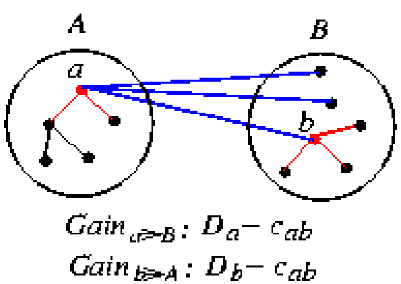

| Step # | Vertex pair | Cost reduction | Cut cost |
|---|---|---|---|
| 0 | - | 0 | 5 |
| 1 | {d, g} | 3 | 2 |
| 2 | {c, f} | 1 | 1 |
| 3 | {b, h} | -2 | 3 |
| 4 | {a, e} | -2 | 5 |

- Questions: How to compute cost reduction? What pairs to be swapped?

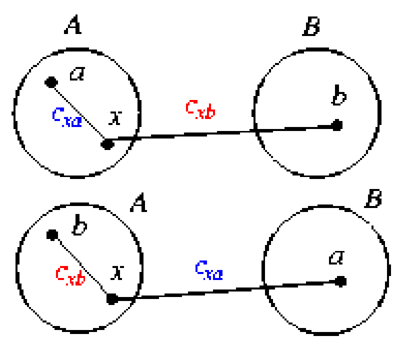
# Properties

- Two sets A and B such that $|A|=n=|B|$ and $A \cap B = \emptyset$ .
- **External cost** of $a \in A$: $E_a = \sum_{v \in B} c_{av}$.
- **Internal cost** of $a \in A$: $I_a = \sum_{v \in A} c_{av}$.
- $D$-value of a vertex $a$: $D_a = E_a - I_a$ (cost reduction for moving $a$).
- Cost reduction (**gain**) for swapping $a$ and $b$: $g_{ab} = D_a + D_b - 2c_{ab}$.
- If $a \in A$ and $b \in B$ are interchanged, then the new $D$-values, $D'$, are given by

$$\begin{aligned} D'_x &= D_x + 2c_{xa} - 2c_{xb}, \forall x \in A - \{a\} \\ D'_y &= D_y + 2c_{yb} - 2c_{ya}, \forall y \in B - \{b\}. \end{aligned}$$



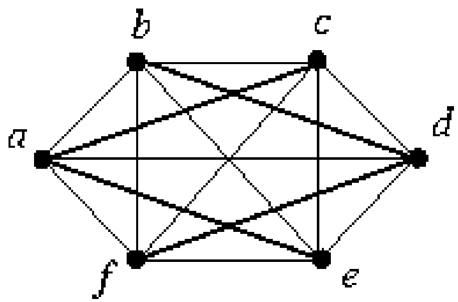

*Internal cost vs. External cost*



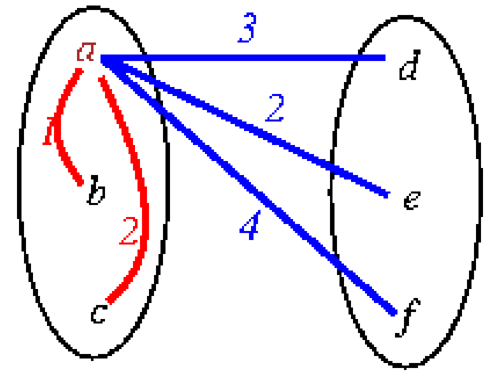

| before swap | after swap | $\triangle C$ |
|---|---|---|
| $-c_{xa}$ | $+c_{xa}$ | $+2c_{xa}$ |
| $+c_{xb}$ | $-c_{xb}$ | $-2c_{xb}$ |

*updating D−values*

# KL Algorithm: A Weighted Example




| | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| a | 0 | 1 | 2 | 3 | 2 | 4 |
| b | 1 | 0 | 1 | 4 | 2 | 1 |
| c | 2 | 1 | 0 | 3 | 2 | 1 |
| d | 3 | 4 | 3 | 0 | 4 | 3 |
| e | 2 | 2 | 2 | 4 | 0 | 2 |
| f | 4 | 1 | 1 | 3 | 2 | 0 |




*costs associated with a*

*Initial cut cost* = (3+2+4)+(4+2+1)+(3+2+1) = 22

- Iteration 1:

$I_a$=1+2=3; $E_a$=3+2+4=9; $D_a$=$E_a$-$I_a$=9-3=6

$I_b$=1+1=2; $E_b$=4+2+1=7; $D_b$=$E_b$-$I_b$=7-2=5

$I_c$=2+1=3; $E_c$=3+2+1=6; $D_c$=$E_c$-$I_c$=6-3=3

$I_d$=4+3=7; $E_d$=3+4+3=10; $D_d$=$E_d$-$I_d$=10-7=3

$I_e$=4+2=6; $E_e$=2+2+2=6; $D_e$=$E_e$-$I_e$=6-6=0

$I_f$=3+2=5; $E_f$=4+1+1=6; $D_f$=$E_f$-$I_f$=6-5=1

# Weighted Example (Cont'd)

- Iteration 1:

  $I_a$=1+2=3; $E_a$=3+2+4=9; $D_a$=$E_a$-$I_a$=9-3=6

  $I_b$=1+1=2; $E_b$=4+2+1=7; $D_b$=$E_b$-$I_b$=7-2=5

  $I_c$=2+1=3; $E_c$=3+2+1=6; $D_c$=$E_c$-$I_c$=6-3=3

  $I_d$=4+3=7; $E_d$=3+4+3=10; $D_d$=$E_d$-$I_d$=10-7=3

  $I_e$=4+2=6; $E_e$=2+2+2=6; $D_e$=$E_e$-$I_e$=6-6=0

  $I_f$=3+2=5; $E_f$=4+1+1=6; $D_f$=$E_f$-$I_f$=6-5=1

- $g_{xy}=D_x+D_y-2c_{xy}$.

  $g_{ad} = D_a + D_d - 2c_{ad} = 6 + 3 - 2 * 3 = 3$

  $g_{ae} = 6 + 0 - 2 * 2 = 2$

  $g_{af} = 6 + 1 - 2 * 4 = -1$

  $g_{bd} = 5 + 3 - 2 * 4 = 0$

  $g_{be} = 5 + 0 - 2 * 2 = 1$

  $g_{bf} = 5 + 1 - 2 * 1 = 4$ (maximum)

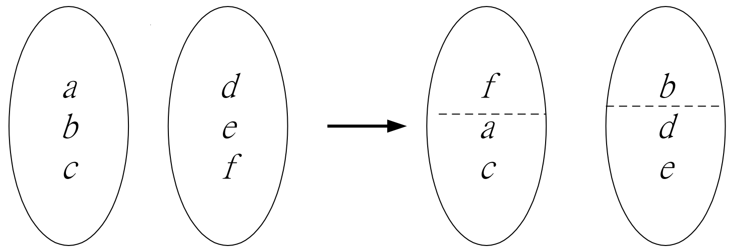
  $g_{cd} = 3 + 3 - 2 * 3 = 0$

  $g_{ce} = 3 + 0 - 2 * 2 = -1$

  $g_{cf} = 3 + 1 - 2 * 1 = 2$

- Swap *b* and *f*! ( $\hat{g}_1$=4 )

# Weighted Example (Cont’d)




- $D_x' = D_x + 2c_{xp} - 2c_{xq}, \forall x \in A - \{p\}$ (swap $p$ and $q$, $p \in A$, $q \in B$)

$$D_a' = D_a + 2c_{ab} - 2c_{af} = 6 + 2*1 - 2*4 = 0$$

$$D_c' = D_c + 2c_{cb} - 2c_{cf} = 3 + 2*1 - 2*1 = 3$$

$$D_d' = D_d + 2c_{df} - 2c_{db} = 3 + 2*3 - 2*4 = 1$$

$$D_e' = D_e + 2c_{ef} - 2c_{eb} = 0 + 2*2 - 2*2 = 0$$

- $g_{xy} = D_x' + D_y' - 2c_{xy}.$

$$g_{ad} = D_a' + D_d' - 2c_{ad} = 0 + 1 - 2*3 = -5$$

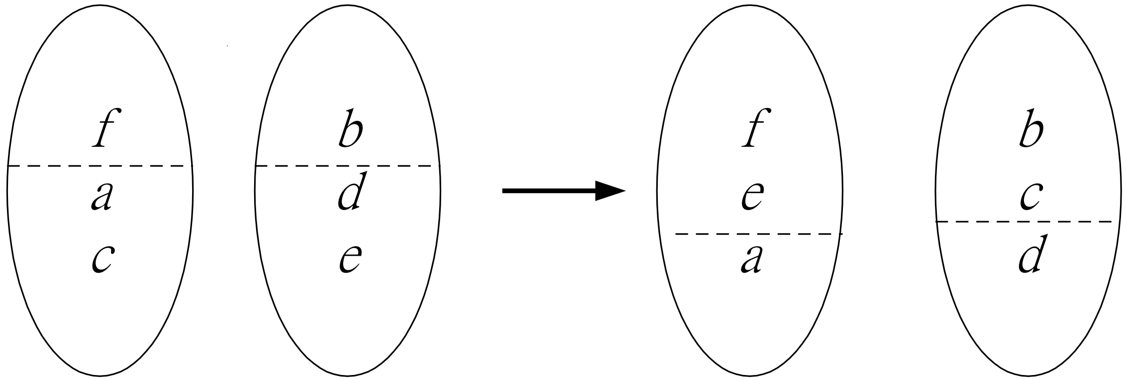
$$g_{ae} = D_a' + D_e' - 2c_{ae} = 0 + 0 - 2*2 = -4$$

$$g_{cd} = D_c' + D_d' - 2c_{cd} = 3 + 1 - 2*3 = -2$$

$$g_{ce} = D_c' + D_e' - 2c_{ce} = 3 + 0 - 2*2 = -1 \text{ (maximum)}$$

- Swap *c* and *e*! ($\hat{g}_2$ = -1 )

# Weighted Example (Cont'd)




- $D''_x = D'_x + 2c_{xp} - 2c_{xq}, \forall x \in A - \{p\}$

$$D''_a = D'_a + 2c_{ac} - 2c_{ae} = 0 + 2*2 - 2*2 = 0$$

$$D''_d = D'_d + 2c_{de} - 2c_{dc} = 1 + 2*4 - 2*3 = 3$$

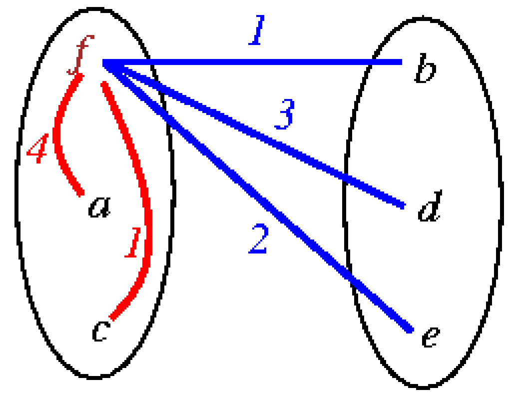
- $g_{xy} = D''_x + D''_y - 2x_{xy}.$

$$g_{ad} = D''_a + D''_d - 2c_{ad} = 0 + 3 - 2*3 = -3 \quad (\hat{g}_3 = -3)$$

- Note that this step is redundant $\left(\sum_{i=1}^{n} \hat{g}_i = 0\right)$
- Summary: $\hat{g}_1 = g_{bf} = 4,\ \hat{g}_2 = g_{ce} = -1,\ \hat{g}_3 = g_{ad} = -3.$
- Largest partial sum max $\sum_{i=1}^{k} \hat{g}_i = 4\ (k = 1) \Rightarrow$ Swap *b* and *f*.

# Weighted Example (Cont'd)

| | a | b | c | d | e | f |
|---|---|---|---|---|---|---|
| a | 0 | 1 | 2 | 3 | 2 | 4 |
| b | 1 | 0 | 1 | 4 | 2 | 1 |
| c | 2 | 1 | 0 | 3 | 2 | 1 |
| d | 3 | 4 | 3 | 0 | 4 | 3 |
| e | 2 | 2 | 2 | 4 | 0 | 2 |
| f | 4 | 1 | 1 | 3 | 2 | 0 |




*Initial cut cost* = (1+3+2)+(1+3+2)+(1+3+2) = 18 (22−4)

- Iteration 2: Repeat what we did at Iteration 1 (Initial cost=22-4=18)

- Summary: $\hat{g}_1 = g_{ce} = -1,\ \hat{g}_2 = g_{ab} = -3,\ \hat{g}_3 = g_{fd} = 4.$

- Largest partial sum= $\max \sum_{i=1}^{k} \hat{g}_i = 0\ (k = 3) \Rightarrow \text{Stop!}$

**Algorithm: Kernighan-Lin**($G$)

**Input:** $G=(V,E),|V|=2n$.

**Output:** Balanced bi-partition $A$ and $B$ with “small” cut cost.

**begin**

Bi-partition $G$ into $A$ and $B$ such that $|V_A|=|V_B|$, $V_A \cap V_B = \emptyset$ , and $V_A \cup V_B = V$.

**repeat**

  Compute $D_v$, $\forall v \in V$.

  **for** $i$=1 **to** $n$ **do**

    Find a pair of unlocked vertices $v_{ai} \in V_A$ *and* $v_{bi} \in V_B$ whose exchange makes the largest decrease or smallest increase in cut cost;

    Mark $v_{ai}$ and $v_{bi}$ as locked, store the gain $\hat{g}_i$ ,and compute the new $D_v$, for all unlocked $v \in V$;

   Find $k$, such that $G_k = \sum_{i=1}^{k} \hat{g}_i$ is maximized;

   **if** $G_k > 0$ **then**

    Move $v_{ai},\ldots,v_{ak}$ from $V_A$ to $V_B$ and $v_{bi},\ldots,v_{bk}$ from $V_B$ to $V_A$ ;

   Unlock $v$, $\forall v \in V$.

**until** $G_k \leq 0$;

**end**

# Time Complexity

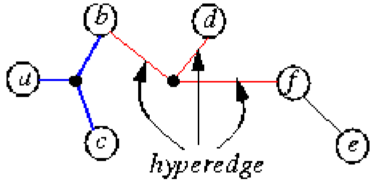
- Line 4: Initial computation of $D$: $O(n^2)$
- Line 5: The **for-**loop: $O(n)$
- The body of the loop: $O(n^2)$
  - Lines 6-7: Step $i$ takes $O(n\text{-}i+1)^2$ time.
- Lines 4-11: Each pass of the repeat loop: $O(n^3)$.
- Suppose the repeat loop terminates after $r$ passes.
- The total running time: $O(rn^3)$.

# Extension of KL Algorithm

- *k*-way partitioning
  1. Partition the graph into *k* equal-sized sets. Apply the KL algorithm for each pair of subsets.
  2. Apply the KL algorithm recursively.

# Drawbacks of KL Algorithm

- KL handles only unit vertex weights.
  - Vertex weights might represent block sizes, different from blocks to blocks.
  - Reducing a vertex with weight $w(v)$ into a clique with $w(v)$ vertices and edges with a high cost increases the size of the graph substantially.
- KL handles only exact bisections.
  - Need dummy vertices to handle the unbalanced problem.
- KL cannot handle hypergraphs.
  - A hypergraph consists of a set of vertices and a set of hyperedges, where each hyperedge $e_i$ corresponds to a subset $N_i$ of distinct vertices with $|N_i| \geq 2$
- The time complexity of a pass is high, $O(n^3)$.

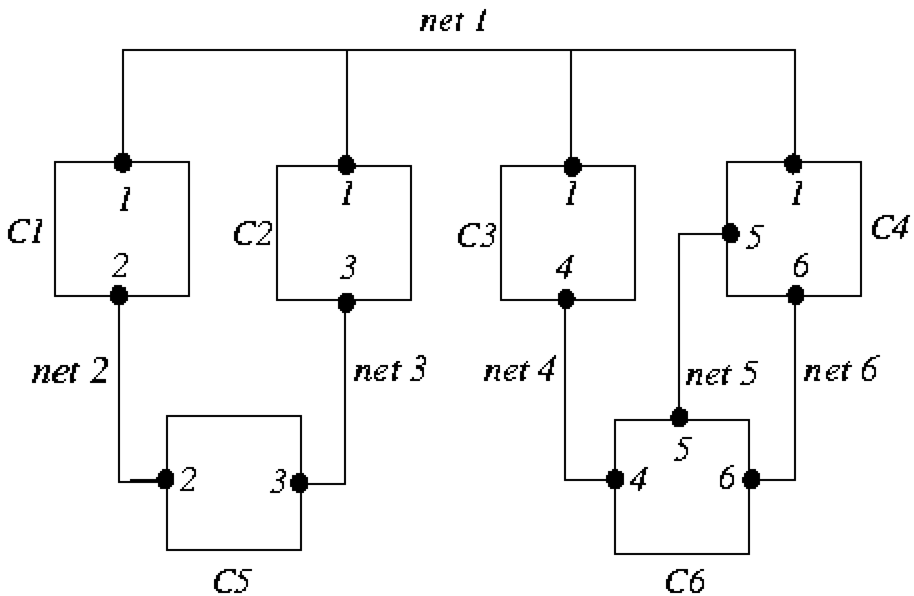


# Fiduccia-Mattheyses (FM) Algorithm

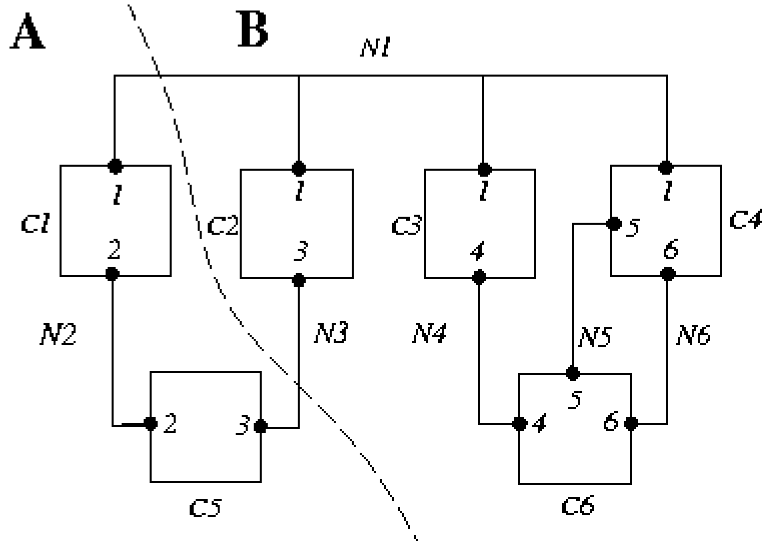
- Fiduccia and Mattheyses. "A linear time heuristic for improving network partitions," 19th Design Automation Conf.,1982.
- Same as KL:
  - Work in passes.
  - Lock vertices after moved.
  - Actually, only move those vertices up to the maximum partial sum of gain.
- Different from KL:
  - Aim at **reducing net-cut costs**; the concept of cut size is extended to hypergraphs.
  - Only a **single vertex** is moved across the cut in a move.
  - Vertices are weighted.
  - Can handle "unbalanced" partitions; a balance factor is introduced.
  - A special data structure is used to select vertices to be moved across the cut to improve running time.
  - **Time complexity of a pass is** $O(P)$, where $P$ is the total # of pins.

# FM: Notation

- $n(i)$: # of cells in Net $i$; e.g., $n(1)=4$.
- $S(i)$: size of Cell $i$.
- $P(i)$: # of pins in Cell $i$; e.g., $p(6)=3$.
- $C$: total # of cells; e.g., $C=6$.
- $N$: total # of nets; e.g., $N=6$.
- $P$: total # of pins; $P = p(1)+\ldots+p(C) = n(1)+\ldots+n(N)$.

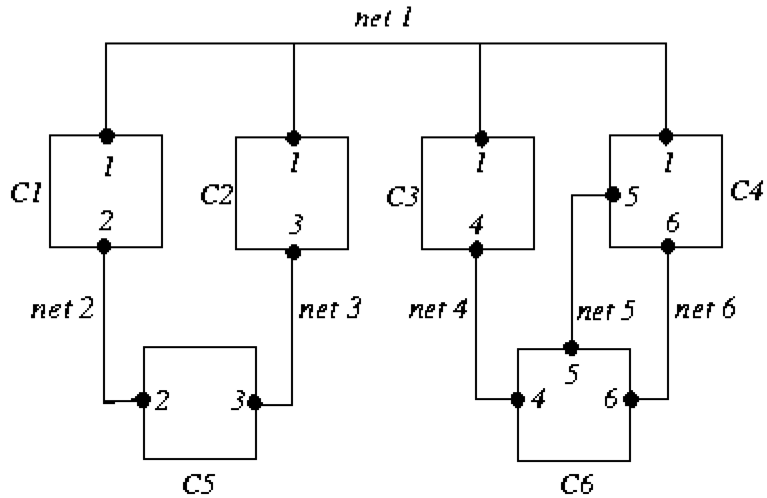


# Cut




- **Cutstate** of a net:
  - Net 1 and Net 3 are **cut**.
  - Net 2, Net 4, Net 5, and Net 6 are **uncut.**
- **Cutset**= {Net 1, Net 3}.
- $|A|$=size of $A$ = $s(1)+s(5)$ ; $|B|=s(2)+s(3)+s(4)+s(6)$.
- Balanced 2-way partitioning: Given a fraction $r$, $0< r <1$, partition a graph into two sets $A$ and $B$ such that
  - $\frac{|A|}{|A|+|B|} \approx r.$
  - Size of the cutset is minimized.

# Input Data Structures




| Cell array | | Net array | |
|---|---|---|---|
| C1 | Nets 1, 2 | Net 1 | C1, C2, C3, C4 |
| C2 | Nets 1, 3 | Net 2 | C1, C5 |
| C3 | Nets 1, 4 | Net 3 | C2, C5 |
| C4 | Nets 1, 5, 6 | Net 4 | C3, C6 |
| C5 | Nets 2, 3 | Net 5 | C4, C6 |
| C6 | Nets 4, 5, 6 | Net 6 | C4, C6 |

- Size of the circuit: $P = \sum_{i=1}^{6} n(i) = 14$
- Construction of the two arrays takes $O(P)$ time.

# Basic Ideas: Balance and Movement

- Only move a cell at a time, preserving "balance."

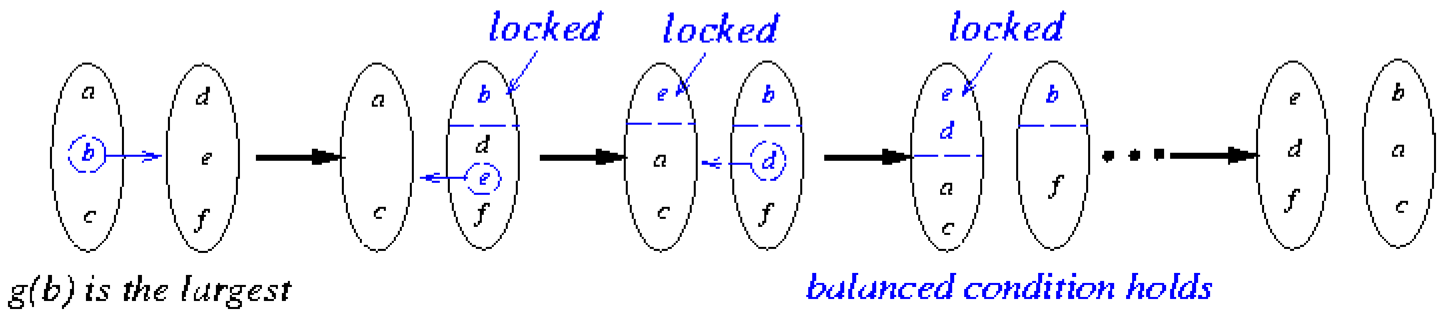
$$\frac{|A|}{|A|+|B|} \approx r$$

$$rW - S_{\max} \le |A| \le rW + S_{\max},$$

when $W$=|$A$|+|$B$|; $S_{\max}$=$\max_i s(i)$.

- $g(i)$: gain in moving cell $i$ to the other set,

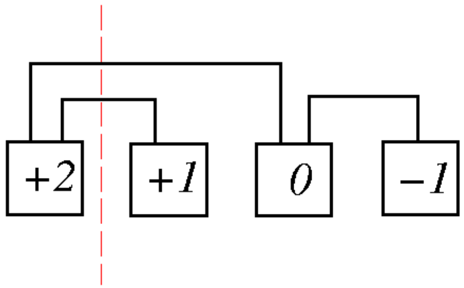
  i.e., size of **old** cutset - size of **new** cutset.

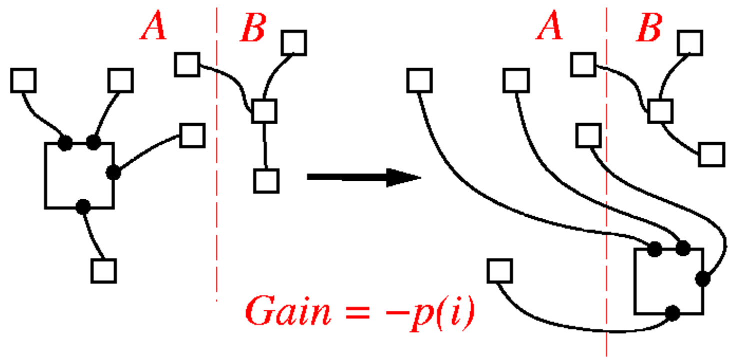



- Suppose $\hat{g}_i$'s: $g(b)$, $g(e)$, $g(d)$, $g(a)$, $g(f)$, $g(c)$ and the largest partial sum is $g(b)+g(e)+g(d)$. Then we should move $b,e,d$ ➔ resulting two sets:{$a,c,e,d$},{$b,f$}

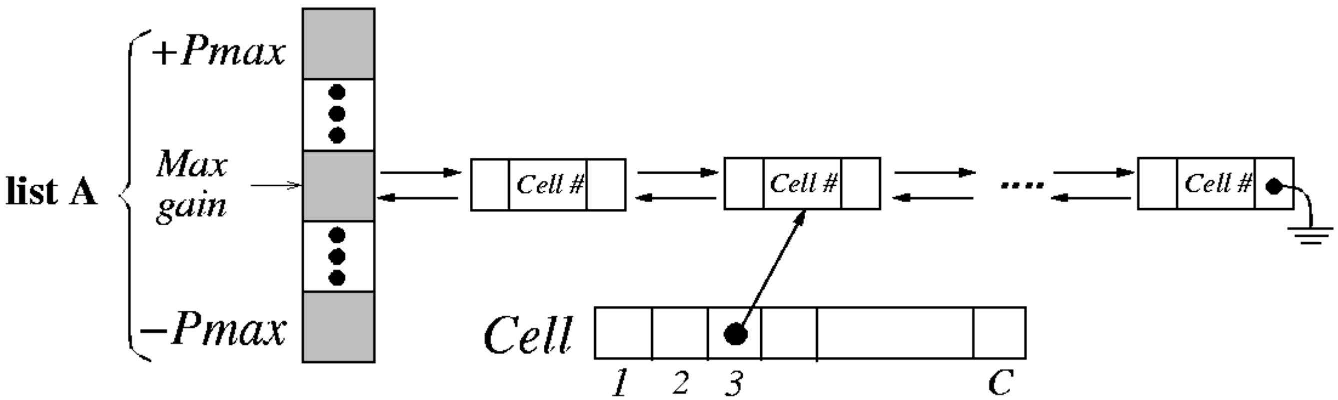
# Cell Gains and Data Structure Manipulation

- $-p(i) \leq g(i) \leq p(i)$



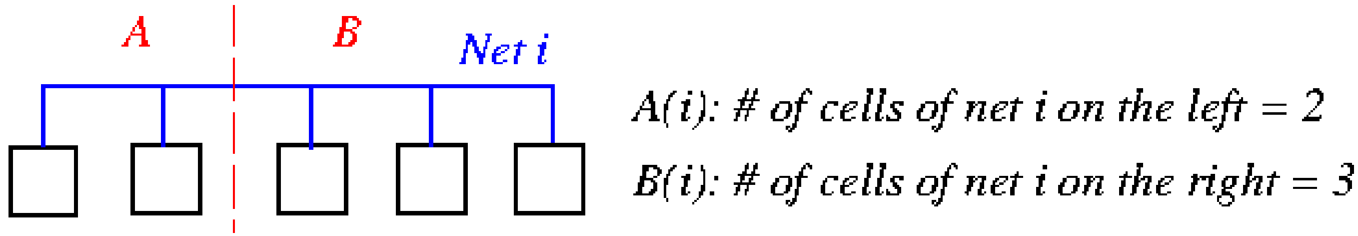

- Two “bucket list” structures, one for set $A$ and one for set $B$ ($P_{max}$=$\max_i p(i)$).



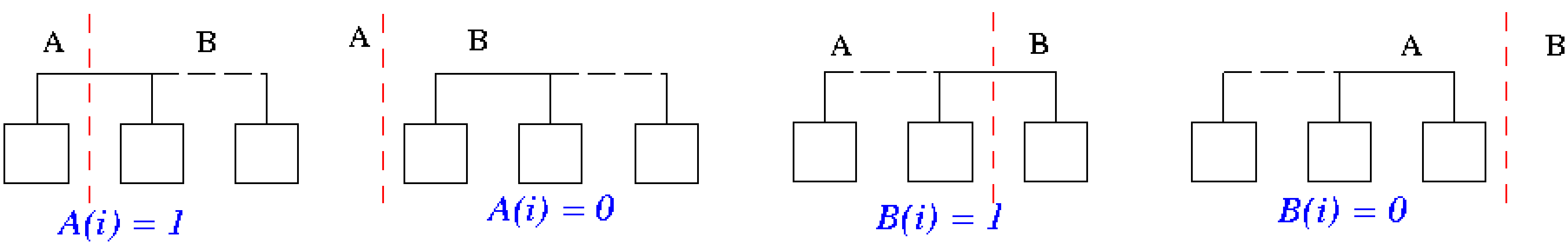

- $O(1)$-time operations: find a cell with Max Gain, remove Cell $i$ from the structure, insert Cell $i$ into the structure, update $g(i)$ to $g(i) + \triangle$, update the Max Gain pointer.

# Net Distribution and Critical Nets

- Distribution of Net $i$: $(A(i), B(i)) = (2, 3)$.
  - $(A(i), B(i))$ for all $i$ can be computed in $O(P)$ time.




- **Critical Nets**: A net is critical if it has a cell which if moved will change its cutstate.
  - 4 cases: $A(i) = 0$ or $1$, $B(i) = 0$ or $1$.

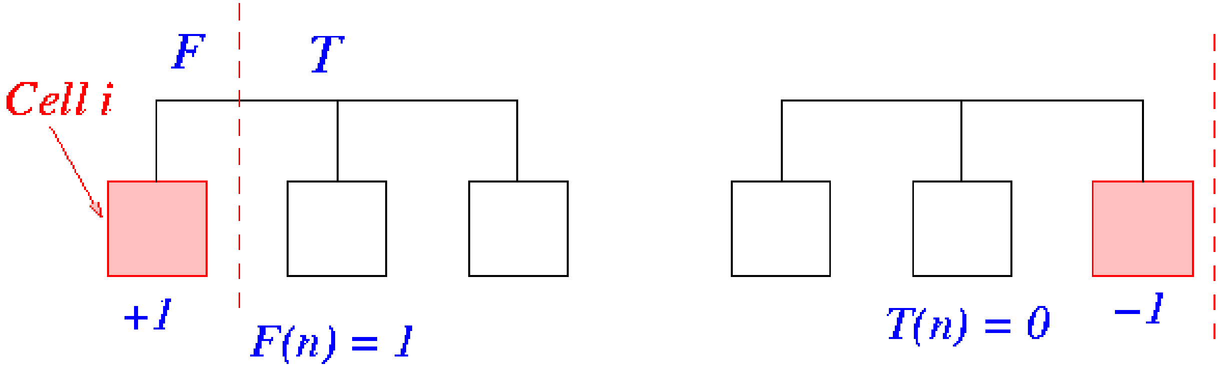



- Gain of a cell depends only on its critical nets.

# Computing Cell Gains

- Initialization of all cell gains requires $O(P)$ time:

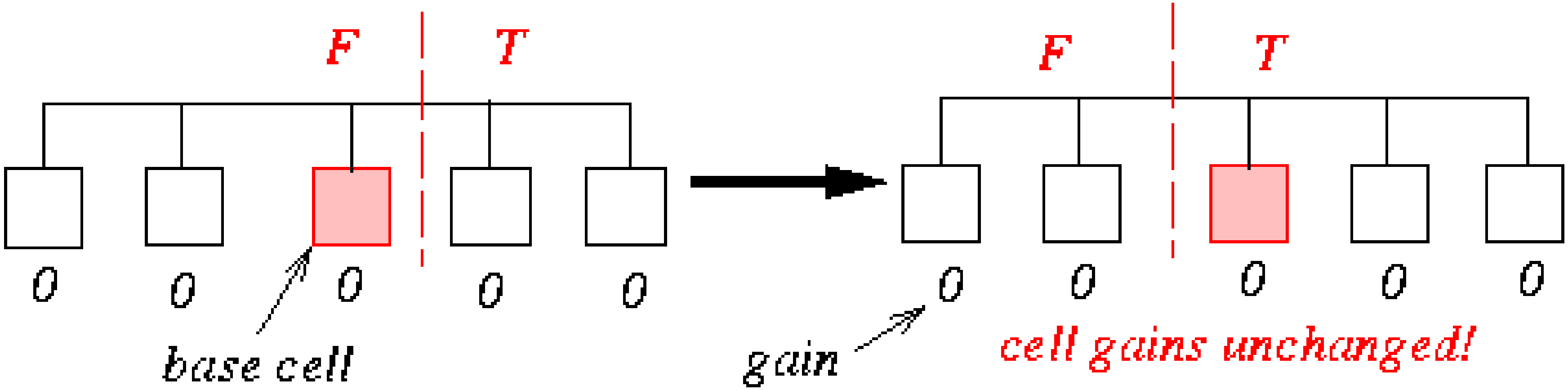
  $g(i) \leftarrow 0$;
  $F \leftarrow$ the "from block" of cell $i$;
  $T \leftarrow$ the " to block" of cell $i$;
  for each net $n$ on Cell $i$ **do**
      **if** $F(n) = 1$ then $g(i) \leftarrow g(i) + 1$;
      **if** $T(n) = 0$ then $g(i) \leftarrow g(i) - 1$;




- Only need $O(P)$ time to maintain all cell gains in one pass.

# Updating Cell Gains

- To update the gains, we only need to look at those nets, connected to the base cell, which are critical before or after the move.
- **Base cell**: The cell selected for movement from one set to the other.



# Algorithm for Updating Cell Gains

**Algorithm: Update_Gain**

1 **begin** /* move base cell and update neighbors' gains */

2 $F$ ← the *Front Block* of the base cell;

3 $T$ ← the *To Block* of the base cell;

4 Lock the base cell and complement its block;

5 **for** each net $n$ on the base cell **do**

/* check critical nets before the move */

6 **if** $T(n) = 0$ then increment gains of all free cells on $n$
**elseif** $T(n) = 1$ **then** decrement gain of the only $T$ cell on $n$, if it is free

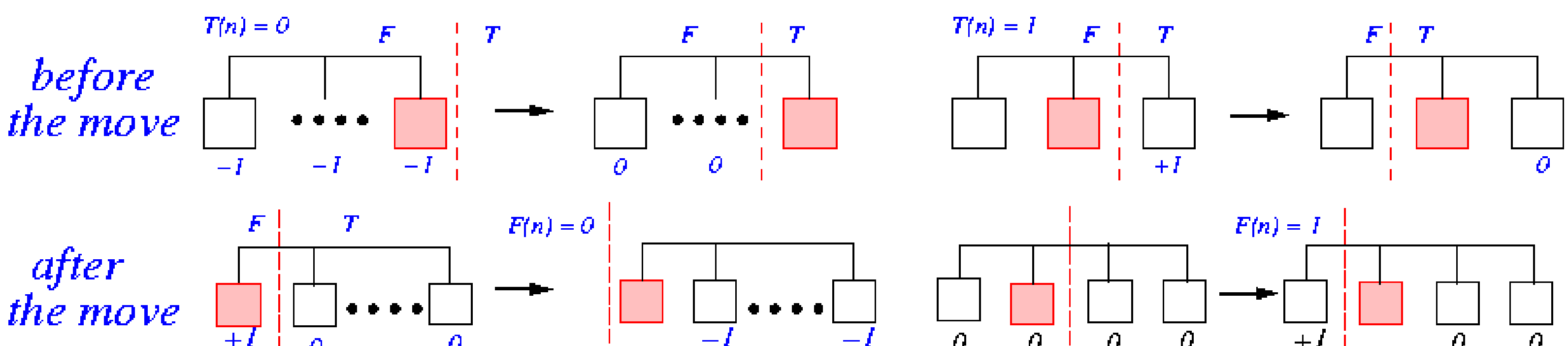
/* change $F(n)$ and $T(n)$ to reflect the move */

7 $F(n) \leftarrow F(n) - 1$; $T(n) \leftarrow T(n) + 1$;

/* check for critical nets after the move */

8 **if** $F(n) = 0$ **then** decrement gains of all free cells on $n$
**elseif** $F(n) = 1$ **then** increment gain of the only $F$ cell on $n$, if it is free

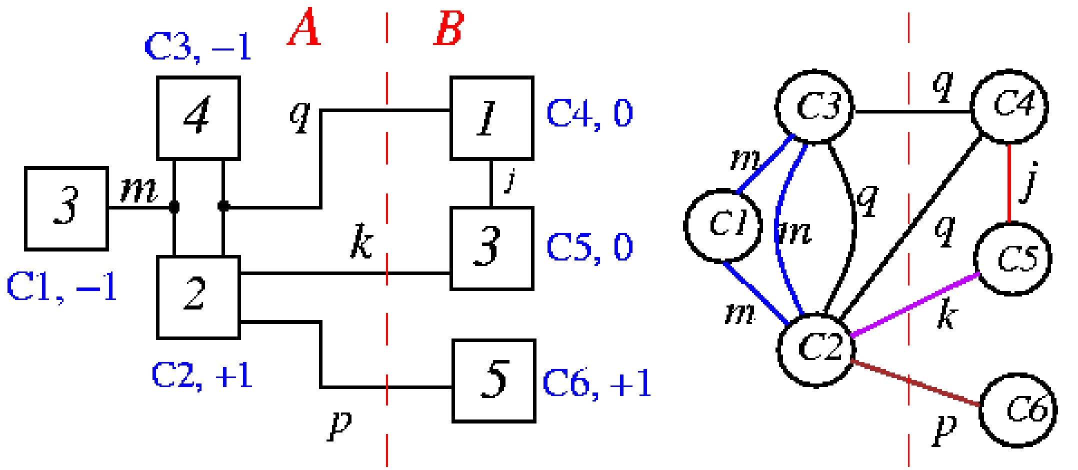
9 **end**



# Complexity of Updating Cell Gains

- Once a net has "locked" cells at both sides, the net will remain cut from now on.
- To update the cell gains, it takes $O(n(i))$ work for Net $i$.
- Total time = $n(1) + n(2) + \ldots + n(N) = O(P)$

# FM: An Example

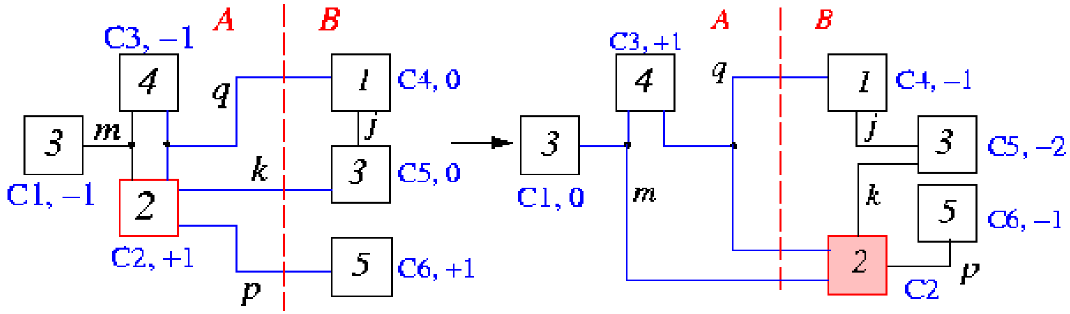



- Computing cell gains: $F(n) = 1 \Rightarrow g(i) + 1$; $T(n) = 0 \Rightarrow g(i) - 1$

| Cell | $m$ | | $q$ | | $k$ | | $p$ | | $j$ | | $g(i)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | $F$ | $T$ | $F$ | $T$ | $F$ | $T$ | $F$ | $T$ | $F$ | $T$ | |
| c1 | 0 | −1 | | | | | | | | | −1 |
| c2 | 0 | −1 | 0 | 0 | +1 | 0 | +1 | 0 | | | +1 |
| c3 | 0 | −1 | 0 | 0 | | | | | | | −1 |
| c4 | | | +1 | 0 | | | | | 0 | −1 | 0 |
| c5 | | | | | +1 | 0 | | | 0 | −1 | 0 |
| c6 | | | | | | | +1 | 0 | | | +1 |

- Balanced criterion: $r|V| - S_{max} \le |A| \le r|V| + S_{max}$. Let $r = 0.4 \Rightarrow |A|=9$, $|V|=18$, $S_{max} = 5$, $r|V| = 7.2 \Rightarrow$ Balanced: $2.2 \le 9 \le 12.2$!
- Maximum gain: $c_2$ and balanced: $2.2 \le 9\text{-}2 \le 12.2 \Rightarrow$ Move $c_2$ from $A$ to $B$ (use size criterion if there is a tie).

# FM: An Example (Cont'd)




- Changes in net distribution:

| Net | Before move $F$ | Before move $T$ | After move $F'$ | After move $T'$ |
|---|---|---|---|---|
| $k$ | 1 | 1 | 0 | 2 |
| $m$ | 3 | 0 | 2 | 1 |
| $q$ | 2 | 1 | 1 | 2 |
| $p$ | 1 | 1 | 0 | 2 |

- Updating cell gains on critical nets (run Algorithm Update_Gain):

| Cells | Gains due to $T(n)$ | | | | Gain due to $F(n)$ | | | | Gain changes | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $k$ | $m$ | $q$ | $p$ | $k$ | $m$ | $q$ | $p$ | Old | New |
| $c_1$ | | +1 | | | | | | | −1 | 0 |
| $c_3$ | | +1 | | | | | +1 | | −1 | +1 |
| $c_4$ | | | −1 | | | | | | 0 | −1 |
| $c_5$ | −1 | | | | −1 | | | | 0 | −2 |
| $c_6$ | | | | −1 | | | | −1 | +1 | −1 |

- Maximum gain: $c_3$ and balanced! ($2.2 \leq 7\text{-}4 \leq 12.2$)

=> Move $c_3$ from A to B (use size criterion if there is a tie).

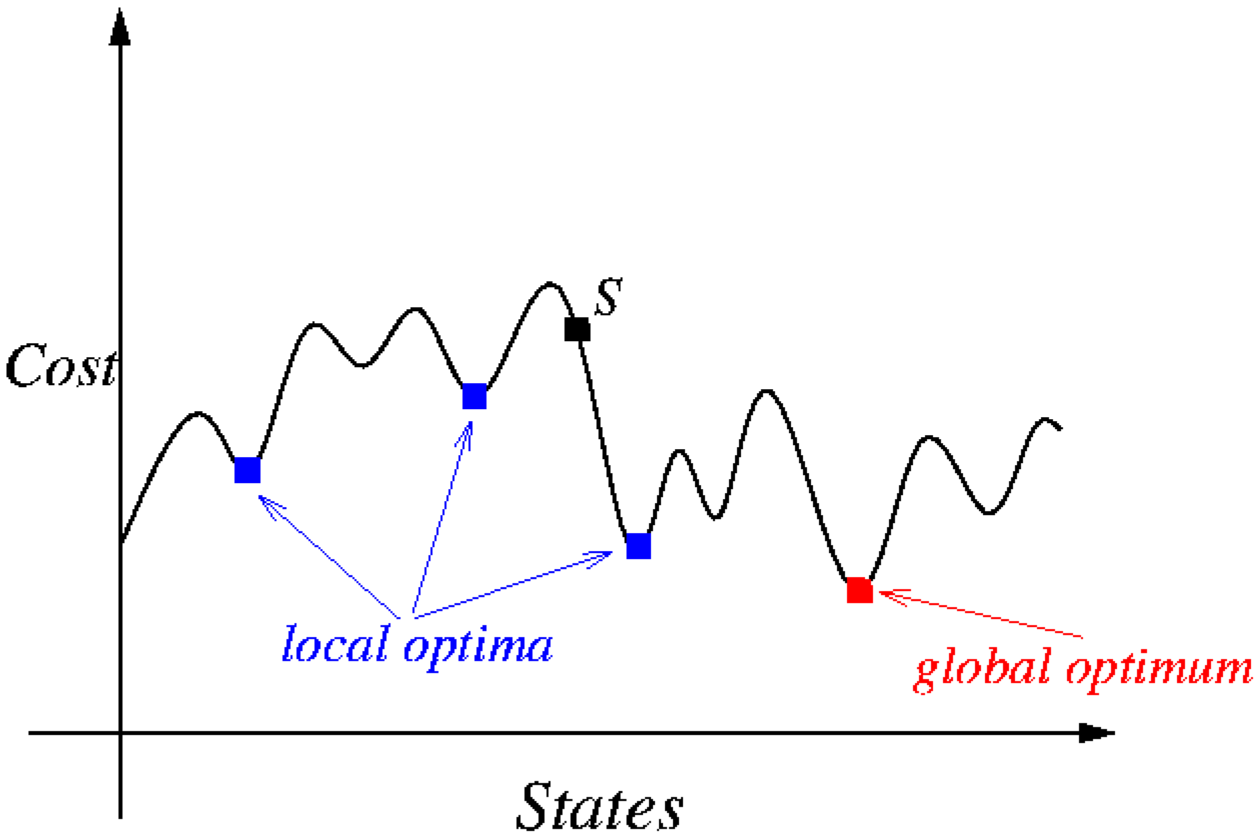
# Summary of the Example

| Step | Cell | Max gain | $\|A\|$ | Balanced? | Locked cell | $A$ | $B$ |
|---|---|---|---|---|---|---|---|
| 0 | - | - | 9 | - | $\emptyset$ | 1, 2, 3 | 4, 5, 6 |
| 1 | $c_2$ | $+1$ | 7 | yes | $c_2$ | 1, 3 | 2, 4, 5, 6 |
| 2 | $c_3$ | $+1$ | 3 | yes | $c_2, c_3$ | 1 | 2, 3, 4, 5, 6 |
| 3 | $c_1$ | $+1$ | 0 | no | - | - | - |
| $3'$ | $c_6$ | $-1$ | 8 | yes | $c_2, c_3, c_6$ | 1, 6 | 2, 3, 4, 5 |
| 4 | $c_1$ | $+1$ | 5 | yes | $c_1, c_2, c_3, c_6$ | 6 | 1, 2, 3, 4, 5 |
| 5 | $c_5$ | $-2$ | 8 | yes | $c_1, c_2, c_3, c_5, c_6$ | 5, 6 | 1, 2, 3, 4 |
| 6 | $c_4$ | 0 | 9 | yes | all cells | 4, 5, 6 | 1, 2, 3 |

- $g_1$=1, $g_2$=1, $g_3$=-1, $g_4$=1, $g_5$=-2, $g_6$=0 => Maximum partial sum $G_k$ = +2, $k$ = 2 or 4.
- Since $k$ = 4 results in a better balanced => Move $c_1$, $c_2$, $c_3$, $c_6$ => $A$={6}, $B$={1,2,3,4,5}.
- **Repeat the whole process until new** $G_k \leq 0$.

# Simulated Annealing

- Kirkpatrick, Gelatt, and Vecchi, “Optimization by simulated annealing,” *Science*, May 1983.



# Simulated Annealing Basics

- Non-zero probability for "up-hill" moves.
- Probability depends on
  1. magnitude of the "up-hill" movement
  2. total search time

$$Prob(S \rightarrow S') = \begin{cases} 1 & \text{if } \Delta C \leq 0 \quad /* \text{ ``}down - hill\text{'' } moves */ \\ e^{-\frac{\Delta C}{T}} & \text{if } \Delta C > 0 \quad /* \text{ ``}up - hill\text{'' } moves */ \end{cases}$$

- $\triangle C = cost(S') - cost(S)$
- $T$: Control parameter (temperature)
- Annealing schedule: $T = T_0, T_1, T_2, \ldots$, where $T_i = r^i T_0$, $r < 1$.

# Generic Simulated Annealing Algorithm

**Algorithm: Simulated_Annealing**

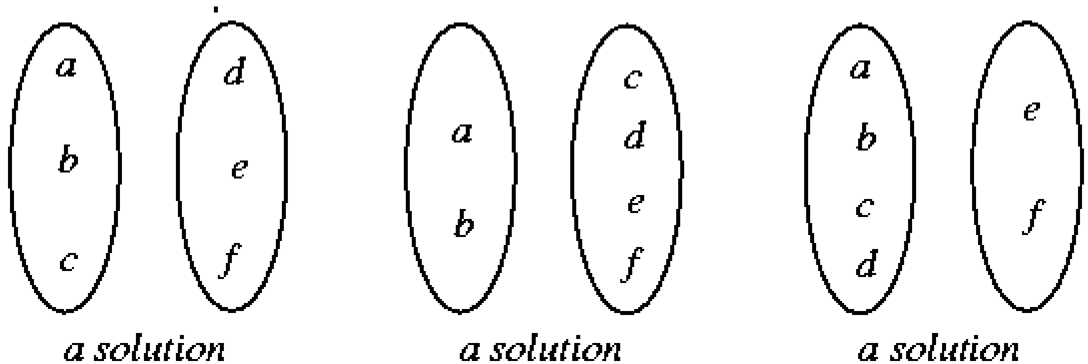
**begin**<br>
Get an initial solution $S$;<br>
Get an initial temperature $T > 0$;<br>
**while** not yet "frozen" **do**<br>
    **for** $1 \leq i \leq P$ **do**<br>
        Pick a random neighbor $S'$ of $S$;<br>
        $\triangle \leftarrow cost(S') - cost(S)$;<br>
        /* down hill move */<br>
        **if** $\triangle \leq 0$ **then** $S \leftarrow S'$<br>
        /* uphill move */<br>
        **if** $\triangle > 0$ **then** $S \leftarrow S'$ with probability $e^{-\frac{\triangle}{T}}$;<br>
    $T \leftarrow rT$ ; /* reduce temperature */<br>
**return** S<br>
**end**

# Basic Ingredients for Simulated Annealing

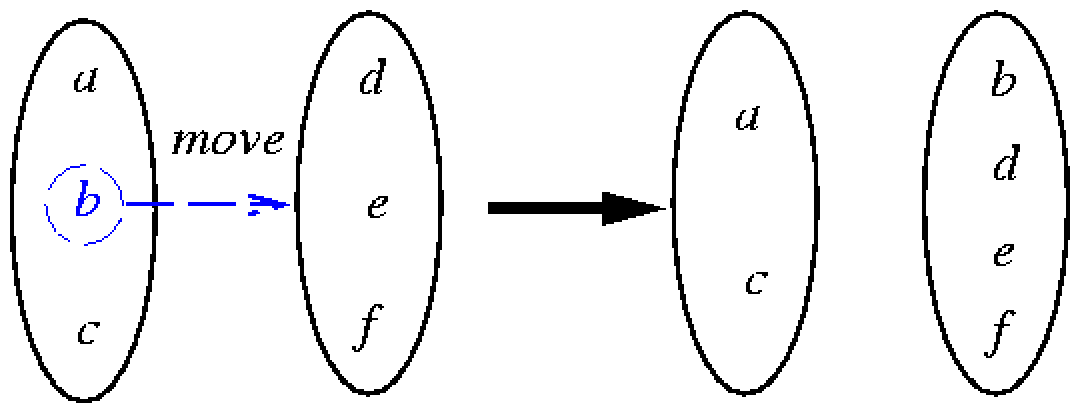
- Solution space
- Neighborhood structure
- Cost function
- Annealing schedule

# Partitioning by Simulated Annealing

- Kirkpatrick, Gelatt, and Vecchi, “Optimization by simulated annealing,” *Science*, May 1983.
- **Solution space**: set of all partitioning solutions



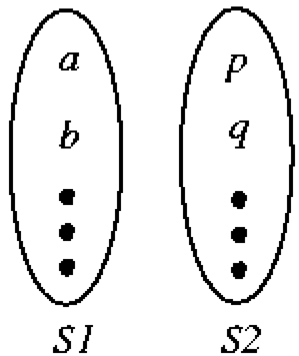

- **Neighborhood structure:**




Randomly move one cell to the other side

# Partitioning by Simulated Annealing (cont'd)

- **Cost function**: $f = C + \lambda B$
  - $C$: the solution cost as used before.
  - $B$: a measure of how balance the solution is
  - $\lambda$: a constant

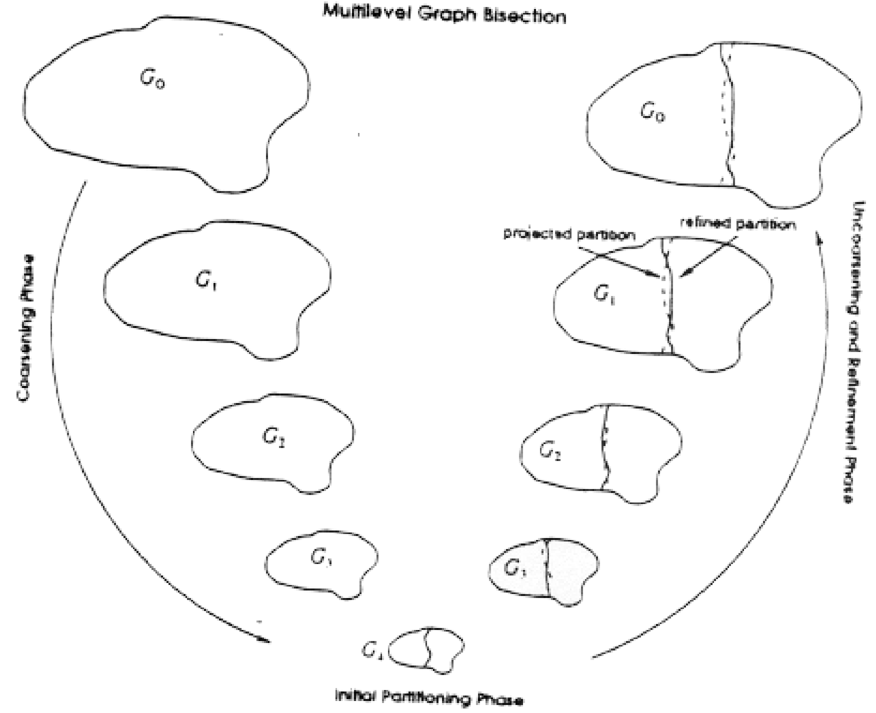



$$B = (|S1| - |S2|)^2$$

- **Annealing schedule:**
  - $T_n = r^n T_0$, $r = 0.9$.
  - At each temperature, either
    1. There are 10 accepted moves/cell on the average, or
    2. # of attempts $\geq$100 * total # of cells.
  - The system is "frozen" if very low acceptances at 3 consecutive temperatures.

# Multilevel Partitioning

- **Three phases** (for bipartitioning)
  - **Coarsening:** construct a sequence of smaller (coarser) graphs.
  - **Initial partitioning:** construct a bipartitioning solution for the coarsest graph.
  - **Uncoarsening & refinement:** the bipartitioning solution is successively projected to the next-level finer graph, and at each level an iterative refinement algorithm (such as KL or FM) is used to further improve the solution.



# hMETIS

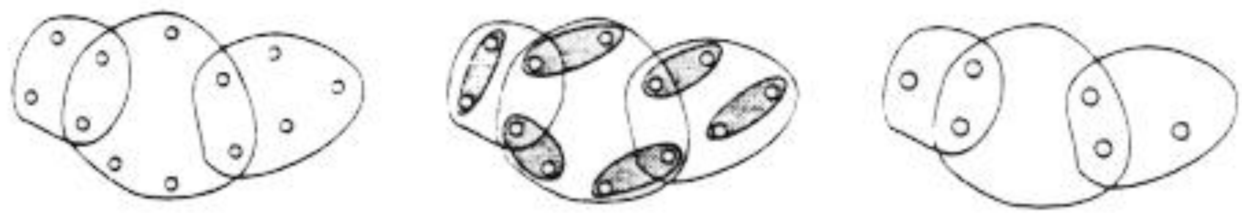
- Kayrpis, Aggarwal, Kumar and Shekhar, "Multilevel hypergraph partitioning: application in VLSI domain," DAC, 1997.
- Three coarsening algorithms:

  - **Edge coarsening:** A maximal matching of the vertices.

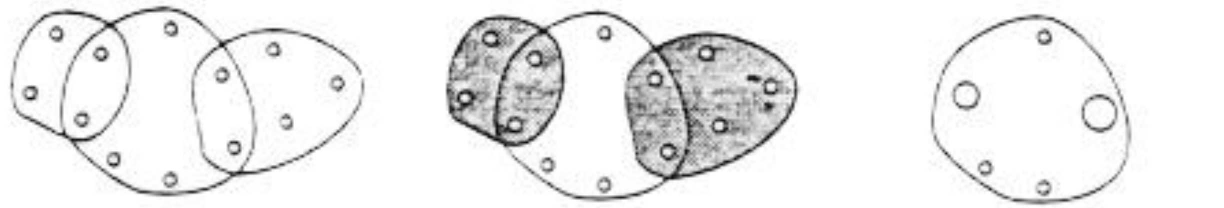
  - **Hyperedge coarsening**: a set of hyperedges is selected, and the vertices belonging to a selected hyperedge are merged into a cluster. (Preference: hyperedges with large weights and hyperedges of small size.)

  - **Modified hyperedge coarsening:** hyperedge coarsening + merging the remaining vertices of each hyperedge into a cluster.

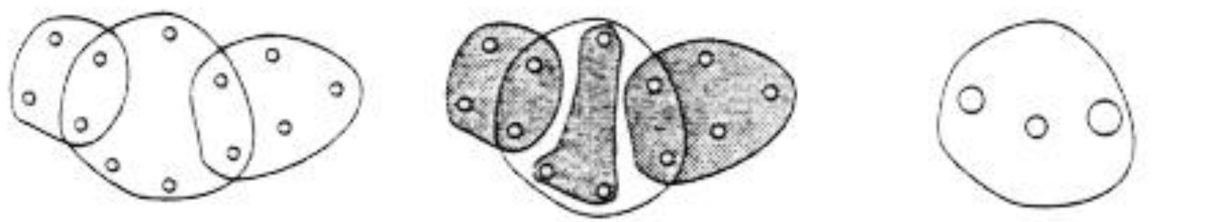
# hMETIS (Cont'd)



(a) Edge Coarsening



(b) Hyperedge Coarsening



(c) Modified Hyperedge Coarsening

# hMETIS (Cont'd)

- Two uncoarsening & refinement algorithms:

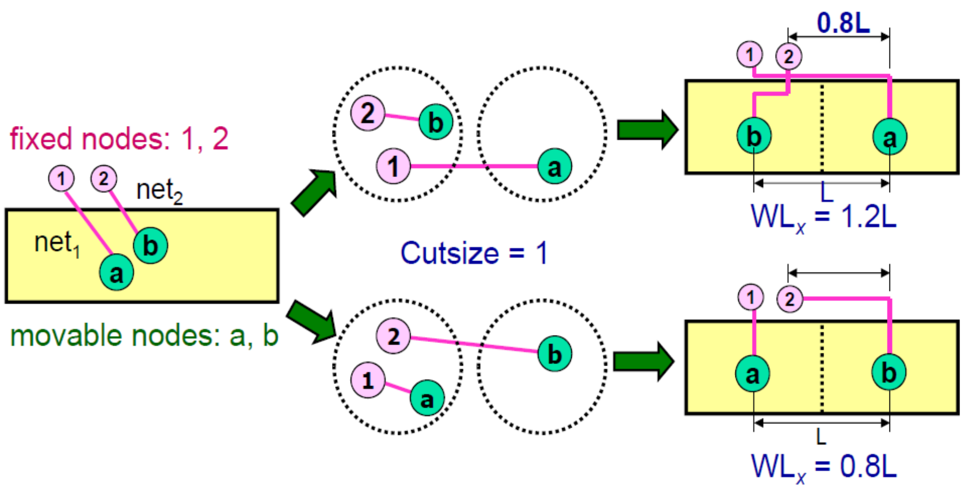
  - FM algorithm with modifications:

    * Restrict the maximum number of passes to 2.
    * Stop each pass when no improvement is made from the first *k* moves.

  - Hyperedge refinement: move groups of vertices between subsets so that an entire hyperedge is removed from the cut set.

# Partitioning for Wirelength Minimization

- Chen, Chang, Lin, "IMF: Interconnection-driven floorplanning for large-scale building-module designs," ICCAD-05
- Minimizing cut size is *not* equivalent to minimizing wirelength (WL)



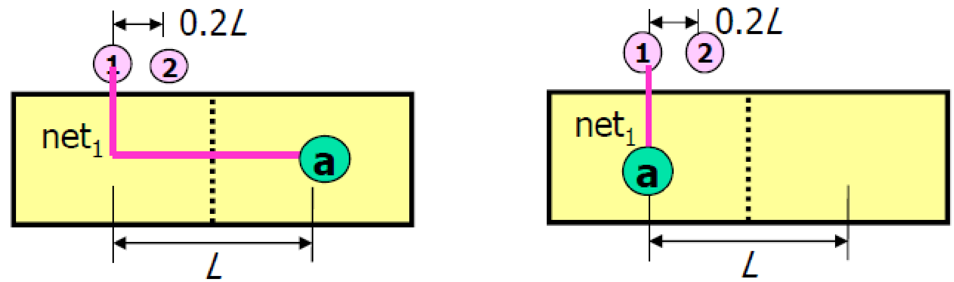

- Problem: **hyperedge weight is a constant value!**
  - Shall map the min-cut cost to wirelength (WL) change
  - Shall assign the hyperedge weight as the value of wirelength contribution if the hyperedge is cut

# Net Weight Assignment

- $net_1$ connects a movable node *a* and a fixed node 1.

Weight($net_1$) = WL($net_1$ is cut) – WL($net_1$ is not cut)

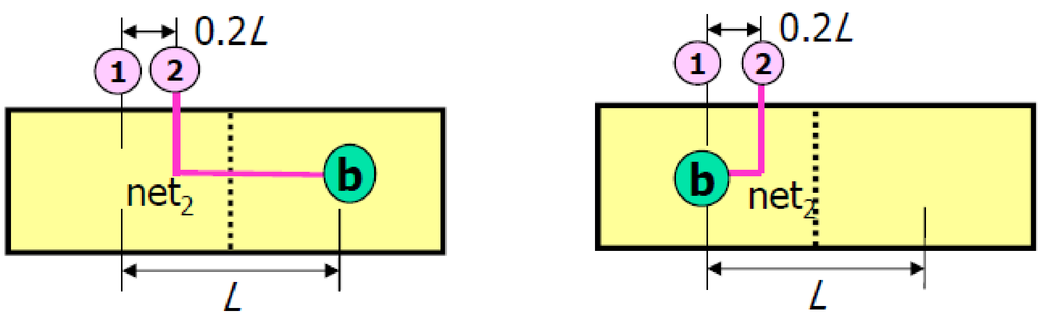
$$= L - 0L = L$$



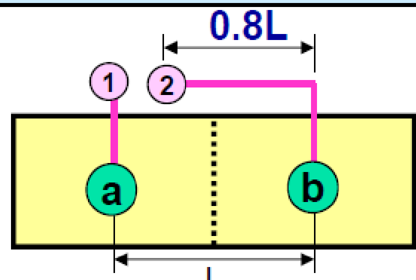
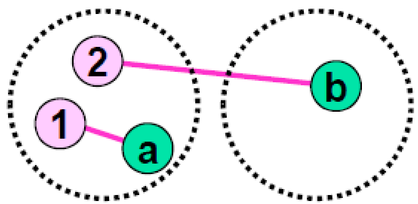
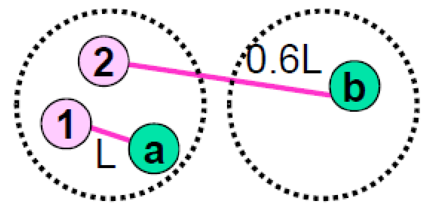
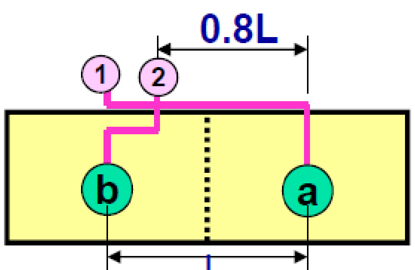
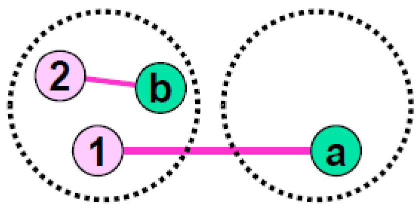
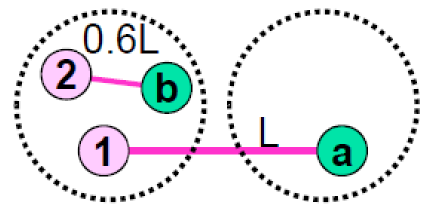

- $net_2$ connects a movable node *b* and a fixed node 2.

Weight($net_2$) = WL($net_2$ is cut) – WL($net_2$ is not cut)

$$= 0.8L - 0.2L = 0.6L$$



# Examples

| Physical Partitions | Traditional Terminal Propagation | Exact Net-Weight Modeling |
|---|---|---|
|   $WL_x = 0.8L$ |   Cutsize = 1 |   Cut weight = 0.6L |
|   $WL_x = 1.2L$ |   Cutsize = 1 |   Cut weight = 1.0L |

**Cut weight is proportitional to the wirelength (WL)**

**WL = Cut weight + 0.2L**

(0.2L is the WL lower bound: placing a & b in the left side)

# Relationship Between WL and Cut Weight

- Theorem: $WL_i = w_{1,i} + n_{cut,i}$
  - $n_{cut,i}$: cut weight for net i
  - $w_{1,j}$: the wirelength lower bound for net i

- Then, we have

Constant

$$min(\sum WL_i) = min\left(\sum\left(w_{1,i} + n_{cut,i}\right)\right) = \sum w_{1,i} + min\left(\sum n_{cut,i}\right)$$

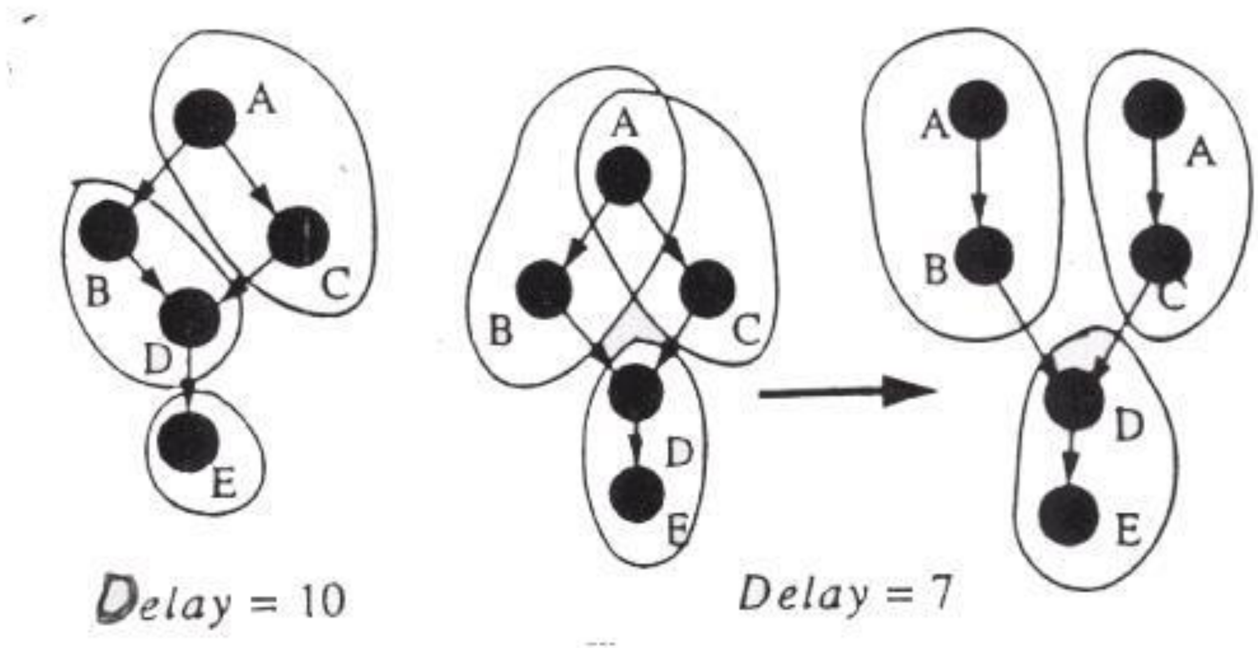
**Finding the minimum wirelength is equivalent to finding the cut weight**

# Clustering for Delay Minimization

- Allow gate duplication.
- Gate duplication may help reduce delay.

$D$=3; $M$=2; $\delta(v)$=1, $w(v)$=1, for each $v$.

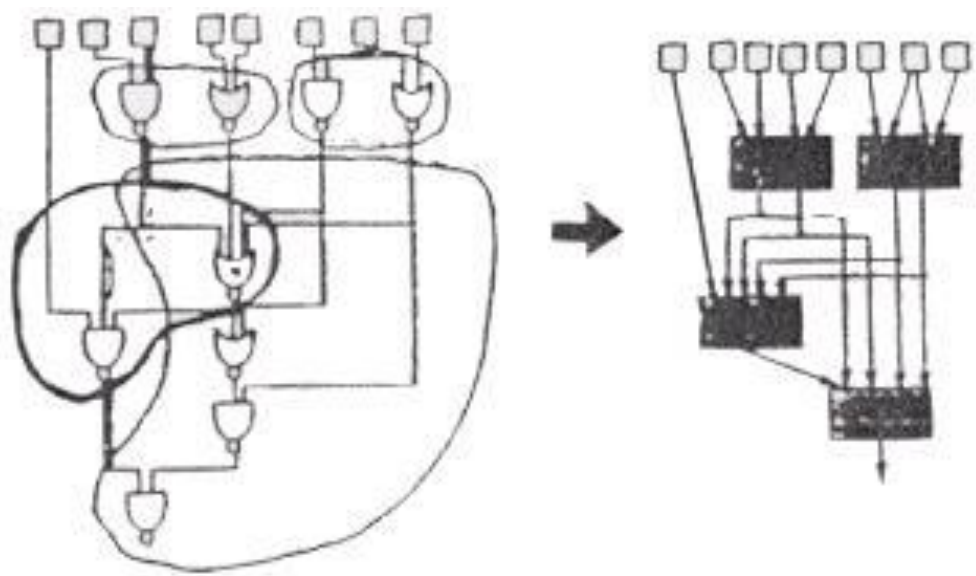
Without gate duplication With gate duplication



# Unit Delay Model

- No gate delay.
- No interconnection delay within a cluster.
- Delay of 1 unit for an interconnection between 2 clusters.
- An optimal algorithm for area constraint only (Lawler, Levitt and Turner, IEEE TC, 1966).
- An optimal algorithm for pin constraint only (Cong and Ding, ICCAD, 1992).

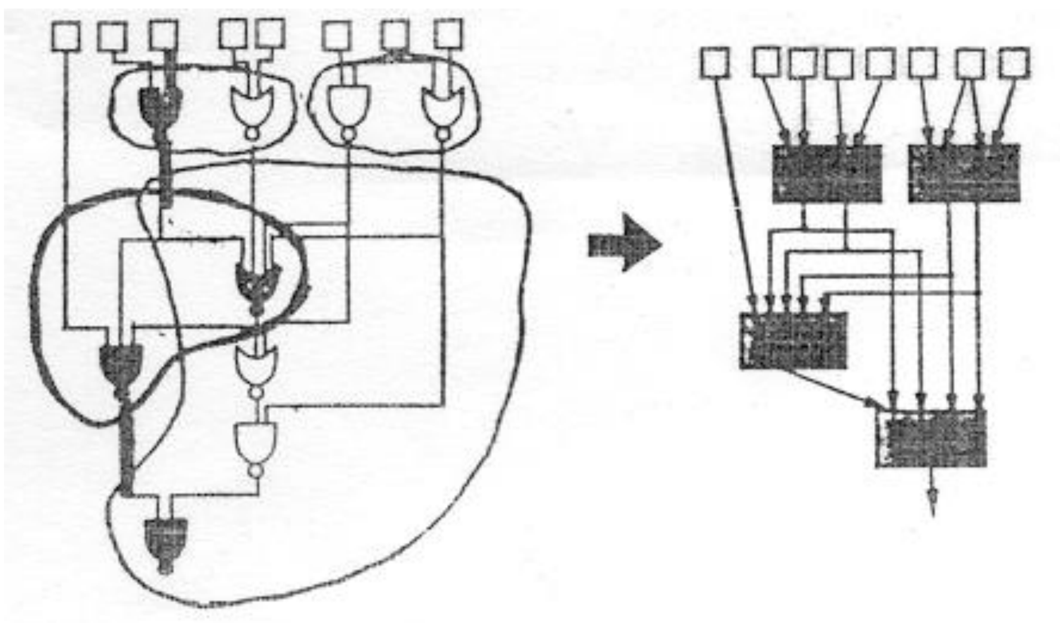
Circuit delay = 3.



# General Delay Model

- Each gate has a delay.
- No interconnection delay within a cluster.
- Delay of *D* units for an interconnection between 2 clusters.
- A heuristic algorithm for area constraint only (Murgai, Brayton and Sangivanni-Vincentelli, ICCAD, 1991).

$D = 2$, $\delta(v) = 1$, circuit delay = 6+4 = 10.



# General Delay Model (Cont'd)

- Rajaraman and Wong, "Optimal clustering for delay minimization," DAC, 1993.
- Optimal algorithm: $O(n^2 \log n + nm)$, where $n$ is # of gates, $m$ is # of interconnections.
- Definitions:

  - $M$: the area constraint on a cluster.

  - $W(C)$: the total area of the gates in cluster $C$.

  - $N$: a given combinational circuit.

  - $N_v$: $v$ and all its *predecessors* in $N$.

  - $\delta(v)$: the delay of $v$.

  - $\Delta(u,v)$: maximum delay along any path from the output of $u$ to the output of $v$, ignoring delays on interconnections.

  - $w(v)$: the area of $v$.

  - $l(v)$: the delay at $v$ in an optimal clustering of $N_v$.

    For each *primary input* $v$, $l(v)=\delta(v)$.

  - $l'(u)=l(u)+\Delta(u,v)$, for each $u$ in $N_v$-$\{v\}$.

# General Delay Model (Cont'd)

- Algorithm: labeling phase + clustering phase.
- **Labeling phase:** compute $l(v)$ for each $v$ in a topological order.

  \- $P$: the set of nodes in $N_v$-$\{v\}$ sorted in non-increasing order in the value of $l'$.

**Algorithm** *Labeling*$(v)$;
**begin**
$done \leftarrow false$;
$cluster(v) \leftarrow \{v\}$;
**while** ($not\ done$)
    Remove the first node $u$ in $P$;
    **if** $(W(cluster(v)) + w(u)) \leq M)$
        $cluster(v) \leftarrow cluster(v) \cup \{u\}$;
        **if** $P$ is empty
            $done \leftarrow true$;
        **endif**
    **else**
        $done \leftarrow true$;
    **endif**
**endwhile**
$l_1(v) \leftarrow max\{l'(x) \mid x \in cluster(v) \cap \mathcal{PI}\}$;
$l_2(v) \leftarrow \cancel{l'(u) + D;}\ max\{l'(u)+D \mid u \in N_v - cluster(v)\}$
$l(v) \leftarrow max\{l_1(v)\ , l_2(v)\}$;
**end**

# General Delay Model (Cont'd)

- **Clustering phase:** generate the clusters based on the information obtained in the labeling phase.
- Overall algorithm:

**begin**<br>
Compute the maximum delay matrix $\Delta$. $\Delta(i, j)$ is the maximum delay along any path from the output of $i$ to the output of $j$;<br>
**for** each $PI$ $i$, **do** $l(i) \leftarrow \delta(i)$;<br>
Sort the non-PI nodes of $N$ in topological order to obtain list $T$;<br>
**while** $T$ is non-empty<br>
    Remove the first node $v$ from $T$;<br>
    Compute $N_v$;<br>
    **for** each node $u \in N_v \backslash \{v\}$ do<br>
        $l'(u) \leftarrow l(u) + \Delta(u, v)$;<br>
    Sort the nodes in $N_v \backslash \{v\}$ in order of decreasing value of $l'$ to form list $P$;<br>
    Call $Labeling(v)$;<br>
**endwhile**<br>
$L \leftarrow \mathcal{PO}$;<br>
$S \leftarrow \phi$;<br>
**while** $L$ is not empty<br>
    Remove a node $v$ from $L$; $N - cluster(v)$<br>
    $S \leftarrow S \cup \{cluster(v)\}$;<br>
    **for** all nodes $x$ in $N$, such that $x$ is adjacent to $y$, for some $y \in cluster(v)$, $L \leftarrow L \cup \{x\}$;<br>
**endwhile**<br>
**end**

# General Delay Model (Cont'd)

- An example: *M*=3, *D*=3