Theory of Computation

Homework 5

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December 23, 2012

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Let t be any negative real number.

$$Pr[X \leq (1-\theta)pn] = Pr[e^{tX} \geq e^{t(1-\theta)pn}] \leq \frac{E[e^{tX}]}{e^{t(1-\theta)pn}}$$

(Markov's inequality, with $k = e^{t(1-\theta)pn}/E[e^{tX}]$)

$$Pr[X \leq (1-\theta)pn] \leq \frac{E[e^{tX}]}{e^{t(1-\theta)pn}} \leq \frac{e^{(e^t-1)pn}}{e^{t(1-\theta)pn}}$$

Let $t = ln(1-\theta) < 0$

$$Pr[X \leq (1-\theta)pn] \leq \frac{e^{(e^t-1)pn}}{e^{t(1-\theta)pn}} = (\frac{e^{-\theta}}{(1-\theta)^{(1-\theta)}})^{pn}$$

$$\begin{aligned}(1-\theta)ln(1-\theta) &= (1-\theta)(-\theta - \frac{\theta^2}{2} - \frac{\theta^3}{3} - ...)\\ &= -\theta + \frac{\theta^2}{2} + (\frac{1}{2} - \frac{1}{3})\theta^3 + (\frac{1}{3} - \frac{1}{4})\theta^4 ... \geq -\theta + \frac{\theta^2}{2}\\ e^{(1-\theta)ln(1-\theta)} &= (1-\theta)^{(1-\theta)} \geq e^{-\theta + \frac{\theta^2}{2}}\end{aligned}$$

$$Pr[X \leq (1-\theta)pn] \leq (\frac{e^{-\theta}}{(1-\theta)^{(1-\theta)}})^{pn} \leq (\frac{e^{-\theta}}{e^{-\theta+\frac{\theta^2}{2}}})^{pn} = e^{\frac{-\theta^2 pn}{2}}$$

2

If L is in BPP, then there is a probabilistic polynomial-time algorithm A for L running in polynomial-time $p(n)$. If we want to know whether a input $x \in \{0,1\}^n$ is in L or not, we need to calculate the probability below:

$$Pr[A(x;r)accepts]$$

$r \in \{0,1\}^{m(n)}$ is nondeterministic choices of computation path. $A(x;r)$ is running algorithm A on input x with computation path r.

$$Pr[A(x;r)accepts] = \frac{\sum_{r \in \{0,1\}^{m(n)}} A(x;r)}{2^{m(n)}}$$

We can compute it in deterministic time $2^{m(n)} \times p(n)$, so $BPP \subseteq EXP$.