Theory of Computation

Homework 4

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1 Problem 1

If there is a reduction from language L to another language $L' \in BPP$ runs in polynomial time. It clearly that $L \in BPP$ because it is decided by the following precise machine N: Run reduction function on input x, then run the machine N' which decides L' on the transformed input. N decides L and fullfills the accepting condiction required for BPP, so $L \in BPP$. Thus BPP is closed under reductions.

2 Problem 2

Let M_1 decides L_1, M_2 decides L_2,$L_1, L_2 \in RP$, we can build a machine $M_\cap$ to decide a input x belongs to a intersection language, for input x, we first simulate $M_1(x)$, if M_1 rejects, $M_\cap$ rejects x, else simulate $M_2(x)$, if $M_2(x)$ rejects, $M_\cap$ rejects, otherwise accepts input. $M_\cap$ accepts input $x \in L_1 \cap L_2$ with probability $\geq \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, rejects $x \notin L_1 \cap L_2$ with probability 1. Running $M_\cap(x)$ 3 times, the accepting probability is $1 - (1 - \frac{1}{4})^3 = \frac{37}{64} \geq \frac{1}{2}$, so $L_1 \cap L_2 \in RP$. So, RP is closed under intersection.