ELE 382: Statistical Signal Processing	Fall 2017

Homework 6

Due date: Monday, Nov. 13 (at the beginning of class)

You are allowed to drop 1 problem without penalty. If you attempt all problems, then the problem with the lowest score will automatically be dropped. In addition, up to 2 bonus points will be awarded to each problem for clean, well-organized, and elegant solutions.

1. Bertsekas 9.20 (*10 points*)

A random variable X is characterized by a normal PDF with mean $\mu_0 = 20$, and a variance that is either $\sigma_0^2 = 16$ (hypothesis H_0) or $\sigma_1^2 = 25$ (hypothesis H_1). We want to test H_0 against H_1, using three sample values x_1, x_2, x_3, and a rejection region of the form

$$R = \{x \mid x_1 + x_2 + x_3 > \gamma\}$$

for some scalar γ. Determine the value of γ so that the probability of false rejection (Type I error) is 0.05. What is the corresponding probability of false acceptance (Type II error)?

2. Bertsekas 9.21 (*10 points*)

A normal random variable X is known to have a mean of 60 and a standard deviation equal to 5 (hypothesis H_0) or 8 (hypothesis H_1).

(a) Consider a hypothesis test using a single sample x. Let the rejection region be of the form

$$R = \{x \mid |x - 60| > \gamma\}$$

for some scalar γ. Determine γ so that the probability of false rejection of H_0 is 0.1. What is the corresponding false acceptance probability? Would the rejection region change if we were to use the LRT with the same false rejection probability?

(b) Consider a hypothesis test using n independent samples $x_1, \cdots, x_n$. Let the rejection region be of the form

$$R = \left\{(x_1, \cdots, x_n) \mid \left|\frac{x_1 + \cdots + x_n}{n} - 60\right| > \gamma\right\},$$

where γ is chosen so that the probability of false rejection of H_0 is 0.1. How does the false acceptance probability change with n? What can you conclude about the appropriateness of this type of test?

(c) Derive the structure of the LRT using n independent samples $x_1, \cdots, x_n$.

3. Bertsekas 9.23 (*10 points*)

The number of phone calls received by a ticket agency on any one day is Poisson distributed. On an ordinary day, the expected value of the number of calls is λ_0, and on a day where there is a popular show in town, the expected value of the number of calls is λ_1, with $\lambda_1 > \lambda_0$. Describe the LRT for deciding whether there is a popular show in town based on the number of calls received. Assume a given probability of false rejection (Type I error), and find an expression for the critical value ξ.

4. Bertsekas 9.24 (*10 points*)

We have received a shipment of light bulbs whose lifetimes are modeled as independent, exponentially distributed random variables, with parameter equal to λ_0 (hypothesis H_0) or equal to λ_1 (hypothesis H_1). We measure the lifetimes of n light bulbs. Describe the LRT for selecting one of the two hypotheses. Assume a given probability of false rejection (Type I error) of H_0 and give an analytical expression for the critical value ξ.