ELE 382: Statistical Signal Processing	Fall 2017

Homework 2

Due date: Monday, Oct. 2, 2017 (at the beginning of class)

You are allowed to drop 1 problem without penalty. In addition, up to 2 bonus points will be awarded to each problem for clean, well-organized, and elegant solutions.

1. Stick breaking (Problem 3.22, Bertsekas) (*10 points*)

We have a stick of unit length, and we consider breaking it in three pieces using one of the following three methods. For each of these methods, what is the probability that the three pieces we are left with can form a triangle?

(a) We choose randomly and independently two points on the stick using a uniform PDF, and we break the stick at these two points

(b) We break the stick at a random point chosen by using a uniform PDF, and then we break the piece that contains the right end of the stick, at a random point chosen by using a uniform PDF.

(c) We break the stick at a random point chosen by using a uniform PDF, and then we break the larger of the two pieces at a random point chosen by using a uniform PDF.

2. Continuous Bayes' rule (Problem 3.34, Bertsekas) (*10 points*)

A defective coin minting machine produces coins whose probability of heads is a random variable P with PDF

$$f_P(p) = \begin{cases} pe^p, & p \in [0,1] \\ 0, & \text{else} \end{cases}$$

A coin produced by this machine is selected and tossed repeatedly, with successive tosses assumed independent.

(a) Find the probability that a coin toss results in heads.

(b) Given that a coin toss resulted in heads, find the conditional PDF of P.

(c) Given that the first coin toss resulted in heads, find the conditional probability of heads on the next toss.

3. The coupon collector's problem (*10 points*)

Recall that in the coupon collector's problem (see lecture notes), we are interested in the number X of packs we need to buy in order to obtain every type of coupon at least once. We have computed the mean $\mathbb{E}[X]$ in class. What is the variance of X?

4. Uncorrelatedness and independence (*10 points*)

Recall the following example in the lecture notes: let $X, Y \in \{-2, -1, 1, 2\}$ be two random variables such that

$$p_{X,Y}(1,1) = 2/5, \qquad p_{X,Y}(-1,-1) = 2/5$$
$$p_{X,Y}(-2,2) = 1/10, \qquad p_{X,Y}(2,-2) = 1/10,$$
$$p_{X,Y}(x,y) = 0 \text{ otherwise}$$

Are X and Y independent? Are they uncorrelated?

5. Correlation coefficient (Problem 4.19, Bertsekas) (*10 points*)

Suppose that a random variable X satisfies

$$\mathbb{E}[X] = 0, \quad \mathbb{E}[X^2] = 1, \quad \mathbb{E}[X^3] = 0, \quad \mathbb{E}[X^4] = 3$$

and let

$$Y = a + bX + cX^2.$$

Find the correlation coefficient $\rho_{X,Y}$.