ELE 382: Statistical Signal Processing Fall 2017

Homework 7

Due date: Monday, Nov. 27 (at the beginning of class)

You are allowed to drop 1 problem without penalty. If you attempt all problems, then the problem with the lowest score will automatically be dropped. In addition, up to 2 bonus points will be awarded to each problem for clean, well-organized, and elegant solutions.

1. Oppenheim & Verghese 8.1 (*10 points*)

Consider the pair of bivariate Gaussian random variables, X and Y, where $\mu_X = 0$. The MMSE estimator for Y in terms of X is $\hat{Y}_{\mathsf{MMSE}}(X) = 2$.

(a) What is $\mathbb{E}[Y]$?

(b) Specify whether X and Y are correlated, uncorrelated, or there isn't enough information to make this determination. Explain.

(c) Specify whether X and Y are independent, dependent, or there isn't enough information to make this determination. Explain.

(d) What is the MMSE estimator of X in terms of Y, $\hat{X}_{\mathsf{MMSE}}(Y)$?

2. Oppenheim & Verghese 8.3 (*10 points*)

Suppose X and Y are zero-mean unit-variance random variables. If the LMMSE estimator $\hat{Y}_{\mathsf{LMMSE}}(X)$ of Y in terms of X is given by

$$\hat{Y}_{\mathsf{LMMSE}}(X) = \frac{3}{4}X.$$

(a) What is its mean square error?

(b) Suppose the random variable Q is defined by $Q = Y + 3$. What is the LMMSE estimator $\hat{Q}_{\mathsf{LMMSE}}(X)$ of Q in terms of X? What is its mean square error?

(c) What is the LMMSE estimator $\hat{X}_{\mathsf{LMMSE}}(Y)$ of X in terms of Y? What is its mean square error?

3. Oppenheim & Verghese 8.7 (*10 points*)

Consider a communication system in which the random variable Y is transmitted through a channel with a random gain W, so that the received variable is $X = WY$. Assume that Y and W are independent, and that both of them are uniformly distributed in the range $[1, 2]$.

(a) Suppose you are sitting at the receiver and want to estimate the transmitted value Y from a measurement of the received value X, using the LMMSE estimator $\hat{Y}_{\mathsf{LMMSE}} = d_1 X + d_2$. Determine what d_1 and d_2 should be, and compute the associated MMSE.

(b) Suppose instead that you are sitting at the transmitter and want to estimate what the received value X will be from a measurement of the transmitted value Y. Find the (unconstrained) MMSE estimator $\hat{X}(Y)$.

4. Oppenheim & Verghese 8.12 (*10 points*)

X and Y are two random variables with unknown PDFs. X is zero-mean. The MMSE estimator $\hat{Y}$ of Y given X is $\hat{Y} = 5$. From the information given, specify whether X and Y are definitely statistically independent, definitely not statistical independent, or if it can't be determined from the information given. Explain.

5. Oppenheim & Verghese 8.13 (*10 points*)

Let X be a noisy measurement of the angular position Y of an antenna: $X = Y + W$, where W denotes the additive noise. Treat Y and W as independent random variables. Suppose Y is uniformly distributed in the interval $[-1, 1]$, and W has the triangular PDF $f_W(w) = [1 - (|w|/2)]/2$ for $|w| \leq 2$ (and $f_W(w) = 0$ elsewhere). Given that $X = 1$, what is the MMSE estimate of Y? What is the corresponding MMSE?