ELE 382: Statistical Signal Processing	Fall 2017

Homework 3

Due date: Monday, Oct. 9, 2017 (at the beginning of class)

You are allowed to drop 1 problem without penalty. In addition, up to 2 bonus points will be awarded to each problem for clean, well-organized, and elegant solutions.

1. Gambling system (Problem 4.22, Bertsekas) (*10 points*)

Consider a gambler who at each gamble either wins or loses his bet with probabilities p and $1-p$, independent of earlier gambles. When $p > 1/2$, a popular gambling system, known as the Kelly strategy, is to always bet the fraction $2p-1$ of the current fortune. Compute the expected fortune after n gambles, starting with x units and employing the Kelly strategy.

2. Dating (Problem 4.23, Bertsekas) (*10 points*)

Pat and Nat are dating, and all of their dates are scheduled to start at 9pm. Nat always arrives promptly at 9pm. Pat is highly disorganized and arrives at a time that is uniformly distributed between 8pm and 10pm. Let X be the time in hours between 8pm and the time when Pat arrives. If Pat arrives before 9pm, their date will last exactly 3 hours. If Pat arrives after 9pm, their date will last for a time that is uniformly distributed between 0 and $3-X$ hours. The date starts at the time they meet. Nat gets irritated when Pat is late and will end the relationship after the second date on which Pat is late by more than 45 minutes. All dates are independent of any other dates.

(a) What is the expected number of hours Nat waits for Pat to arrive?

(b) What is the expected duration of any particular date?

(c) What is the expected number of dates they will have before breaking up?

3. (Problem 4.24, Bertsekas) (*10 points*)

A retired professor comes to the office at a time which is uniformly distributed between 9am and 1pm, performs a single task, and leaves when the task is completed. The duration of the task is exponentially distributed with parameter $\lambda(y) = 1/(5-y)$, where y is the length of the time interval between 9am and the time of his arrival.

(a) What is the expected amount of time that the professor devotes to the task?

(b) What is the expected time at which the task is completed?

4. Generating random variables (*10 points*)

Given $X \sim \mathsf{Unif}(0,1)$, find the function $g(x)$ such that $Y = g(X) \sim \mathcal{N}(2,4)$. Plot $g(x)$ (using whatever programming language you like). Generate 100 samples of Y by first generating 100 samples of X and then using the function $g(x)$ to find the corresponding samples of Y. Find the empirical CDF, that is, the weighted cumulative sum of the histogram of the samples of Y. Compare the empirical CDF to the "true"

CDF of Y by plotting them on the same graph. Now generate 5000 samples of Y and find the empirical CDF. Submit the code you used and the plots.

5. Covariance matrices (*10 points*)

Which of the following matrices can be a covariance matrix? Justify your answer. Either construct a random vector $\mathbf{X}$ with the given covariance matrix as a function of the i.i.d. zero mean unit variance random variables Z_1, Z_2, Z_3, or establish a contradiction as was done in lecture.

$$\text{(a)} \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix} \qquad \text{(b)} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \qquad \text{(c)} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix} \qquad \text{(d)} \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 3 \end{bmatrix}$$