An Upper Bound on Multi-hop Transmission Capacity with Dynamic Routing Selection

Yuxin Chen, *Student Member, IEEE*, Jeffrey G. Andrews, *Senior Member, IEEE*

Abstract

This paper studies the capacity bounds of random interference-limited wireless network employing multi-hop transmission under QoS constraints. We are interested in the metric "transmission capacity", which measures the maximum contention density achievable under outage constraints. Instead of employing predetermined routing, potential paths are allowed to be selected from a pool of randomly deployed relay set dynamically. By modeling the location of all transmitters as Poisson point process, we derive a closed-form bound on the connection of outage probability and expected number of potential paths. This has been used to derive an upper bound on transmission capacity with respect to the outage constraint ε and the number of hops $m+1$ in non-asymptotic regime. This capacity bound characterizes an ideal diversity gain assuming strong incoherence among potential paths, which implies that randomness in the location of relay sets and dynamically changing channel states should be exploited to achieve higher aggregate throughput. In fact, our analysis also indicates that predetermined location aware routing cannot achieve optimality of capacity in interference-limited networks. This analytic framework has also been extended to incorporate retransmissions with a maximum number of allowed attempts. The Poisson field assumption motivates us to assume a general exponential form of single hop success probability, and we briefly examine the feasibility under different channel models including Rayleigh and Nakagami fading and non-fading environment.

I. Introduction

In the distributed wireless network with random node locations, how to determine the exact fundamental limits of network capacity remains an open and complicated problem. A variety

Y. Chen and J. G. Andrews are with the Department of Electrical and Computer Engineering, University of Texas at Austin, Austin TX 78712, USA (email:{yxchen, jandrews}@mail.utexas.edu). The contact author is J. G. Andrews. This research has been supported by the DARPA Information Theory for Mobile Ad Hoc Networks (IT-MANET) program. Manuscript date: November 30, 2009.

of communication technique have been designed to improve the aggregate throughput. Among them, multihop routing is inherently a widely advocated technique that promises better coverage and cooperative diversity gain in large random networks, but it is typically not well optimized and the performance limits in many different scenarios are left unknown. The reason is that the interference that arises from multiple attempts for each end-to-end session is difficult to characterize exactly in a tractable way. Additionally, among all multi-hop routing strategies, nearest-neighbor routing has been well propagated as a simple routing strategy that achieves order-optimal throughput with respect to the number of nodes n in an asymptotic regime [1][2], and this basically performs well among the class of predetermined routing selection. However, nearest-neighbor routes are not necessarily the best choice with respect to all dynamic channel states. In fact, a randomly deployed set of potential relays and dynamic fading channels may give a massive diversity gain by providing more potential routes. Especially in dense networks, randomness in the location of relay nodes provide significant path diversity that largely increases the probability for successful reception. Here, we are interested in how these potential choices can improve the success probability for each session, and how best we can use the massive diversity gain to improve the aggregate throughput.

In this paper, we assume that the node locations are a realization of Poisson process in order to investigate the throughput statistically. Despite its oversimplification for most realistic networks, this model provides a tractable way in characterizing how the end-to-end success probability vary from different multihop routing strategies and how much throughput gain we can get from them. We aim to study how multi-hop routing with the assistance of a pool of randomly deployed relays impacts the capacity scaling in non-asymptotic regime. We are interested in how much gain we can get on the average throughput under quality of service (QoS) constraints. For this purpose, we use the transmission capacity framework developed in [3], and extend it to fit end-to-end throughput analysis. Considering uncoordinated routing selection, we aim at determining the fundamental limits for a general class of routing strategies instead of simply predetermined selection. It can be expected that the massive diversity gain resulting from the randomness and dynamical states potentially provides significant throughput improvement. Although we do not design a routing protocol that can approach the bound, these perspectives can become guidelines for practical protocol design.

A. Related Work and Motivation

There has been significant work on finding the limiting scaling behavior of ad hoc networks with the number of nodes n. Gupta and Kumar [1] pioneered the studies of the asymptotic scaling behavior of end-to-end throughput by showing that the maximum aggregate throughput scales as $\Theta(\sqrt{n})$ in arbitrary networks. The feasibility of this throughput in random networks has also been shown [4] by relaying all information via crossing paths constructed in through the network. Özgür et. al. [5][2] extended this framework to more general operating regimes. Their findings have shown that nearest neighbor multihop transmission is order optimal in power limited regime, while multihopping across clusters with distributed MIMO can achieve order-optimal throughput in bandwidth-limited and power-inefficient regime. However, the main shortcoming of this line of works is that most of these results hold only for sufficiently large n, which fails to indicate how capacity scales in normal conditions. Moreover, these work do not explicitly give us the intuition how other network parameters imposed by a specific transmission strategy affect the throughput.

The analytical framework of stochastic geometry provides an alternative path to study network throughput. Baccelli et. al. [6] first investigated how to optimize the spatial density and forward progress towards the destinations in the multi-hop context. They suggest that it is advisable to select next hop relays that optimize forward progress in long distance transmission. The connection between spatial contention density and outage probability constraints has been further studied by Weber et. al. [3] with transmission capacity metric. This framework attempts to determine the maximum concurrent contentions that the network can support while simultaneously meeting the QoS requirement for each node. The original works focus on simplified path-loss effect, and has been extended to consider fading and shadowing [7]. This framework allows closed-form expressions of achievable throughput to be derived in non-asymptotic regimes, which are useful in examining how various communication technique and design parameters affect the scaling of aggregate throughput [8][9]. See [10] for a summary. One limitation of these works is that transmission capacity is investigated in the context of single-hop transmission. Wireless nodes are assumed to perform independent transmission with no routing or relays employed. Recently, [11][12] began to investigate the throughput scaling in two-hop decentralized wireless networks with opportunistic relay selection under different channel gain distribution and relay deployment.

Both of them agreed that location aware opportunistic relay selection can provide diversity gain by exploiting randomness. However, more general multi-hop capacity is basically hard to develop because of its intractability.

Following this line of research, recent work [13] extended the point-to-point transmission capacity to an end-to-end metric. Their findings follow the scaling law capacity results $(\Theta(\sqrt{n}))$ but provide an exact characterization of preconstant in terms of salient network parameters. Its novel analytic framework allows them to derive the optimal tradeoff between the number of hops and success probability as well as showing that multi-hop is preferable than single-hop in power limited regime. It should be noted that its strong assumption that all relays are equally spaced along a straight line, while convenient for mathematical analysis and constituting a best case, contradicts fundamentally the Poisson node distribution. Moreover, Reference [13] focuses on a simple class of nearest neighbor routing, which is a reasonable choice among all predetermined routing strategies that do not adapt to the dynamic changes of channel states. However, the failure of nearest neighbor routes does not exclude the opportunities of success reception from other routes. In fact, the pool of randomly located relays with dynamically varying channels provides more potential routes, which in turn promises a massive diversity gain as the number of hops increase within a reasonable range. Recent work [14] also investigated multihop strategy in Poisson process model, whose focus is to characterize the delay and stability in an end-to-end analysis. This work is again based on a predetermined routing selection, and fails to consider more general dynamic routing and diversity gain.

The above-mentioned diversity gain has been utilized to design opportunistic routing mechanism [15][16] that allows any node that overhears the packets to participate in forwarding. Reference [16] is the first to investigate the capacity improvement of an opportunistic routing scheme compared with other classical predetermined routing in Poisson field. However, the performance gain here is based on simulation without an exact mathematical characterization. On the other hand, in the context of predetermined routing, various routing protocols like longest edge routing [17] and nearest-neighbor routing [18] have been studied. These work again focuses on the comparison and tradeoffs between different routing protocols instead of finding fundamental throughput bounds with multi-hop transmissions.

B. Contributions

This work determines the closed-form upper bound on transmission capacity as a function of outage constraint ε and the number of relays m for a general class of multi-hop routing strategies. Instead of predetermined routing, dynamic selection from random relay sets under varying channel states are taken into consideration in order to get diversity gain. We have derived a lower bound on the end-to-end outage probability, which can be expressed as an exponential function with respect to the expected number of potential paths. This result implies that higher throughput can be achieved when the correlation between the states of different hops is reduced and more randomness and opportunism is introduced. This can be extended to obtain a success probability bound for multicast or multi-path transmission. We further derive the closed-form expected number of relay combinations both for single transmission in each hop and for two kinds of retransmission approach, i.e., to perform transmission in each hop according to a specified number of allowed attempts. The basic approach is to map all relay combinations to higher dimensional space and focusing on the level set with respect to success probability function.

The above results have been used to show our capacity bounds. We prove that in general case with uncoordinated routing, an ideal diversity gain allows the throughput to exhibit linear scaling in the number of relays m as long as the density of relay nodes exceed the threshold imposed by outage constraint ε. This diversity gain requires strong incoherence among different paths, which may degrade for large m since longer routes are more likely to be correlated or share common links. Unlike in a single-hop scenario where network throughput grows linearly with ε in low outage regime, the multihop capacity bound is less sensitive to ε especially for large m. Besides, we briefly show that all predetermined routing strategies without further information like channel state information (CSI) may fail to perform better than single hop transmission in the interference limited network, simply because the throughput improvement cannot make up for the increase of interference. Hence, exploiting the randomness is important to obtain higher throughput. It should be noted that all routing strategies here are assumed to be uncoordinated and there is no centralized coordination. Throughout this paper, we assume a general exponential form of success probability, and this model is shown to be feasible for several commonly used channel models.

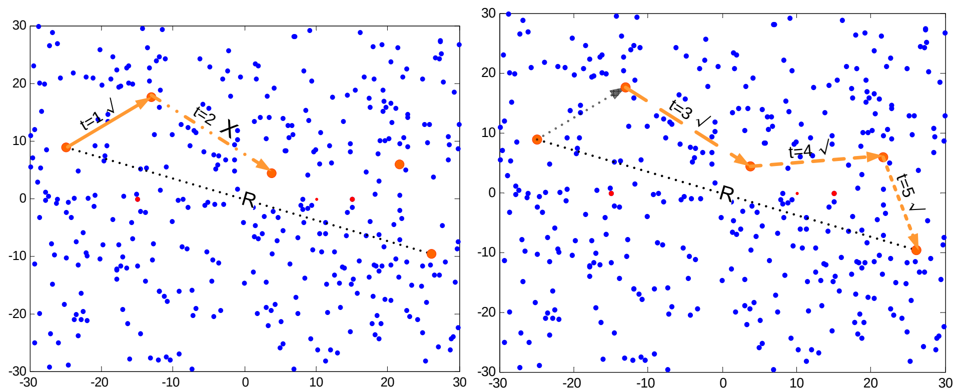

Figure 1. The above $\mathcal{S} - \mathcal{D}$ pair use 5 attempts in total to complete transmission with the assistance of 3 relays in 5 subslots. The second hop fails in subslot 2, but retransmission succeeds in subslot 3. In fact, this end-to-end session consumes 5 resources (time slots) to transmit 1 packet.

II. Models and Preliminaries

A. Models and Assumptions

We assume that new packets are generated per unit time slot and the locations of all active transmitters are a realization of HPPP Ξ_t of intensity λ_t. In addition, a set of relays are also randomly deployed in the plane with homogeneous Poisson distribution independent of Ξ_t. We consider a fixed-portion model, i.e., the relay set is of spatial density $\frac{1-\gamma}{\gamma}\lambda_t$, where $\gamma \in (0,1)$ is assumed to be a fixed constant. In other words, if the locations of all wireless nodes are assumed to be an HPPP Ξ with intensity λ, then the set of active transmitters is of intensity $\lambda_t = \gamma\lambda$. Denote by ε as the target outage probability, transmission capacity [3] in uncoordinated single-hop setting can be defined as:

$$C(\varepsilon) = (1-\varepsilon) \max_{\Pr(\text{SIR}<\beta)\leq\varepsilon} \lambda_t. \tag{1}$$

This primary metric is the spatial contention density in each time slot thinned by the success probability $1-\varepsilon$, which determines the maximum expected throughput per unit area.

Now each session utilizes k attempts with the assistance of the fixed-portion relay set. The k transmissions can be performed in an arbitrary k orthogonal slots, for example, the unit time slot can be divided into k equal subslots and the source and relays take turn in forwarding within these subslots. We assume that the total bandwidth is W, and unit throughput is achieved when the entire bandwidth is used for transmission. Therefore, the transmission capacity metric can be modified as follows:

$$C_m(\varepsilon) = (1-\varepsilon) \max_{\Pr(\mathrm{SIR}<\beta)\leq\varepsilon} \frac{\lambda_t}{k}. \tag{2}$$

This follows because each hop requires a time slot, so the overall throughput must be normalized by k. When no retransmissions are allowed, we have $k = m + 1$ with m relays; if we consider M allowed attempts (including retransmissions) in total for a single session, then $k = M$.

Slivnyak's theorem [19] suggests that an entire homogeneous network can be characterized by a typical single transmission. Conditioning on a typical pair, the spatial point process is still homogeneous and with the same statistics. It is also assumed that all transmitters employ equal amounts of power, and the network is assumed to be interference-limited, i.e. noise power is negligible compared to interference power. Additionally, every end-to-end destination node is assumed to be located a distance R from the source node for simplicity without losing intuition. Relays can be picked from any node in the feasible region which depends on different routing and searching algorithm. In this paper, we consider the effect of both path loss and fading. For point-to-point transmission from node i to node j at a distance r_{ij}, the requirement for successful reception in this hop can be expressed in terms of the following signal-to-interference ratio (SIR) constraint:

$$\mathrm{SIR}_{ij} = \frac{\left\|h_{ij}\right\|^2 r_{ij}^{-\alpha}}{\sum_{k\neq i}\left\|h_{kj}\right\|^2 r_{kj}^{-\alpha}} \geq \beta \tag{3}$$

where α denotes the path loss exponent, β the SIR threshold, and h_{ij} the fading factor experienced by the path from i to j. In this paper, distinct links are assumed to experience identically independent fading, which is reasonable in rich scattering environment.

B. Single Hop Success Probability

In a large random ad hoc network, the locations of wireless nodes are commonly modeled as homogeneous Poisson point process (HPPP). The statistics of interference combining the effect of path loss and fading [6][9] have been investigated. It can be noted that Poisson distribution often suggests an exact or approximate exponential form for the probability for successful single-hop reception, i.e. given that the packet is transmitted from node i to next hop receiver j over distance r_{ij} and contention density λ, the probability that the received SIR stays above the target β can be expressed as:

$$g_0(r_{ij}, \lambda_t) = G\exp(-\lambda_t K r_{ij}^2), \tag{4}$$

where G and K are variables which depends on specific channel models and which are independent of r_{ij} and λ_t. In this subsection, we will examine the feasibility of the exponential form assumption under a couple of commonly used channel models, including Rayleigh fading, Nakagami fading and simplified path loss model.

1) Rayleigh fading: Baccelli et. al. [6] has derived the probability for successful reception under Rayleigh fading as:

$$g_0(r_{ij}, \lambda_t) = \exp\left(-\lambda_t r_{ij}^2 \beta^{2/\alpha} C(\alpha)\right), \tag{5}$$

where $C(\alpha) = 2\pi\Gamma(\frac{2}{\alpha})\Gamma(1-\frac{2}{\alpha})/\alpha$ with $\Gamma(z) = \int_0^\infty t^{z-1}\exp(-t)\mathrm{d}t$ being the Gamma function. Hence, the coefficients under Rayleigh fading can be given as:

$$K_{\mathrm{RF}} \;=\; \beta^{\frac{2}{\alpha}} C(\alpha); \quad G_{\mathrm{RF}} \;=\; 1. \tag{6}$$

2) Nakagami fading: Nakagami fading is considered the more general fading distribution, whose power distribution can be expressed in terms of fading parameter $m_0 \geq 0.5$ as:

$$f_Z(z) = \frac{m_0^{m_0} z^{m_0-1}}{\Gamma(m_0)} \exp(-m_0 z). \tag{7}$$

In both low-outage and high-outage regimes, the success probability under Nakagami fading can be expressed as a exponential function with the following coefficients:

$$\begin{cases} \text{low outage regime: } K_{\mathrm{NF}} = \Omega_{m_0}\beta^{\frac{2}{\alpha}}, \quad G_{\mathrm{NF}} = 1 \\ \text{high outage regime: } K_{\mathrm{NF}} = \Omega_{m_0}\beta^{\frac{2}{\alpha}}, \quad G_{\mathrm{NF}} = 1 + \displaystyle\sum_{k=1}^{m_0-1}\sum_{l=1}^{k} \frac{l!}{k!}\left(-\frac{2}{\alpha}\right)^l \Upsilon_{k,l} \end{cases} \tag{8}$$

Both the derivation of these coefficients and the definition of Ω_{m_0} and $\Upsilon_{k,l}$ can be found in Appendix A. Since practical system typically require low outage probability, our analysis may still work to a certain extent.

3) Path Loss Model (non-fading): We do not generally know the closed-form formula of the success probability with only simplified path loss model. One approach is letting $m_0 \to \infty$ with Nakagami fading, which converges to a path-loss-only model. In addition, bounds can be developed to illustrate the exponential form explicitly. By partitioning the set of interferers into dominating and non-dominating nodes, an upper bound can be obtained as $g_0^{\mathrm{ub}}(r_{ij}, \lambda_t) = \exp(-\lambda_t \pi \beta^{\frac{2}{\alpha}} r_{ij}^2)$. Reference [3] has illustrated by simulation the tightness of this bound, which suggests that there is some constant C_{PL} such that

$$g_0(r_{ij}, \lambda_t) = \exp\left(-\lambda_t C_{\mathrm{PL}} \pi \beta^{\frac{2}{\alpha}} r_{ij}^2\right). \tag{9}$$

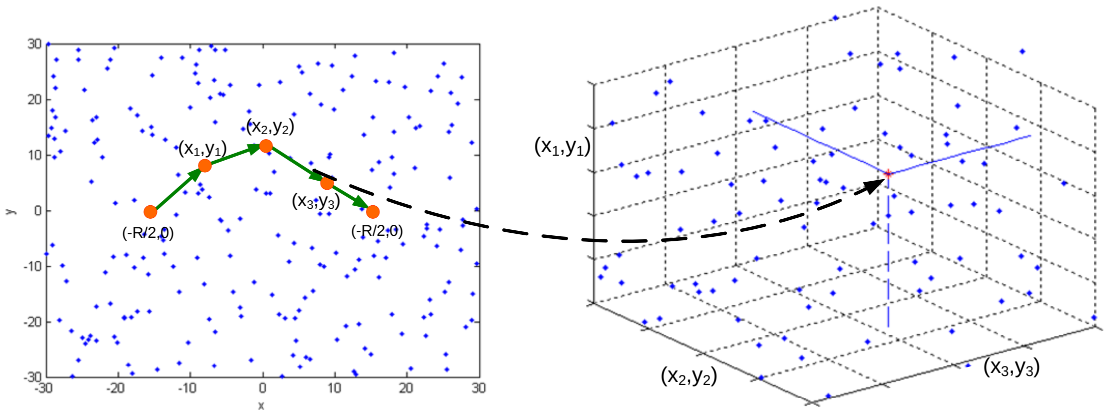

Figure 2. In the left plot, the $\mathcal{S}-\mathcal{D}$ pair use 3 relays to assist the transmission, which can be matched to a point in high-dimensional space as plotted in the right plot. In fact, a precise plot requires drawing on 6 dimensional space with each relay accounting for 2 dimensions, but the right plot may help explain the mapping intuitively.

Therefore, the coefficients in this model can be expressed as:

$$K_{\mathrm{PL}} = \pi\beta^{\frac{2}{\alpha}}, \quad G_{\mathrm{PL}} = 1. \tag{10}$$

III. General Capacity Bound

A. General Outage Probability Analysis

First, we want to bound the outage probability regardless whether the single hop success probability has exponential form, and then use the exponential form to derive closed-form solutions. For a typical $\mathcal{S}-\mathcal{D}$ pair, we suppose that it employs m relays. We will build a connection between the outage probability and the expected number of relay sets that can connect the source and destination. Suppose that there is a transmission pair with source and destination located at $(-R/2, 0)$ and $(R/2, 0)$, respectively. Conditional on this typical pair, the spatial point process is still a HPPP with the same statistics. In m relay case with the $i^{th}(1 \leq i \leq m)$ relay located at (x_i, y_i), let $Z_m = (x_1, y_1, \ldots, x_m, y_m)$ denote the location of the specific relay set. From Slivnyak's theorem, conditional on a typical transmission pair or finite number of nodes, the rest of the point process is still homogeneous Poisson process with the same spatial density (we ignore singular points here). Therefore, all relay combinations form a homogeneous point process in a $2m$-dimensional space $\mathcal{R}^{2m}$. We assume the effective spatial density to be

$\tilde{\lambda}$, which may be different for different routing protocols or scheduling approaches. Assume that each relay combination Z_m can successfully assist in communications between the end-to-end transmission pair with probability $g_m(Z_m, \lambda_t)$, if we call the relay set that can successfully complete forwarding a potential relay set, then the expected number of potential relay sets in a hypercube B can be expressed as:

$$\begin{aligned}\lim_{v_{2m}(B)\to 0} \mathbb{E}(N_B) &= \lim_{v_{2m}(B)\to 0} \mathbb{E}\left(\sum_{Z_m \in B} \mathbb{I}(Z_m \text{ is a potential relay set})\right) \\ &= \tilde{\lambda}^m g_m(Z_m, \lambda_t) v_{2m}(B). \end{aligned} \tag{11}$$

where $v_{2m}(B)$ denotes the Lebesgue measure of B and $\mathbb{I}(\cdot)$ denotes indicator function. Let N_m be the number of existing potential relay sets with the assistance of m relays. It should be noted that $\mathbb{E}(N_m)$ characterizes the expected number of different routes that can successfully forward the packets for each $\mathcal{S}-\mathcal{D}$ pair. It should be noted that a larger $\mathbb{E}(N_m)$ typically gives rise to lower outage, since more successful routes can be expected to find. The following theorem provides an outage probability bound for all success probability function $g_m(\cdot)$.

Theorem 1. *Assume that all end-to-end transmissions are achieved via $m+1$ hops with m relays. The outage probability can be lower bounded as:*

$$p_{out}^{(m)} \geq \exp(-\mathbb{E}(N_m)) \tag{12}$$

Proof: Let the high-dimensional feasible region $\mathcal{D}$ for relay sets be the allowable range to select relays from determined by different routing protocols and design parameters. Denote by $\mathcal{A}$ the event that there is no relay set within $\mathcal{D}$ that can successfully complete forwarding. Ignoring the edge effect, we attempt to approximately divide $\mathcal{D}$ into n disjoint hypercubes $\mathcal{D}_i (1 \leq i \leq n)$ each of equal volume. For sufficiently large n, this approximation is exact. Let $\mathcal{A}_i (1 \leq i \leq n)$ be the event that there exists no potential relay set within $\mathcal{D}_i$ that can complete forwarding. Hence, $\mathcal{A} = \bigcap_{i=1}^n \mathcal{A}_i$. Consider two realization of higher-dimensional point process ω and ω', and let $\omega \preceq \omega'$ if ω' can be obtained from ω by adding points. If an event $\mathcal{A}_i$ is said to be decreasing if for every $\omega \preceq \omega'$, $\mathbb{I}_{\mathcal{A}_i}(\omega) \leq \mathbb{I}_{\mathcal{A}_i}(\omega')$, then it can be noted that $\mathcal{A}_i (1 \leq i \leq n)$ are all decreasing events. The Harris-FKG inequality [20] yields

$$\mathbb{P}(\mathcal{A}) = \mathbb{P}(\bigcap_{i=1}^n \mathcal{A}_i) \geq \prod_{i=1}^n \mathbb{P}(\mathcal{A}_i). \tag{13}$$

Consider the hypercube $\mathcal{D}_i$ as $[(x_1, y_1, \ldots, x_m, y_m), (x_1 + \delta x_1, y_1 + \delta y_1, \ldots, x_m + \delta x_m, y_m + \delta y_m)]$ when $\delta x_i \to 0, \delta y_i \to 0$. Define $Z_i = (x_1, y_1, \ldots, x_m, y_m)$. Since this is a simple point process, we can approximate the void probability as follows if $v_{2m}(\mathcal{D}_i)$ is small or n is sufficiently large:

$$\begin{aligned}\mathbb{P}(\mathcal{A}_i) &\approx 1 - g_m(Z_i, \lambda_t) \prod_{i=1}^{m} \Big(1 - \exp(-\tilde{\lambda}\delta x_i \delta y_i)\Big) \\ &\approx 1 - g_m(Z_i, \lambda_t) \prod_{i=1}^{m} \tilde{\lambda}\delta x_i \delta y_i \\ &\approx \exp\Big(-\tilde{\lambda}^m g_m(Z_i, \lambda_t) v_{2m}(\mathcal{D}_i)\Big) \end{aligned} \tag{14}$$

In fact, for any n, we can find some constant $\delta_{1n}, \delta_{2n} \in (0, 1)$ that bound all $\mathbb{P}(\mathcal{A}_i)$ tightly as follows:

$$\exp\Big(-(1+\delta_{1n})\tilde{\lambda}^m g_m(Z_i, \lambda_t) v_{2m}(\mathcal{D}_i)\Big) \le \mathbb{P}(\mathcal{A}_i) \le \exp\Big(-(1-\delta_{2n})\tilde{\lambda}^m g_m(Z_i, \lambda_t) v_{2m}(\mathcal{D}_i)\Big) \tag{15}$$

It can be seen that $\delta_{1n}, \delta_{2n} \to 0$ when $n \to \infty$. Therefore, the product of the probability n regions can be bounded as:

$$\exp\left(-(1+\delta_{1n})\sum_{i=1}^{n} \tilde{\lambda}^m g_m(Z_i, \lambda_t) v_{2m}(\mathcal{D}_i)\right) \le \prod_{i=1}^{n} \mathbb{P}(\mathcal{A}_i) \le \exp\left(-(1-\delta_{2n})\sum_{i=1}^{n} \tilde{\lambda}^m g_m(Z_i, \lambda_t) v_{2m}(\mathcal{D}_i)\right) \tag{16}$$

Let $n \to \infty$, then δ_{1n}, $\delta_{2n} \to 0$, we can get the lower bound of outage probability as follows:

$$\begin{aligned}\mathbb{P}(\mathcal{A}) &\ge \lim_{n\to\infty} \prod_{i=1}^{n} \mathbb{P}(\mathcal{A}_i) \\ &= \exp\left(-\tilde{\lambda}^m g_m(Z_i, \lambda_t) \lim_{n\to\infty} \sum_{i=1}^{n} v_{2m}(\mathcal{D}_i)\right) \\ &= \exp(-\mathbb{E}(N_m)) \end{aligned} \tag{17}$$

■

It is immediate to find that the lower bound can only be obtained when all potential relay sets forms a Poisson point process, i.e., all potential points are independently scattered in the high-dimensional region. This indicates that lower correlation between different possible routes reduces the outage probability in essence by maximizing diversity. We can expect that this bound is tight and reasonable for small m (e.g. the bound is exact for single relay case) but may be loose for large m. This is because for a fixed pool of relays, the correlation between different routes increases when the number of relays m grows, i.e. for large m, many possible routes may share a couple of links. In addition, the performance bound can also be extended to incorporate multi-path or multicast transmissions in the following corollary.

Corollary 1. *Let $A_{m,i}(i = 1, 2, \ldots, r)$ be the event that there exists a $m+1$ hop path that can successfully transmit packets to receiver k_i and $I_r(A_m)$ be the event that there exists r edge-disjoint paths connecting the transmitter and r receivers respectively. If each destination node is located at distance R from the source node, then the probability of $I_r(A_m)$ can be bounded as:*

$$\mathbb{P}(I_r(A_m)) \leq \ \left(1 - \exp\left(-\mathbb{E}\left[N_m\right]\right)\right)^r \tag{18}$$

Proof: Let e_{kl} be an edge between k and l and let it be open if $\text{SIR}_{kl} \geq \beta$ and closed otherwise. For each edge e_{kl} in the graph, we further replace it by r parallel edges $e_{kl}^j(j = 1, \cdots, r)$, each of which being open with the same probability $\mathbb{P}\left(\text{SIR}_{kl} \geq \beta\right)$ independent of each other. Clearly, the event $I_r(A_m)$ in this new graph occurs more likely than the original graph because it has more edges. If r edge-disjoint paths exist in the new graph, then we can find a distinct path connecting the transmitter and receiver k_i using $\{e_{kl}^i \in E\}$ for each path. Existence of these r paths without edge disjoint constraints can be treated as mutually independent, which yields:

$$\begin{aligned}\mathbb{P}(I_r(A_m)) &\leq \prod_{i=1}^{r} \mathbb{P}(A_{m,i}) \\ &\leq \ \left(1 - p_{\text{out}}^{(m)}\right)^r \\ &\leq \left[1 - \exp\left(-\mathbb{E}\left[N_m\right]\right)\right]^r\end{aligned} \tag{19}$$

■

This corollary can be used to bound the performance for both the multicast scenario with r distinct receivers and for a $\mathcal{S}-\mathcal{D}$ pair using multi-path transmissions. In a dense network with relatively small r, inequality (19) is a reasonable approximation because we can obtain more choices of potential paths that are not overlapped with each other. Again, the expected number of different routes $\mathbb{E}\left[N_m\right]$ plays an important role here, which will be exactly calculated in the following subsection.

B. Capacity Bound for Exponential Single-Hop Success Probability

Now we begin to concentrate on the success probability of exponential forms. When no retransmissions are adopted, if a specific route is selected for packet delivering over m relays with hop distances r_1, $r_2, \ldots,$ r_{m+1}, respectively, the probability for successful reception can be found as the product of each hop's success probability:

$$g_m(r_1, \cdots, r_{m+1}, \lambda_t) = \prod_{i=1}^{m+1} g_0(r_i, \lambda_t) = G^{m+1} \exp(-\lambda_t K \sum_{i=1}^{m+1} r_i^2). \tag{20}$$

Here, we assume independence among the success probability of each hop. This is reasonable because the orthogonality among either different subslots or different subchannels typically guarantees low correlation among different hops.

Conditional on a typical transmission pair with source and destination located at $(-R/2, 0)$ and $(R/2, 0)$, the spatial point process is still a HPPP with the same statistics. In the m relay case with the $i^{th}(1 \leq i \leq m)$ relay located at (x_i, y_i), let $Z_m = (x_1, y_1, \ldots, x_m, y_m)$ denote the locations of the specific relay set, then we can define the corresponding distance statistics:

$$d_m(Z_m) \triangleq (x_1 + \frac{R}{2})^2 + \sum_{i=1}^{m-1} (x_i - x_{i+1})^2 + (x_m - \frac{R}{2})^2 + y_1^2 + \sum_{i=1}^{m-1} (y_i - y_{i+1})^2 + y_m^2. \tag{21}$$

This is the sum of squares of hop distances. Hence, the routing success probability for a specific set of relays with location Z_m can be explicitly expressed as:

$$g_m(Z_m, \lambda_t) \triangleq G^{m+1} \exp(-\lambda_t K d_m(Z_m)). \tag{22}$$

In fact, an arbitrary set of relays will have positive probability for successful forwarding. However, for those relay sets with large $d_m(Z_m)$, the communication process becomes extremely fragile and difficult to maintain due to the low reception probability and large distance. Practical protocols usually attempt to search potential routes inside a locally finite area instead of from the infinite space since the longer routes are very unlikely to be an efficient one. In order to leave the analysis general, we impose a constraint $d_m(Z_m) \leq D_m$ for the m relay case, where $D_m \to \infty$ reverts to the unconstrained distance case. We will later show that a reasonably small constraint D_m is sufficient to achieve an aggregate rate arbitrarily close to capacity upper bound.

Moreover, since we use $m+1$ hop transmission, in each subslot, each node is used as a relay by other source-destination pairs with probability γ. Therefore, the pool of relays in each hop can be treated as the original point process Ξ with each point being deleted with probability γ.

Hence, the location of all relay sets in $\mathscr{R}^{2m}$ can be viewed as a realization of a point process with effective spatial density $\lambda^m(1-\gamma)^m$. This leads to the following theorem.

Lemma 1. *If all end-to-end transmissions are achieved via $m+1$ hops with m relays, with the constraint $d_m(Z_m) \leq D_m$ ($Z_m \in \mathscr{R}^{2m}$), the expected number of potential relay sets can be computed as:*

$$\mathbb{E}(N_m) = \frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m(m+1)} \left\{ \exp\left(-\frac{\lambda\gamma K R^2}{m+1}\right) - \exp\left(-\lambda\gamma K D_m\right) \sum_{0}^{m-1} \frac{1}{i!}\left(\lambda\gamma K\left(D_m - \frac{R^2}{m+1}\right)\right)^i \right\}, \tag{23}$$

Proof: The key point in the proof is that the isosurface of $d_m(Z_m)$ forms a high-dimensional elliptical surface, which provides a tractable closed-form solution. See Appendix B for details.

■

This result indicates that larger m typically provides more diversity, because it provides more possible combinations of different relays, and the dynamically changing channel states provide more opportunities for us to find a potential route. A larger feasible range for route selection D_m also increases the expectation, but since the impact of D_m mainly exhibit as an exponentially vanishing term, it can be expected that a fairly small range is enough to approach the limits. What's more, this analytic framework can be extended to consider retransmissions in the following two scenarios. First, the best-effort retransmission protocol requires that each hop adopts k retransmissions regardless of the results of each transmissions. The following lemma provides more general results for best-effort protocols by allowing each hop to adopt a different number of retransmissions. Second, instead of specifying retransmissions for any hop, we bound the maximum number of total allowed attempts to M. The following lemma provides closed-form results for these two scenarios.

Lemma 2. *Assume that all end-to-end transmissions are achieved via $m+1$ hops with m relays. In best effort retransmission setting, if the i^{th} $(1 \leq i \leq m+1)$ hop is retransmitted k_i times, then the expected number of potential relay sets can be given as:*

$$\mathbb{E}_{\mathbf{k}}(N_m) \;=\; \frac{mG^{m+1}\pi^m(1-\gamma)^m \exp\left(-\frac{\lambda\gamma K R^2}{\sum_{i=1}^{m+1} 1/k_i}\right)}{\gamma^m K^m \left(\prod_{i=1}^{m+1} k_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{k_i}\right)} \tag{24}$$

If the $\mathcal{S}-\mathcal{D}$ transmission allows M attempts in total without specifying the number of retransmissions for each hop, then the expected number of potential relay sets can be given *as:*

$$\mathbb{E}_{\mathbf{k}^T\cdot\mathbf{1}\preceq M}(N_m)=\sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}\leq M\\ \mathbf{k}\succeq\mathbf{1}}}(-1)^{\mathbf{k}^T\cdot\mathbf{1}}\sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\leq M\\ \mathbf{j}\succeq\mathbf{k}}}\prod_{l=1}^{m+1}\binom{j_l-1}{k_l-1}\frac{mG^{m+1}\pi^m(1-\gamma)^m\exp\left(-\frac{\lambda\gamma KR^2}{\sum_{i=1}^{m+1}1/k_i}\right)}{\gamma^m K^m\left(\prod_{i=1}^{m+1}k_i\right)\left(\sum_{i=1}^{m+1}\frac{1}{k_i}\right)} \tag{25}$$

Proof: See Appendix C. ■

This lemma shows that without specifying a predetermined route, the best effort strategy may be optimized when each hop uses the same amount of retransmissions by noting (24) is maximized for equal k_i, simply because each hop is equivalent in best-effort setting. But if only the total allowable attempts is specified, then the number of potential alternatives largely depends on M and is not uniquely optimized by equal amount of attempts allocated for each hop.

The above results on the expected number of potential relay sets immediately yield:

Corollary 2. *Assume that all end-to-end transmissions are achieved via $m+1$ hops with m relays. If single transmission is adopted in each hop, then the outage probability under constraints $d_m(Z_m)\leq D_m$ ($Z_m\in\mathscr{R}^{2m}$) can be computed as:*

$$p_{out}^{(m)}\geq\exp\left\{-\frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m(m+1)}\left\{\exp\left(-\frac{\lambda\gamma KR^2}{m+1}\right)-\exp\left(-\lambda\gamma KD_m\right)\sum_{0}^{m-1}\frac{1}{i!}\left(\lambda\gamma K\left(D_m-\frac{R^2}{m+1}\right)\right)^i\right\}\right\} \tag{26}$$

If each hop adopts $k_i(1\leq i\leq m+1)$ retransmissions, the outage probability can be computed as:

$$p_{out,\mathbf{k}}\geq\ \exp\left(-\frac{mG^{m+1}\pi^m(1-\gamma)^m\exp\left(-\frac{\lambda\gamma KR^2}{\sum_{i=1}^{m+1}1/k_i}\right)}{\gamma^m K^m\left(\prod_{i=1}^{m+1}k_i\right)\left(\sum_{i=1}^{m+1}\frac{1}{k_i}\right)}\right) \tag{27}$$

If the transmission adopts M retransmission in total, the outage probability can be given as:

$$p_{out,M}\geq\ \exp\left(-\sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}\leq M\\ \mathbf{k}\succeq\mathbf{1}}}(-1)^{\mathbf{k}^T\cdot\mathbf{1}}\sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\leq M\\ \mathbf{j}\succeq\mathbf{k}}}\prod_{l=1}^{m+1}\binom{j_l-1}{k_l-1}\frac{mG^{m+1}\pi^m(1-\gamma)^m\exp\left(-\frac{\lambda\gamma KR^2}{\sum_{i=1}^{m+1}1/k_i}\right)}{\gamma^m K^m\left(\prod_{i=1}^{m+1}k_i\right)\left(\sum_{i=1}^{m+1}\frac{1}{k_i}\right)}\right) \tag{28}$$

This corollary provides a closed-form lower bound on the end-to-end outage probability. Especially for sufficiently large D_m in non-retransmission case, the lower bound reduces to:

$$p_{\text{out}}^{(m)}\geq\exp\left\{-\frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m(m+1)}\exp\left(-\frac{\lambda\gamma KR^2}{m+1}\right)\right\}, \tag{29}$$

which gives a clear characterization for low-coherence routing selections. As expected, multi-hop routing with the help of randomly deployed relays improves the success probability by providing massive diversity. Randomness in both relay locations and channel states brings more opportunities that we can exploit.

It should be noted that unlike the single hop scenario [3], our bound for outage probability is not globally monotonically increasing with λ if $D_m \nrightarrow \infty$. For sufficiently large but not infinite D_m, the outage probability can be approximated through a first-order Taylor expansion in low density regime:

$$
\begin{aligned}
p_{\text{out}}^{(m)}(\lambda) \;&\geq \exp\left\{-\frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m(m+1)}\left[\exp(-\frac{\lambda\gamma}{m+1}KR^2)-\exp(-\lambda\gamma K D_m)\right]\right\} \\
&\approx\; 1-\lambda\frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^{m-1}K^{m-1}(m+1)}(D_m-\frac{R^2}{m+1}),
\end{aligned} \tag{30}
$$

which indicates large outage probability in low density regime. This significantly high outage arises from the exceeding difficulty to guarantee a relay in the sparse network. The detailed monotonicity can be further examined by looking at the function $f(\lambda)=\exp(-a\lambda)-\exp(-b\lambda)$ where $b>a>0$. Its derivative can be computed as:

$$
f'(\lambda)=\exp(-b\lambda)\left\{b-a\exp\left[(b-a)\lambda\right]\right\}. \tag{31}
$$

The maximum value of $f(\lambda)$ occurs at $\lambda_0=\frac{1}{b-a}\ln\frac{b}{a}$, and $f(\lambda)$ is monotonically increasing at $(0,\lambda_0]$ and decreasing at (λ_0,∞). Using this property, we can see that

$$
\min p_{\text{out}}^{(m)}(\lambda)\geq\exp\left\{\frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m(m+1)}\left[\left(\frac{R^2}{(m+1)D_m}\right)^{\frac{(m+1)D_1}{(m+1)D_1-R^2}}-\left(\frac{R^2}{(m+1)D_1}\right)^{\frac{R^2}{(m+1)D_1-R^2}}\right]\right\}, \tag{32}
$$

where the minimizing λ is:

$$
\lambda_0=\frac{1}{\gamma K\left(D_m-\frac{R^2}{m+1}\right)}\ln\frac{(m+1)D_m}{R^2}. \tag{33}
$$

Hence, $p_{\text{out}}^{(m)}(\lambda)$ is monotone in both $[0,\lambda_0]$ and (λ_0,∞). Taking the inverse over (λ_0,∞) will yield the bounds on maximum contention density.

Corollary 3. *If each hop adopts single transmission, the transmission capacity can be bounded as:*

$$
C_m(\varepsilon)\;\leq\;\frac{m\ln\frac{G\pi(1-\gamma)}{K\gamma}+\ln G-\ln(m+1)-\ln\ln\frac{1}{\varepsilon}}{KR^2}(1-\varepsilon)\triangleq C_m^{ub}(\varepsilon), \tag{34}
$$

where $\varepsilon \geq \exp(-\frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m(m+1)})$*. If best effort retransmissions is adopted with each hop utilizing* k_i *retransmissions, the transmission capacity can be bounded as:*

$$C_m(\varepsilon) \;\leq\; \frac{m\ln\frac{G\pi(1-\gamma)}{K\gamma} + \ln(Gm) - \ln\left[\left(\prod_{i=1}^{m+1} k_i\right)\left(\sum_{i=1}^{m+1}\frac{1}{k_i}\right)\right] - \ln\ln\frac{1}{\varepsilon}}{KR^2\sum_{i=1}^{m+1} k_i}\left(\sum_{i=1}^{m+1}\frac{1}{k_i}\right)(1-\varepsilon), \tag{35}$$

where $\varepsilon \geq \exp\left(-\frac{mG^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m\left(\prod_{i=1}^{m+1} k_i\right)\left(\sum_{i=1}^{m+1}\frac{1}{k_i}\right)}\right)$.

Proof: Let $D_m \to \infty$, then integral part in (59) becomes Gamma function:

$$\lim_{D_m\to\infty}\int_0^{\lambda\gamma K(D_m-\frac{R^2}{m+1})} x^{m-1}\exp(-x)\mathrm{d}x = \Gamma(m) = (m-1)!$$

Setting the outage probability equal to ε, (26) can be simplified as:

$$\varepsilon \geq \exp\left\{-\frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m(m+1)}\exp(-\frac{\lambda\gamma}{m+1}KR^2)\right\}. \tag{36}$$

Notice that the effective spatial density is $\lambda\gamma/(m+1)$ and that ε is monotonically increasing with respect to λ, we can immediately derive

$$\frac{\lambda}{m+1} \leq \frac{m\ln\frac{G\pi(1-\gamma)}{K\gamma} + \ln G - \ln(m+1) - \ln\ln\frac{1}{\varepsilon}}{KR^2}, \tag{37}$$

which yields (34). Furthermore, in order to make the transmission capacity well-defined, i.e. $C_m(\varepsilon) > 0$, we will have the constraint for outage probability stated in the corollary.

The derivation of capacity in best-effort setting is exactly the same. ■

In the case where each hop use single transmission, when $D_m \nrightarrow \infty$ but is reasonably large, the outage probability can be approximated using L'Hôspital's rule:

$$\varepsilon \geq \exp\left\{-\frac{G^{m+1}\pi^m(1-\gamma)^m}{\gamma^m K^m(m+1)}\left[\exp(-\frac{\lambda\gamma}{m+1}KR^2) - \exp(-\frac{\lambda\gamma}{m+1}KD_m)\right]\right\}. \tag{38}$$

By simple manipulation, the upper bound $\overline{C_m^{\mathrm{ub}}}(\varepsilon)$ on transmission capacity with D_m constraint becomes:

$$\overline{C_m^{\mathrm{ub}}}(\varepsilon) + \Theta\{\exp\{-\overline{C_{\mathrm{ub}}}(\varepsilon)K(m+1)D_m\}\} = C_m^{\mathrm{ub}}(\varepsilon) \tag{39}$$

which means the gap between the general bound and the bound with distance constraints will decay exponentially fast with D_m.

It should be noted that if we only impose the maximum number of allowable attempts M, it is difficult to get closed-form capacity bound. But since the outage bound is monotonically increasing with λ, it allows us to get numerical solution.

Our novel analytic framework gives us a simple and clear closed-form upper bound to characterize the maximum achievable throughput in a general class of routing strategy. Although this is derived under the assumption of equivalent $\mathcal{S}-\mathcal{D}$ separation distance and equal number of hops for each end-to-end transmission pair, the result gives us some intuition on how the throughput grows with number of hops, portion of relay set, and allowable outage probability in an ideal low-coherent routing selection strategy. We leave detailed analysis of this result in next section.

IV. Numerical Analysis and Discussions

Corollary 3 shows that the capacity improvement from multihop routing can be remarkable. Since $\Theta(\ln(m+1))$ is negligible compared with $\Theta(m)$ when m grows, the transmission capacity bound exhibits near linear scaling behavior with respect to the number of hops $m+1$, which can be seen from the Fig. 3. This gain arises from the increasing diversity due to the increment of m, because the more hops we take, the more choices of potential routes we can get to guarantee the successful assignment of routes. However, this gain may not be able to obtain for large m, because longer potential routes may be more likely to be correlated or share common links especially in high-interference environment. Besides, employing a large number of hops may impose huge overhead to update current CSI, and how to design routing protocols that can guarantee the randomness and uncorrelation among different choices to achieve higher throughput should be carefully considered. In fact, if we cannot take advantage of the randomness and opportunism, the resulting throughput may be significantly worse than the theoretical order because of the degradation of cooperative diversity. Hence, employing a large feasible m in practice is a tradeoff that should be carefully compared and determined.

Another interesting result is that if m falls in feasible region, a practical outage constraint ε will not affect the throughput a great deal since double logarithm dramatically reduce its impact. For instance, when $\varepsilon = 10^{-10}$, the resulting $\ln\ln\frac{1}{\varepsilon}$ is approximately 3.14. It can also be illustrated in Fig. 3: when we decrease ε from 10^{-2} to 10^{-4}, the capacity only experience a constant shift (e.g., relatively small compared with the density with $m = 10$). This is quite

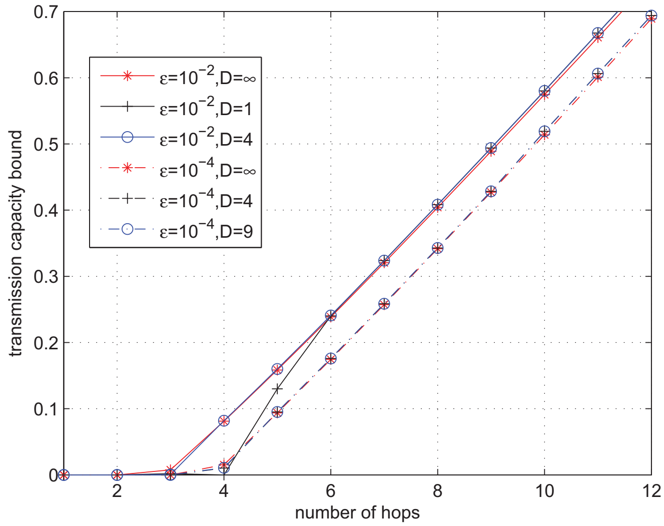

Figure 3. Transmission capacity bound vs. the number of nodes for $\varepsilon = 10^{-2}$ and $\varepsilon = 10^{-4}$ when each hop employs single transmission. For transmit distance $R = 1$, different distance constraints including $D = 1, D = 4, D = 9, D = \infty$ are plotted. Other parameters are: $\alpha = 5, \beta = 10, \gamma = 0.1$.

different from single-hop scenario, where the throughput exhibits linear scaling with ε in low outage regime [3].

Moreover, the corollary implies that increasing the portion of relay nodes will result in a logarithmic increase of capacity as $\ln \frac{1-\gamma}{\gamma}$. A large enough pool of relay sets is necessary to guarantee multi-hop selection, but bringing in too many relays may only allow limited throughput increment but incur substantial resource overhead. The analysis of Corollary 3 also indicates that the gap between constrained maximum density and the unconstrained transmission capacity is subject to exponential decay with respect to $(m+1)D_m$. From Fig. 3, we also notice that the capacity converges to optimal bound with D more quickly when ε is smaller. Hence, searching for the routes in a locally finite region can already approach arbitrarily close to the limit with a properly selected D_m.

This capacity order can be a guideline for practical design. Although we do not provide a protocol design approach to approach this capacity bound, the randomness and dynamic routing selection is of great importance. In fact, if we only adopt location-aware predetermined routing, it is not likely to get throughput close to the bound in the interference-limited networks. A simple argument can be given as follows. Considering a typical source-destination pair, the outage probability can be bounded as:

$$\begin{aligned} 1 - p_{\text{out}}^{(m)} &= G \exp(-\lambda\gamma K d_m(Z_M)) \\ &\leq \; G \exp\left(-\lambda\gamma K \frac{R^2}{m+1}\right) \end{aligned} \tag{40}$$

The equality can be achieved if and only if the m relays are equally spaced along the line segment between source and destination. In fact, from the properties of Poisson random process, this is almost surly unlikely to occur, thus resulting in strict inequality. Setting $\lambda\gamma(1-\varepsilon)/(m+1)$ to $C_m^{\text{ub}}(\varepsilon)$, we can immediately get an upper bound:

$$C_m^{\text{ub}}(\varepsilon) = \frac{1-\varepsilon}{KR^2} \ln \frac{G}{1-\varepsilon}, \tag{41}$$

It is worth noting that this bound is exactly equal to single hop case. This suggests that no throughput gain can be obtained using predetermined routing in the interference limited network compared with single hop direct communication. This is different from power-limited regime [13] where multi-hop strategy can promise better throughput than single-hop transmission by mitigating the impact of high-power noise. Therefore, how to exploit the multi-route diversity arising from randomness to improve throughput is quite useful. One possible way to obtain higher throughput is to introduce a set of central monitors that instead of transmitting data packets, monitor and schedule all sessions. Although this may be a bit apart from decentralized uncoordinated networks, introducing a certain degree of centralization may provide efficiency of network operating.

Additionally, our model fits well with Rayleigh fading and non-fading environment, and may also be suitable in examining general Nakagami fading in extreme outage regime, which is reasonable in many real world designs. Care should be exercised if we want to consider more channel models like shadowing effect, because usually it does not lead to an exponential form. In addition, all these work consider uncoordinated routing strategies. The capacity with centralized scheduling or with other distributed strategy like game theoretic method are left for future work.

APPENDIX A

SINGLE HOP SUCCESS PROBABILITY UNDER NAKAGAMI FADING

The single-hop success probability with Nakagami fading can be developed as:

$$
\begin{aligned}
g_0(r_{ij}, \lambda_t) &= \int_0^{\infty} \Pr\left(\frac{z r_{ij}^{-\alpha}}{t} \geq \beta\right) f_{I_\Phi}(t) dt \\
&= \sum_{k=0}^{m_0-1} \frac{(-m_0 \beta r_{ij}^{\alpha})^k}{k!} \mathcal{L}_{I_\Phi}^{(k)}(m_0 \beta r_{ij}^{\alpha}),
\end{aligned} \tag{42}
$$

where $\mathcal{L}_{I_\Phi}(s)$ is the Laplace transform of the general Poisson shot noise process, and $\mathcal{L}_{I_\Phi}^{(k)}(s)$ denotes the k^{th} derivative of $\mathcal{L}_{I_\Phi}(s)$. The closed-form formulas of them are given by [9] as:

$$
\mathcal{L}_{I_\Phi}(s) = \exp\left\{-\lambda_t \Omega_{m_0} \left(\frac{s}{m_0}\right)^{\frac{2}{\alpha}}\right\} \tag{43}
$$

$$
\mathcal{L}_{I_\Phi}^{(k)}(s) = \frac{\exp\left\{-\lambda_t \Omega_{m_0} \left(\frac{s}{m_0}\right)^{\frac{2}{\alpha}}\right\}}{(-s)^k} \sum_{j=1}^{k} \left[-\frac{2\lambda_t \Omega_{m_0}}{\alpha}\left(\frac{s}{m_0}\right)^{\frac{2}{\alpha}}\right]^j \Upsilon_{k,j} \tag{44}
$$

where $\Upsilon_{k,j}$ is a constant defined in [9], and

$$
\Omega_{m_0} = \frac{2\pi}{\alpha} \sum_{k=0}^{m_0-1} \binom{m}{k} B(k + \frac{2}{\alpha}, m_0 - k - \frac{2}{\alpha}) \tag{45}
$$

with $B(a, b)$ denoting Beta function. By manipulation, we have:

$$
g_0(r_{ij}, \lambda_t) = \exp\left\{-\lambda_t \Omega_{m_0} \beta^{\frac{2}{\alpha}} r_{ij}^2\right\} \left\{1 + \sum_{k=1}^{m_0-1} \sum_{l=1}^{k} \frac{1}{k!} \left[-\frac{2\lambda_t \Omega_{m_0} \beta^{\frac{2}{\alpha}} r_{ij}^2}{\alpha}\right]^l \Upsilon_{k,l}\right\} \tag{46}
$$

Generally speaking, this does not have an expected exponential form. But we can simplify the expression in certain cases. For small single-hop outage constraint ε, we have $\lambda_t \Omega_{m_0} \beta^{\frac{2}{\alpha}} r_{ij}^2 \ll 1$, therefore $g_0(r_{ij}, \lambda_t)$ can be approximated as:

$$
g_0(r_{ij}, \lambda_t) \approx \exp\left\{-\lambda_t \Omega_{m_0} \beta^{\frac{2}{\alpha}} r_{ij}^2\right\}. \tag{47}
$$

In contrast, for large single-hop outage regime, i.e. $\lambda_t \Omega_{m_0} \beta^{\frac{2}{\alpha}} r_{ij}^2 \gg 1$, employing L'Hospital's rule yields:

$$
g_0(r_{ij}, \lambda_t) \approx \left\{1 + \sum_{k=1}^{m_0-1} \sum_{l=1}^{k} \frac{l!}{k!} \left(-\frac{2}{\alpha}\right)^l \Upsilon_{k,l}\right\} \exp\left\{-\lambda_t \Omega_{m_0} \beta^{\frac{2}{\alpha}} r_{ij}^2\right\}. \tag{48}
$$

We summarize them as follows:

$$\begin{cases} \text{low outage regime: } K_{\text{NF}} = \Omega_{m_0}\beta^{\frac{2}{\alpha}}, \quad G_{\text{NF}} = 1 \\ \text{high outage regime: } K_{\text{NF}} = \Omega_{m_0}\beta^{\frac{2}{\alpha}}, \quad G_{\text{NF}} = 1 + \sum_{k=1}^{m_0-1}\sum_{l=1}^{k}\frac{l!}{k!}\left(-\frac{2}{\alpha}\right)^{l}\Upsilon_{k,l} \end{cases} \tag{49}$$

Since practical system typically require low outage probability, our analysis may still work to a certain extent.

Appendix B
Proof of Lemma 1

Proof: It can be noted that the isosurface of $d_m(Z_m) = a$ has the following coordinate geometry form:

$$X_{\text{sum}} + Y_{\text{sum}} \;=\; a, \tag{50}$$

where

$$\begin{aligned} X_{\text{sum}} &= (x_1 + \frac{R}{2})^2 + \sum_{i=1}^{m-1}(x_{i+1} - x_i)^2 + (x_m - \frac{R}{2})^2, \\ Y_{\text{sum}} &= y_1^2 + \sum_{i=1}^{m-1}(y_{i+1} - y_i)^2 + y_m^2. \end{aligned}$$

If we treat x_i, y_i $(1 \leq i \leq m-1)$ as mutually orthogonal coordinates, then (50) forms a quadratic surface in $2m$-dimensional space. See Fig. 4 for an illustration when $m = 1$. From the properties of quadratic forms, the x part and y part of (50) can be expressed as:

$$\begin{cases} X_{\text{sum}} \;=\; (\mathbf{CX} - \mathbf{R_x})^{\mathbf{T}}\mathbf{\Lambda_x}(\mathbf{CX} - \mathbf{R_x}) + t_m R^2, \\ Y_{\text{sum}} \;=\; (\tilde{\mathbf{C}}\mathbf{Y})^{\mathbf{T}}\mathbf{\Lambda_y}(\tilde{\mathbf{C}}\mathbf{Y}). \end{cases} \tag{51}$$

where $\mathbf{C}, \tilde{\mathbf{C}}$ are orthogonal matrices, $\mathbf{\Lambda_x}, \mathbf{\Lambda_y}$ are diagonal matrices, $\mathbf{R_x}$ has the explicit form of $[k_1, k_2, ..., k_{n-1}]^T R$, and t_m is a constant that will be determined in the sequel. Here, the orthogonal transformation of $\mathbf{X}$ $(\mathbf{Y})$ by $\mathbf{C}$ $(\tilde{\mathbf{C}})$ and translation transformation by $\mathbf{R_x}$ only result in rotation, flipping or translation of the quadratic surface without changing the shape of it. Since the corresponding quadratic terms of X_{sum} and Y_{sum} have equivalent coefficients, we have

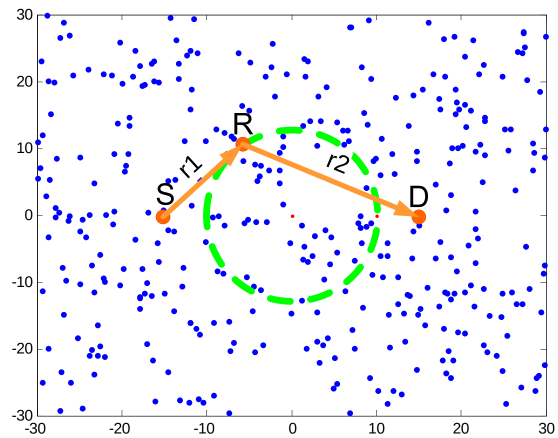

Figure 4. In single relay scenario, the contour of end-to-end success probability $g_1 = G^2 \exp\left(-\lambda_t K\left(r_1^2 + r_2^2\right)\right)$ is exactly a circle. The plot is a realization of Poisson point process of $\lambda = 0.1$/unit area. The source S and destination D are a distance 30 apart with relay R on the dotted circle satisfying $r_1^2 + r_2^2 = 500$.

$\mathbf{\Lambda_m} \triangleq \mathbf{\Lambda_x} = \mathbf{\Lambda_y}$. Denote the symmetric quadratic-form matrix corresponding to Y_{sum} as $\mathbf{A_m}$, then $\mathbf{A_m}$ is the following tridiagonal matrix of dimension m:

$$\mathbf{A_m} = \begin{pmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ 0 & -1 & 2 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & 2 \end{pmatrix}. \tag{52}$$

In fact, $\mathbf{\Lambda_m}$ is the canonical form of $\mathbf{A_m}$ with its eigenvalues on the main diagonal. Through the transformation, $X_{\text{sum}}, Y_{\text{sum}}$ can be brought to the explicit form:

$$\begin{cases} X_{\text{sum}} = \sum_{i=1}^{m} \lambda_i \tilde{x}_i^{\,2} \; + \; t_m R^2, \\ Y_{\text{sum}} = \sum_{i=1}^{m} \lambda_i \tilde{y}_i^{\,2}. \end{cases} \tag{53}$$

where $\tilde{x}_i, \tilde{y}_i$ are the new orthogonal coordinates and λ_i is the eigenvalue of $\mathbf{A_m}$. From its definition, X_{sum} is positive definite, and the following minimum value can be obtained if and only if m relays are placed equidistant along the line segment between the source and destination:

$$X_{\text{sum}} \;\geq\; \frac{(|\frac{R}{2} + x_1| + |x_2 - x_1| + \dots + |x_m - \frac{R}{2}|)^2}{m+1} \;\geq\; \frac{R^2}{m+1}. \tag{54}$$

Therefore, $t_m = \frac{1}{m+1}$. Now, (50) can be brought to:

$$\sum_{i=1}^{m} \lambda_i \tilde{x}_i^{\,2} \;+\; \sum_{i=1}^{m} \lambda_i \tilde{y}_i^{\,2} \;=\; a - \frac{R^2}{m+1}. \tag{55}$$

From the positive definiteness of $\mathbf{A_m}$, $\lambda_\mathbf{i} \; > 0$ for all i, i.e., the above equation forms the surface of a $2m$-dimensional ellipsoid. The Lebesgue measure of the ellipsoid can be written as:

$$V_m(a) = \frac{\pi^m (a - \frac{R^2}{m+1})^m}{m! \prod_{i=1}^{m} \lambda_i} = \frac{\pi^m (a - \frac{R^2}{m+1})^m}{m! \det(\mathbf{A_m})}. \tag{56}$$

We also need to determine $\det(\mathbf{A_m})$, which can be computed by the Laplace expansion of the determinant:

$$\det(\mathbf{A_m}) = 2\det(\mathbf{A_{m-1}}) - \det(\mathbf{A_{m-2}}). \tag{57}$$

Solving this recursive form with the initial value $\det(\mathbf{A_1}) = 2$ and $\det(\mathbf{A_2}) = 3$ yields:

$$\det(\mathbf{A_m}) = m + 1 \quad \Rightarrow \quad V_m(a) = \frac{\pi^m (a - \frac{R^2}{m+1})^m}{(m+1)!}. \tag{58}$$

Now, we can compute the outage probability. Integrating along different isosurface with $g(Z_m, \lambda\gamma) \;=\; \exp(-\lambda\gamma K a)$, and define $B_\lambda \;=\; \lambda\gamma K$ and $H \;=\; \frac{\pi(1-\gamma)}{\gamma K}$, we can compute the average number of potential relay sets:

$$\begin{aligned}
\mathbb{E}(N_m) \;&=\; \int_{\frac{R^2}{m+1}}^{D_m} \lambda^m (1-\gamma)^m \frac{\mathrm{d}V_m(a)}{\mathrm{d}a} G^{m+1} \exp(-B_\lambda a) \mathrm{d}a \\
&=\; G^{m+1} \lambda^m (1-\gamma)^m \int_{\frac{R^2}{m+1}}^{D_m} \frac{m \pi^m (a - \frac{R^2}{m+1})^{m-1}}{(m+1)!} \exp(-B_\lambda a) \mathrm{d}a \\
&=\; \frac{m G^{m+1} H^m \exp(-\frac{B_\lambda R^2}{m+1})}{(m+1)!} \int_0^{B_\lambda (D_m - \frac{R^2}{m+1})} x^{m-1} \exp(-x) \mathrm{d}x \qquad (59) \\
&=\; \frac{m G^{m+1} H^m \exp(-\frac{B_\lambda R^2}{m+1})}{(m+1)!} \left\{ \exp(-x) \sum_{0}^{m-1} \frac{(m-1)!}{i!} x^i \right\}_{B_\lambda (D_m - \frac{R^2}{m+1})}^{0} \\
&=\; \frac{G^{m+1} H^m}{m+1} \left\{ \exp(-\frac{B_\lambda R^2}{m+1}) - \exp(-B_\lambda D_m) \sum_{0}^{m-1} \frac{1}{i!} \left(B_\lambda (D_m - \frac{R^2}{m+1}) \right)^i \right\}. \qquad (60)
\end{aligned}$$

$\blacksquare$

Remark 1. *It is worth noting that a relay set may contain the same location for different relays. This can be interpreted as employing the same node in different frequency bands for forwarding, although this is not common in practical routing. We notice that these sets form* $\binom{m}{2}$ *hyperplanes* *in the $2m$ dimensional hyperspace, which are of measure 0. Hence, even if we require distinct relays and take the integral over feasible regions, we will still get the same results.*

Appendix C
Proof of Lemma 2

Proof: We follow similar spirit as in Theorem 1. Since we consider retransmission, the isosurface is now determined by:

$$d_m^*(Z_m) \triangleq \sum_{i=0}^{m+1} k_i^2. \tag{61}$$

Therefore, we refine X_{sum}, Y_{sum} as:

$$\begin{aligned} X_{\text{sum}}^* &= k_1(x_1 + \frac{R}{2})^2 + \sum_{i=1}^{m-1} k_{i+1}(x_{i+1} - x_i)^2 + k_{m+1}(x_m - \frac{R}{2})^2, \\ Y_{\text{sum}}^* &= k_1 y_1^2 + \sum_{i=1}^{m-1} k_{i+1}(y_{i+1} - y_i)^2 + k_{m+1} y_m^2. \end{aligned}$$

Cauchy-Schwarz inequality indicates that:

$$\left(\sum_{i=1}^{m+1} k_i r_i^2\right)\left(\sum_{i=1}^{m+1} \frac{1}{k_i}\right) \geq \left(\sum_{i=1}^{m+1} r_i\right)^2 = R^2$$

$$\Rightarrow \quad X_{\text{sum}}^* \geq \frac{R^2}{\sum_{i=1}^{m+1} \frac{1}{k_i}}. \tag{62}$$

Therefore, the Lebesgue measure of the ellipsoid $d_m^*(Z_m) \leq a$ can be calculated as:

$$V_m(a)^* = \frac{\pi^m \left(a - \frac{R^2}{\sum_{i=1}^{m+1} 1/k_i}\right)^m}{m! \det(\mathbf{A_m^*})}. \tag{63}$$

where $\mathbf{A_m^*}$ is the canonical form corresponding to Y_{sum}^* and can be written as:

$$\mathbf{A_m^*} = \begin{pmatrix} k_1 + k_2 & -k_2 & 0 & \dots & 0 \\ -k_2 & k_2 + k_3 & -k_3 & \dots & 0 \\ 0 & -k_3 & k_3 + k_4 & \dots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \dots & k_m + k_{m+1} \end{pmatrix}. \tag{64}$$

By Laplace expansion of the determinant, we get:

$$\det(\mathbf{A_m^*}) = (k_m + k_{m+1}) \det(\mathbf{A_{m-1}^*}) - k_m^2 \det(\mathbf{A_{m-2}^*}). \tag{65}$$

It can be shown by induction that:

$$\det(\mathbf{A}^*_{\mathbf{m}}) = \left(\prod_{i=1}^{m+1} k_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{k_i}\right) \tag{66}$$

Define $\mathbf{k} = (k_1, \cdots, k_m)^T$. Then the average number of relay sets when retransmitting k_i times in the i^{th} hop can be obtained as:

$$\begin{aligned}
E_{\mathbf{k}}(N_m) &= \int_{\frac{R^2}{\sum_{i=1}^{m+1} 1/k_i}}^{\infty} \lambda^m (1-\gamma)^m \frac{\mathrm{d}V_m^*(a)}{\mathrm{d}a} G^{m+1} \exp(-\lambda\gamma K a)\mathrm{d}a \\
&= G^{m+1}\lambda^m(1-\gamma)^m \int_{\frac{R^2}{\sum_{i=1}^{m+1} 1/k_i}}^{\infty} \frac{\pi^m (a - \frac{R^2}{\sum_{i=1}^{m+1} 1/k_i})^{m-1}}{(m-1)!\left(\prod_{i=1}^{m+1} k_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{k_i}\right)} \exp(-\lambda\gamma K a)\mathrm{d}a \\
&= \frac{G^{m+1}\pi^m(1-\gamma)^m \exp\left(-\frac{\lambda\gamma K R^2}{\sum_{i=1}^{m+1} 1/k_i}\right)}{\gamma^m K^m (m-1)!\left(\prod_{i=1}^{m+1} k_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{k_i}\right)} \int_0^{\infty} x^{m-1}\exp(-x)\mathrm{d}x \qquad (67) \\
&= \frac{m G^{m+1}\pi^m(1-\gamma)^m \exp\left(-\frac{\lambda\gamma K R^2}{\sum_{i=1}^{m+1} 1/k_i}\right)}{\gamma^m K^m \left(\prod_{i=1}^{m+1} k_i\right)\left(\sum_{i=1}^{m+1} \frac{1}{k_i}\right)} \qquad (68)
\end{aligned}$$

In addition, we can impose the constraint of the maximum number of retransmission M without specifying the number of retransmission for each hop. For a typical relay sets with each hop of distance $r_i (1 \le i \le m+1)$, we denote by p_i the success probability of hop i in any time slot. In the M time slot, successful reception occurs when there exists $m+1$ slots $t_i (1 \le i \le m+1)$ that satisfies: (1) the i^{th} hop transmission is successful in t_i; (2) for $\forall 1 \le i < j \le m+1$, we have $1 \le t_i < t_j \le M$. We use greedy approach here to search for all possible scenarios for successful reception. In fact, successful reception for each configuration can be determined by the smallest $t = (t_1, \cdots, t_{m+1})$ that satisfies the above two requirement. By "smallest" we mean there is no $\tilde{t} \preceq t$ that meets the requirement. This can be converted to finding the interval $(t_1 - 1, t_2 - t_1 - 1, \cdots, t_{m+1} - t_m - 1)$, or rather, finding a vector $\mathbf{j} = (j_1, \cdots, j_{m+1})$ such that $\mathbf{j} \succeq 0$ and $\mathbf{j}^T \cdot \mathbf{1} \le M - m - 1$. Hence, the success probability with $m+1$ hop routing can be calculated as:

$$
\begin{aligned}
g_{(m,M)}(r_1,\cdots,r_{m+1}) &= \left(\prod_{i=1}^{m+1} p_i\right)\left\{\sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\le M-m-1\\ \mathbf{j}\succeq 0}}\prod_{i=1}^{m+1}(1-p_i)^{j_i}\right\}\\
&= \left(\prod_{i=1}^{m+1} p_i\right)\left\{\sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\le M-m-1\\ \mathbf{j}\succeq 0}}\sum_{0\preceq\mathbf{k}\preceq\mathbf{j}}(-1)^{\mathbf{k}^T\cdot\mathbf{1}}\prod_{l=1}^{m+1}\binom{j_l}{k_l}\prod_{i=1}^{m+1}p_i^{k_i}\right\}\\
&= \sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}M-m-1\\ \mathbf{k}\succeq 0}}(-1)^{\mathbf{k}^T\cdot\mathbf{1}}\sum_{\substack{\mathbf{j}\succeq\mathbf{k}\\ \mathbf{j}^T\cdot\mathbf{1}\le M-m-1}}\prod_{l=1}^{m+1}\binom{j_l}{k_l}\prod_{i=1}^{m+1}p_i^{k_i+1}\\
&= \sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}\le M\\ \mathbf{k}\succeq\mathbf{1}}}(-1)^{\mathbf{k}^T\cdot\mathbf{1}}\sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\le M\\ \mathbf{j}\succeq\mathbf{k}}}\prod_{l=1}^{m+1}\binom{j_l-1}{k_l-1}\prod_{i=1}^{m+1}p_i^{k_i}
\end{aligned}
$$

Therefore, we can obtain:

$$
\begin{aligned}
E_{\mathbf{k}^T\cdot\mathbf{1}\preceq M}(N_m) &= \sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}\le M\\ \mathbf{k}\succeq\mathbf{1}}}(-1)^{\mathbf{k}^T\cdot\mathbf{1}}\sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\le M\\ \mathbf{j}\succeq\mathbf{k}}}\prod_{l=1}^{m+1}\binom{j_l-1}{k_l-1}E_{\mathbf{k}}(N_m)\\
&= \sum_{\substack{\mathbf{k}^T\cdot\mathbf{1}\le M\\ \mathbf{k}\succeq\mathbf{1}}}(-1)^{\mathbf{k}^T\cdot\mathbf{1}}\sum_{\substack{\mathbf{j}^T\cdot\mathbf{1}\le M\\ \mathbf{j}\succeq\mathbf{k}}}\prod_{l=1}^{m+1}\binom{j_l-1}{k_l-1}\frac{mG^{m+1}\pi^m(1-\gamma)^m\exp\left(-\frac{\lambda\gamma KR^2}{\sum_{i=1}^{m+1}1/k_i}\right)}{\gamma^m K^m\left(\prod_{i=1}^{m+1}k_i\right)\left(\sum_{i=1}^{m+1}\frac{1}{k_i}\right)}
\end{aligned}
\tag{69}
$$

■

REFERENCES

[1] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Transactions on Information Theory*, vol. 46, no. 2, pp. 388–404, Mar 2000.

[2] A. Ozgur, R. Johari, D. Tse, and O. Leveque, "Information theoretic operating regimes of large wireless networks," *submitted to IEEE Transactions on Information Theory*, April 2008.

[3] S. Weber, X. Yang, J. G. Andrews, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with outage constraints," *IEEE Transactions on Information Theory*, vol. 51, no. 12, pp. 4091–4102, December 2005.

[4] M. Franceschetti, O. Dousse, D. N. C. Tse, and P. Thiran, "Closing the gap in the capacity of wireless networks via percolation theory," *IEEE Transactions on Information Theory*, vol. 53, no. 3, pp. 1009–1018, March 2007.

[5] A. Ozgur, O. Leveque, and D. Tse, "Hierarchical cooperation achieves optimal capacity scaling in ad hoc networks," *IEEE Transactions on Information Theory*, vol. 53, no. 10, pp. 3549–3572, Oct. 2007.

[6] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "An aloha protocol for multihop mobile wireless networks," *IEEE Transactions on Information Theory*, vol. 52, no. 2, pp. 421–436, Feb. 2006.

[7] S. Weber, J. Andrews, and N. Jindal, “The effect of fading, channel inversion, and threshold scheduling on ad hoc networks,” *IEEE Transactions on Information Theory*, vol. 53, no. 11, pp. 4127–4149, Nov. 2007.

[8] S. Weber, J. Andrews, X. Yang, and G. de Veciana, “Transmission capacity of wireless ad hoc networks with successive interference cancellation,” *IEEE Transactions on Information Theory*, vol. 53, no. 8, pp. 2799–2814, Aug. 2007.

[9] A. Hunter, J. Andrews, and S. Weber, “Transmission capacity of ad hoc networks with spatial diversity,” *IEEE Transactions on Wireless Communications*, vol. 7, no. 12, pp. 5058–5071, December 2008.

[10] S. Weber, J. G. Andrews, and N. Jindal, “A tutorial on transmission capacity,” *submitted to IEEE Transactions on Communications, http://arxiv.org/abs/0809.0016*.

[11] M. Kountouris and J. G. Andrews, “Throughput scaling laws for wireless ad hoc networks with relay selection,” *IEEE 69th Vehicular Technology Conference, 2009.*, pp. 1–5, April 2009.

[12] R. K. Ganti and M. Haenggi, “Analysis of Uncoordinated Opportunistic Two-Hop Wireless Ad Hoc Systems,” *IEEE International Symposium on Information Theory*, 2009.

[13] J. G. Andrews, S. Weber, M. Kountouris, and M. Haenggi, “Random access transport capacity,” *Submitted to IEEE Trans. on Wireless Communications*, September 2009. [Online]. Available: http://arxiv.org/abs/0909.5119

[14] K. Stamatiou, F. Rossetto, M. Haenggi, T. Javidi, J. Zeidler, and M. Zorzi, “A delay-minimizing routing strategy for wireless multi-hop networks,” *7th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks, 2009.*, pp. 1–6, June 2009.

[15] S. Biswas and R. Morris, “Opportunistic routing in multi-hop wireless networks,” *SIGCOMM Comput. Commun. Rev.*, vol. 34, no. 1, pp. 69–74, 2004.

[16] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, “On the performance of time-space opportunistic routing in multihop mobile ad hoc networks,” *6th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks and Workshops, 2008*, pp. 307–316, April 2008.

[17] S. Weber, N. Jindal, R. Ganti, and M. Haenggi, “Longest edge routing on the spatial aloha graph,” *IEEE Global Telecommunications Conference, 2008.*, pp. 1–5, 30 2008-Dec. 4 2008.

[18] M. Haenggi, “On routing in random Rayleigh fading networks,” *IEEE Transactions on Wireless Communications*, vol. 4, no. 4, pp. 1553–1562, July 2005.

[19] W. K. D. Stoyan and J. Mecke, *Stochastic Geometry and Its Applications, 2nd Edition*. John Wiley and Sons, 1996.

[20] R. Meester and R. Roy, *Continuum percolation*, ser. Cambridge Tracts in Mathematics. Cambridge University Press, 1996.

Some comments:

1. The reason that I use $g_0(r_{ij}, \lambda_t)$ instead of $p_{\text{s}}(r_{ij}, \lambda_t)$ is: (a) g as a connection probability is commonly used in random connection model in continuum percolation, e.g. [20]; (b) It looks cleaner to use g_m to denote success probability with m relays compared to $p_{\text{s},m}$.

2. r_{ij} is not R in my paper: R is the distance between the end-to-end source and destination, whereas r_{ij} denotes the distance between any transmitter i and receiver j, e.g. from the transmitter to its next hop relay.