ELE 382: Statistical Signal Processing Fall 2017

Homework 8

Due date: Wednesday, Dec. 13 (at the beginning of class)

You are allowed to drop 1 problem without penalty. If you attempt all problems, then the problem with the lowest score will automatically be dropped. In addition, up to 2 bonus points will be awarded to each problem for clean, well-organized, and elegant solutions.

1. Bertsekas 9.7 (*10 points*)

Derive the ML estimator of the parameter of a Poisson random variable based on i.i.d. observations X_1, $\cdots$, X_n. Is the estimator unbiased?

2. Bertsekas 9.8 (*10 points*)

We are given i.i.d. observations $X_1, \cdots, X_n$ that are uniformly distributed over the interval $[0, \theta]$. What is the ML estimator of θ? Is it unbiased or asymptotically unbiased? Can you construct alternative estimators that are unbiased?

3. Bertsekas 9.9 (*10 points*)

We are given i.i.d. observations X_1, $\cdots$, X_n that are uniformly distributed over the interval $[\theta, \theta + 1]$. Find the ML estimator.

4. Poisson processes (*10 points*)

Let $\{N(t), t \geq 0\}$ be a Poisson process with rate λ. Calculate $\mathbb{E}[N(t)]$ and $\mathbb{E}[N(t)N(t+s)]$.

5. Moving-average process. (*10 points*)

Let $\{X_n \mid n \geq 1\}$ be a discrete-time white Gaussian noise process, that is, $X_1, X_2, X_3 \ldots$ are i.i.d. random variables with $X_n \sim \mathcal{N}(0, N)$. Consider the *moving-average* process $\{Y_n \mid n \geq 2\}$ defined by

$$Y_n = \tfrac{2}{3} X_{n-1} + \tfrac{1}{3} X_{n-2}\,, \quad n \geq 2\,.$$

Let $X_0 = 0$. Find the mean $\mathbb{E}[Y_n]$ and autocorrelation function $R_Y(m, n) = \mathbb{E}[Y_m Y_n]$ for the process $\{Y_n\}$.