Homework VII
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Q.1 A graph with no links is a trivial D-Map.
True.
A graph is said to be D-Map of a distribution if all CI satisfied by distribution is reflected on the graph, of course graph with no links will reflect any conditional independency.

Q.2.
a- A conditionally independent of D give {B, C} - True.
b- E marginally independent of F — No
c- Delete edge to make A independent of C.
$A \rightarrow C$.

Q.3. Tables for p(a), p(c|a), p(b|c) by marginalizing and conditioning the joint distribution from the given table :-

a	p(a)
0	0.6
1	0.4

a	c	p(c|a)
0	0	0.4
0	1	0.6
1	0	0.6
1	1	0.4

c	b	p(b|c)
0	0	0.8
0	1	0.2
1	0	0.4
1	1	0.6

Multiplying the three distribution together we recover the joint distribution p(a,b,c) given in the table, thereby allowing us to verify the validity of the decomposition $p(a,b,c) = p(a) * p(c|a) * p(b|c)$.

We can express the distribution using the graph.

$$a \rightarrow c \rightarrow b$$

Q.4 Given graphical model with 4 binary variables

a) $P(S=1|V=1)$ is $\dfrac{P(S=1, V=1)}{P(V=1)}$.

$$= \frac{1}{P(V=1)} \sum_{R=0}^{1} \sum_{G=0}^{1} P(V=1) * P(G) * P(R|V=1, G) * P(S=1|G)$$

$$= \sum_{RG} P(G) * P(R|V=1, G) * P(S=1|G)$$

$$= \sum_{G} P(G) * P(S=1|G) \sum_{R} P(R|V=1, G)$$

$$= \sum P(G) * P(S=1|G) * 1$$

$$= P(G=0) * P(S=1|G=0) + P(G=1) * P(S=1|G=1)$$

$$= \alpha(1-\gamma) + (1-\alpha) * (1-\beta)$$

$$= \alpha - \alpha\gamma + 1 + \alpha\beta - \alpha - \beta$$

$$= 1 - \alpha\gamma + \alpha\beta - \beta$$

b. $P(S=1|V=1)$ and $P(S=1|V=0)$ are same because they are independent of V.

c. MLE can be estimated by couting events. Thus, $\alpha = 1/3$ $\beta = 0$ and $\gamma = 1$.

V and S are independent given nothing
V and S are independent given G
V and S are independent given R & G
V and S are dependent given R.

Q.5

a. The CPDs for nodes 1,2,3 have 1 free parameter each (since they are Bernoulli). $P(H|X_{1:3})$ has 8 parameters one per conditioning case.

$P(X_i|H)$ for $i = 4:6$ are $2*2$ tables, but due to the sum of one constraint, only have 2 free parameters.

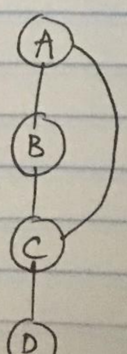
So total of $3\times1+8+3\times2 = 17$.

b. For the graph on the right, the CPDs for nodes 1,2,3 have 1 free parameter each (since they are Bernoulli). $P(X_4|X_{1:3})$ has 8 parameters, one per conditioning case. $P(X_5|X_{1:4})$ has 16 parameters.

$P(X_6|X_{1:5})$ has 32 parameters.

In total, $3+8+16+32 = 59$ parameters.

Q.6. The functions after instantiating evidence variable are $P(B|A)$, $P(C|A,B)$, $P(D|C)$ and function of D. The induced graph along the ordering ABCD is shown below. The no of children for A, B, C & D are 2, 1, 1 and 0 resp., so the width of the tree is 2. The complexity of the variable elimination algorithm is $O(n\ \exp(w+1))$, where n is no. of non-evidence variables and 'w' is the width of the ordering. Therefore the complexity is $O(4\exp(3))$.



Q.7 The functions after instantiating evidence variable are $P(B|A)$, $P(C|B)$, $P(D|C)$, $P(A)$ and function of D.

The induced graph along the ordering B C D A is shown below.

The number of children for B, C, D and A are 2, 2, 1 and 0 respectively. So the tree-width is 2.

The complexity of the variable elimination algorithm is $O(n \exp(w+1))$, where n is no. of non-evidence variables and w is the width of the ordering.

Thus, the complexity is $O(4 \exp(3))$.