$X_1 \quad \cdots\cdots \quad X_M$

$P(\underline{X_1}, X_2 \cdots\cdots X_M)$

$2^M - 1$

$P(X_1)\, P(X_2|X_1)\, P(X_3|X_2) \cdots P(X_M|X_{M-1})$

$X_1 \to X_2 \to X_3 \to \cdots\; X_{M-1} \to X_M$

$X_1 \to 1$

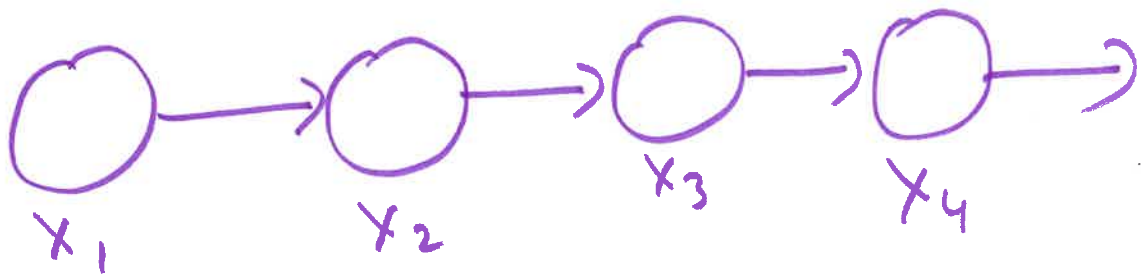
$X_2 \to P(X_2|X_1) \qquad 2$

$1 + 2(M-1)$

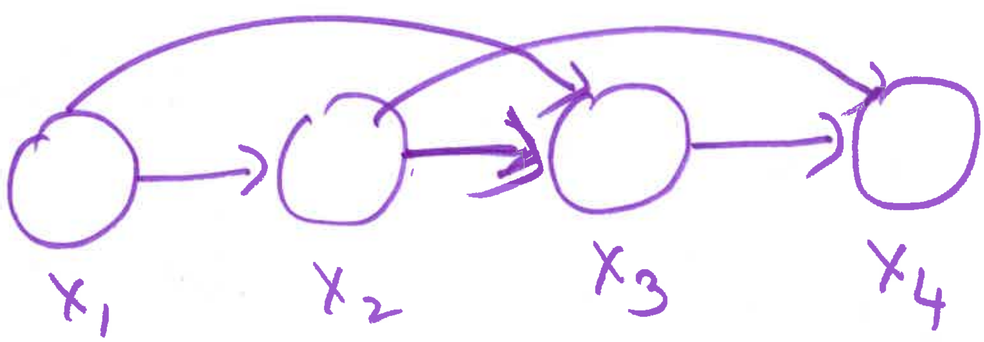
$P(X_2 = 0 | X_1 = 0)$

$P(X_2 = 0 | X_1 = 1)$

Markov Model

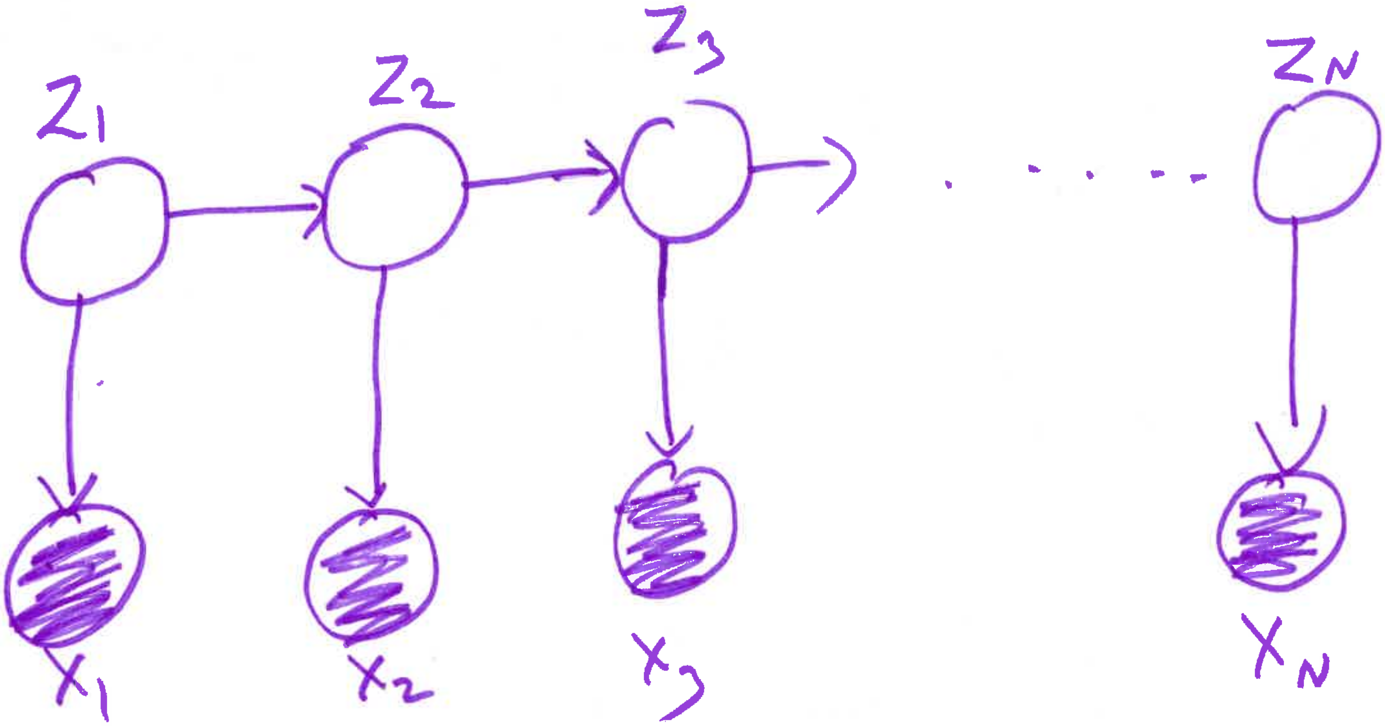


First Order



Second order

Hidden Markov Model



$$p(Z_1, Z_2, \ldots, Z_N, X_1, X_2 \ldots x_N)$$

$$= \underbrace{p(Z_1)}_{\text{Starting}} \underbrace{\prod_{n=1}^{N} \underline{p(x_n \mid z_n)}}_{\text{Emission probabilities}} \underbrace{\prod_{n=2}^{N} p(z_n \mid z_{n-1})}_{\text{Transition Prob}}$$

Memoryless HMM

$$P\left(Z_{n+1} = k \mid \text{"Whatever happened so far"}\right)$$

$$= P\left(Z_{n+1} = k \mid X_1 \cdots X_N, Z_1 \cdots Z_n\right)$$

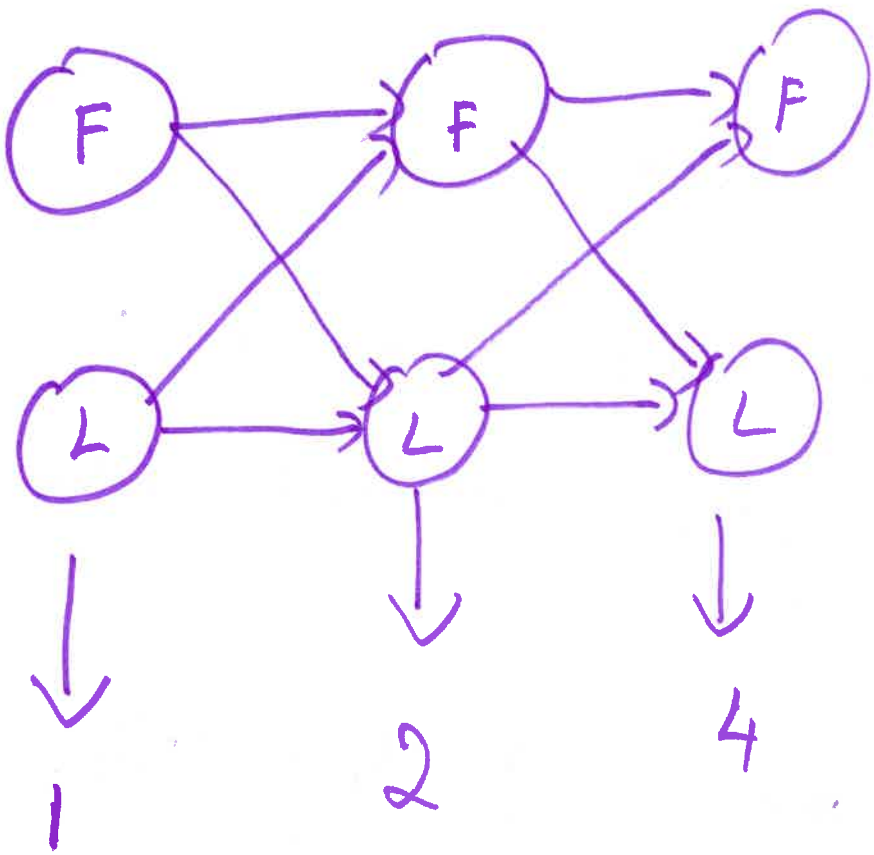
$$= P\left(Z_{n+1} = k \mid Z_n\right)$$

$$P\left(X_n = b \mid X_1 \cdots X_{n-1}, Z_1 \cdots \underline{Z_n}\right)$$

$$= P\left(X_n = b \mid Z_n\right)$$

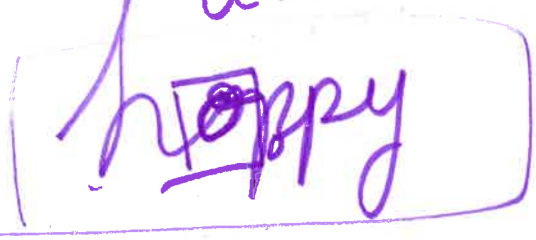
$$P\left(X \mid Z_n\right)$$

Lattice Diagram



Handwriting

o = 0.5
a = 0.3



happy = 0.3
hoppy = 0.2
boppy = 0.1

26^5

anjum

$$\max_{\{z_1 \cdots z_N\}} P(x_1\, x_2 \cdots\cdots x_N, \underbrace{z_1, z_2 \cdots\cdots z_N}_{})$$

Viterbi Algorithm

To find most probable sequence of hidden state given observed state

$$\underbrace{V_k(i)}_{} = \max_{\{z_1 \cdots z_{i-1}\}} P(x_1, x_2 \cdots x_{i-1}, x_i, z_1, z_2 \cdots, z_{i-1}, z_i = k)$$

$$V_{\underline{l}}(i+1) = \max_{\{z_1 \cdots z_{i+1}\}} P(x_1\, x_2 \cdots\cdots x_{i+1}, z_1 \cdot z_2 \cdots z_i, z_{i+1} = l)$$

$$= \max_{\{z_1 \cdots z_{i+1}\}} P(\underline{x}_{i+1}, z_{i+1} = l \mid x_1 \cdots x_i, z_1 \cdots z_i)\, P(x_1 \cdots x_i, z_1 \cdots z_i)$$

$$P(X_{i+1}, Z_{i+1} = \ell \mid Z_i = k)$$

$$= \max_{k} \; \cancel{(Z_1 \ldots Z_i)} \; \boxed{\max_{z_1 \cdots z_i} P(x_1, x_2 \ldots x_i \;\; z_1 \ldots z_i = k)}$$

$$= P(X_{i+1} \mid Z_{i+1}, \underline{Z_i = k}) \; P(Z_{i+1} \mid Z_i = k)$$

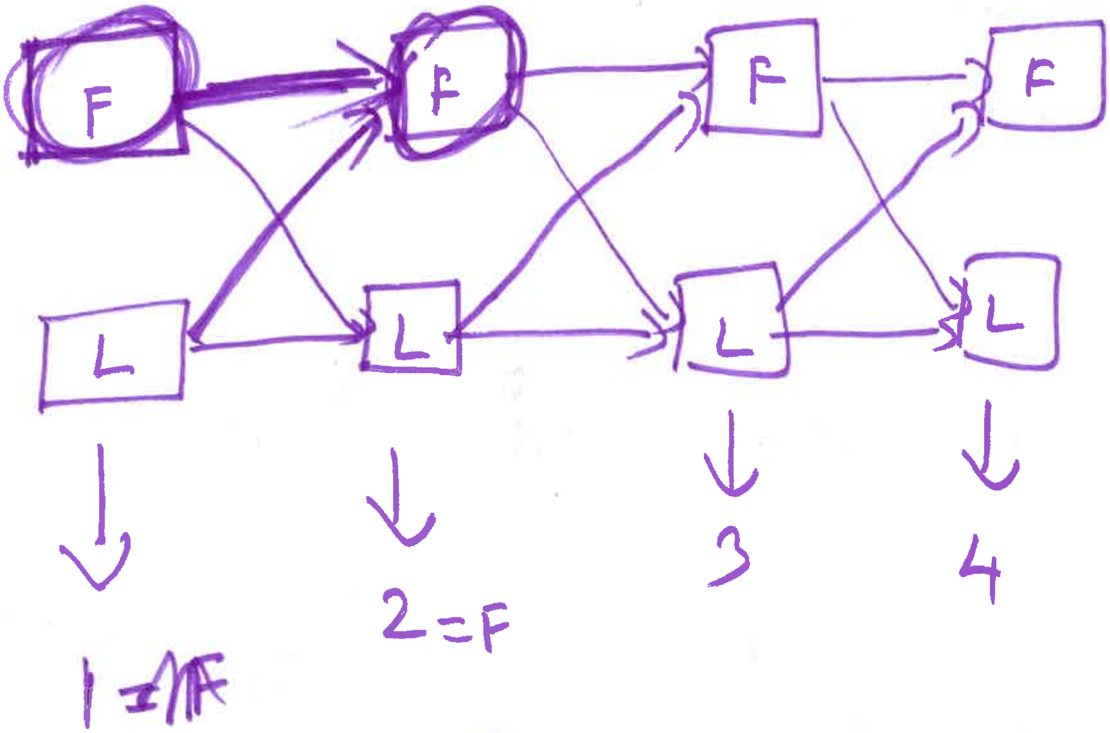
$$= \max_{k} \underline{P(X_{i+1} \mid Z_{i+1})} \; P(Z_{i+1} \mid Z_i = k) \; \boxed{\max_{(z_1 \cdots z_{i-1})} P(x_1 \cdots x_i, z_1 \cdots z_i = k)}$$

$$V_\ell(i+1) = P(X_{i+1} \mid Z_{i+1}) \max_{k} P(Z_{i+1} \mid Z_i = k) \; V_k(i)$$

$$V_\ell(i+1) = \cancel{\text{Em}} \; Em_{i+1}(\ell) \max_{k} a_{k\ell} V_k(i)$$

FFFF
FFFL
FFLL
FFLF

$$X = 1 \quad 2 \quad 3 \quad 4$$



$$P(Z_1 = F) = 0.5$$

$$V_F(1) = e_F(1) \cdot P(Z_1 = F) = \frac{1}{6} \times 0.5 = 0.08333$$

$$V_L(1) = e_L(1) \cdot P(Z_1 = L) = \frac{1}{10} \cdot 0.5 = 0.05$$

5

$$V_F(2) = e_F(2) \cdot Max\left[\underline{V_F(1)\, a_{FF}}, \; V_L(1)\, a_{LF} \right);$$

$$V_L(2) = e_L(2) \cdot Max\left[V_F(1)\, a_{FL}, \; V_L(1)\, a_{LL} \right]$$

$$V_F(2) = 0.01319$$

$$V_F(2) = \frac{1}{6} \times Max\left[\underline{0.95 \cdot 0.08333}, \; 0.05 \cdot 0.05 \right]$$

$$= \underline{0.01319}$$

$$V_L(2) = 0.00475$$

$V_F(3) = 0.0020885$

$V_L(3) = 0.000\,4513$

$V_F(4)$ ✓

$V_L(4)$

$\underbrace{FFFF}$

$$P(Z_n = k \mid X_1 \cdots X_N)$$

$$\gamma(Z_n) = P(Z_n \mid X)$$

$$P(Z_n \mid X) = \frac{P(X \mid Z_n)\, P(Z_n)}{P(X)}$$

$$= \frac{P(X_1\, X_2 \cdots X_N \mid Z_n)\, P(Z_n)}{P(X)}$$

$$= \frac{P(X_1\, X_2 \cdots X_n \mid Z_n)\, P(X_{n+1} \cdots X_N \mid Z_n)\, P(Z_n)}{P(X)}$$

6

$$= \frac{\boxed{P(x_1, x_2 \cdots x_n, z_n)}\ \boxed{P(x_{n+1} \cdots x_N | z_n)}}{P(x)}$$

$$= \frac{\alpha(z_n)\ \beta(z_n)}{P(x)}$$

$$\alpha(z_n) = P(x_1\, x_2 \cdots x_n, z_n)$$

$$= P(x_1, x_2 \cdots x_n | z_n)\ P(z_n)$$

$$= P(x_n | z_n)\ \underbrace{P(x_1, x_2 \cdots x_{n-1} | z_n)\ P(z_n)}_{= P(x_1, x_2 \cdots x_{n-1}, z_n)}$$

$$= P(x_n | z_n) \sum_{\underline{z_{n-1}}} P(x_1\, x_2\, x_3 \cdots x_{n-1}, z_n, \underline{z_{n-1}})$$

$$= P(x_n|z_n) \sum_{z_{n-1}} P(x_1 x_2 \cdots x_{n-1}, z_n, z_{n-1})$$

$$= P(x_n|z_n) \sum_{z_{n-1}} P(x_1 \cdots x_{n-1}, z_n | z_{n-1}) \, P(z_{n-1})$$

$$P(x_n|z_n) \sum_{z_{n-1}} P(x_1 x_2 \cdots x_{n-1}|z_{n-1}) \, P(z_n|z_{n-1}) \, P(z_{n-1})$$

$$= P(x_n|z_n) \sum_{z_{n-1}} \underbrace{P(x_1 x_2 \cdots x_{n-1}, z_{n-1})}_{\alpha(z_{n-1})} P(z_n|z_{n-1})$$

$$= P(x_n|z_n) \sum_{z_{n-1}} P(z_n|z_{n-1}) \, \alpha(z_{n-1})$$

$$\beta(Z_n) = \sum_{Z_{n+1}} \beta(Z_{n+1})\, P(X_{n+1} \mid Z_{n+1})\, P(Z_{n+1} \mid Z_n)$$

$$\beta(Z_N) = 1$$

$$\alpha(Z_1) = \cancel{0}\; e(x_1) \cdot \cancel{\phi}\, \text{Skrg } 2,1$$

$$\underbrace{P(Z_n, Z_{n+1} \mid X)} \qquad P(Z_n \mid X)$$

(8)

$$X_1 \quad - \quad \cdot \quad \cdot \quad \cdot \quad X_N \quad X_{N+1}$$

$$P(X_{N+1} \mid X) = \sum_{Z_{n+1}} P(X_{N+1} \mid Z_{n+1}, X)\, P(Z_{N+1} \mid X)$$

$$= \sum_{Z_{N+1}} P(X_{N+1} \mid Z_{N+1})\, P(Z_{N+1} \mid X)$$

$$= \sum_{Z_{N+1}} P(X_{N+1} \mid Z_{N+1}) \sum_{Z_N} P(Z_{N+1}, Z_N \mid X)$$

$$= \sum_{Z_{N+1}} P(X_{N+1} \mid Z_{N+1}) \sum_{Z_N} P(Z_{N+1} \mid Z_N)\, \underbrace{P(Z_N \mid X)}_{\frac{\alpha(Z_N)}{P(X)}}$$

$$\alpha(z_N) = p(z_N, X)$$

$$p(z_N \mid X) = \frac{p(z_N, X)}{p(X)}$$

$$p(z_N \mid X) = \frac{\alpha(z_N)}{p(X)}$$

$$p(x_{N+1} \mid X) = \frac{\sum_{z_{N+1}} p(x_{N+1} \mid z_{N+1}) \sum_{z_n} p(z_{N+1} \mid z_N)\, \alpha(z_N)}{p(X)}$$