# SVM for Regression

# SVM Recall

Two-class classification problem using linear model:

$$y(x) = w^T \phi(x) + b$$

# Regularized Error Function

In linear regression, we minimize the error function:

$$\frac{1}{2}\sum_{n=1}^{N}\{y_n - t_n\}^2 + \frac{\lambda}{2}\|w\|^2$$

Replace the quadratic error function by Є-insensitive error function:

$$C\sum_{n=1}^{N}E_{\varepsilon}(y(x_n) - t_n) + \frac{1}{2}\|w\|^2$$

An example of Є-insensitive error function:

$$L_{\epsilon}(y) = \begin{cases} 0 & \text{for} \quad |f(\mathrm{x}) - y| < \varepsilon \\ |f(\mathrm{x}) - y| - \varepsilon & \text{otherwise} \end{cases}$$

# Slack Variables

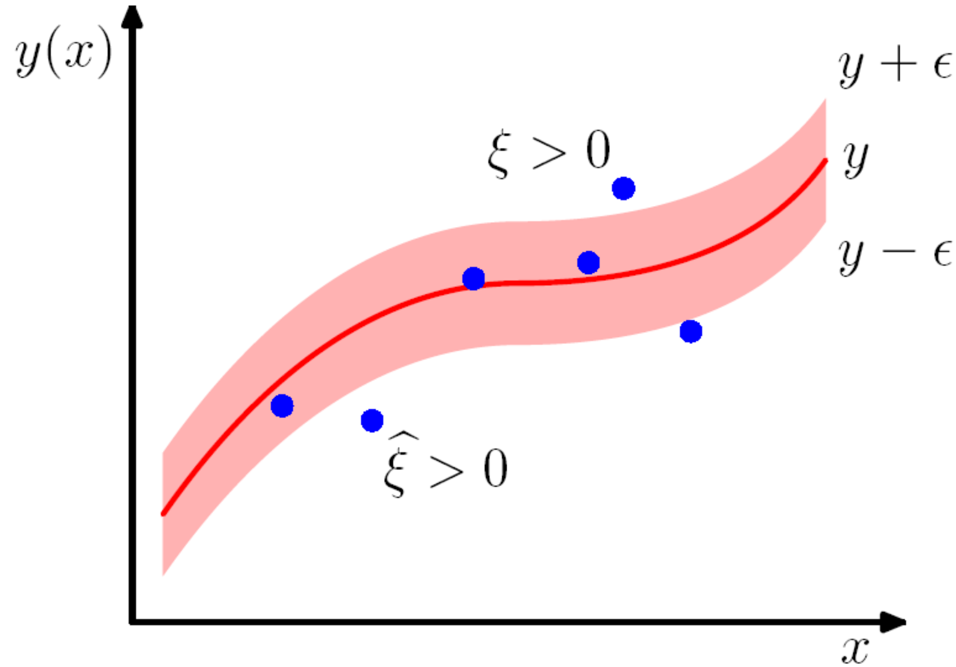
For a target point to lie inside the tube:

$$y_n - \in \leq t_n \leq y_n + \in$$

Introduce slack variables to allow points to lie outside the tube:

$$t_n \leq y(x_n) + \in + \xi_n$$

$$t_n \geq y(x_n) - \in - \xi_n^-$$



# Error Function for Support Vector Regression

Minimize:

$$C\sum_{n=1}^{N}(\xi_n + \xi_n^{-}) + \frac{1}{2}\|w\|^2$$

Subject to:

$$\xi_n \geq 0 \qquad \text{and} \qquad t_n \leq y(x_n) + \in + \xi_n$$

$$\xi_n^{-} \geq 0 \qquad\qquad t_n \geq y(x_n) - \in - \xi_n^{-}$$

# Lagrangian

Minimize:

$$L = C\sum_{n=1}^{N}(\xi_n + \xi_n^-) + \frac{1}{2}\|w\|^2 - \sum_{n=1}^{N}(\mu_n\xi_n + \mu_n^-\xi_n^-) - \sum_{n=1}^{N}a_n(\in + \xi_n + y_n - t_n) - \sum_{n=1}^{N}a_n^-(\in + \xi_n^- - y_n + t_n)$$

$$\frac{\partial L}{\partial w} = 0 \Rightarrow w = \sum_{n=1}^{N}(a_n - a_n^-)\phi(x_n)$$

$$\frac{\partial L}{\partial b} = 0 \Rightarrow \sum_{n=1}^{N}(a_n - a_n^-) = 0$$

$$\frac{\partial L}{\partial \xi_n} = 0 \Rightarrow a_n + \mu_n = C$$

$$\frac{\partial L}{\partial \xi_n^-} = 0 \Rightarrow a_n^- + \mu_n^- = C$$

# Dual Form of Lagrangian

Maximize:

$$W(a,a^{-}) = -\frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{N}(a_n - a_n^{-})(a_m - a_m^{-})k(x_n, x_m) - \in \sum_{n=1}^{N}(a_n + a_n^{-}) + \sum_{n=1}^{N}(a_n - a_n^{-})t_n$$

$$0 \leq a_n \leq C$$

$$0 \leq a_n^{-} \leq C$$

Prediction can be made using:

$$y(x) = \sum_{n=1}^{N}(a_n - a_n^{-})k(x, x_n) + b$$

# How to determine b?

Karush-Kuhn-Tucker (KKT) conditions:

$$a_n(\in + \xi_n + y_n - t_n) = 0$$

$$a_n^-(\in + \xi_n^- - y_n + t_n) = 0$$

$$(C - a_n)\xi_n = 0$$

$$(C - a_n^-)\xi_n^- = 0$$

Support vectors are points that lie on the boundary or outside the tube

$$b = t_n - \in - w^T \phi(x_n) = t_n - \in - \sum_{m=1}^{N} (a_m - a_m^-) k(x_n, x_m)$$