$VC(H) = 4$

VC (Rectangle in n dimension) $= 2n$

a b c

Tail to Tail

$$
\begin{tikzcd}
a & c \arrow[l] \arrow[r] & b
\end{tikzcd}
$$

$$p(a,b,c) = p(c)\, p(a|c)\, p(b|c)$$

$a \perp\!\!\!\perp b \mid \emptyset$? a & b marginally independent

$a \perp\!\!\!\perp b \mid c$? a & b independent given c

$$p(a,b) = \sum_{c} p(a,b,c)$$

if $p(a,b) = p(a) \cdot p(b)$ then $a \perp\!\!\!\perp b \mid \emptyset$

$$P(a,b) = \sum_c P(a,b,c)$$

$$= \sum_c P(a,b,c)$$

$$= \sum_c P(c)\, P(a|c)\, P(b|c)$$

$$\neq P(a)\, P(b)$$

$$a \not\perp\!\!\!\perp b \mid \emptyset \qquad a \leftarrow c \rightarrow b$$

if $\underline{\underline{P(a,b|c) = P(a|c)\,P(b|c)}}$ then

$a \perp\!\!\!\perp b \mid c$ is true

$$P(a, b \mid c) = \frac{P(a, b, c)}{P(c)}$$

$$P(a, b \mid c) = \frac{\cancel{P(c)}\, P(a \mid c)\, P(b \mid c)}{\cancel{P(c)}}$$

$$P(a, b \mid c) = P(a \mid c)\; P(b \mid c)$$

$$a \perp\!\!\!\perp b \mid c \quad \checkmark$$

Head to Tail

$$a \longrightarrow c \longrightarrow b$$

$$p(a,b,c) = p(a)\, p(c|a)\, p(b|c)$$

$a \perp\!\!\!\perp b \mid \emptyset$? No $\qquad$ $a \perp\!\!\!\perp b \mid c$?

$$p(a,b) = \sum_c p(a,b,c)$$

$$= \sum_c p(a)\, p(c|a)\, p(b|c)$$

$$= p(a) \sum_c p(b|c)\, p(c|a)$$

$$= p(a)\, p(b|a)$$

$$\neq p(a)\, p(b)$$

$$a \not\!\perp\!\!\!\perp b \mid \emptyset$$

$a \perp\!\!\!\perp b \mid c$? Yes

$$P(a, b \mid c) = \frac{P(a, b, c)}{P(c)}$$

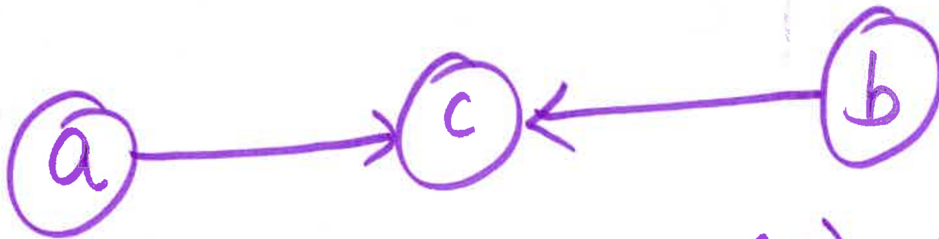
$$= \frac{\boxed{P(a) \; P(c \mid a)} \; P(b \mid c)}{P(c)}$$

$$= \frac{P(a, c)}{P(c)} \; P(b \mid c)$$

$$= P(a \mid c) \; P(b \mid c)$$

$a \perp\!\!\!\perp b \mid c$ ✓

Head to Head



$$p(a,b,c) = p(a)\,p(b)\,p(c|a,b)$$

$a \perp\!\!\!\perp b \mid \emptyset$? Yes $\quad$ $a \perp\!\!\!\perp b \mid c$? No

$$p(a,b) = \sum_c p(a,b,c)$$

$$= \sum_c p(a)\,p(b)\,p(c|a,b)$$

$$= p(a)\,p(b) \boxed{\sum_c p(c|a,b)} = 1$$

$$= p(a)\,p(b)$$

$$a \perp\!\!\!\perp b \mid \emptyset$$

$$P(a, b \mid c) = \frac{P(a, b, c)}{P(c)}$$

$$= \frac{P(a)\, P(b)\, P(c \mid a, b)}{P(c)}$$

$$\neq P(a \mid c)\, P(b \mid c)$$

$$a \not\perp\!\!\!\perp b \mid c$$

$a \leftarrow c \rightarrow b$

$a \rightarrow c \rightarrow b$

$a \perp\!\!\!\perp b \mid c$

$a \not\perp\!\!\!\perp b \mid \theta$

θ = null

$a \rightarrow c \leftarrow b$

$a \perp\!\!\!\perp b \mid \theta$

$a \not\perp\!\!\!\perp b \mid c$

$$P(G, F, B)$$

$$P(G) = \sum_{F,B} \underline{P(G, F, B)}$$

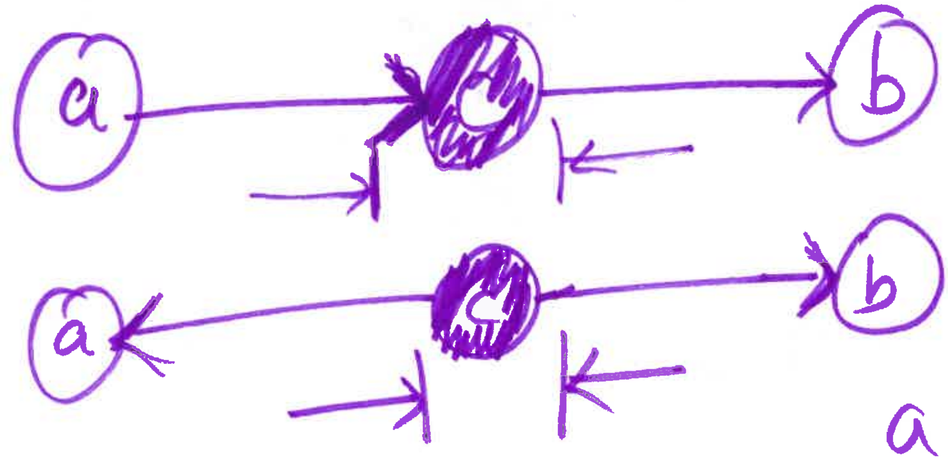
$$= \sum_{FB} P(G \mid F, B)\, P(F)\, P(B)$$

$$P(G=0 \mid F=0) = \frac{P(G=0, F=0)}{P(F=0)}$$

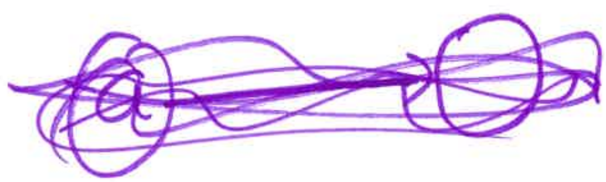
$$P(G=0, F=0) = \sum_{B} P(G=0, F=0, B)$$

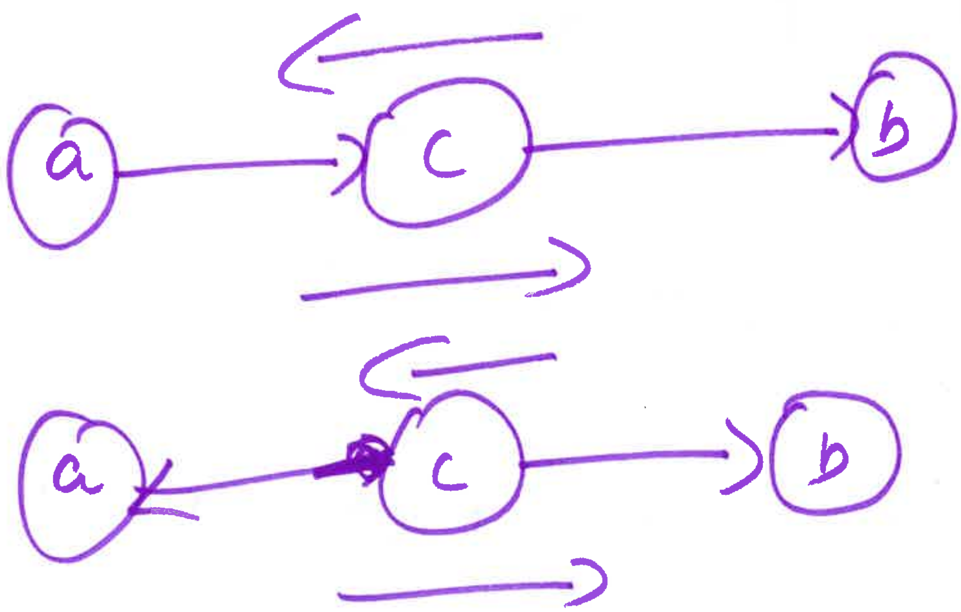
$P(F=0 \mid B=0, G=0)$

Flow indicates dependency

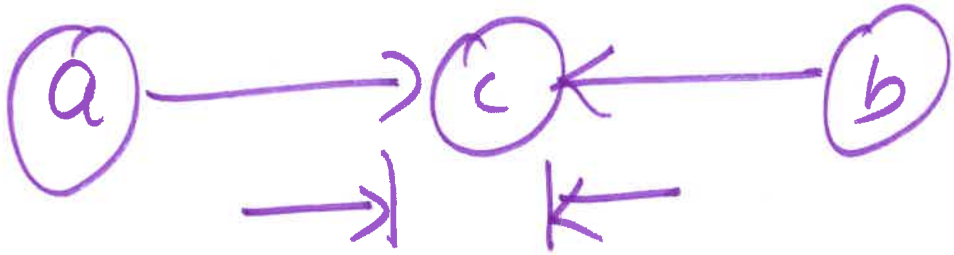


$a \perp\!\!\!\perp b \mid c$





$a \not\perp\!\!\!\perp b \mid \emptyset$



$a \perp\!\!\!\perp b \mid \emptyset$ $\qquad$ $a \not\!\perp\!\!\!\perp b \mid c$

$\underline{a \perp\!\!\!\perp b} \mid \emptyset$ ✓

$p(a, b, c, e, f)$

$p(a, b) = p(a)\ p(b)$

Markov ~~blanket~~ blanket for Bayesian networks

ϕ

$\bigcirc = X_i$

$$P\left(X_i \mid X_j \ (j \neq i)\right)$$

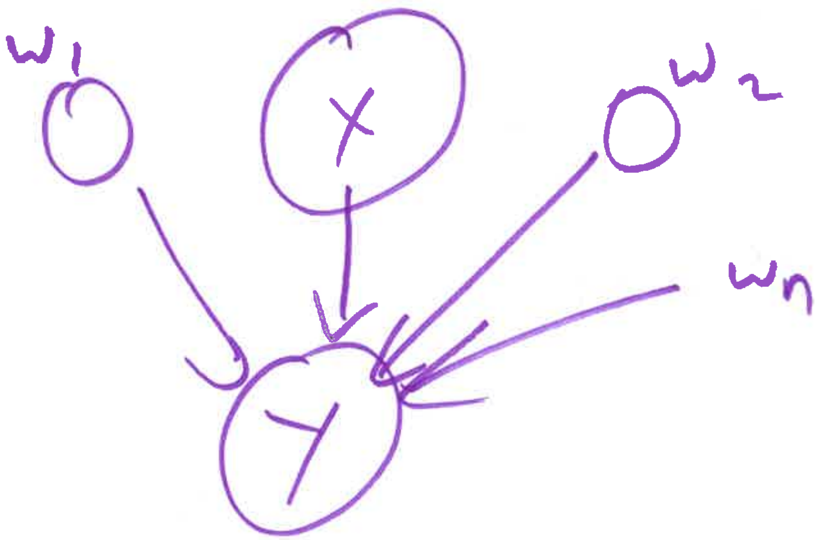
$$= \frac{P(X_1, X_2 \ \dots \ X_M)}{P(X_j \ \ j \neq i)}$$

$$P(X_j) \quad j \neq i \quad = \sum_{X_i} P(X_1, X_2 \cdots X_n)$$

$$\Big| \ \underbrace{X_j X_i X_j}$$

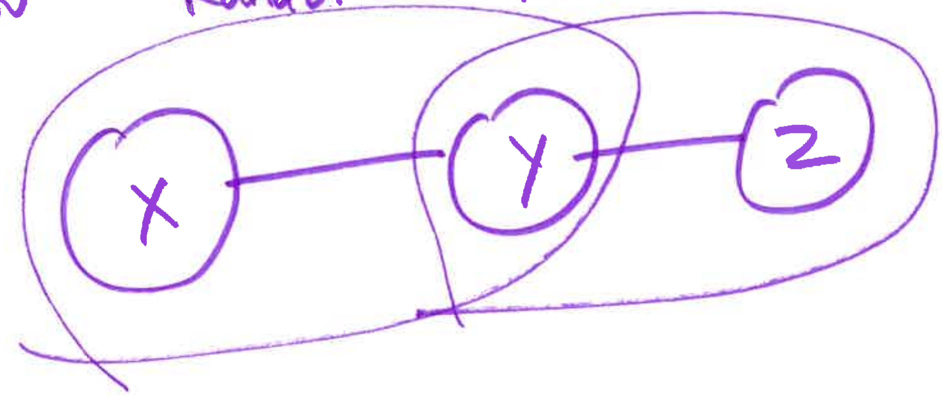
$$X_i \mid X_p$$

$=$



$$P(Y \mid \boxed{X}, w_1 \cdots w_N)$$

Markov Random Field



$$P(x,y,z) = \frac{1}{\boxed{Z}}\,\psi_1(x,y)\cdot\psi_2(y,z)$$

x $\quad\quad \psi_1$

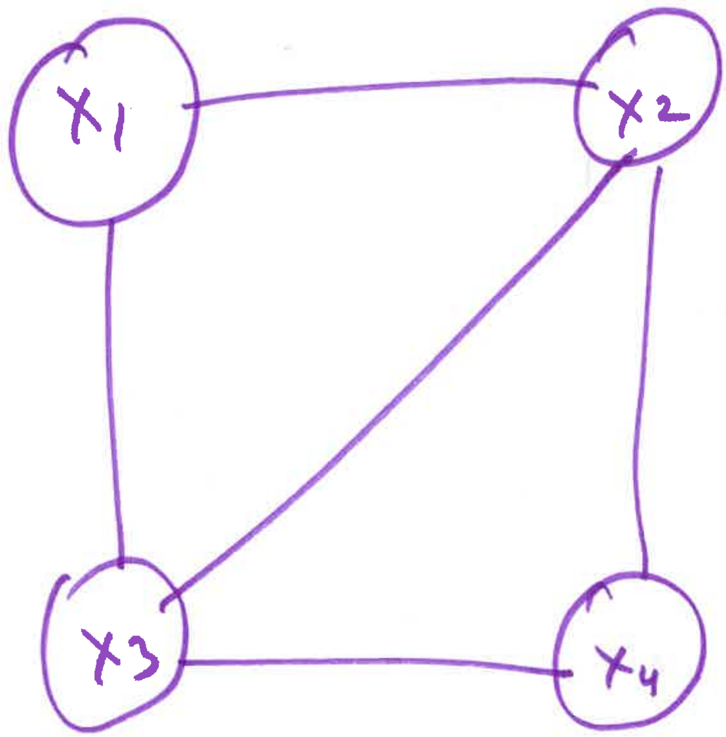
0, 1, 2

$$\psi_1(x=0) = 50 = 50/100 = 0.5$$

$$\psi_1(x=1) = 25 \quad 25/100 = 0.25$$

$$\psi_1(x=2) = 25 \quad 25/100 = 0.25$$

$$\overline{\boxed{100}}$$



$$P(x_1, x_2, x_3) = \frac{\psi_1(x_1, x_2, x_3)\ \psi_2(x_2, x_3, x_4)}{Z}$$

~~Ising Model~~

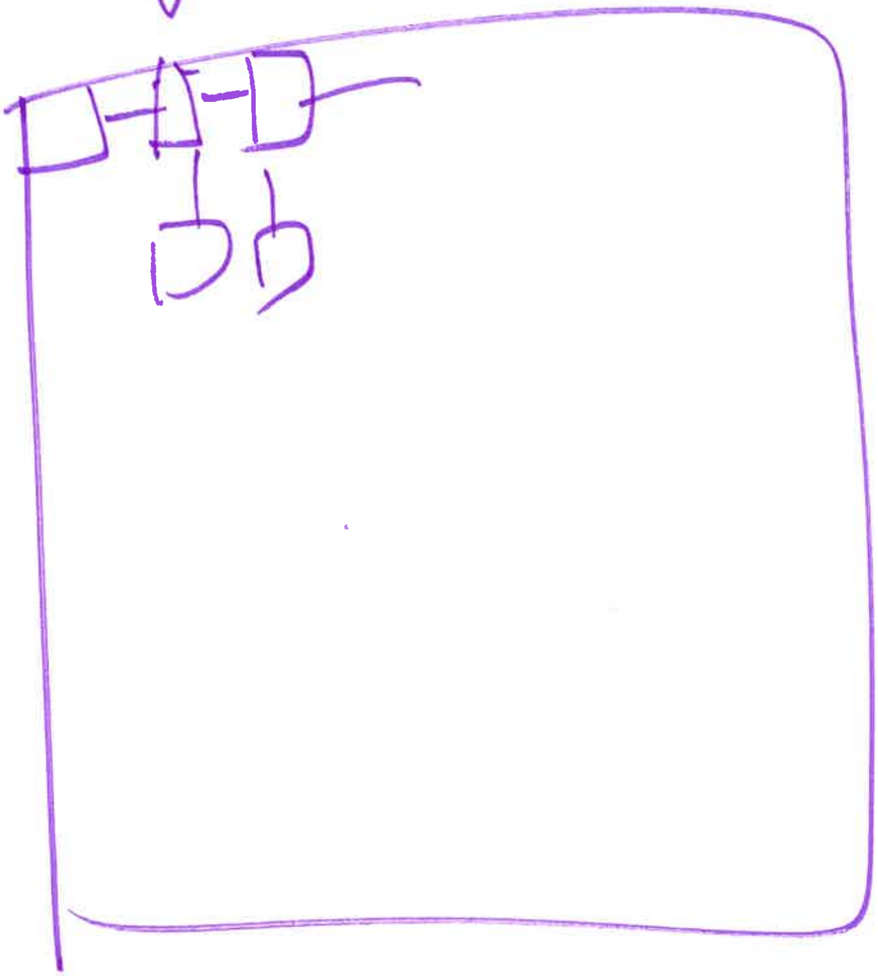
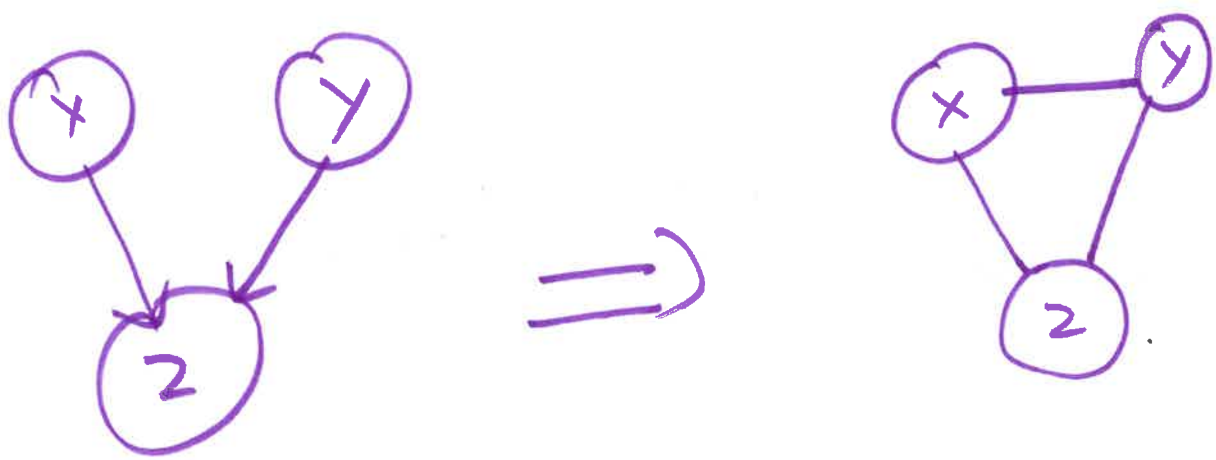
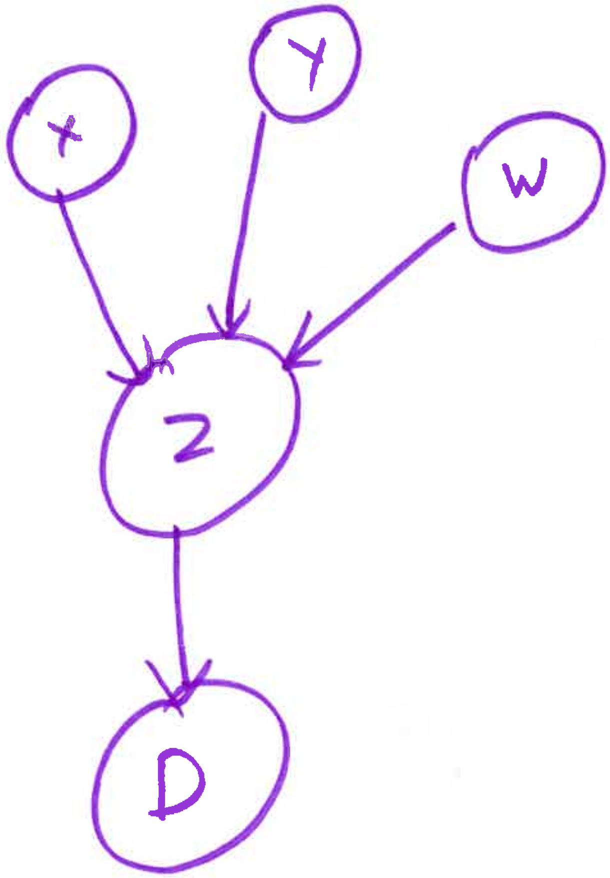


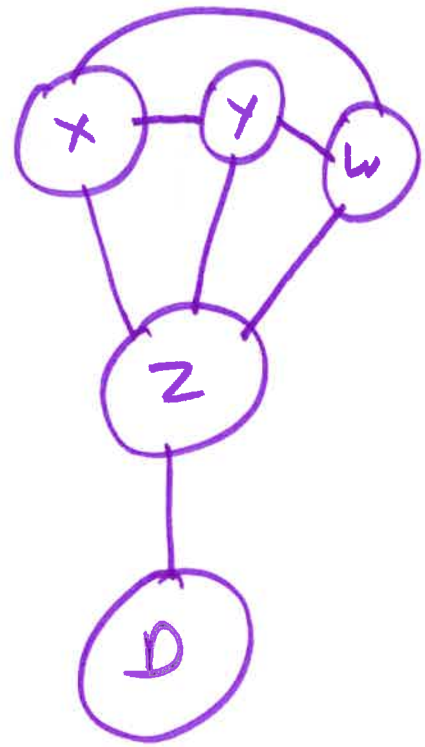
Image De-Noising

Moralization





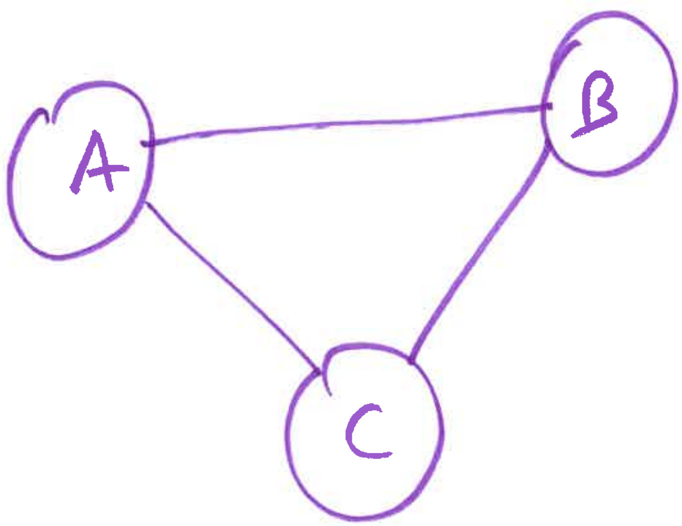
$\Rightarrow$

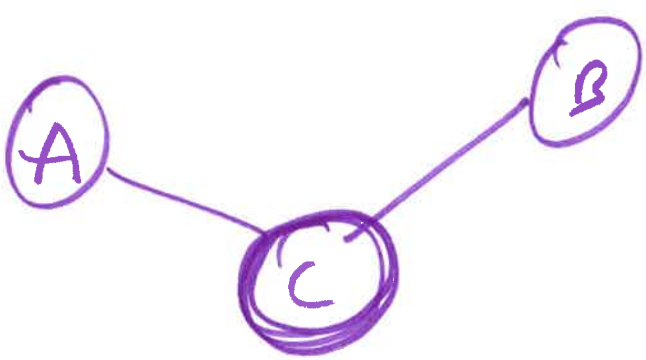


Joint Probability decomposes into marginals & conditional

$$P(a, b, c) = P(a)\, P(b)\, P(c)$$

$$= P(a|b)\, P(b|c)\, P(c)$$

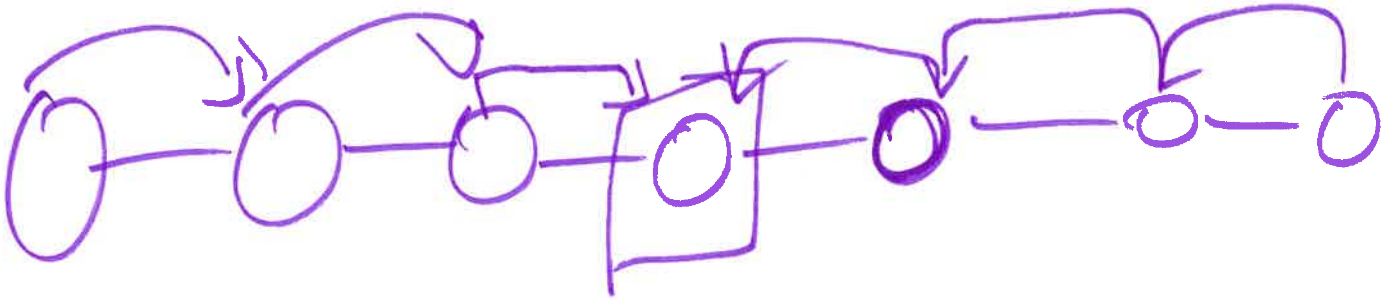




$A \perp\!\!\!\perp B \mid C$

$A \not\!\perp\!\!\!\perp B \mid Q$

Message passing Algorithm



$P(X_n)$

$X_n = 0$ 200

$X_n = 1$ 100

$X_n = 2$ 500

$X_n = 3$ 400

$X_n = 4$ 300