

Homework 3 Machine Learning

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Q.1.

- ① The MLE estimate for both the coin and thumbtack is the same but the MAP estimate is not.

TRUE

The MLE estimate for both depend on the outcome of the experiment. $\alpha_H = 60$ & $\alpha_T = 40$.

$$\text{MLE}_{\text{coin}} = \frac{\alpha_H}{\alpha_H + \alpha_T} = \frac{60}{60+40} = 0.6$$

The MAP estimates are not because the beta priors are different.

- ② The MAP estimate of the parameter θ (probability of landing heads) for the coin is greater than the MAP estimate of θ for the thumbtack.

FALSE

$$\text{MAP estimate for the coin} = \frac{\alpha_H + \beta_H - 1}{\alpha_H + \beta_H + \alpha_T + \beta_T - 2}$$

$$\text{For coin} \Rightarrow \frac{60 + 100 - 1}{60 + 100 + 40 + 100 - 2} = \frac{159}{298} = 0.53356$$

$$\text{MAP estimate for the thumbtack} = \frac{60 + 1 - 1}{60 + 1 + 40 + 1 - 2} = \frac{60}{100} = 0.6$$

Thus

$$\text{MAP}_{\text{coin}} < \text{MAP}_{\text{thumbtack}}$$

Q.3. Naive Bayes is a linear classifier

TRUE. Only under certain circumstances. When X is Gaussian & variance is class independent.

Q.4. $X \rightarrow Y$

$X = \langle X_1, X_2 \rangle$. $X_1 \rightarrow$ boolean $X_2 \rightarrow$ continuous variables seq.

7 parameters

2 - ^{for} Boolean X_1 -

one for $P(X_1|Y=1)$ & other for $P(X_1|Y=0)$.

4 - ^{for} Gaussian X_2 . $\begin{cases} P(X_2|Y=1) \in \mathbb{R} \\ P(X_2|Y=0) \in \mathbb{R} \end{cases}$

1 \rightarrow for $P(Y)$.

$$P(Y|X) = \frac{P(X|Y) \cdot P(Y)}{P(X)}$$

$$= \frac{P(X_1, X_2|Y) \cdot P(Y)}{P(X)}$$

$$= \frac{P(X_1|Y) \times P(X_2|Y) \times P(Y)}{P(X)}$$

Q.5

F1	F2	F3	Category
a	c	a	+
c	a	c	+
a	a	c	-
b	c	a	-
c	c	b	-

$$P(\text{Class } +) = \frac{2}{5} \quad P(\text{Class } -) = \frac{3}{5}$$

$$P(F1 = a \mid \text{Class } +) = \frac{1}{2} \quad P(F1 = a \mid \text{Class } -) = \frac{1}{3}$$

$$P(F1 = b \mid \text{Class } +) = 0 \quad P(F1 = b \mid \text{Class } -) = \frac{1}{3}$$

$$P(F1 = c \mid \text{Class } +) = \frac{1}{2} \quad P(F1 = c \mid \text{Class } -) = \frac{1}{3}$$

$$P(F2 = a \mid \text{Class } +) = \frac{1}{2} \quad P(F2 = a \mid \text{Class } -) = \frac{1}{3}$$

$$P(F2 = b \mid \text{Class } +) = 0 \quad P(F2 = b \mid \text{Class } -) = 0$$

$$P(F2 = c \mid \text{Class } +) = \frac{1}{2} \quad P(F2 = c \mid \text{Class } -) = \frac{2}{3}$$

$$P(F3 = a \mid \text{Class } +) = \frac{1}{2} \quad P(F3 = a \mid \text{Class } -) = \frac{1}{3}$$

$$P(F3 = b \mid \text{Class } +) = 0 \quad P(F3 = b \mid \text{Class } -) = \frac{1}{3}$$

$$P(F3 = c \mid \text{Class } +) = \frac{1}{2} \quad P(F3 = c \mid \text{Class } -) = \frac{1}{3}$$

$$F1 = a \quad F2 = c \quad F3 = b$$

$$P(F1 = a \mid \text{Class } +) * P(F2 = c \mid \text{Class } +) * P(F3 = b \mid \text{Class } +) * P(\text{Class } +)$$

$$= \frac{1}{2} \times \frac{1}{2} \times 0 \times \frac{2}{5} = 0$$

$$P(F1 = a \mid \text{Class } -) * P(F2 = c \mid \text{Class } -) * P(F3 = b \mid \text{Class } -) * P(\text{Class } -)$$

$$= \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} \times \frac{3}{5} = \frac{2}{45}$$

Therefore, Naive Bayes would classify the given case in (Class -)

6. Naive Bayes -

Person	height	weight	foot size
male	6	180	12
male	5.92	190	11
male	5.58	170	12
female	5.92 5	100	6
female	5.5	150	8
female	5.42	150	7
female	5.75	130	9
male	5.72	165	10

To Classify \rightarrow 6, 130, 8

Solⁿ $P(M) = 0.5$ & $P(F) = 0.5$. This prior probability could be based on our knowledge of freq. in the larger population or on frequency in the training set.

stats derived from ^{given} training data :-

	Mean Height	Mean Wt	Mean ft size	Variance ht	Variance wt	Variance foot
Male	5.855	176.25	11.25	0.035	0.0122	0.916
Female	5.4175	132.50	7.5	0.097	0.055	1.667

$$\text{Posterior (Male)} = \frac{P(\text{male}) P(\text{ht}|\text{male}) p(\text{wt}|\text{male}) p(\text{ft size}|\text{male})}{\text{Evidence}}$$

$$\text{evidence} = P(\text{male}) p(\text{ht}|\text{male}) \cdot p(\text{wt}|\text{male}) \cdot p(\text{ft.sz}|\text{male}) \\ + P(\text{female}) p(\text{ht}|\text{female}) p(\text{wt}|\text{female}) p(\text{ft.sz}|\text{female})$$

$$p(\text{ht}|\text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(6-\mu)^2}{2\sigma^2}} \approx 1.5789 \leftarrow \text{probability density because ht is a continuous variable}$$

$$p(\text{wt}|\text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(130-\mu)^2}{2\sigma^2}} = 5.9881 \times 10^{-6}$$

$$p(\text{ft.size}|\text{male}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(8-\mu)^2}{2\sigma^2}} = 1.3112 \times 10^{-3}$$

$$\text{posterior numerator (male)} = 6.1984 \times 10^{-9}$$

$$P(\text{female}) = 0.5$$

$$p(\text{ht}|\text{female}) = 2.23 \times 10^{-1}$$

$$p(\text{wt}|\text{female}) = 1.67 \times 10^{-2}$$

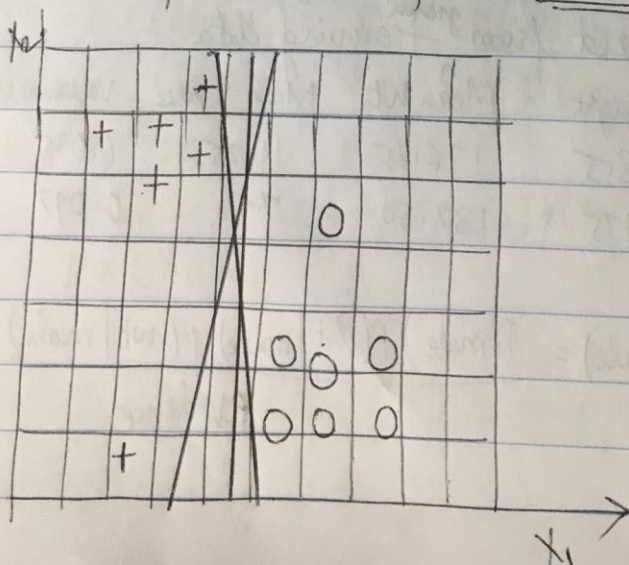
$$p(\text{ft.size}|\text{female}) = 2.86 \times 10^{-1}$$

$$\text{posterior numerator (female)} = 5.3778 \times 10^{-4}$$

Since numerator (male) is less than the one in female case, we predict sample is FEMALE.

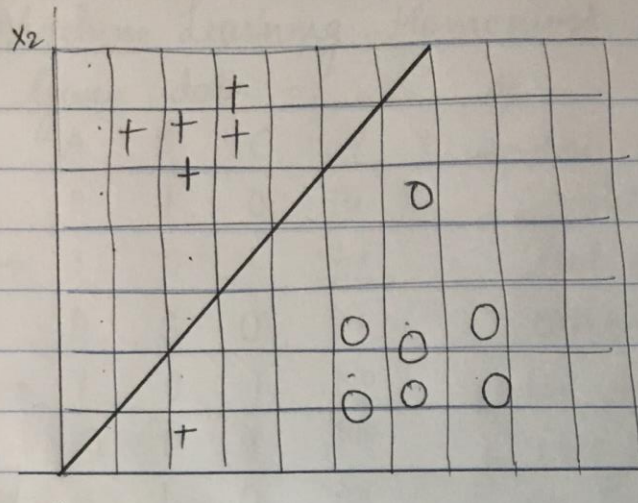
Q.7.

a).



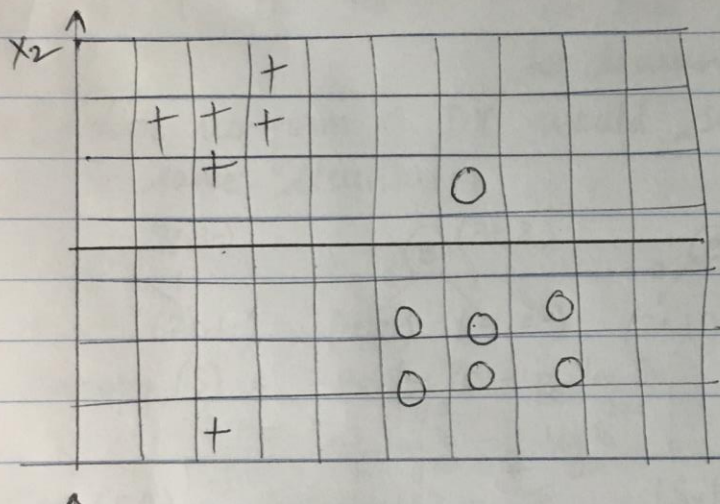
Classification errors = 0

(b)



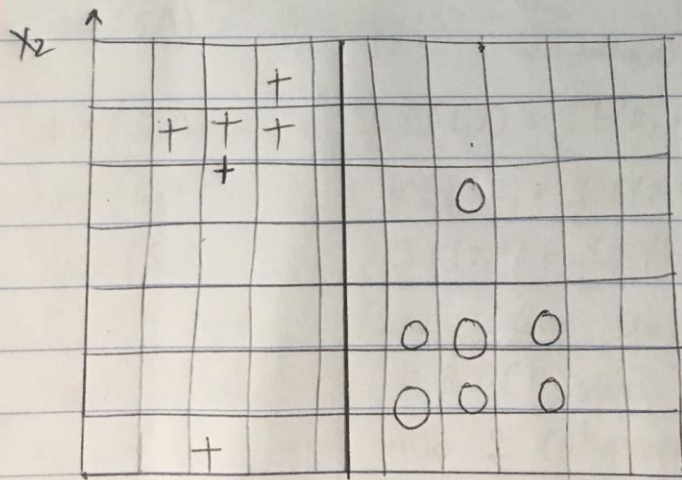
Classification errors
= 1.

(c)



classification errors = 2

(d)



classification errors = 0