

Max individual probabilities & combined probabilities are different

$$\max_{0,1} P(X) = 0$$

$$\max_{00,01,10,11} P(X,Y) = 10$$

$$2^5$$

$x_1 \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad x_N$

$P(\boxed{x_n})$

$P(x_1 \cdot \cdot \cdot x_N)$

~~happy~~

$P(\text{ch}[2] = 'o') = ?$ forward
backward

$\max_{z_1 \dots z_N} P(x_1 \dots x_N, z_1 \dots z_N) = ?$ Viterbi
algorithm

$$P(\underline{x_n}) = \sum_{\substack{x_1, \dots, x_{n-1} \\ x_{n+1}, \dots, x_N}} P(x_1, x_2, \dots, x_N)$$

x_n

1



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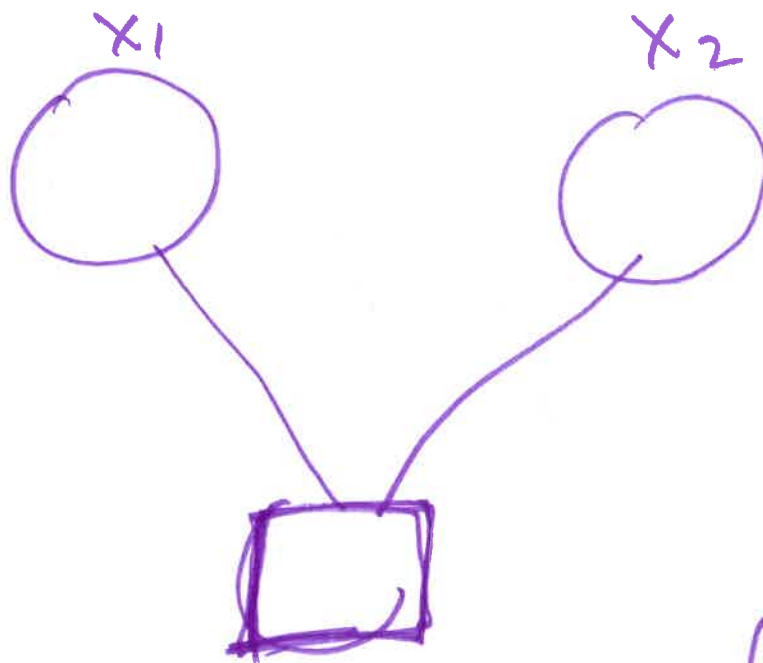
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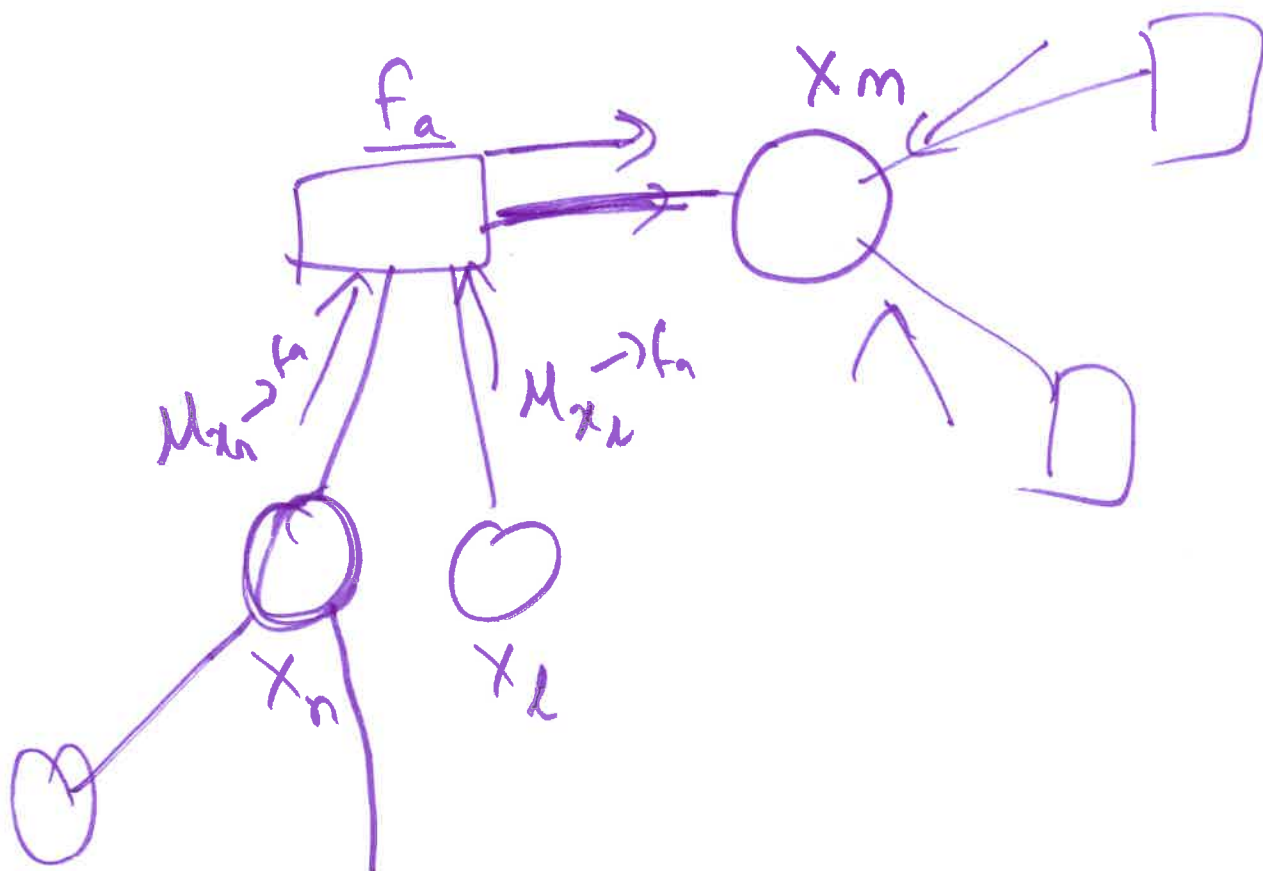


$z =$

$$\psi(x_1, x_2)$$



$$f_a = f(x_n, x_l, x_m)$$



$$M[D] = \left| \underline{\text{Domain of } D - 1} \right| \cdot$$

$$(\text{Domain of } C) \cdot (\text{Domain of } D)$$

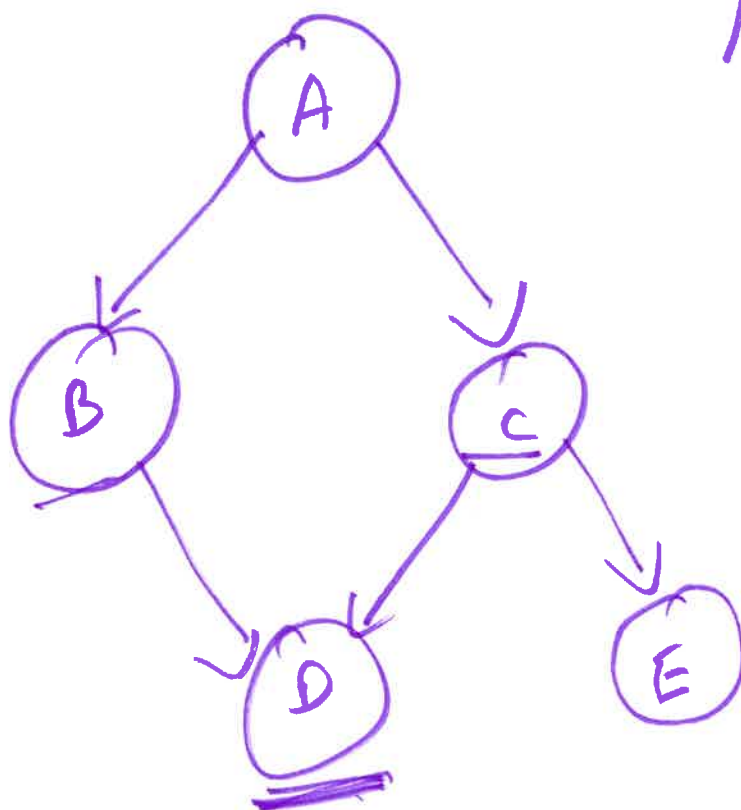
$$= (2-1) (2) (2) = 4$$

$$P(E=T) = \sum_{\substack{ABCD \\ 2^{16}}} P(A, B, C, D, E=T)$$

$$P(E=T) = \sum_{ABCD} P(A, B, C, D, E=T)$$

$$= \sum_{\boxed{ABCD}} P(A) P(D|A) \underbrace{P(C|A)}_{P(C|A)} P(D|B, C) P(E|C)$$

$$ab + ac = a(b+c)$$



$$M[A] = 1$$

$$M[B] = 2$$

$$M[C] = 2$$

$$M[D] = 4$$

$$M[E] = 2$$

$$\underline{\underline{11}}$$

$$P(A, B, C, D, E) = \frac{P(A) P(B|A)}{P(C|A) P(D|C, B) \underline{P(E|C)}}$$

$$P(A=0) \Rightarrow 1$$

$$P(B|A)$$

$$P(\underline{B=0} | \underline{A=0}) = 1 - P(\underline{B=1 | A=0})$$

$$P(\underline{B=0 | A=1}) = 1 - P$$

[illegible]

$$P(D|B,C)$$

$$\phi(B,C,D)$$

$$P(D=\text{true}, E=\text{true}) = \sum_{ABC} P(A,B,C,D=T, E=T)$$

$$= \sum_{ABC} P(A) P(B|A) P(C|A) P(D=T|B,C) P(E=T|C)$$

$$= \sum_A P(A) \sum_B P(B|A) \sum_C P(C|A) P(\check{D}=T, \check{B}, C) P(\check{E}=T|C)$$

$$\begin{aligned}
 &= \sum_A P(A) \sum_B P(B|A) \sum_C \overbrace{P(C|A)}^{\phi(A,c)} \underbrace{\frac{P(D=T|B,c)}{P(F=T|c)}}_{\underbrace{\phi(B,c)}_1} \\
 &\quad \underbrace{\phi_2(A,B,c)} \\
 &\quad \underbrace{\phi_3(A,B)} \\
 &\quad \underbrace{\phi_4(A,B)} \\
 &\quad \underbrace{\phi_5(A)} \\
 &\quad \underbrace{\phi_6(A)} \\
 &\quad \underbrace{\phi_7(\cdot)}
 \end{aligned}$$

$$P(E=T|C)$$

C	$\phi_0(C)$
0	0
1	0.7

$$P(D=T|B, C)$$

B	C	
0	0	0
0	1	0.8
1	0	0.9
1	1	0.95

$$P(C|A)$$

A	C	
0	0	0.91
0	1	0.11
1	0	0.2
1	1	0.8

$$\phi_2(A, B, C)$$

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$$0 \times 0 \times 0.9 = 0$$

$$0.7 \cdot 0.8 \times 0.1 = 0.056$$

$$0 = 0$$

$$0.7 \cdot 0.95 \times 0.1 = 0.0665$$

$$0 = 0$$

$$0.7 \cdot 0.8 \cdot 0.8 = 0.448$$

$$0 = 0$$

$$0.7 \cdot 0.95 \cdot 0.8 = 0.532$$

A	B	$\Phi_3(A, B)$	A	B	$P(B A)$
0	0	0.056	0	0	0.25
0	1	0.0665	0	1	0.75
1	0	0.448	1	0	0.8
1	1	0.532	1	1	0.2

A	B	$\Phi_4(A, B)$
0	0	$0.056 \cdot 0.25 = 0.0140$
0	1	$0.0665 \cdot 0.75 = 0.0499$
1	0	$0.448 \cdot 0.8 = 0.3584$
1	1	$0.1064 = 0.532 \times 0.2$

A	$\phi_5(A)$
0	0.0639
1	0.4648

A	$P(A)$
0	0.4
1	0.6

A	$\phi_6(A)$
0	$0.4 \cdot 0.0639 = 0.02556$
1	$0.6 \cdot 0.4648 = 0.27888$

$$\begin{aligned}\phi_7(\cdot) &= 0.02556 + 0.27888 \\ &= 0.30444\end{aligned}$$

$$P(D=T, E=T) = 0.30444$$

$$O(5 \cdot \underline{2^3})$$

$$O(n \cdot \exp(\underline{w}))$$

————— X —————

$$P(D=T, E=T)$$

$$\sum_{ABC} \frac{P(A) P(B|A) P(C|A) P(D=T|\overset{BC}{\cancel{AB}})}{P(E=T|C)}$$

$$\sum_A \underline{\underline{\phi(A, B, C)}}$$

$$P(E=T | A=T)$$

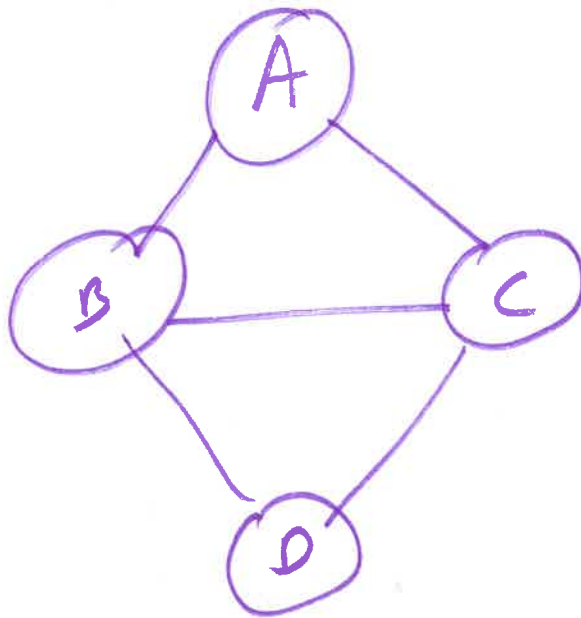
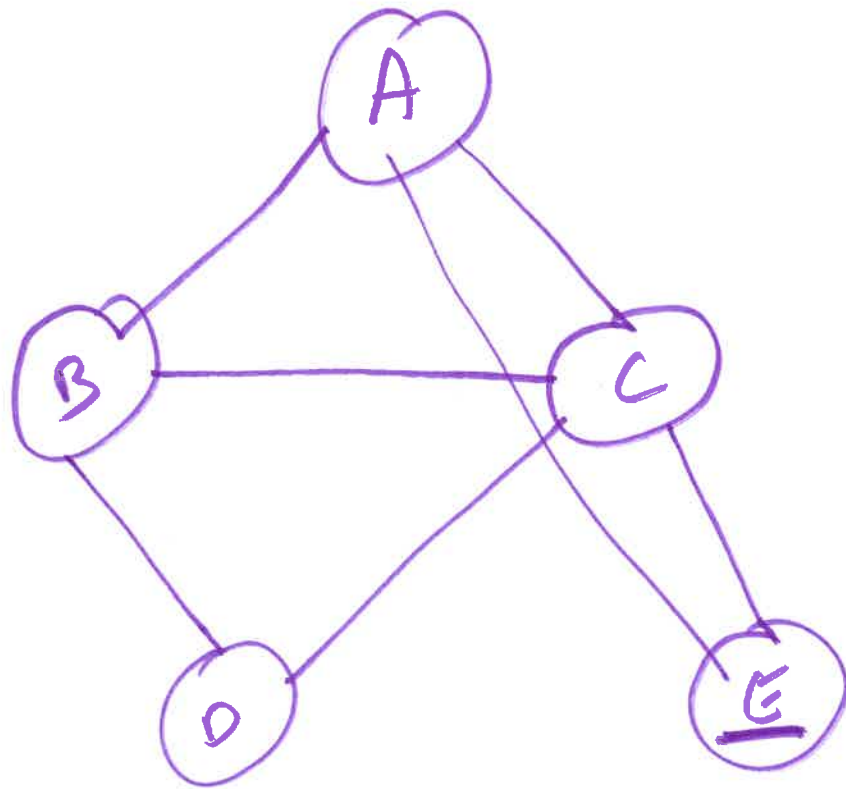
$$= \frac{P(\overset{\checkmark}{E=T}, A=T)}{P(A=T)\checkmark}$$

Moral Graph = Primal Graph

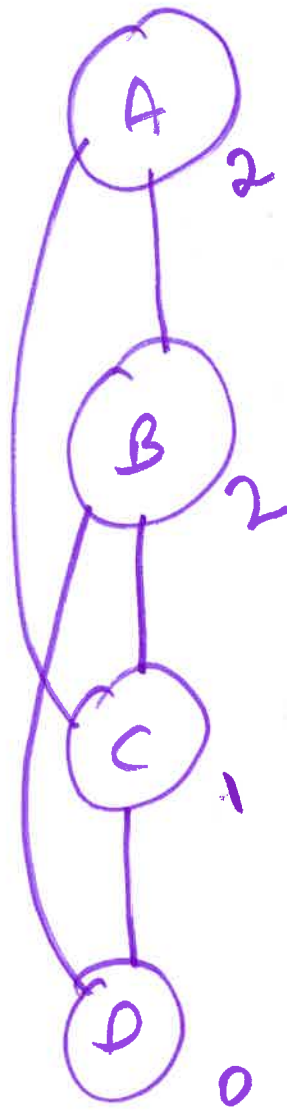
Moralization

Take directed graph & convert
into undirected

$P(E=T)$



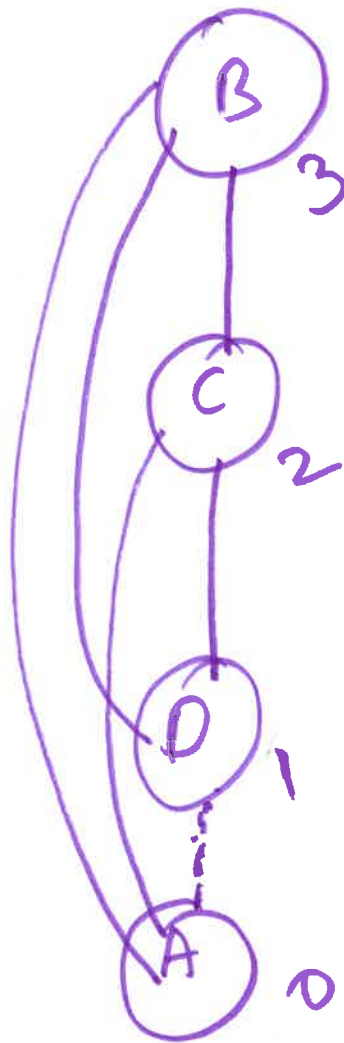
A B C D



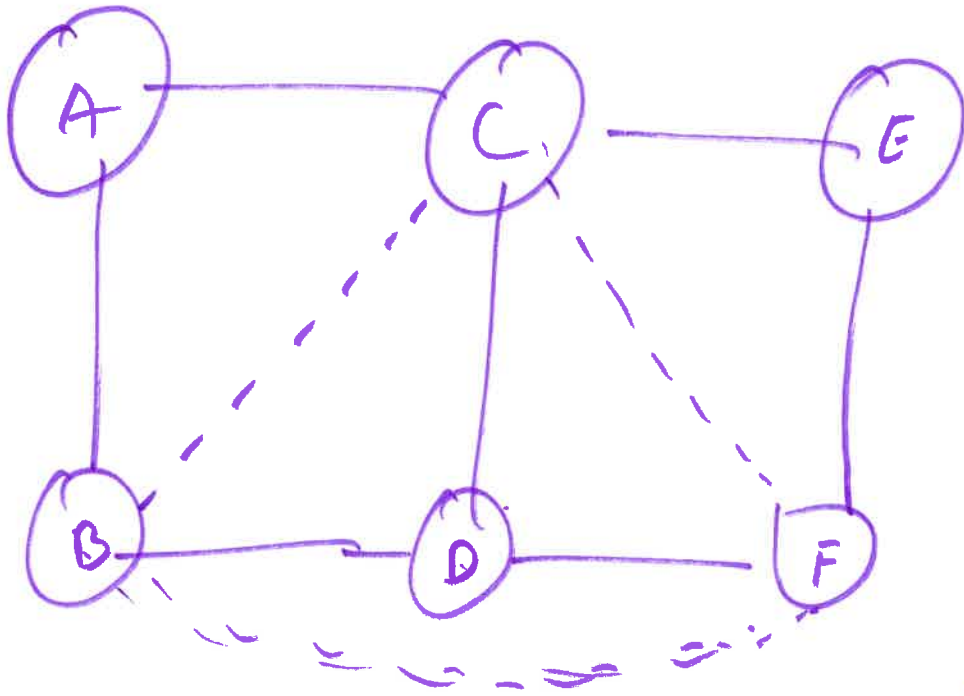
width of Graph

$$= \underline{\underline{2}}$$

$$O(\underline{n} \exp(w+1))$$



$$w(a) = 3$$



A E D F B C

$$P(E=T)$$

$$P(F=T)$$

