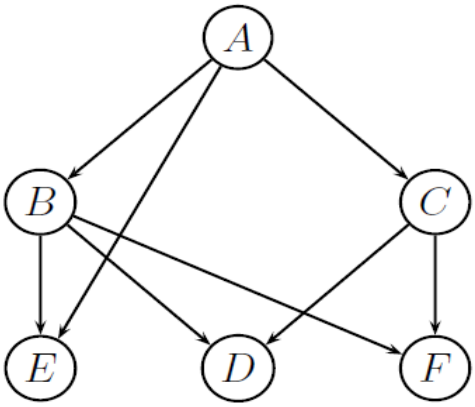
Problem Set – Graphical Models

1. A graph with no links is a trivial D-Map. True/False

2. Consider the Bayesian network given below

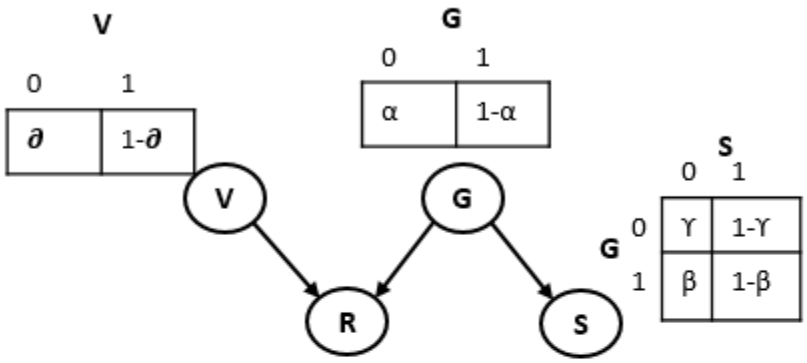


a. Is A conditionally independent of D give {B,C}.
b. Is E marginally independent of F
c. Which edge would you delete to make A independent of C.

3. Evaluate the distribution p(a), p(b|c) and p(c|a) corresponding to the joint distribution given in the Table. Hence show by direct evaluation that p(a,b,c) = p(a) p(c|a) p(b|c). Draw the corresponding directed graph.

a	b	c	p(a, b, c)
0	0	0	0.192
0	0	1	0.144
0	1	0	0.048
0	1	1	0.216
1	0	0	0.192
1	0	1	0.064
1	1	0	0.048
1	1	1	0.096

4. Consider the directed graphical model in following figure with 4 binary variables.



R

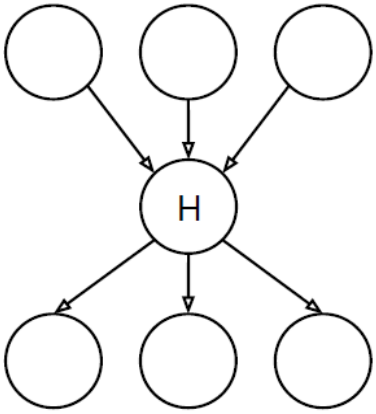
VG	0	1
00	0.6	0.4
01	0.3	0.7
10	0.2	0.8
11	0.1	0.9

a. Write down the expression for P(S=1|V=1) in terms of α, β, ϒ, $\boldsymbol{\partial}$.
b. Write down the expression for P(S=1|V=0) . Is it the same or different to P(S=1|V=1? Explain why.
c. Find the maximum likelihood estimate of α, β, ϒ using the following dataset, where each row is a training case.

V	G	R	S
1	1	1	1
1	1	0	1
1	0	0	0

5. Hidden variables in DGMs:

a. Consider the following graphical model, where we number nodes left to right, top to bottom.



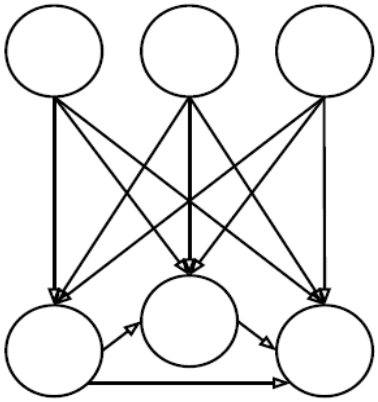
The graph defines the joint as

$$P(X_1, X_2, X_3, X_4, X_5, X_6) = \sum_h P(X_1)P(X_2)P(X_3)p(H = h|X_1X_2X_3)P(X_4|H = h)P(X_5|H = h)P(X_6|H = h)$$

where we have marginalized over the hidden variable H.

Assuming all nodes are binary, how many parameters does this model have?

b. Consider the following graph and its joint distribution (again we number nodes from left to right and from top to bottom)



$$P(X_1, X_2, X_3, X_4, X_5, X_6) = P(X_1)P(X_2)P(X_3)\, P(X_4|X_1, X_2, X_3)P(X_5|X_1, X_2, X_3, X_4)P(X_6|X_1, X_2, X_3, X_4, X_5)$$

Assuming all nodes are binary, how many parameters does this model have?

6. What is the complexity of computing $P(E = e)$ using variable elimination in the following Bayesian network along the ordering (A, B, C, D) The edges in the Bayesian network are $A \rightarrow B, A \rightarrow C, B \rightarrow C, C \rightarrow D \; and \; D \rightarrow E$.

7. What is the complexity of computing $P(E = e)$ using variable elimination in the following Bayesian network along the ordering (B, C, D, A). The edges in the Bayesian network are $A \rightarrow B, B \rightarrow C, C \rightarrow D \; and \; D \rightarrow E$.

8. Consider a Bayesian networks with edges $A \rightarrow B$ and $A \rightarrow C$, and parameters which are given below

$P(A = 1) = 0.9$

$P(B = 1|A = 1) = 0.1, \qquad P(B = 1|A = 0) = 0.6$

$P(C = 1|A = 1) = 0.7, \qquad P(C = 1|A = 1) = 0.3$

Consider the dataset given below

A	B	C
0	1	?
0	1	1
?	0	1
1	1	?
1	0	?
0	0	0
1	1	1

Assume the CPTs are the CPTs at some iteration of EM. You have to derive new set of parameters after running one iteration of EM.

a. Show the calculation involved in E-step.
b. Show the calculations involved in M-step and give the new CPTs.