1 Introduction and Examples

1.1 Value of the stochastic Solution

cf. 1_1.jl

1.2 Price effect

cf. 1_1.jl

1.3 Binary first stage

Set

- F: fields, index i
- P: products, index j

Parameter

- $field_i$: size of field i
- $plant_j$: unit price of planting product j
- buy_j: unit price of buying product j
- $sell_j$: unit price of selling product j
- $cattle_j$: amount of product j to keep for the cattle
- $yield_j$: yield of product j for an unit of a field
- $capBeet$: max quantity of beet to be sold at a favorable price

Decision

- y_j= "amount of product j bought", $\forall j = 1...|P| - 1$
  sugar beets can't be bought on the market
- w_j= "amount of product j sold", $\forall j = 1...|P| + 1$
  the last one being the sugar beets sold at a lower price
- $x_{ij} = \begin{cases} 1 & \text{"if field } i \text{ is planted with product } j\text{"} \\ 0 & \text{else} \end{cases} \quad , \forall i = 1...|F|,\ \forall j = 1...|P|$

Mathematical model

$$\min \sum_{j=1}^{|P|} \sum_{i=1}^{|F|} plant_j \cdot field_i \cdot x_{ij} + \mathcal{Q}(x) \tag{1}$$

where x = $(x_{ij})_{i=1...|F|, j=1...|P|}$

s.t.:

$$\sum_{j=1}^{|P|} x_{ij} \leq 1 \qquad \forall i = 1...|F| \tag{2}$$

$$x_{ij} \in \{0,1\} \qquad \forall i = 1...|F|, \forall j = 1...|P| \tag{3}$$

with the recourse function $\mathcal{Q}$ defined as:

$$\mathcal{Q} = \sum_{j=1}^{|P|-1} buy_j \cdot y_j - \sum_{j=1}^{|P|+1} sell_j \cdot w_j \tag{4}$$

s.t.

$$y_j - w_j \geq cattle_j - yield_j \cdot (\sum_{i=1}^{|F|} field_i \cdot x_{ij}) \qquad \forall j = 1...|P| - 1 \tag{5}$$

$$-w_j - w_{j+1} \geq cattle_j - yield_j \cdot (\sum_{i=1}^{|F|} field_i \cdot x_{ij}) \qquad j = |P| \tag{6}$$

$$w_j \leq capBeet \qquad j = |P| \tag{7}$$

$$w_j \geq 0 \qquad \forall j = 1...|P| + 1 \tag{8}$$

$$y_j \geq 0 \qquad \forall j = 1...|P| - 1 \tag{9}$$

1.4 Integer second stage

Parameter

- $order$: order of magnitude of a contract, numerically here $order$=100
- $field$: total size of the field

Decision x_j= "amount of field used to produce product j", $\forall j = 1...|P|$

Model

$$\min \sum_{j=1}^{|P|} plant_j \cdot x_j + \mathcal{Q}(x) \tag{1}$$

where x = $(x_j)_{j=1...|P|}$
s.t.:

$$\sum_{j=1}^{|P|} x_j \leq field \tag{2}$$

$$x_j \geq 0 \qquad \forall j = 1...|P| \tag{3}$$

with the recourse function $\mathcal{Q}$ defined as:

$$\mathcal{Q} = \sum_{j=1}^{|P|-1} buy_j \cdot y_j - \sum_{j=1}^{|P|+1} sell_j \cdot w_j \tag{4}$$

s.t.

$$\begin{aligned}
order \cdot (y_j - w_j) &\geq cattle_j - yield_j \cdot x_j & \forall j = 1...|P| - 1 \quad (5)\\
-order \cdot (w_j + w_{j+1}) &\geq cattle_j - yield_j \cdot x_j & j = |P| \quad (6)\\
order \cdot w_j &\leq capBeet & j = |P| \quad (7)\\
w_j &\in \mathbb{N} & \forall j = 1...|P| + 1 \quad (8)\\
y_j &\in \mathbb{N} & \forall j = 1...|P| - 1 \quad (9)
\end{aligned}$$

1.5 Computation

cf. 1_1.jl

1.6 Multistage program

3-stage problem :

- 1st stage is planting products
- 2nd stage is buying, selling grown products and planting for the next year
- 3rd stage is selling grown products of year 2 and buying

Set and Parameter

- H: time horizon
- T: time scale, index t, $T = 1, ..., H$

here H=2

Decision

- x_i^t = "amount of field planted with product i at time t", $\forall i \in P, \forall t = 1, ..., H$
- y_j^t= "amount of product j bought at time t", $\forall j = 1...|P| - 1, \forall t = 1, ..., H$
  sugar beets can't be bought on the market
- w_j^t= "amount of product j sold at time t", $\forall j = 1...|P|+1, \forall t = 1, ..., H$
  the last one being the sugar beets sold at a lower price

Model

$$\min \sum_{j=1}^{|P|} plant_j \cdot x_j^1 + \mathcal{Q}_1(x^1) \tag{1}$$

where $x^1 = (x_j^1)_{j=1...|P|}$ s.t.:

$$\sum_{j=1}^{|P|} x_j^1 \leq field \tag{2}$$

$$x_j^1 \geq 0 \qquad \forall j = 1...|P| \tag{3}$$

with the recourse function $\mathcal{Q}_1$ defined as:

$$\mathcal{Q}_1 = \mathrm{E}_{\xi^1} \mathrm{Q}_1(x^1, \xi^1) \tag{4}$$

and

$$\mathrm{Q}_1(x^1, \xi^1) = \sum_{j=1}^{|P|-1} buy_j \cdot y_j^1 - \sum_{j=1}^{|P|+1} sell_j \cdot w_j^1 + \sum_{j=1}^{|P|} plant_j \cdot x_j^2 + \mathcal{Q}_2(x^2) \tag{5}$$

s.t.:

$$\sum_{j=1}^{|P|} x_j^2 \leq field \tag{6}$$

$$x_{|P|}^2 \leq field - x_{|P|}^1 \tag{7}$$

$$y_j^1 - w_j^1 \geq cattle_j - yield_j \cdot x_j^1 \qquad \forall j = 1...|P|-1 \tag{8}$$

$$-w_{|P|}^1 - w_{|P|+1}^1 \geq cattle_{|P|} - yield_{|P|} \cdot x_{|P|}^1 \tag{9}$$

$$w_{|P|}^1 \leq capBeet \tag{10}$$

$$w_j^1 \geq 0 \qquad \forall j = 1...|P|+1 \tag{11}$$

$$y_j^1 \geq 0 \qquad \forall j = 1...|P|-1 \tag{12}$$

$$x_j^2 \geq 0 \qquad \forall j = 1...|P| \tag{13}$$

$$\tag{14}$$

$$\mathcal{Q}_2 = \mathrm{E}_{\xi^2} \mathrm{Q}_2(x^2, \xi^2) \tag{15}$$

and

$$\mathrm{Q}_2(x^2, \xi^2) = \sum_{j=1}^{|P|-1} buy_j \cdot y_j^2 - \sum_{j=1}^{|P|+1} sell_j \cdot w_j^2 \tag{16}$$

s.t.

$$y_j^2 - w_j^2 \geq cattle_j - yield_j \cdot x_j^2 \qquad \forall j = 1...|P|-1 \tag{17}$$

$$-w_{|P|}^2 - w_{|P|+1}^2 \geq cattle_{|P|} - yield_{|P|} \cdot x_{|P|}^2 \tag{18}$$

$$w_{|P|}^2 \leq capBeet \tag{19}$$

$$w_j^2 \geq 0 \qquad \forall j = 1...|P|+1 \tag{20}$$

$$y_j^2 \geq 0 \qquad \forall j = 1...|P|-1 \tag{21}$$

Equation 7 links the sugar beets planting quantity between the 2 years. Using the 2 recourse functions $\mathcal{Q}_1$ and $\mathcal{Q}_2$, decisions are taken while knowing what was the result of the previous steps.

If there is uncertainty on the yield, sale and bying prices, then the associated parameters becomes variables of the scenarios $\xi^t = \{\xi_k^t\}_{1 \leq k \leq K}$, with K being the number of scenarios:
$buy_j^t(\xi_k^t)$, $sell_j^t(\xi_k^t)$, $yield_j^t(\xi_k^t)$

1.7 Risk aversion

Plan for the worst-case, i.e. maximize the profit for the worst case due to risk aversion.

a) The worst situation corresponds to scenario 3 (below average yields), hence the agent applies the first-stage optimal solution of this scenario to all cases. i.e. surfaces in acres : (wheat, corn, sugar beets) = (100, 25, 375).
The computation results in the following objective values, from the worst case to the best case:
59950 86600 113250, i.e. the risk aversion expected profit = 86600 (see 1_1.jl).
To compare it with results from the optimal decisions of expected profit, i.e. decisions were (170, 80, 250):
48820 109350 167000, i.e. the standard expected profit = 108390.
One can notice the value obtained in the worst case is better in the risk averse case, however in the 2 other scenarios, values are significantly lower, resulting in an average worse value. The loss in expected profit with this solution: 21790

b) Add a constraint of a lower bound for the objective in the worst case, in the recourse function Q add:

$$\sum_{j=1}^{|P|-1} buy_j(\xi) \cdot y_j - \sum_{j=1}^{|P|+1} sell_j(\xi) \cdot w_j \leq -LowerBound - \sum_{j=1}^{|P|} plant_j \cdot x_j \quad (1)$$

The equation is like this because sell - buy - plant = profit, hence buy + plant - sell = - profit, as we want profit $\geq$ LowerBound $\iff$ - profit $\leq$ - LowerBound

c)

1.8 Data fluctuations