Generalized Hough Transform

16-385 Computer Vision

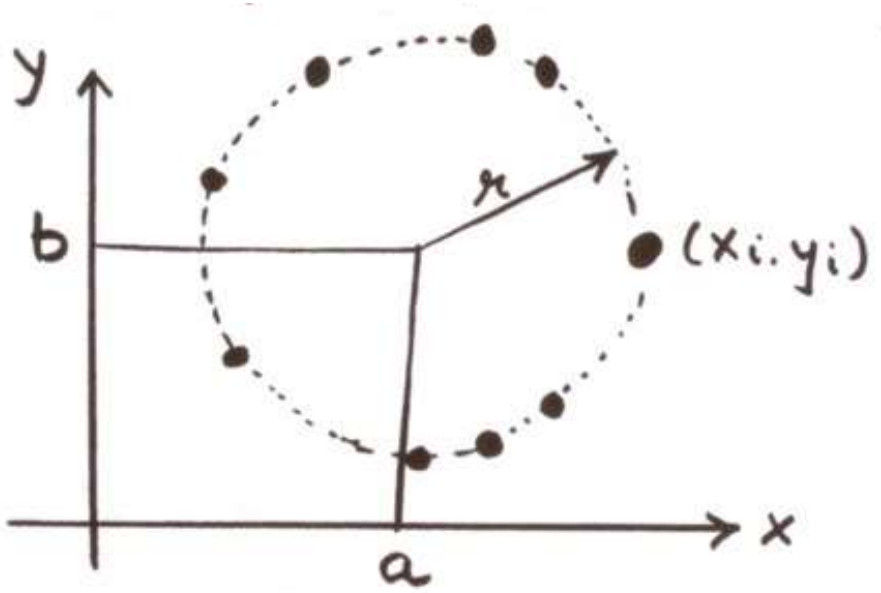
Hough Circles

Finding Circles by Hough Transform

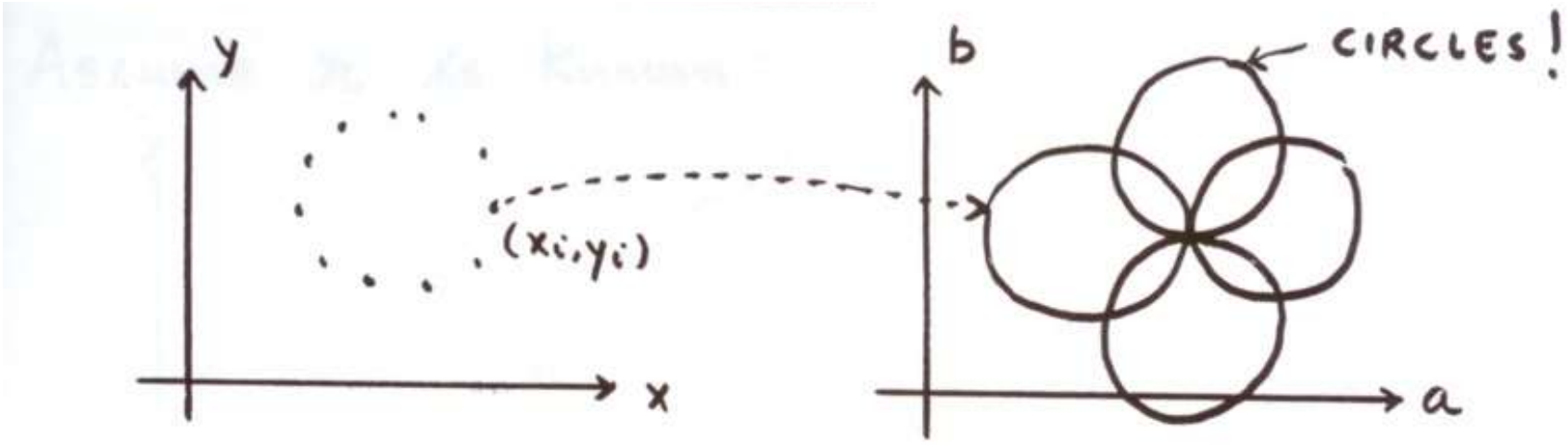
Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$

If radius is known: (2D Hough Space)



Accumulator Array $A(a,b)$



parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

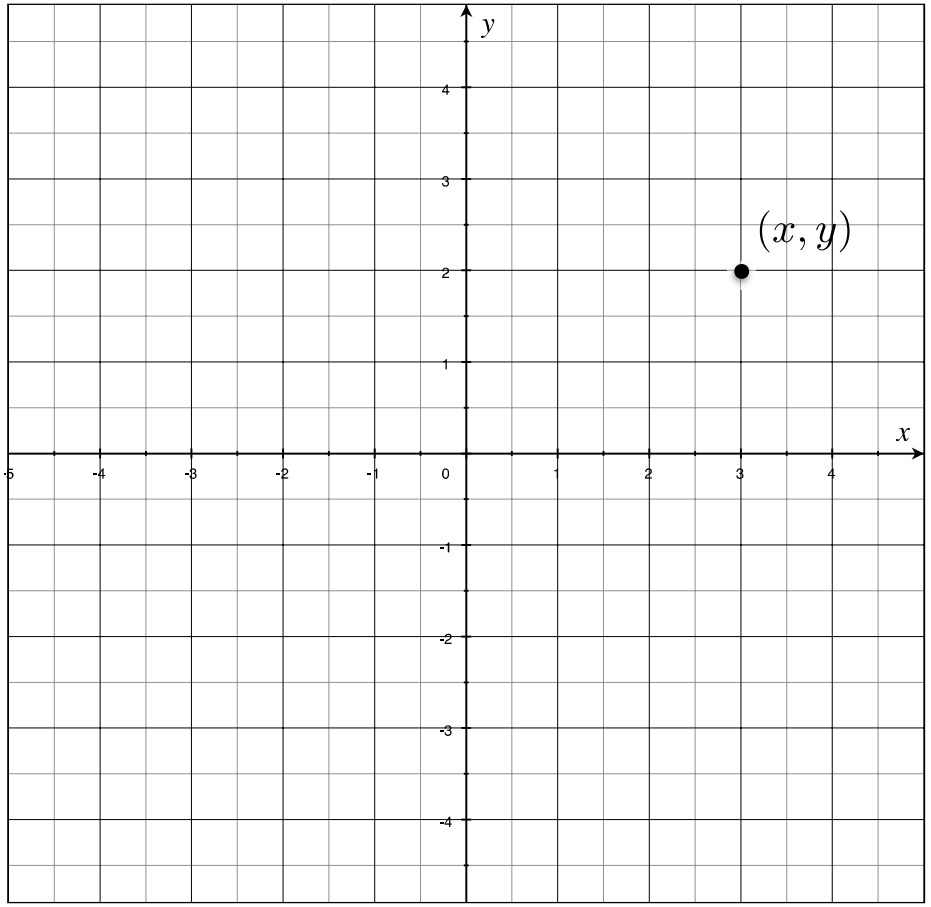
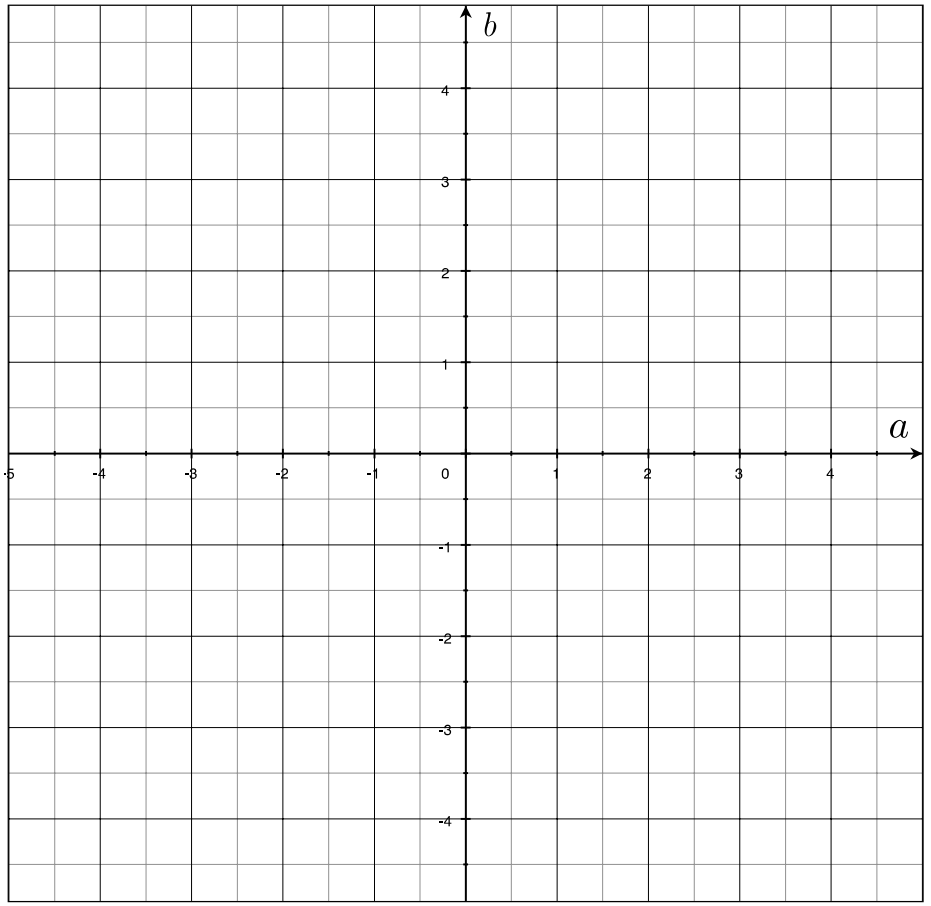


Image space

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables

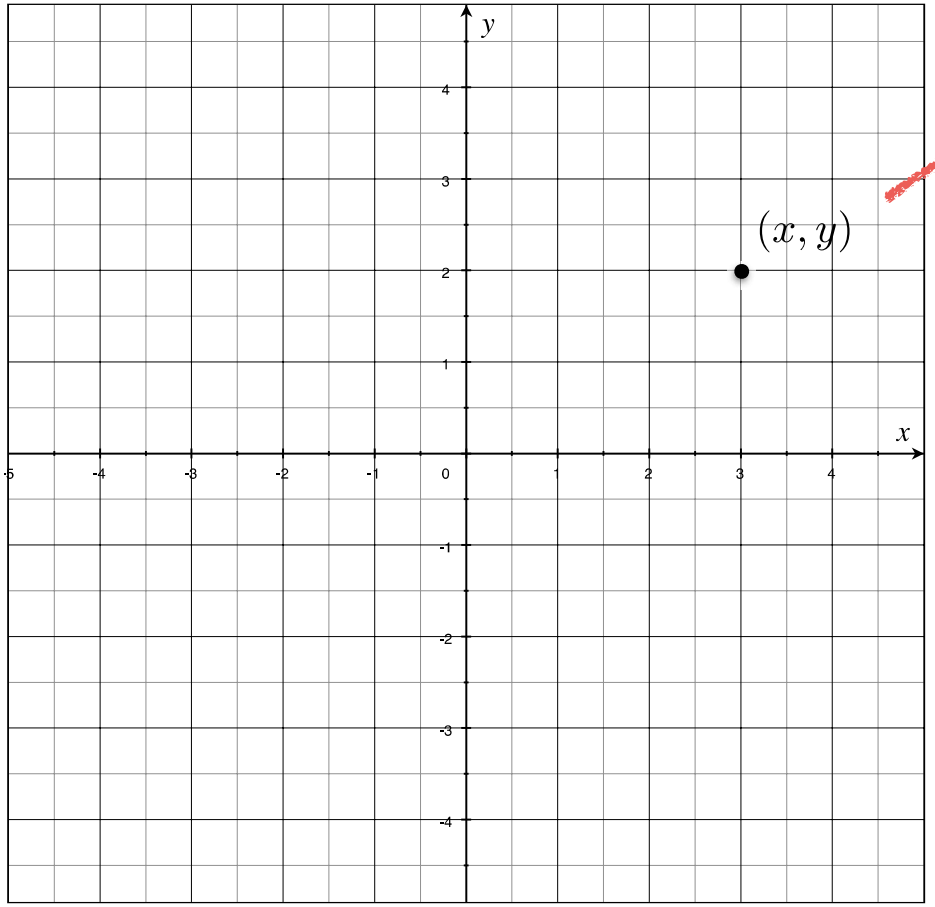


Parameter space

parameters

$$(x-a)^2+(y-b)^2=r^2$$

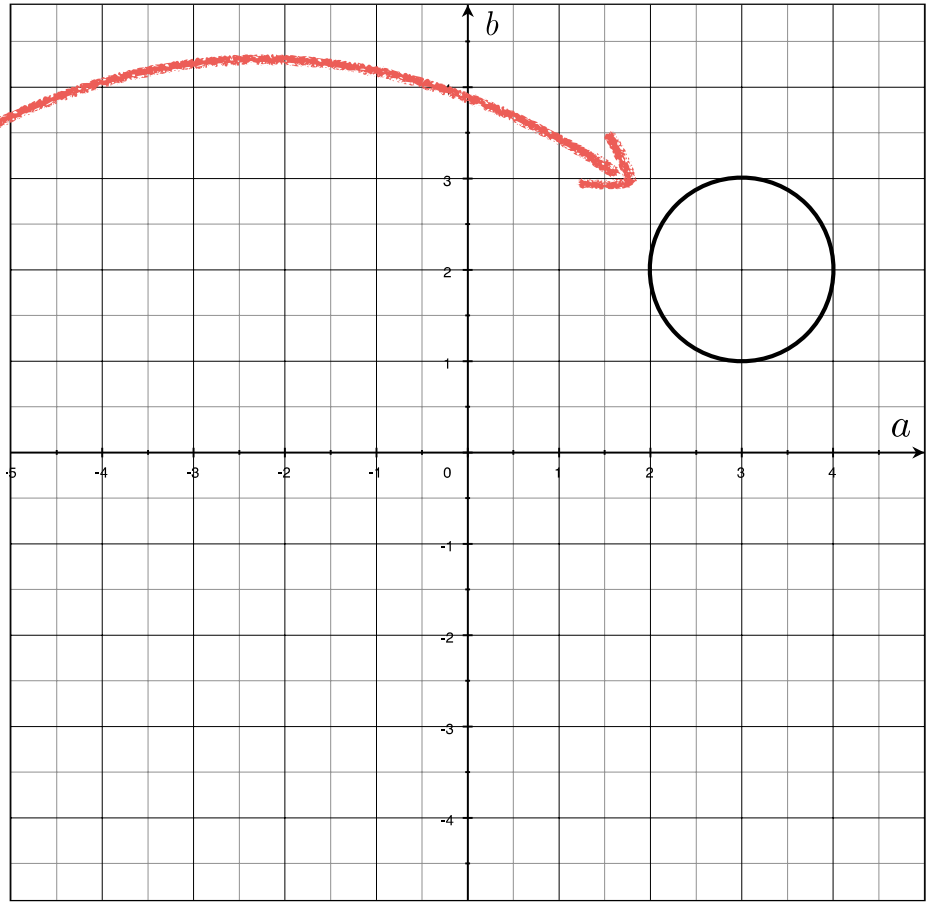
variables



parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables



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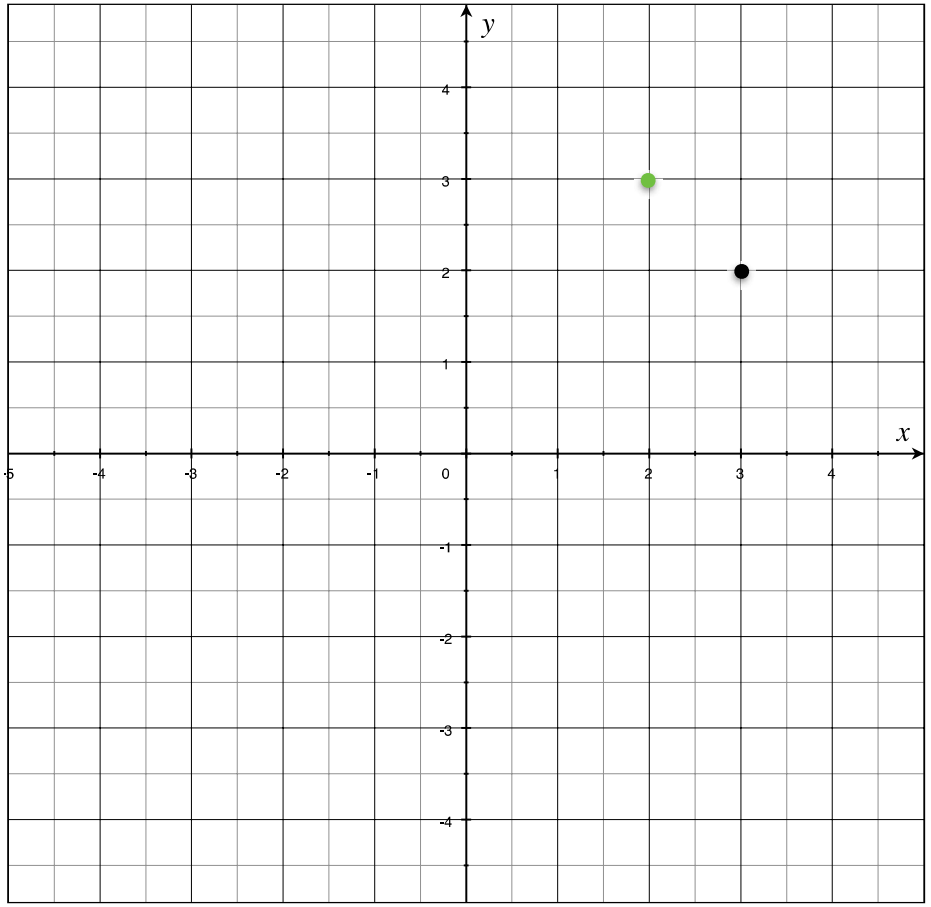

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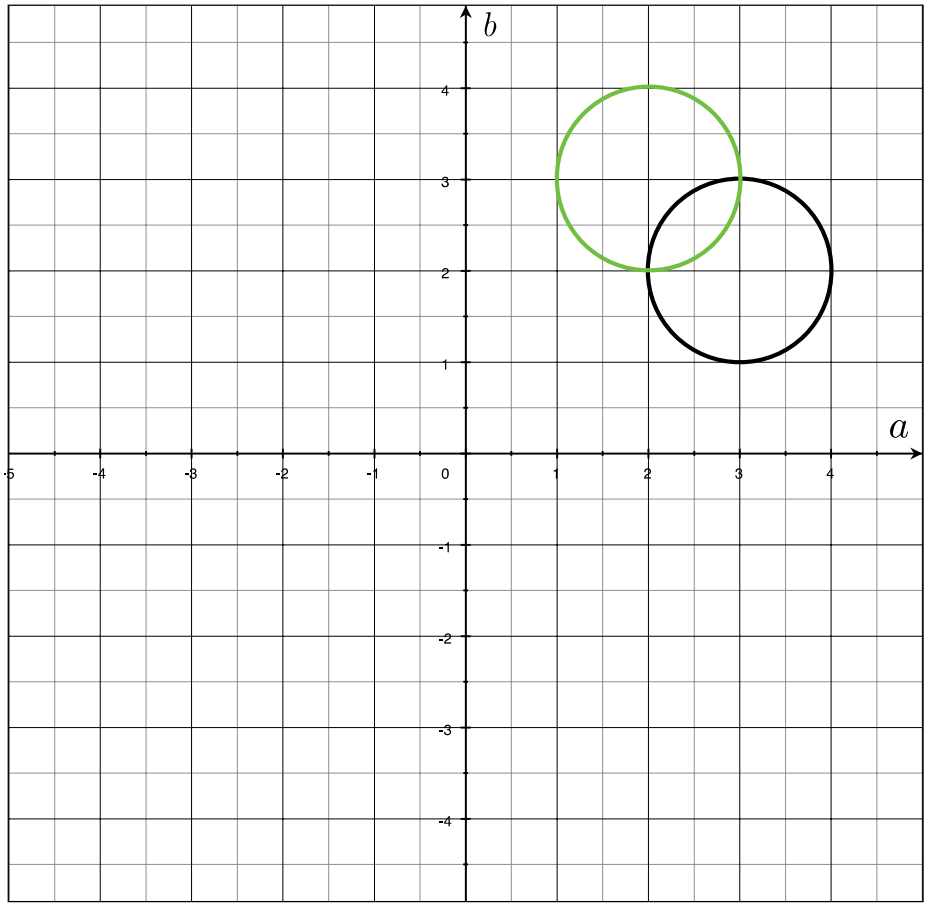

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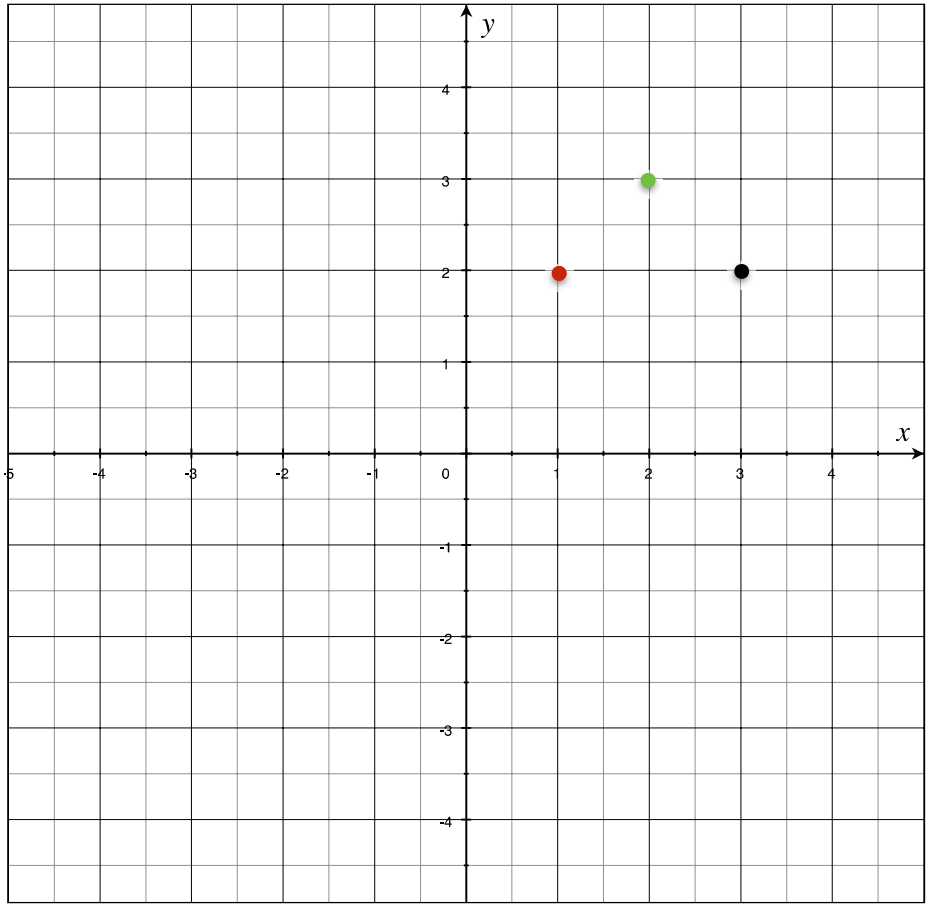

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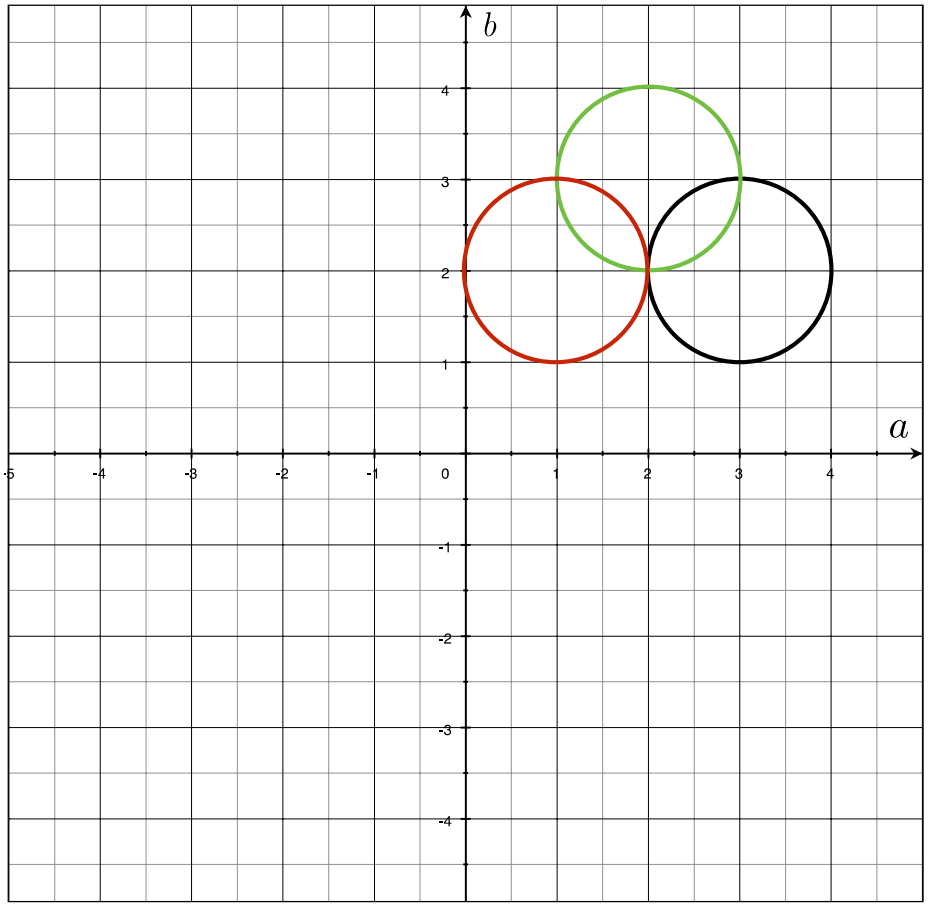

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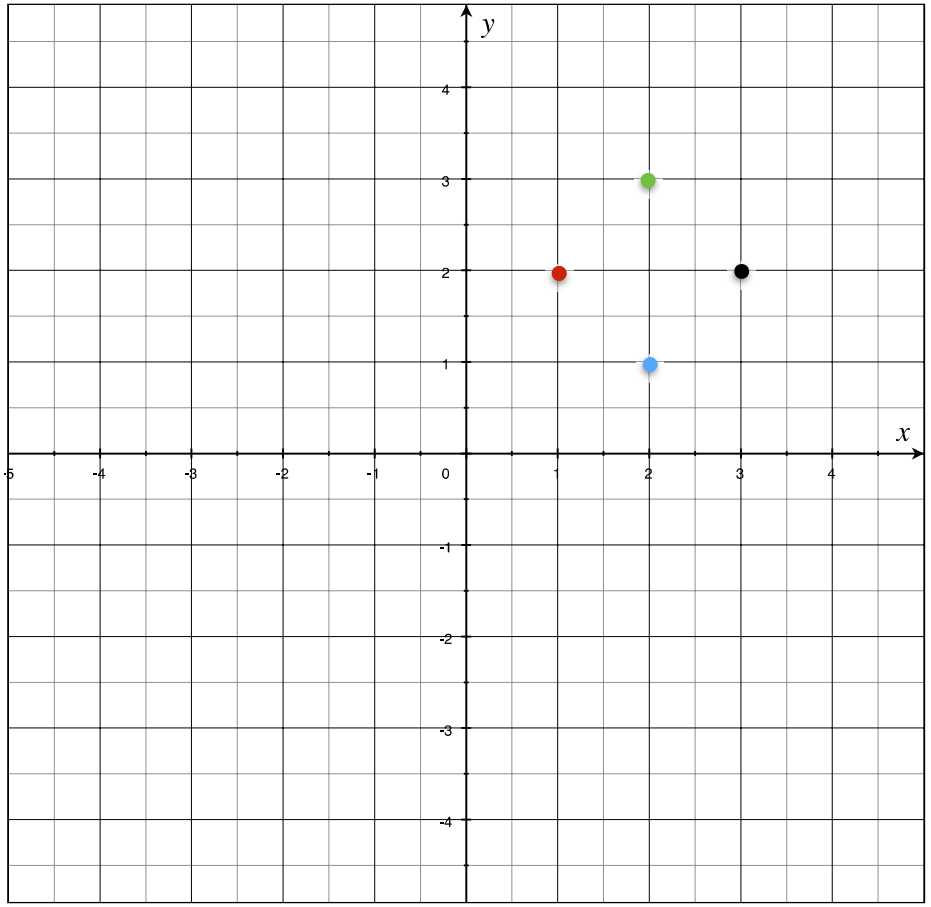

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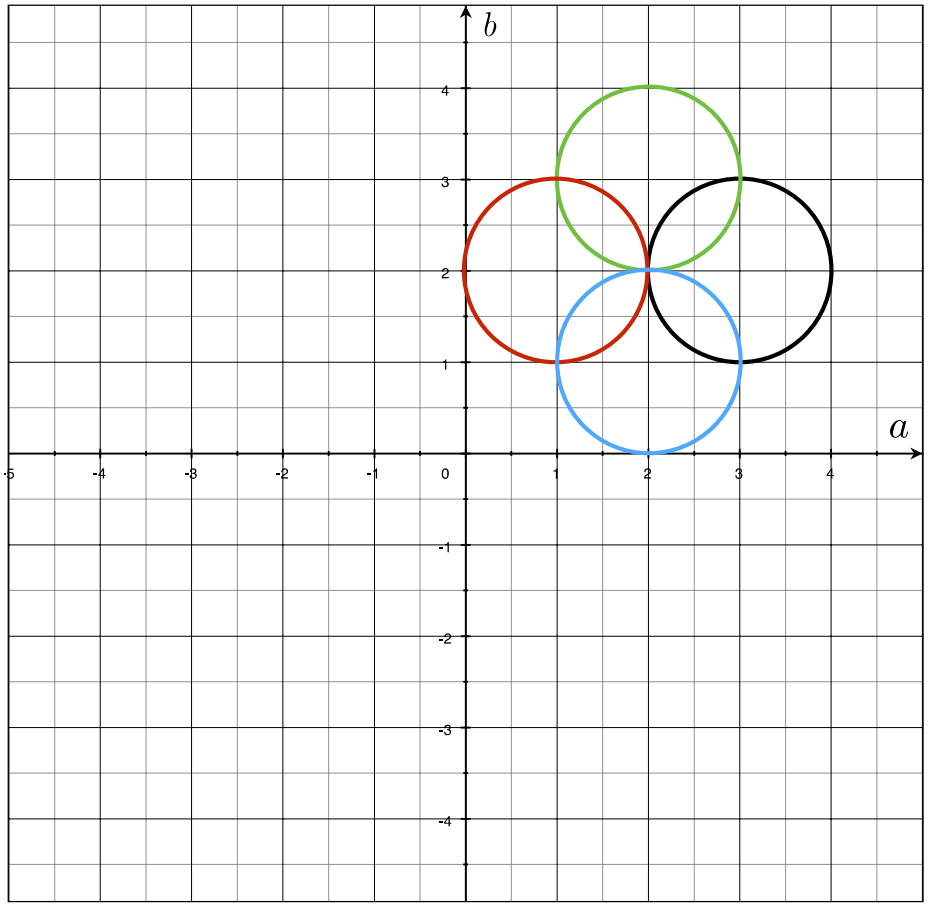
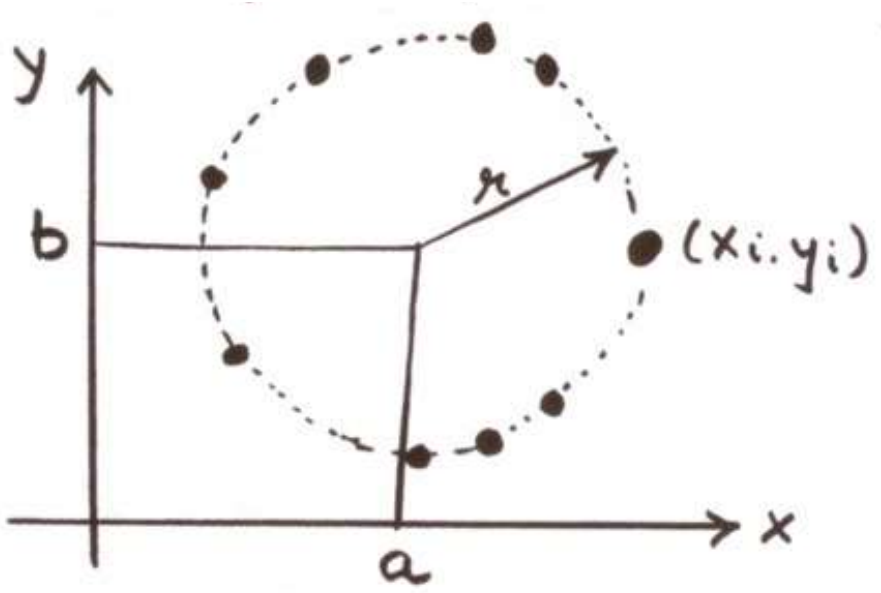


Finding Circles by Hough Transform

Equation of Circle:

$$(x_i - a)^2 + (y_i - b)^2 = r^2$$



If radius is not known: 3D Hough Space!

Use Accumulator array $A(a, b, r)$

What is the surface in the hough space?

Finding Circles by Hough Transform

Equation of Circle:

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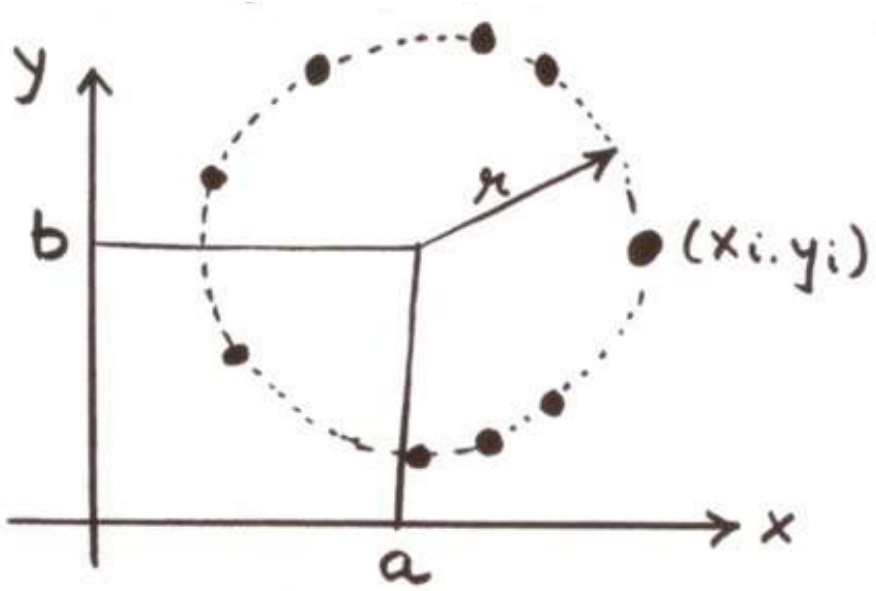

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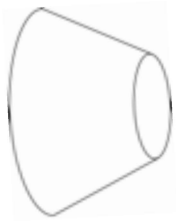
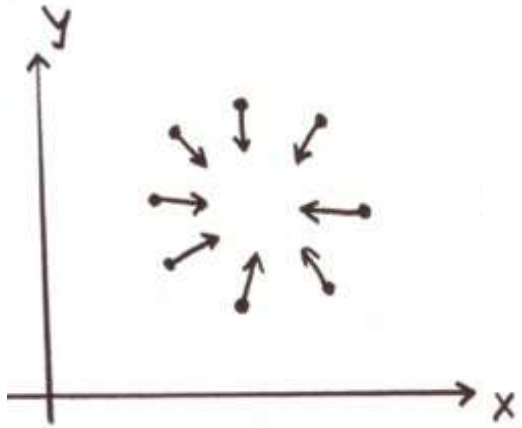


Using Gradient Information

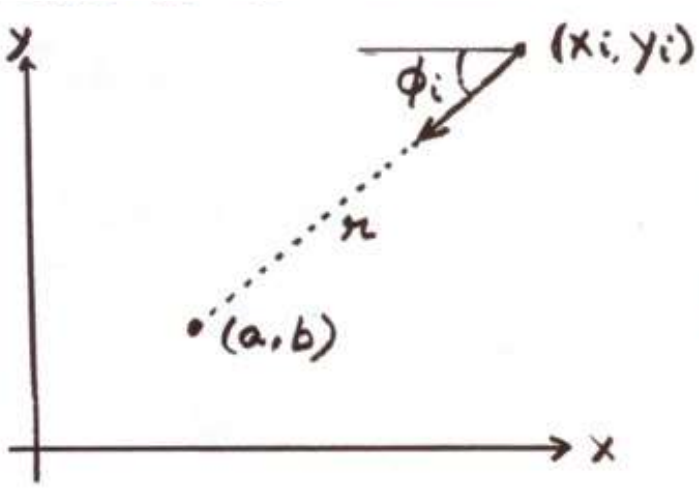
Gradient information can save lot of computation:

Edge Location (x_i, y_i)

Edge Direction ϕ_i



Assume radius is known:



$$a = x - r\cos\phi$$

$$b = y - r\sin\phi$$

Need to increment only one point in accumulator!!

parameters

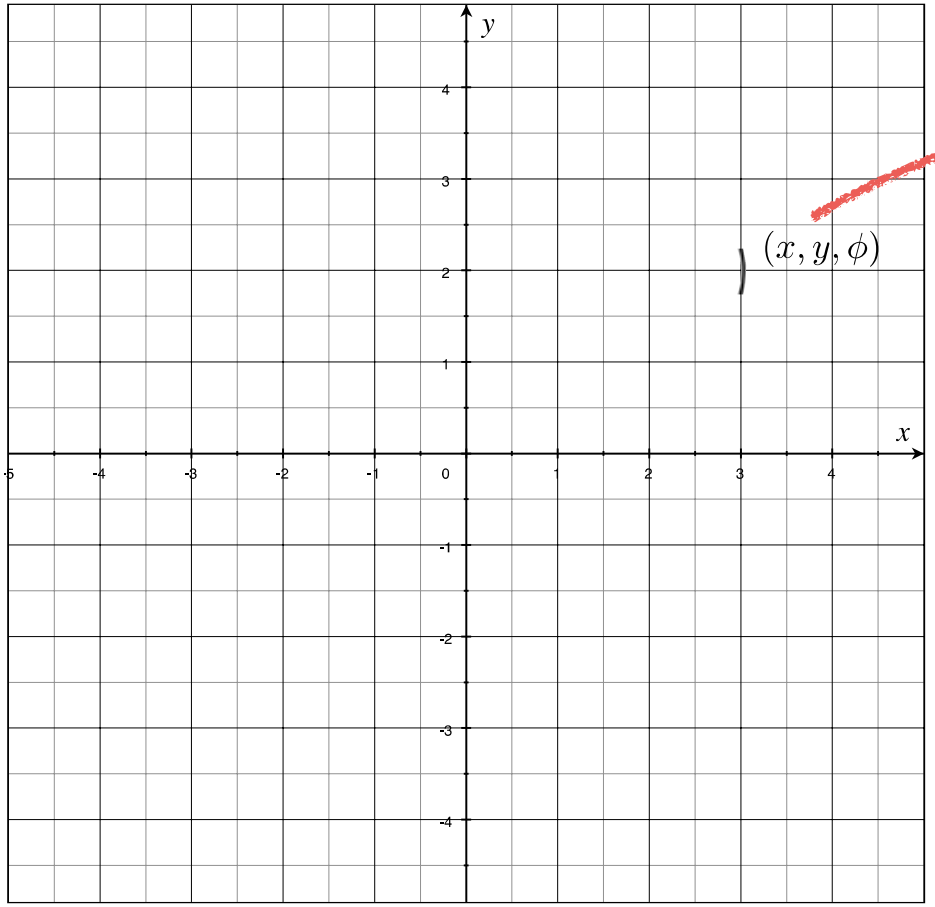
$$(x-a)^2+(y-b)^2=r^2$$

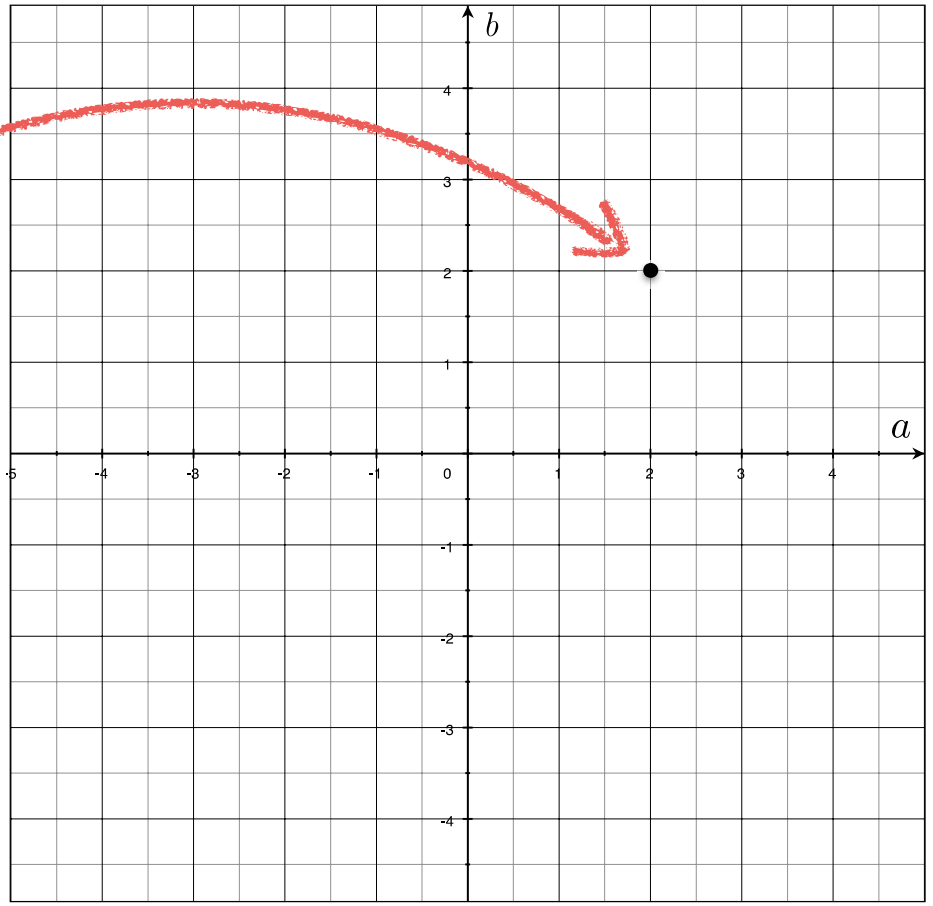
variables

parameters

$$(x-a)^2+(y-b)^2=r^2$$

variables





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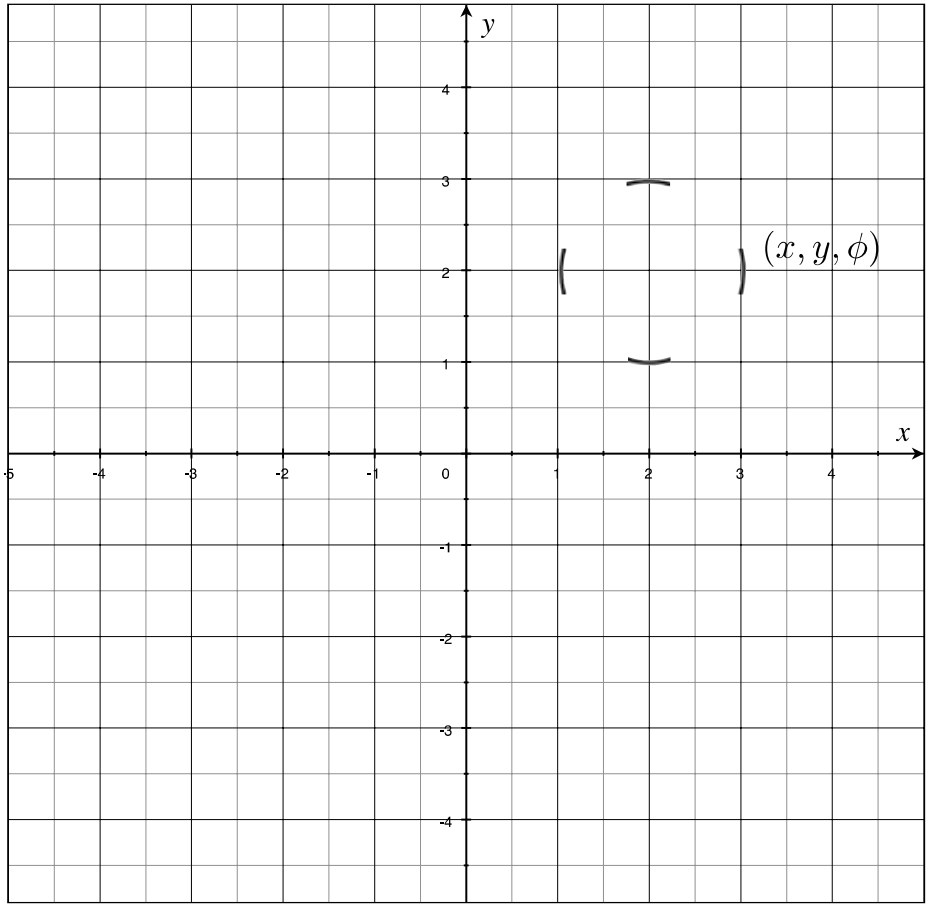

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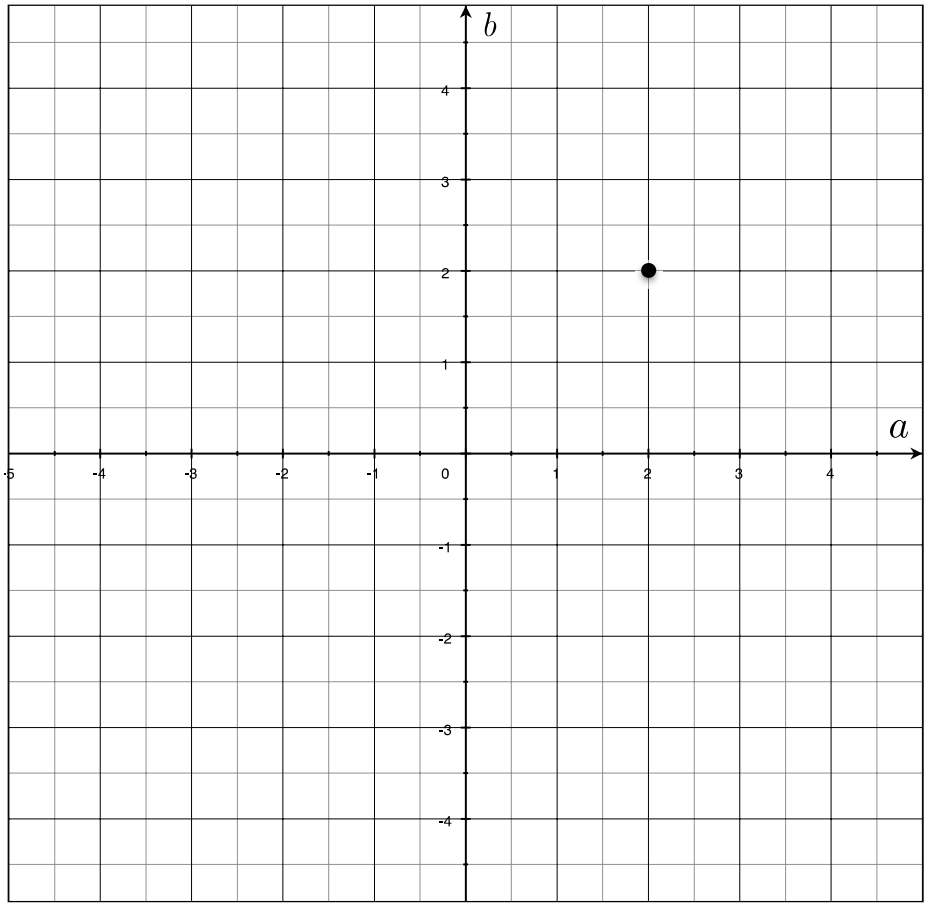
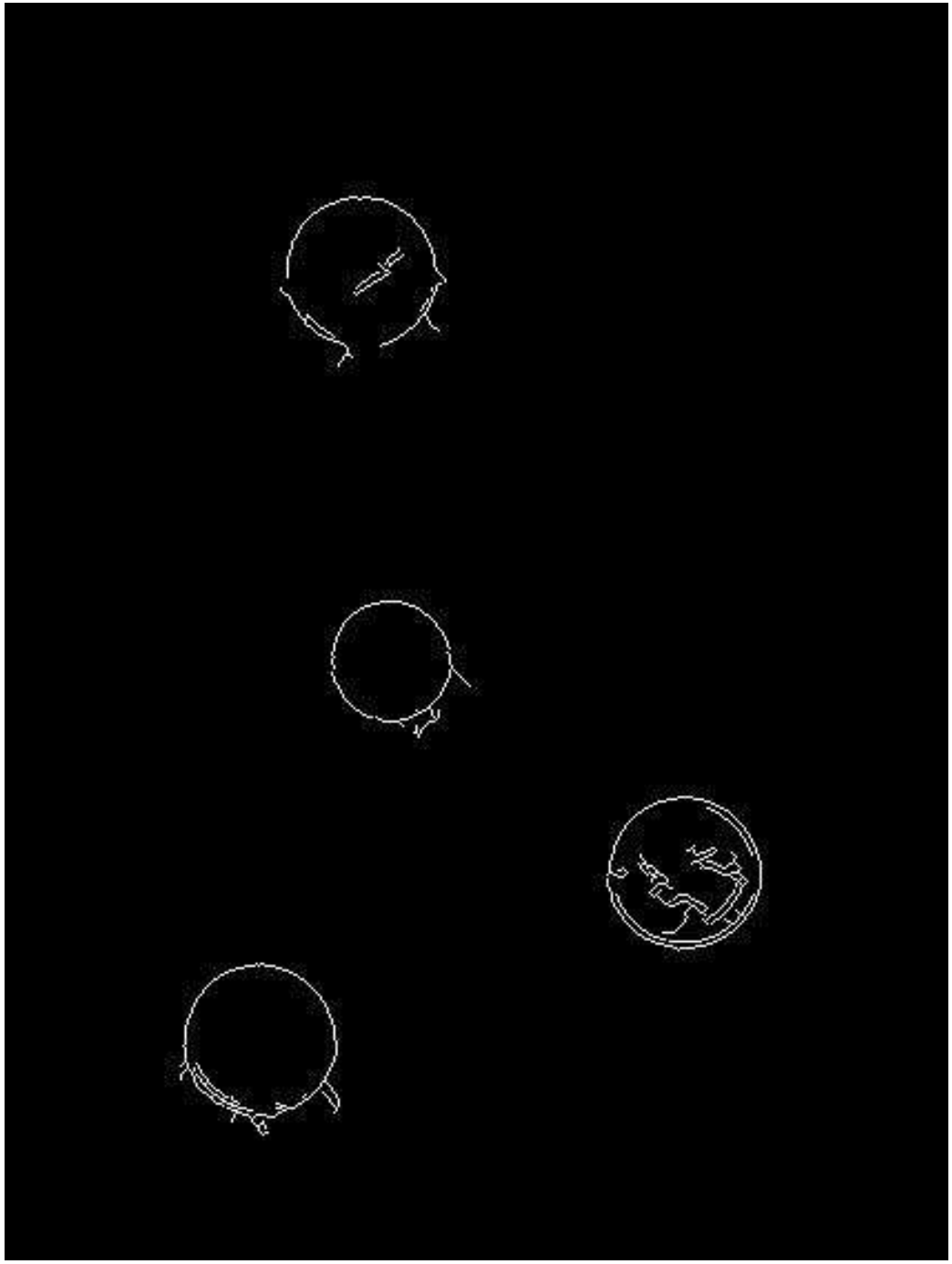
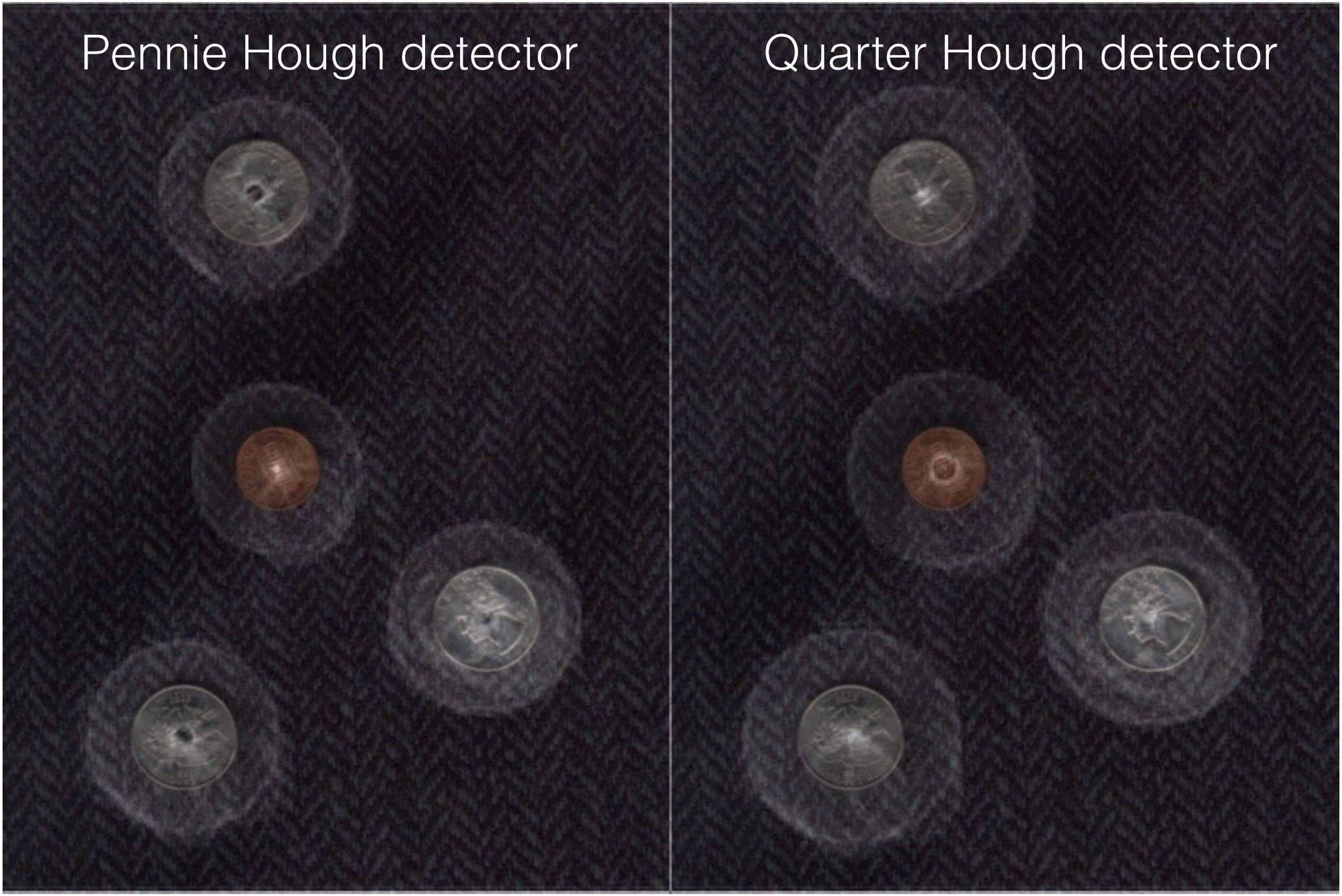
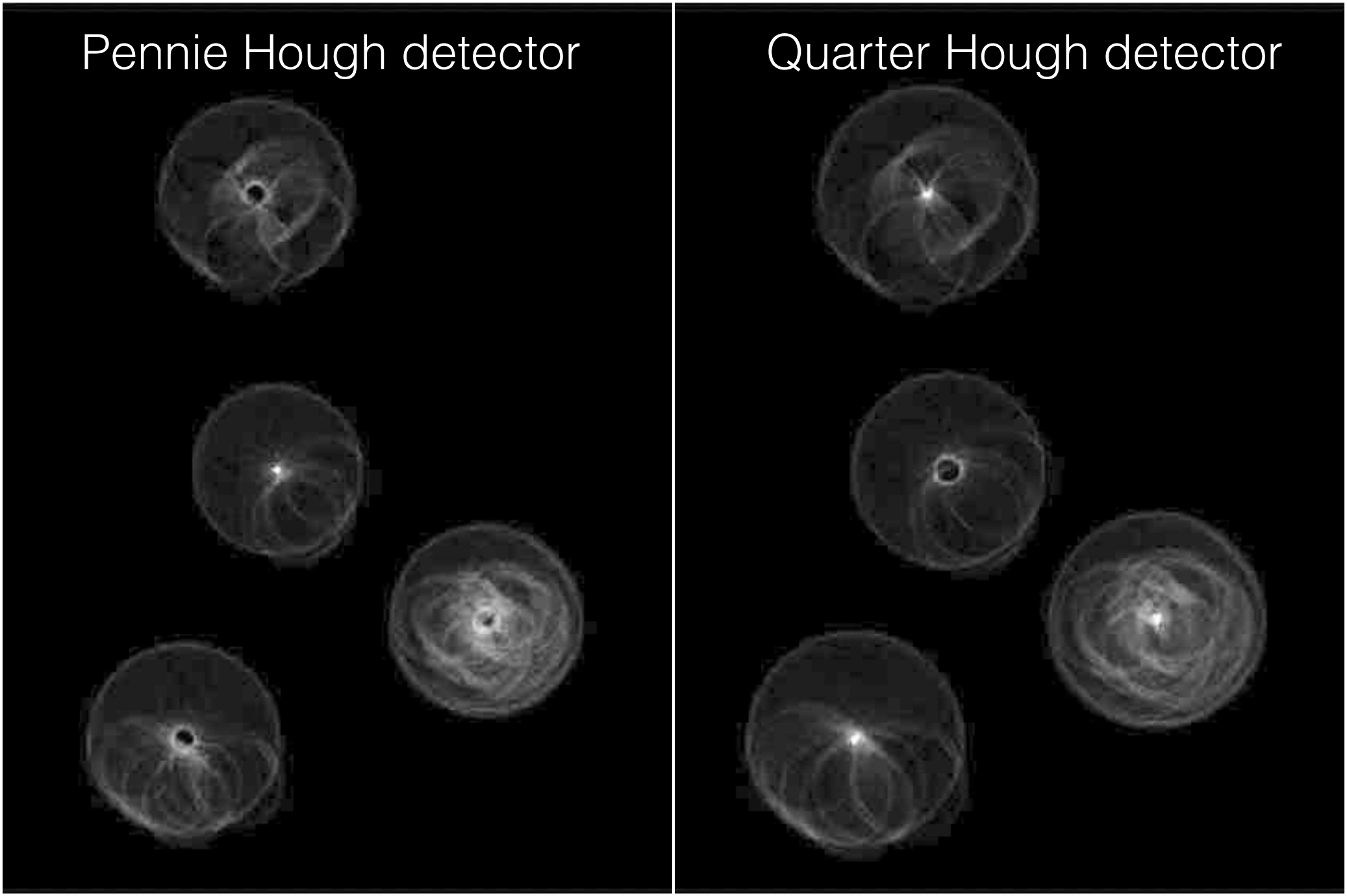


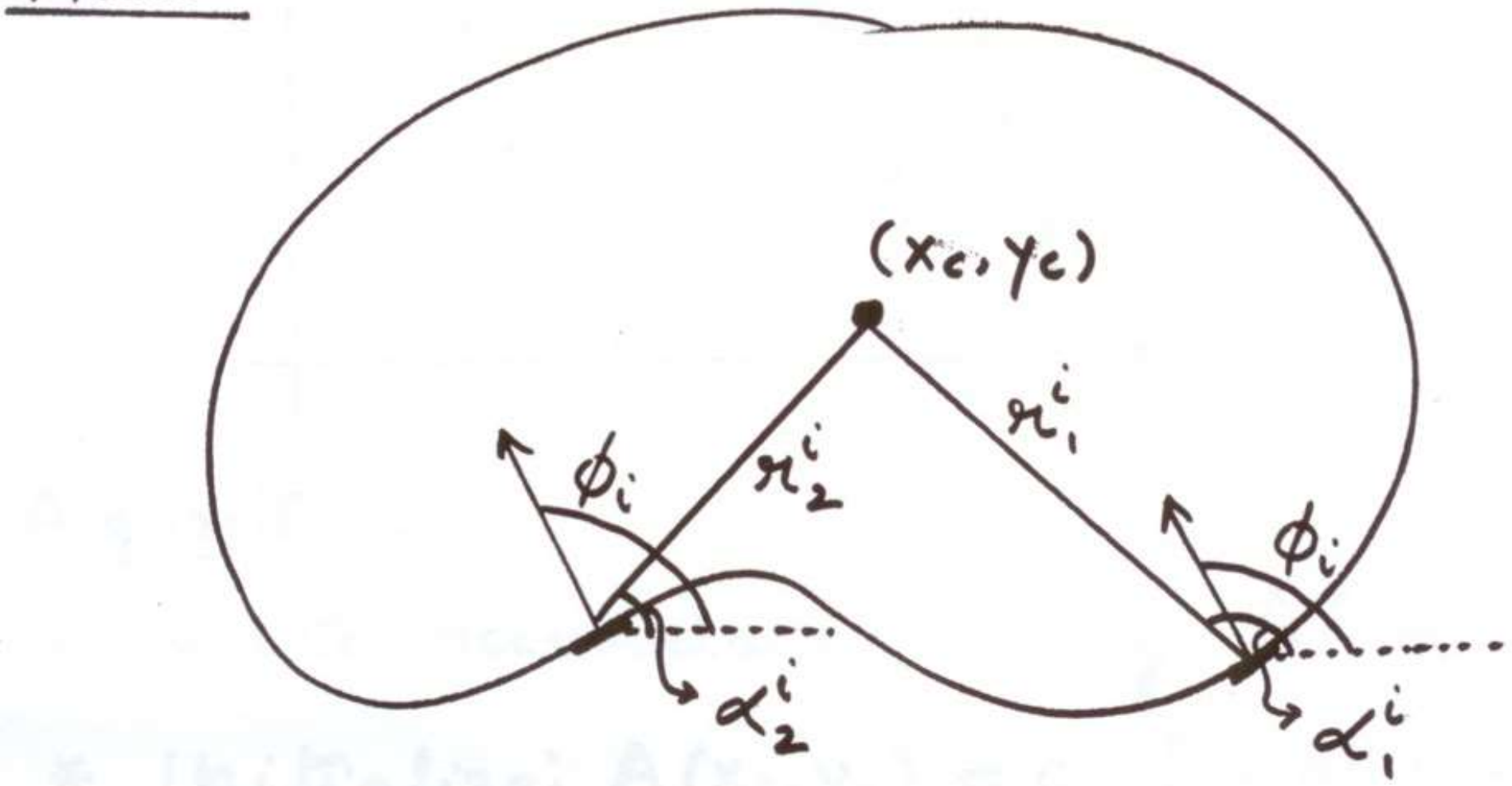



Pennie Hough detector
Quarter Hough detector


Pennie Hough detector
Quarter Hough detector

Can you use Hough Transforms for other objects, beyond lines and circles?

Generalized Hough Transform



ϕ - Table

Edge Direction	$\bar{\pi} = (\pi, \alpha)$
ϕ_1	$\bar{\pi}^1_1, \bar{\pi}^1_2, \bar{\pi}^1_3$
ϕ_2	$\bar{\pi}^2_1, \bar{\pi}^2_2$
ϕ_i	$\bar{\pi}^i_1 \vdots \bar{\pi}^i_2$
ϕ_n	$\bar{\pi}^n_1, \bar{\pi}^n_2$

Generalized Hough Transform

Find Object Center (x_c, y_c) given edges (x_i, y_i, ϕ_i)

Create Accumulator Array $A(x_c, y_c)$

Initialize: $A(x_c, y_c) = 0 \quad \forall (x_c, y_c)$

For each edge point (x_i, y_i, ϕ_i)

For each entry $\bar{r}_k^i$ in table, compute:

$$x_c = x_i + r_k^i \cos\alpha_k^i$$

$$y_c = y_i + r_k^i \sin\alpha_k^i$$

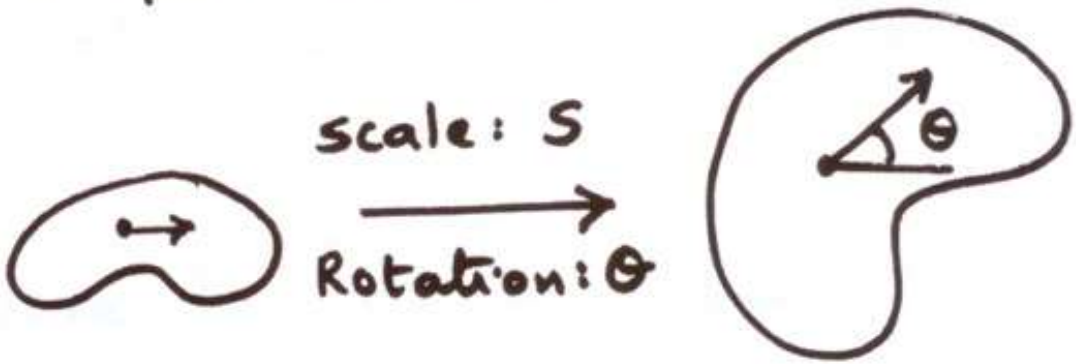
Increment Accumulator: $A(x_c, y_c) = A(x_c, y_c) + 1$

Find Local Maxima in $A(x_c, y_c)$

Scale & Rotation:

Use Accumulator Array:

$$A[x_c, y_c, S, \theta]$$



Use:

$$x_c = x_i + r^i_k S \cos(\alpha^i_k + \theta)$$

$$y_c = y_i + r^i_k S \sin(\alpha^i_k + \theta)$$

$$A(x_c, y_c, S, \theta) = A(x_c, y_c, S, \theta) + 1 .$$

Do you have to use edge detectors
to vote in Hough Space?

A. Train phase:

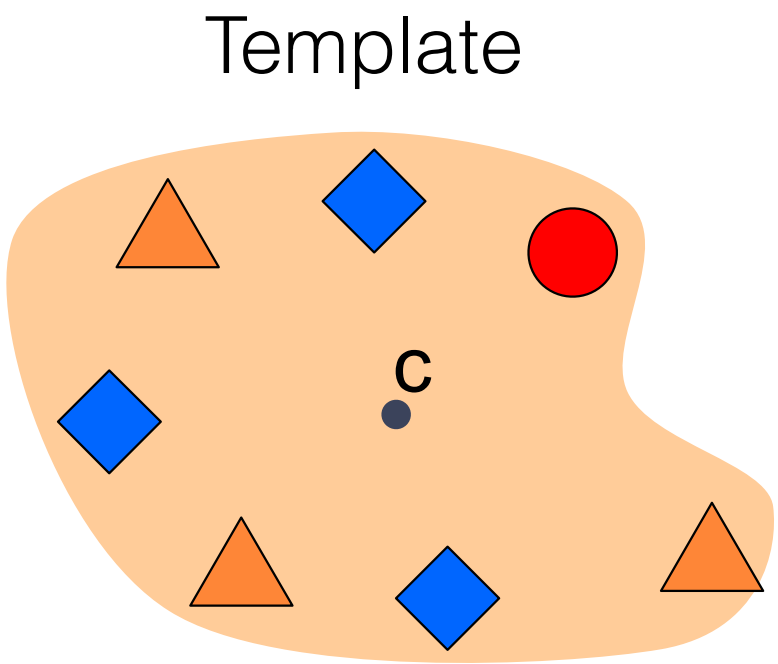
   1. Get features

   2. Store all displacements of feature from center

B. Test phase:

   1. Get features & lookup displacements

   2. Vote for center location



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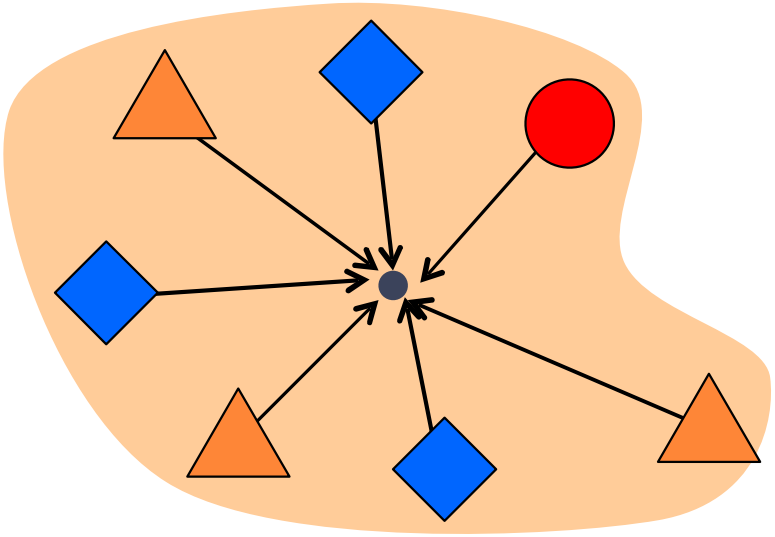
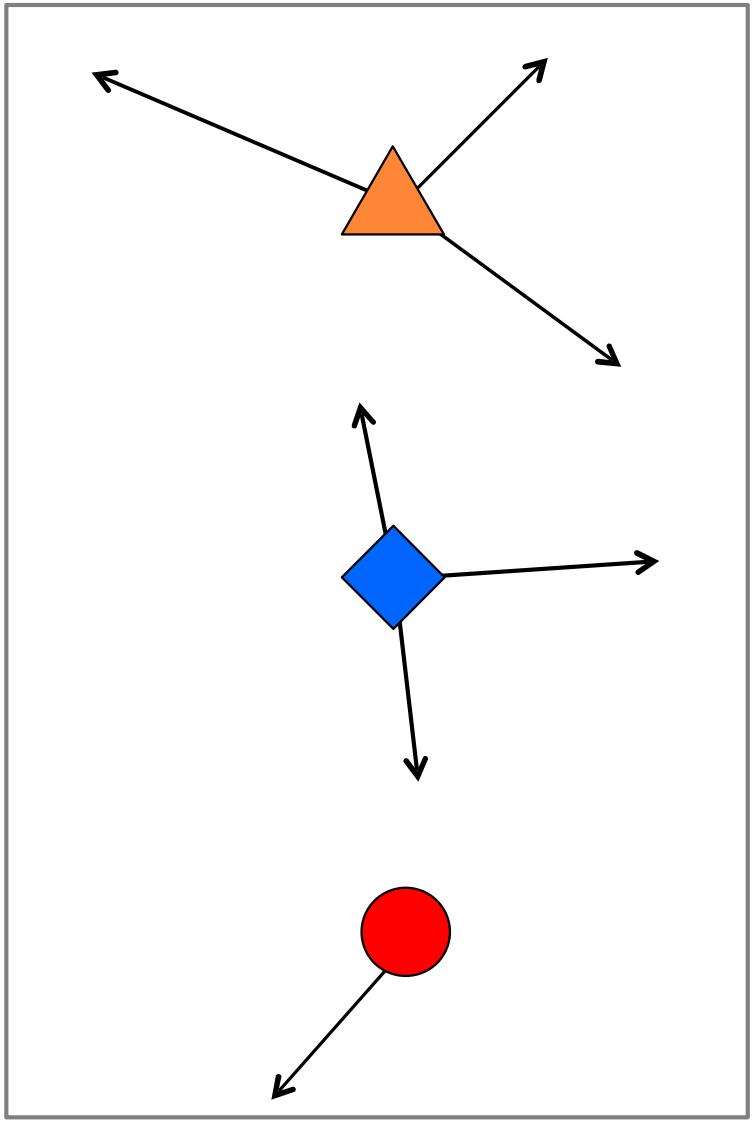
Template





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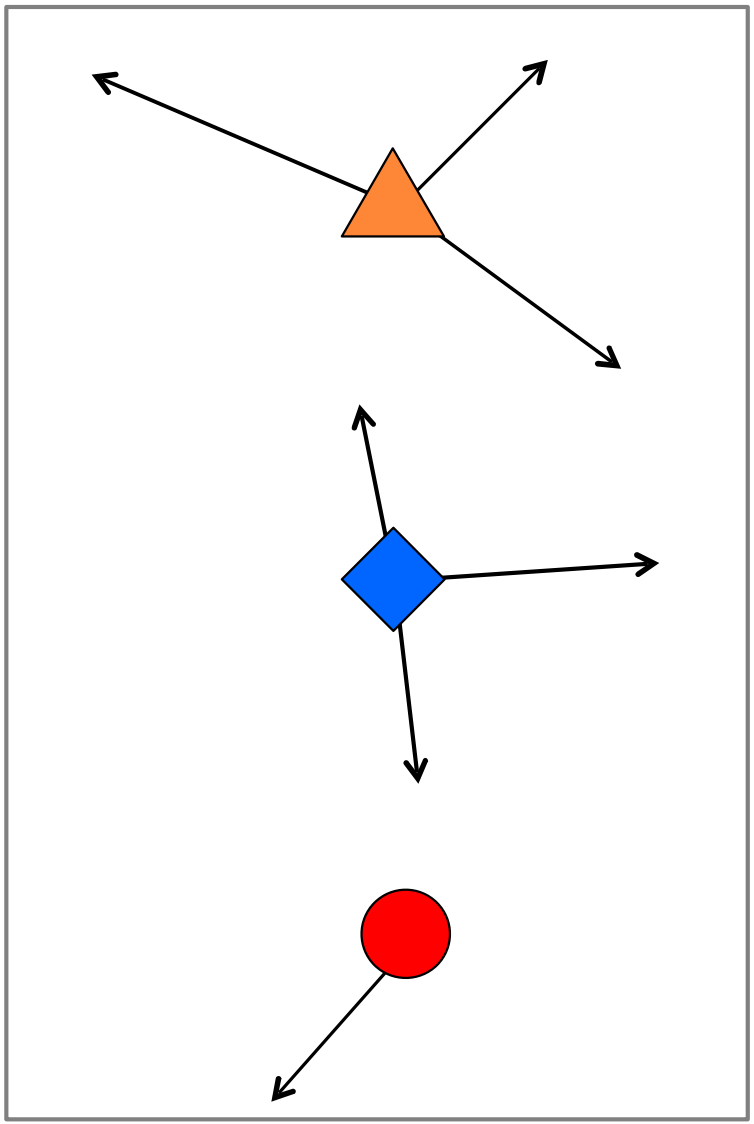
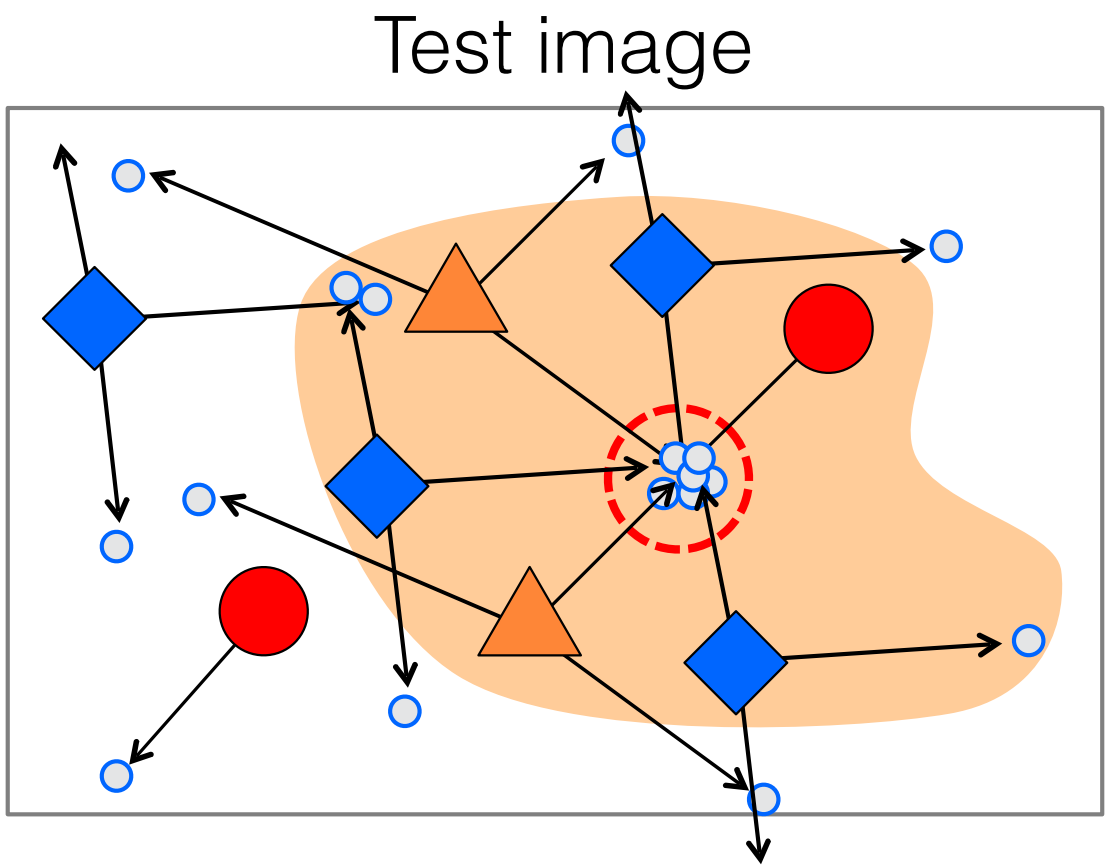
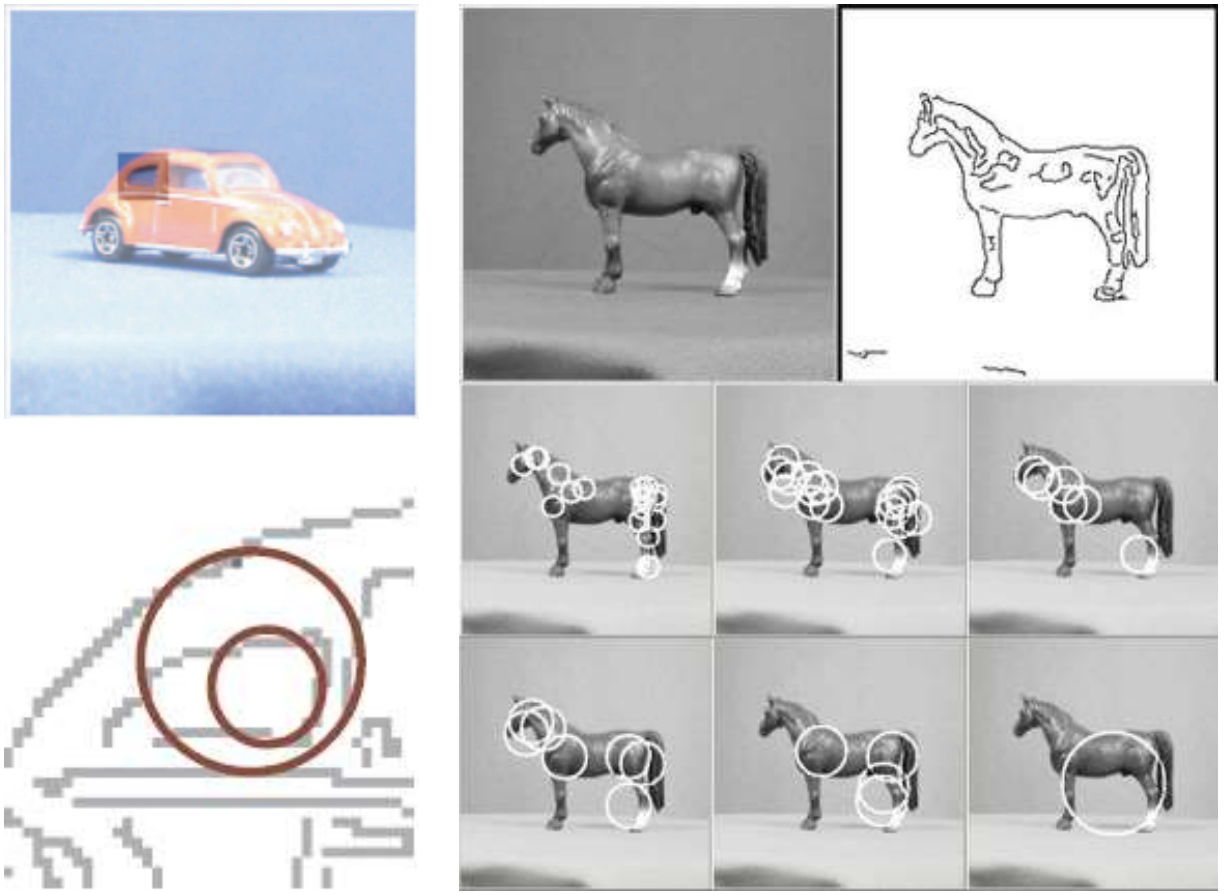




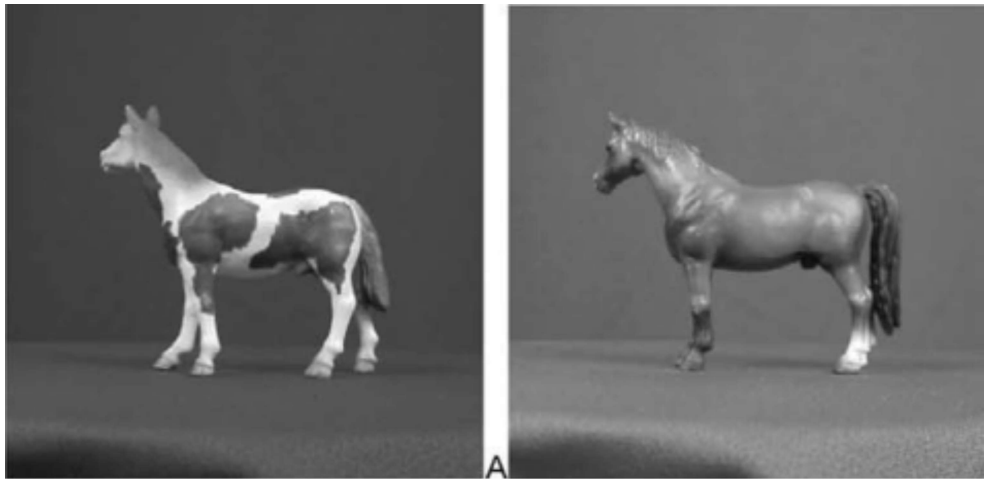
Application of Hough Transforms

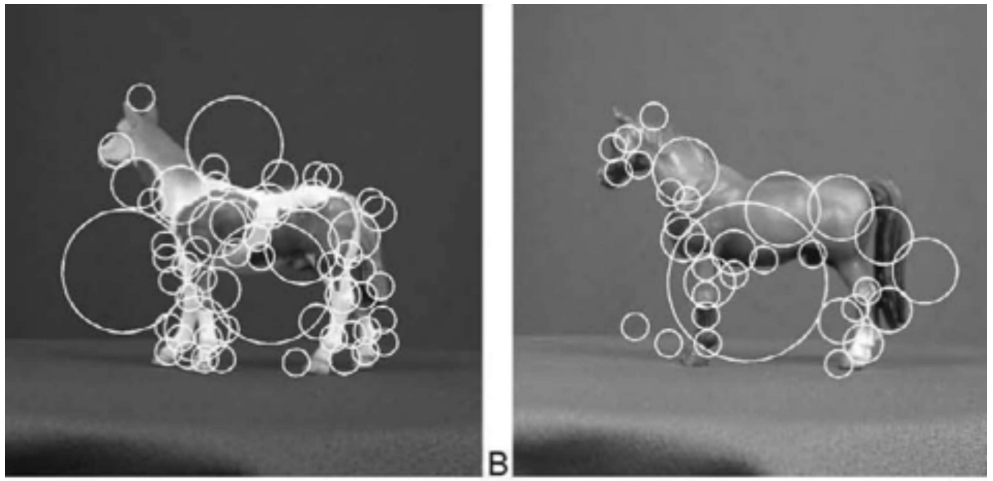
Detecting shape features



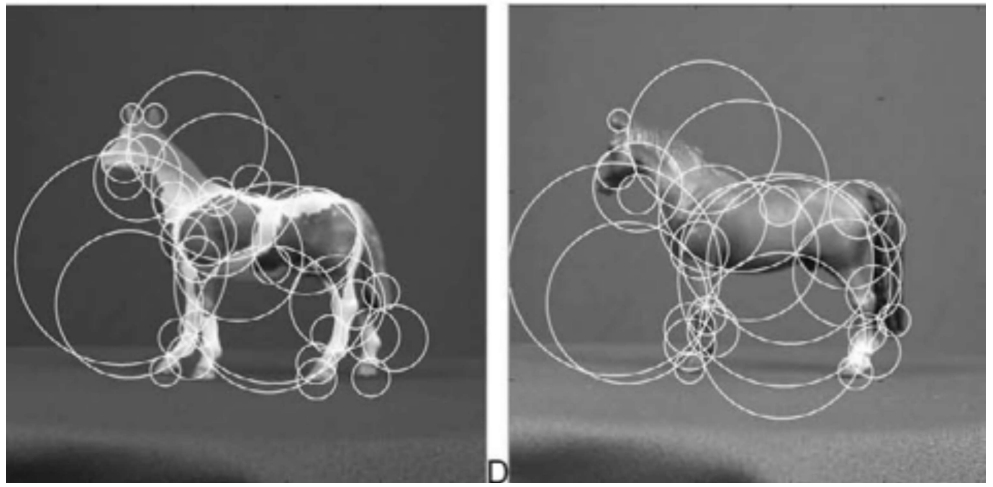
F. Jurie and C. Schmid, Scale-invariant shape features for recognition of object categories, CVPR 2004

Original images



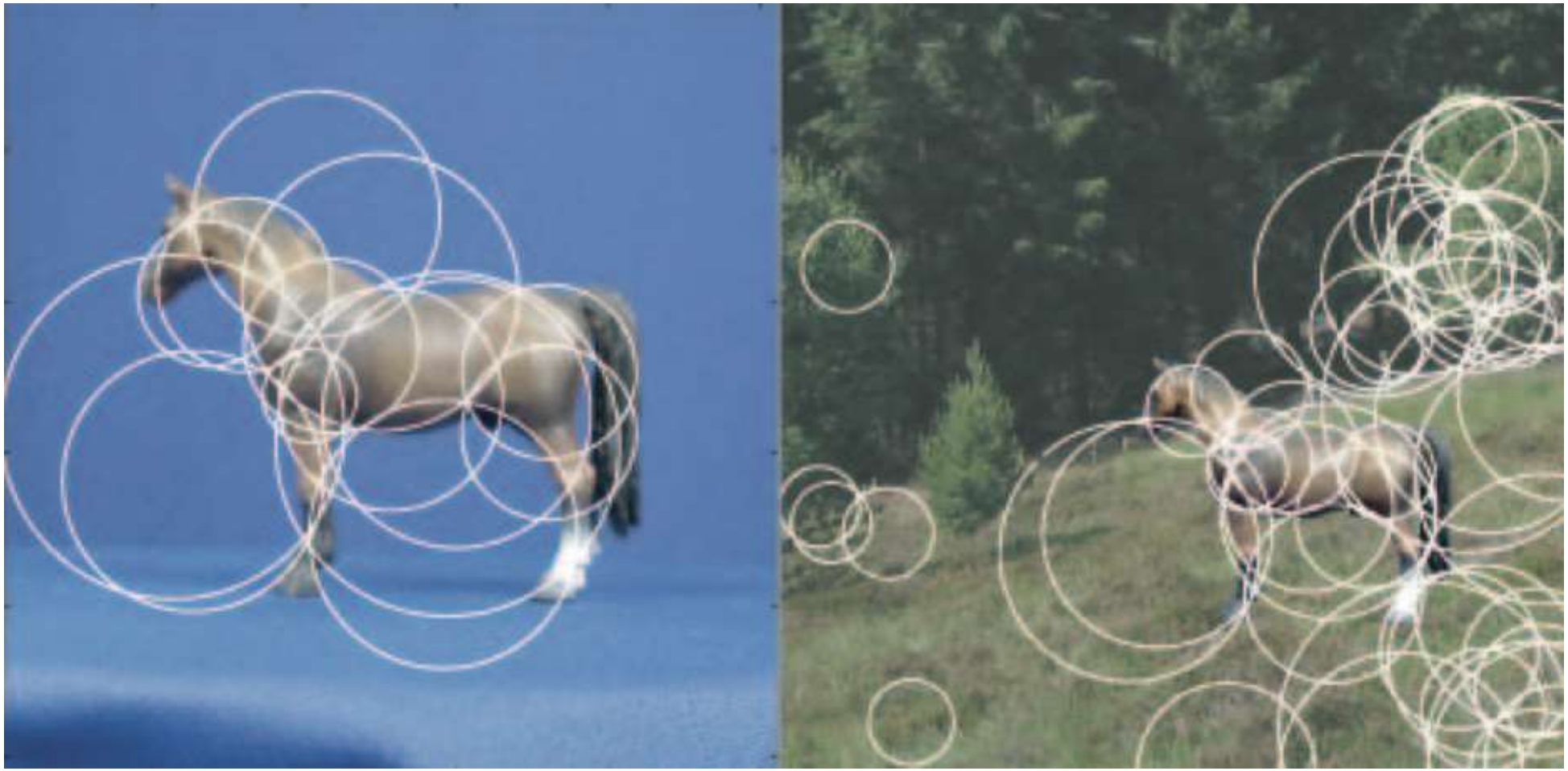


Laplacian circles



Hough-like circles

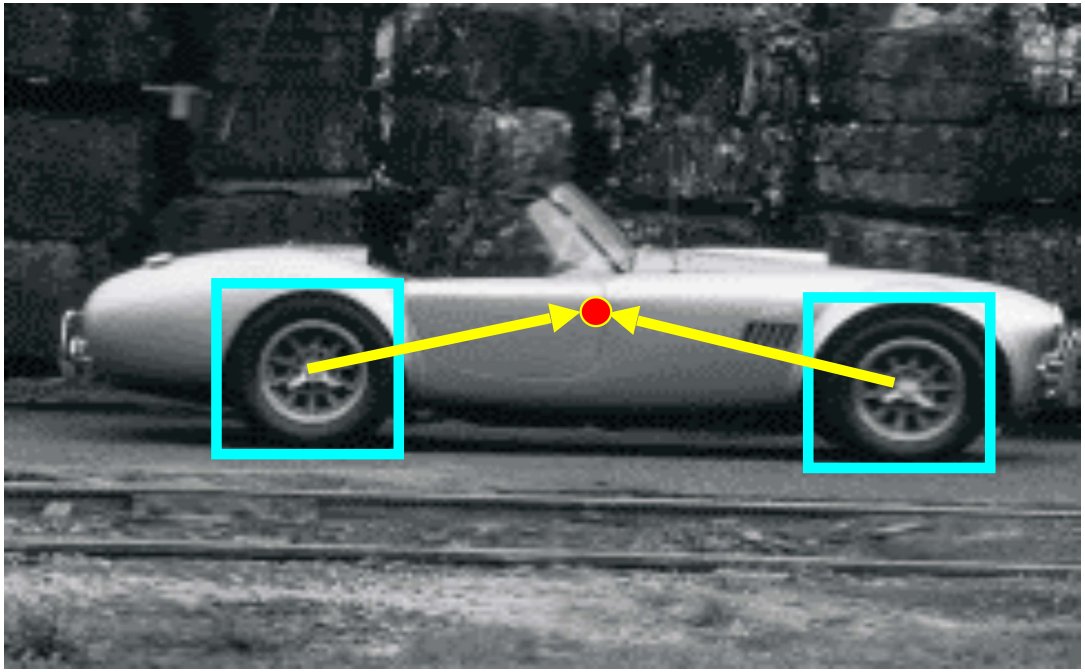
Which feature detector is more consistent?



Robustness to scale and clutter

Object detection

Index displacements by “visual codeword”



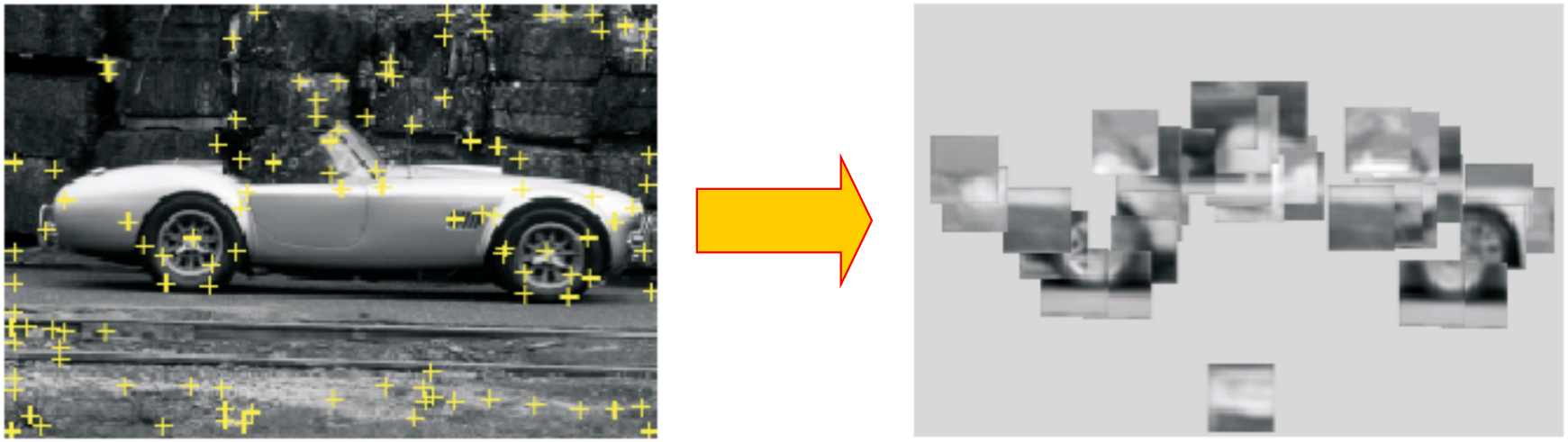
training image

visual codeword with displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, Combined Object Categorization and Segmentation with an Implicit Shape Model,
ECCV Workshop on Statistical Learning in Computer Vision 2004

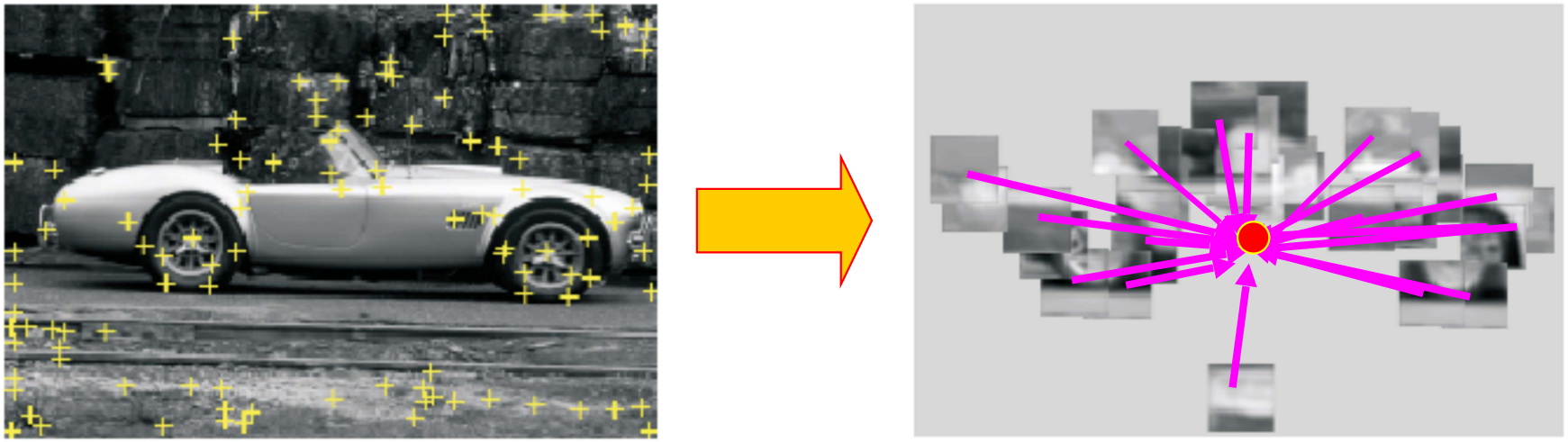
Train phase

1. get features

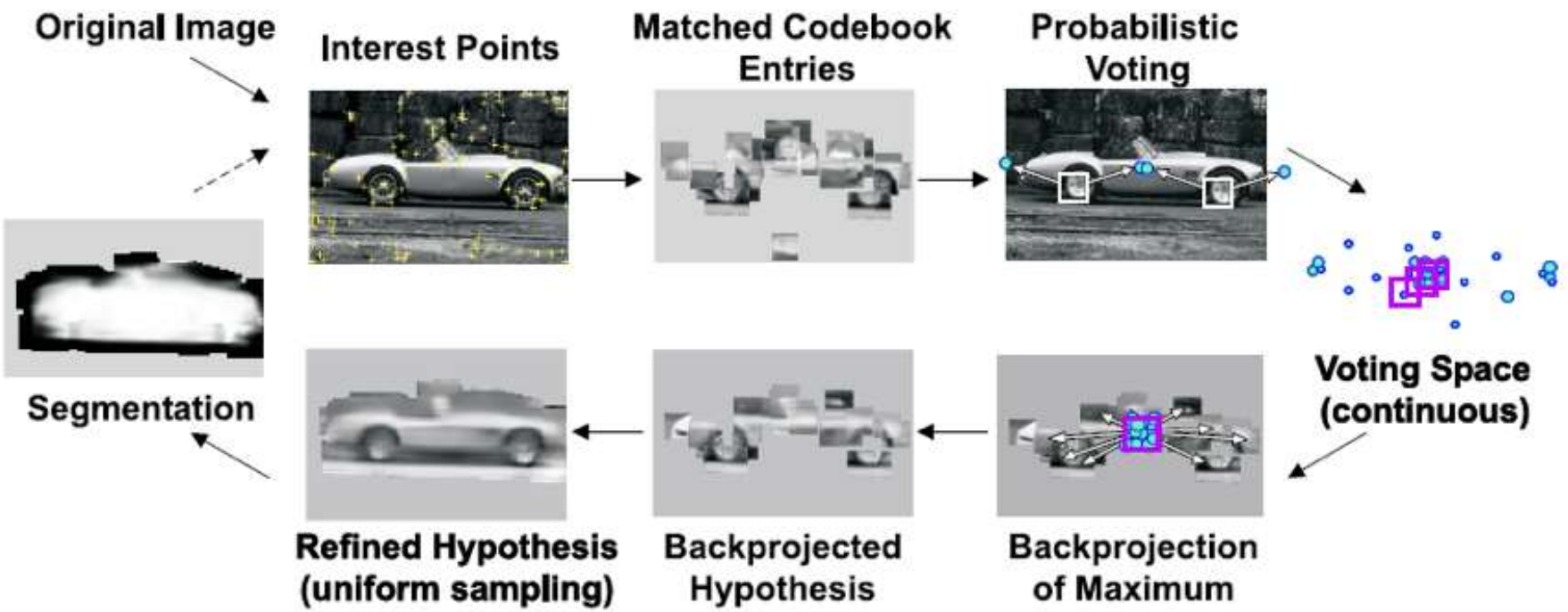


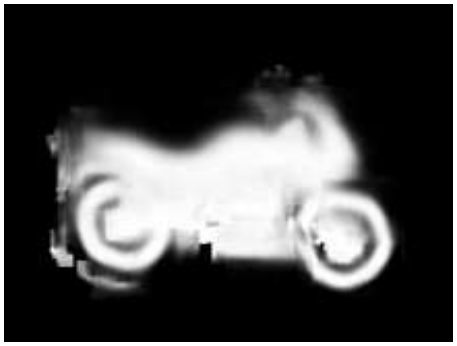
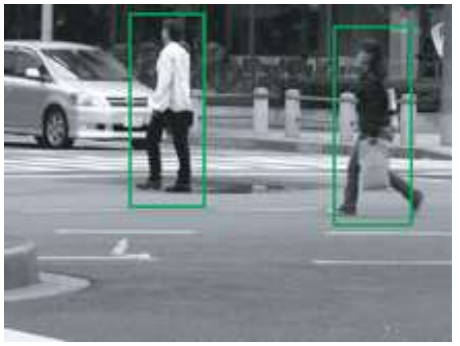
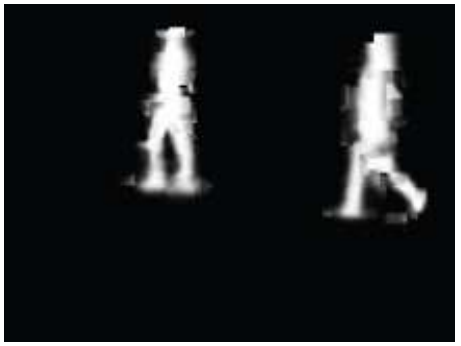
Train phase

2. store displacements

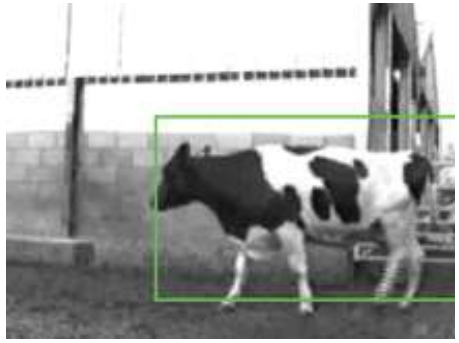


Test phase


Original Image
Interest Points
Matched Codebook Entries
Probabilistic Voting
Voting Space (continuous)
Backprojection of Maximum
Backprojected Hypothesis
Refined Hypothesis (uniform sampling)
Segmentation




The Hough transform …

Deals with occlusion well? 

Detects multiple instances? 

Robust to noise? 

Good computational complexity? 

Easy to set parameters?