XY model simulation with Metropolis Monte Carlo algorithm

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October 26, 2017

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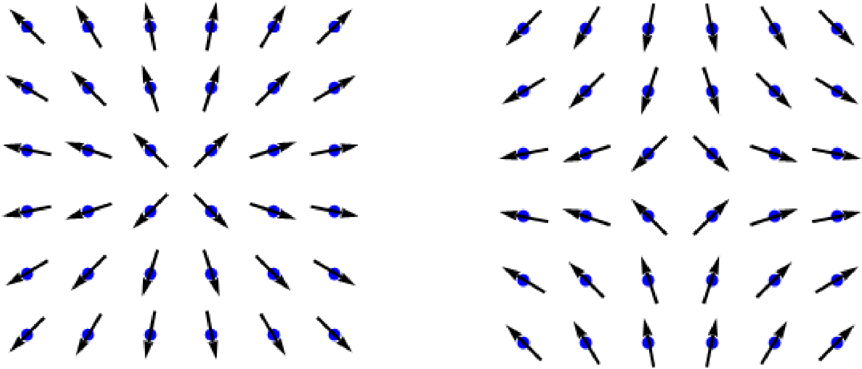
- Explains the exisctence of two-dimensional superconductors

Vortices and anti-vortices



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- In long run simulates XY model

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- $T_c = \frac{\kappa}{2k_B}$, k_B – Boltzmann constant, κ – system parameter.

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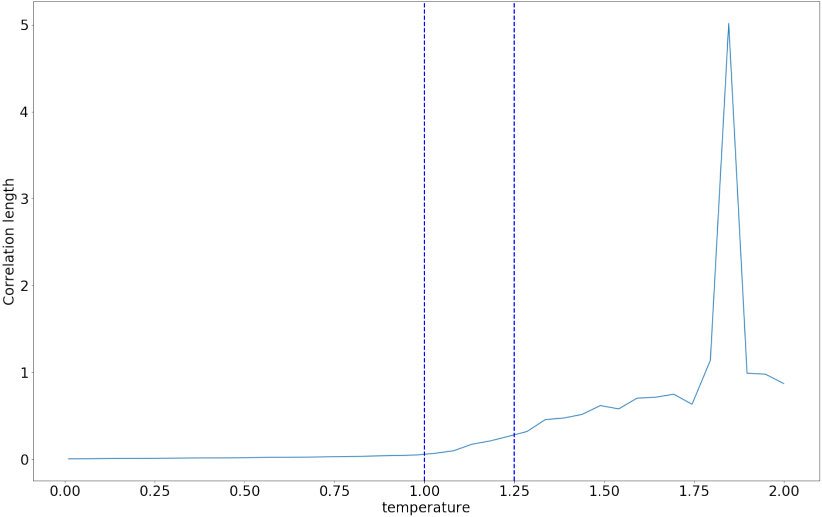
- 10 millions iterations for 40 values of temperature for 20×20 lattice to approximate T_c

Correlation lengths



Specific heat