Inventors and Firm Innovation:

Evidence from the World War I Draft

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Abstract

New knowledge is produced by firms together with individual inventors. However, the importance of inventors in firm innovation has not been examined. I investigate the firm's innovation rates when the firm exogenously lose inventors using the draft and military service during WWI. I find that the loss of inventor team members decreases the firm's inventions, but the loss of inventors outside the firm working in the same industry increases the firm's inventions. The latter effect is driven by frontier firms. New ideas sprout from knowledge frontiers primarily when there exists a vacancy in the space of ideas.

JEL classification: J24, N42, O31

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I Introduction

The interest in knowledge spillovers goes back to at least Bernard in the 12th century, who noted, "We are like dwarfs on the shoulders of giants, and thus we are able to see more and farther than the latter." This same sentiment was echoed in 1675 by Newton. While it is clear that each generation creates knowledge that is at least in part inspired by the ideas of previous thinkers, new ideas might also be developed as a response to the "vacuum" that results when those who have dominated the field disappear. This "vacuum" phenomenon was first formally suggested by Max Planck in 1950 and interest in knowledge creation and spillovers has persisted until today. However, we still know little about the process and dynamics of knowledge production (Jones, 2005).

The study of knowledge production is difficult in that firms play a foundational role in creating new ideas and inventions, whereas most of natural experiments documented in the causal literature are conducted and read by individual knowledge producers, such as academic scientists. Firms account for 82 to 85 percent of U.S. patents between 2006 to 2016 (National Science Board, 2018), and these patents result in ripple effects on economy-wide innovation. To develop innovations, firms typically hire or work with highly-skilled workers, such as inventors, who hold patents. These workers face challenges to their labor supply, including the possibility of migration, health shocks, government conscription, etc. Even though an inventor's human capital is presumably the most important input to a firm's innovation production function, there are few studies of how a supply shock of inventors propagates into a firm's innovation activity. As a result, research has yet to address the interactions between inventors and firm innovation. We therefore know little about which firms produce new knowledge that generate spillovers, or which firms depend heavily on knowledge spillovers to produce new innovations. In this paper, I introduce new matched inventor-firm data and use WWI draft to examine the effect of supply shocks of various types of inventors on firm-level innovation.

The potential effects of a labor supply of inventors on firm innovation are not a priori obvious. Given the possibility of human capital externalities, a reduction in inventor labor supply may have a more than proportionate impact on firm innovation. Conversely, given the presence of diminishing marginal returns, a reduction in inventor labor supply may have a less than proportionate impact on firm innovation. For example, it is unsure if a firm's innovation activity will increase or decrease in response to a decreasing labor supply of inventors outside the firm working in the same industry.

It is possible that losing inventors who work directly with a firm or generate knowledge spillovers could decrease a firm's overall invention production. Alternatively, losing inventors who compete with or work with rivals of a firm could increase the chance that the firm increases its invention. These two possibilities arise from the fact that a firm's innovation activity is affected in different ways depending on which of the three types of inventors are leaving: (1) team members working directly with the firm; (2) geographically close inventors; or (3) inventors working on similar topics to those the firm is working on (Borjas and Doran, 2015a). The effects of innovator loss on firm innovation could vary depending on the position of the firm relative to the knowledge frontier of the firm's particular field.

In this paper, I introduce a novel use of empirical data to examine the effects of innovator loss on firm innovation. I make use of a documented large labor supply shock that affected some inventors but not others: the United States draft during World War I. Due to the specificity of the age groups affected by the draft, some pools of inventors were heavily depleted during the war while other pools of inventors were left relatively unaffected. These differential effects occurred across all three types of inventors: team members at a firm, inventors working in the same county, and inventors working on similar topics. As a result, the dataset I use in this study could be used to differentiate among the separate effects of losing inventors within a firm's team, within a geographic location, or within an area in the space of ideas on a firm's innovation production.

I construct a unique data set by matching records among the patent database, WWI records, and the 1920 Census. The linked data set contains information on the number of patent applications per year, WWI draft registrations, WWI military service records, geographic locations, and inventors' characteristics from the complete count 1920 Census. Using variation in the proportion of inventors in draftable age groups, I use a difference-in-difference setup to identify the impact of losing inventors on a firm's invention. Specifically, I estimate innovation rates of firms that are likely and unlikely to lose inventors due to the

WWI draft and military service between 1917 and 1918. I provide evidence of the validity of the instruments from a placebo experiment demonstrating that inventors in undraftable age groups are unlikely to impact a firm's invention production.

I find evidence that the effect of supply shocks to inventors on firm innovation depends heavily on both the space in which the shock takes place and how close the firm is to the knowledge frontier. In particular, a 10 percent decrease in inventor team members within a firm decreases the number of patent applications assigned to the firm by 12 percent per year. In contrast, a 10 percent decrease in inventors outside a firm working on similar topics to that firm increases patents assigned to that firm by 8 percent. This increase in patents is driven by firms that are highly innovative prior to the WWI draft, while less innovative firms decrease innovation rates in response to the loss of inventors in the space of ideas. Taken together, this evidence suggests that knowledge spillovers have different effects within a firm than they do between firms. Within a firm, knowledge spillovers are powerful enough to negate any diminishing marginal returns to the firm's pool of inventors. A net decline in patents within a firm suggests that knowledge spillovers are an important source of innovation rather than diminishing marginal returns. Between firms, knowledge spillovers primarily work through transfer of ideas from highly innovative firms to their less innovative peers. Among highly innovative firms already at the knowledge frontier themselves, knowledge spillovers in the space of ideas are not a substantial determinant of knowledge production.

I also investigate the effect of supply shocks on the innovation rates of individual inventors who did not serve in the military. Consistent with the effects at the firm level, losing peers who collaborated with an individual inventor decreases the productivity of that inventor, and highly innovative inventors are less affected by spillover in the space of ideas. Furthermore, I disaggregate the effects of losing inventors by the quality of inventors. I find that within a firm, highly innovative knowledge producers disproportionately affected the invention production of the firm as a whole.

The results presented in this paper contribute to two bodies of literature. First, these results add to a growing number of studies that explore how a supply shock of knowledge producers affect knowledge spillovers. The results here suggest that authors who examine the process of knowledge production but ignore the roles of firms in that process could be

missing an important source of knowledge creation and spillovers that may have implications for economy-wide innovations. The existing literature on supply shocks and knowledge spillovers is motivated by different theoretical perspectives. One view considers human capital externalities in which the creation of new ideas generates positive externalities. This provides evidence that losing peers has a negative impact on knowledge creation. Other studies have demonstrated that the inflow of knowledge producers creates positive externalities. Another perspective follows the law of diminishing marginal returns in which losing peers has a positive impact on the rate of knowledge production. This hypothesis suggests that knowledge producers take advantage of the decrease in competition within their field. In this paper, I provide evidence that addresses both human capital externalities and the presence of diminishing returns by distinguishing the separate effects of losing inventors in a firm's team, within a geographic location, and within a particular idea space.

Furthermore, this paper builds on a rich body of literature that examines the determinants of firm innovation. Despite the importance of human capital in firm innovation, we know little about how a labor supply of inventors affects a firm's innovation activity. One group of early studies of this topic explored a relationship between R&D investment and firm innovation,⁴ while another group of papers related competition to firm innovation

¹ For example, Waldinger (2010) finds that PhD students suffer after superstar scientists who trained them emigrated. Azoulay, Graff Zivin, and Wang (2010); Jaravel, Petkova, and Bell (2018) provide evidence that the loss of knowledge producers causes a decline in the productivity of collaborators. Iaria, Schwarz, and Waldinger (2018) examine the impact of the collapse of international scientific cooperation and find a decrease in new knowledge and technology. Examples of collaboration would include Wuchty, Jones, and Uzzi (2007); Jones (2009).

² For example, Moser, Voena, and Waldinger (2014) document knowledge spillovers in which immigrant researchers attract new researchers to their fields and encourage innovation. Borjas, Doran, and Shen (2018) find that the influx of Chinese students into the U.S. increases the output of Chinese-American "advisors who advise the Chinese students. Bernstein, Diamond, McQuade, and Pousada (2019) find that immigrant collaborators create strong positive spillovers. Doran, Gelber, and Isen (2016) find that firm-level invention does not increase when a firm receives high-skilled immigrants. Further examples of knowledge spillovers in specific geographic locations include Jaffe, Trajtenberg, and Henderson (1993); Keller (2002); Thompson and Fox-Kean (2005); Thompson (2006); Singh (2005); Ellison, Glaeser, and Kerr (2010); Belenzon and Schankerman (2013); Moretti (2019).

³ For example, Waldinger (2012) finds that researchers who were left behind after their colleagues emigrated did not decrease their productivity. Similarly, Borjas and Doran (2012, 2015b) examine the output of American mathematicians when Soviet mathematicians immigrated into the U.S. These authors find that American mathematicians in fields that received the influx of Soviet mathematicians experienced a decrease in the productivity and moved away from such fields. Furthermore, Azoulay, Fons-Rosen, and Graff Zivin (2019) argue that the loss of knowledge producers provides an opportunity for non-collaborators in a field.

⁴ See, for example, Acs and Audretsch (1988, 1987); Acs, Audretsch, and Feldman (1994)

following Schumpeter (1942).⁵ Recent studies investigate various determinants of firm innovation other than R&D.⁶ Acemoglu (2010) developed the theoretical model that explains how labor affects technology advances, but it is an open empirical question of how inventors' human capital affects a firm's knowledge production. This paper, therefore, advances our understanding of how knowledge is generated by connecting the literature that addresses knowledge spillovers with that of firm innovation.

I begin by describing the context of the WWI draft in Section II. I use Section III to provide data and Section IV to present empirical strategies. I report the results in Section V and investigate the channels though which supply shocks affect firm-level innovation in Section VI. I provide a conclusion to the study in Section VII.

II Historical Context

World War I began in Europe on July 28, 1914, with the U.S. entering the war on April 6, 1917. Only 73,000 volunteers enlisted in response to an immediate call for volunteers, a number far short of the goal of one million in the first six weeks after the call. The Selective Service Act, which manages conscription in the U.S., was enacted one month after this initial call for volunteers. In 1917 and 1918, all men between the ages of 18 and 45 were required to register. Approximately 24 million men, nearly 98 percent of the population of men aged 18 to 45, completed draft registration cards during three rounds of registrations. As a result of three draft lotteries, about 2.8 million men served in the military for the years of 1917 and 1918. Figure A.1 shows how many persons were engaged in military service over time (Kendrick, 1961). The labor force in the public sector thus significantly increased at the time of the WWI draft (Figure B.1).⁷ Military expenditures in Figure B.2 also surged when the U.S. entered the war, with one fifth of U.S. resources spent on the war effort (Rockoff, 2004).

Not all men who registered for the draft during WWI served in the military. Further,

⁵ For example, see Gilbert (2006); Cohen (2010).

⁶ For example, Autor, Dorn, Hanson, Pisano, and Shu (2019) quantify how foreign competition influences domestic innovation. Aghion, Bergeaud, Lequien, and Melitz (2018) measure the effect of export shocks on innovation. Examples include Akcigit and Kerr (2018); Atkeson and Burstein (2018).

⁷ Online Appendix Figure B and Table B are available from https://sites.google.com/site/chungeunyoon

not all men who served in the military registered for the draft, since some men were already serving during registration. Table A.1 shows the number of men who registered, were drafted, served, and information about their status as patent holders (U.S. Provost Marshal General, 1919). Specifically, about 10 million men aged 21 to 30 registered in the first draft registration on June 5, 1917, and about one million men who had turned 21 registered in the second draft registration on June 5, 1918. The third registration on September 12, 1918 was intended for all men aged 18 to 45 who had not registered in the first or second registration.

Because WWI ended on 11 November, 1918, the majority of men who were drafted and served through draft lotteries came from the first and second registrations. Fewer than 200,000 men were inducted from the third registration despite about 13 million men having registered in the third round. Registrants provide their name, age, address, birth date, citizenship status and occupation on their draft registration card (Figure A.2).

Three draft lotteries randomly determined the draft order for registrants if they did not provide a valid excuse as to why they were not able to serve in the military. Most of registrants who were drafted eventually served in the military unless they claimed for exemption and their claim was granted. Fewer than 350,000 men who were selected were successful in gaining exemption.

The first national draft lottery held on July 20, 2017. The Secretary of War Newton D. Baker drew 258, the first draft number in the lottery (Figure 1). Each registrant in every local board throughout the country whose number was 258 was given an order number of 1. This process was repeated until 10,500 numbers were drawn. According to this order number, all registrants were required to appear before the local board for a physical examination or to claim exemption. The numbers in the second and third registration were drawn in the same manner from the second and third draft lottery, respectively.

The first national draft lottery held on July 20, 2017. The Secretary of War Newton D. Baker drew the first draft number in the lottery as shown on the photo in Figure 1. The first number drawn was 258 shown in Figure 1 for which each registrant in every local board throughout the country whose number is 258 was given an order number of 1. This process was repeated until 10,500 numbers were drawn. Every serial number in the draft registration cards from the first registration were given an order number. According to

this order number, all registrants were required to appear in the local board for physical examination or his claim to exemption. In the same fashion as the first draft lottery, the numbers in the second and third registration were drawn from the second and third draft lottery respectively.

The fact that different age groups were drafted during different times provides a possible identification strategy based on which portion of inventors in a firm was in different age categories. In the next section, I describe how the draft data could be used to identify the age of inventors within particular firms who served in the military.

III Data and Matching

My empirical analysis examines how the number of patent applications from firms is affected by the negative inventor supply shocks due to the WWI draft. First, I use a PATSTAT database provided by European Patent Office (2017) that contains characteristics of each patent application, such as the inventor's full name and year of application. This database draws from more than 40 patent authorities including the United States Patent and Trademark Office (USPTO). The PATSTAT database also contains information on the field (International Patent Classification, IPC) of each patent application, thus allowing me to measure the supply shock in the space of ideas. To construct firm-level patent data, I use patents by "company" in a type attribute of inventors in the PATSTAT database. The data allows me to identify when individual inventors file a patent application together with which firms.

To determine how innovation rates are impacted when firms lose inventors that were geographically close to the firm, I need to know where inventors were living when the WWI draft occurred and where the firms were located. I used data from Doran and Yoon (2019) in which patent data are merged into census data at the individual level. A fuzzy matching procedure performs a match between the patent database and the complete 1920 U.S. Census with the full names of individuals. In the 1920 Census, 43 percent of the U.S. population was had a unique first name, middle name, and last name combination. To increase the probability that the fuzzy matching procedure is precise, I only consider the population with a unique name between the ages of 18 and 80 (Doran and Yoon, 2019). I also only

consider patents matched between the years 1910 and 1918 in regression specifications, thus reducing the probability of the results being caused by those who died or migrated. Because a person could move geographically between 1917 and 1918, the identifying assumption when measuring the supply shock within a geographic space is that a person with a unique name observed in 1919 from the complete count the 1920 Census lived at the same place between the years 1917 and 1918 when the WWI draft occurred. As a proxy for a firm's location, I conduct a matching procedure between the PATSTAT database and the HistPat database (Petralia et al., 2016), which provides the geography of patents by the USPTO from 1790 to 1975. I calculate the location of each firm by the most frequently reported location in the HistPat database.

The data linked to the census provides the age and location of inventors. By constructing this novel linked data, I thus identify which firms worked with which inventors at which ages, where firms and individual inventors were located, and when and in what field inventors filed a patent application.

I use the platform FamilySearch for collecting records on draft registration and veteran service. FamilySearch publicly shares a large collection of historical records. In particular, I collect information that was recorded on WWI draft registration cards from the three registrations rounds. Figure A.2 shows an example of the draft registration card from the first registration. Though information in each registration was slightly different, registrants provided name in full, age in years, home address, date of birth, citizenship status, and occupation or employer's name on their draft registration card. Information on full name and birth date is digitized and publicly available from FamilySearch.org.⁸

I collect information by conducting a matching procedure on the web. I first create a donor pool of draftable inventors who were active during the WWI draft. This pool is limited to inventors who were not foreign-born, were 18 to 45 years old between 1917 and 1918, and who had any patent applications before 1917. I conduct a match between their full name and birth year in the patent database merged into the 1920 Census and those in WWI draft registration cards on FamilySearch (2019a). I find that about 96 percent of the inventors

⁸Ancestry.com, another large platform containing genealogical and historical records, also provides more complete sources but prohibits automatic access tools and it is not publicly available.

between the ages of 18 and 45 were registered, a registration rate that is close to the overall reported rate of 98 percent. I also collect WWI veteran records from FamilySearch (2019b) following the same procedure. FamilySearch provides information on full name and birth year of veterans who served during WWI.

Draft registration differs from draft and service in the military. Men who registered could voluntarily serve even though they were not drafted. Further, men who were drafted could evade the draft or fail the physical examination. In Table A.1, I report the populations within each of these categories. Since draft records were not available, three different groups (registered and served, registered but not served, or neither registered nor served) could be identified after a matching procedure between the patent database and WWI draft registration and service records. I use these data to create supply shocks of inventors who served in the military. These merged data also allows me to measure the effects of inventors who did not serve on innovation rates, specifically inventors who registered but did not serve or who neither registered nor served while their peers did serve.

Table 1 provides descriptive statistics for the firm panel data. The sample consists of 29,504 firms which had at least one patent application prior to the WWI draft and had no patent applications belonging to the arms industry, including firms that manufactured weapons, ammunition, or explosives. Additionally, war-related patents are not counted as the outcome of the number of patent applications. Specifically, I use the International Patent Classification (IPC) in the patent database to identify which patents belong to the arms industry. There could be characteristics of individual firms that could change how a particular firm experienced the effects of the WWI on innovation rates. For example, firms that produced weapons during WWI could have experienced a large increase in their output and thereby increase their inventions when WWI began or when the U.S. entered the war. To address this issue, I use the sample of firms not related to arms industry and the outcome of patents not related to such industry.

Firms significantly decreased their innovation rates during the WWI draft. Consistent with the sample, USPTO administrative data shows a sharp decline at the time of WWI draft

⁹The following IPC codes are relevant for the arms industry: F41A, F41B, F41C, F41F, F41G, F41H, F41J, F42B, F42C, F42D, B63G, C06B, G21J.

in patent applications (Figure B.3). This decrease could be due to the fact that the nation's resources were devoted to the war effort and that 2.8 million men, including inventors, were induced into the military during this time. The number of inventors represents the number of inventors per firm in each of three distinct categories between the years 1910 and 1918 defined as follows: inventors in collaboration space filed a patent application together with a firm, inventors in geographic space lived in the county where a firm was located, inventors in idea space filed a patent application classified in a particular field where a firm had a patent. The average firm worked with 3 team members, and had 556 inventors outside the firm in the same county and 1,134 inventors outside the firm working on the same topics.

In the next section, I outline empirical strategies that deal with the possibility of endogenous military service by inventors and potential confounding effects of WWI.

IV Identification

4.1 Estimating effects

I begin the empirical analysis by measuring the shocks to the labor supply, disaggregated by the three types of inventors. Specifically, I define the supply shocks firms encountered as falling into three distinct categories: the network of collaboration, the space of geography, and the space of ideas.

I first calculate the supply shock of team members who worked for a firm or collaborated with the firm in collaboration space. Firm j may have had team members who had any patent applications with firm j for the pre-WWI period. Some of team members of firm j were drafted and inducted into the military, and some of them did not serve. I use the linked patent data that provides the information on the service records. In particular, let P_{jC1} be the number of patents between the years 1910 and 1916 by firm j and inventors who worked with firm j and served in the military, and let P_{jC0} be the number of pre-WWI patents by firm j inventors who worked with firm j and did not serve. The collaboration-specific service rate is then defined as

$$S_{jC} = \frac{P_{jC1}}{P_{jC0} + P_{jC1}} \tag{1}$$

The variable S_{jC} measures the supply shock experienced by firm j when they lost their network of collaborators due to the WWI draft. The supply shock in collaboration space is used to measure a direct impact of an inventor's human capital on a firm's patenting capacity. The share of team members who served in the military is weighted by the number of pre-WWI patents assigned to the firm and inventors. This is because the shock depends on how close the firm and inventors are and on inventor productivity. I assume that there is no supply shock if the firm did not work with any individual inventors before the shock. i.e., the supply shock has a value of zero if the denominator is zero.

To measure the supply shock in geographic space at the firm level, I need to identify a geographic location of firm j and the associated inventors. I track the location of firms at the county level by using the information on the geography of patents from the HistPat that covers patents by the USPTO from 1790 to 1975. To identify the location of inventors, I use the information on the 1920 Census merged into the patent data. I assume a person observed in the 1920 Census lived in the same geographic location between the years 1917 and 1918.

Then, let P_{G1} be the number of pre-WWI patents by inventors who served and lived in county G where firm j is located, and let P_{G0} be the number of pre-WWI patents by inventors who did not serve in county G. The geographic-specific service rate at the firm level is then defined by

$$S_{jG} = \frac{P_{G1}}{P_{G0} + P_{G1}} \tag{2}$$

The variable S_{jG} measures the supply shock that firm j located in county G faced at the time of the WWI draft. Figure 2 shows the degree of the supply shock in each U.S. county.

The supply shock in the space of ideas is measured using the information on the international patent classification (IPC), available in the PATSTAT database, which specifies the field of each patent application. I assign each of the patent applications to a weighted set of patent classifications. Let $patent_{jf}$ be the number of pre-WWI patents in field f by firm j, and $patent_{j}$ be the total number of pre-WWI patents by firm j. Then, let P_{f1} be the number of pre-WWI patents in field f by inventors who served, and let P_{f0} be the number of pre-WWI patents in field f by inventors who did not serve. The field-specific service rate,

calculated by the field composition of firm j, is defined as

$$S_{jF} = \sum_{f} \frac{patent_{jf}}{patent_{j}} \frac{P_{f1}}{P_{f0} + P_{f1}}$$

$$\tag{3}$$

The variable S_{jF} measures the supply shock that firm j encountered when they lost inventors in a similar field.

Using the measure of the supply shocks and panel data set of firm-level outcomes, I investigate the effects of the inventor supply shocks firm outcomes with difference-in-differences specifications in the following regression model:

$$Y_{jt} = \beta_1(S_{jC} \times T_t) + \beta_2(S_{jG} \times T_t) + \beta_3(S_{jF} \times T_t) + \theta X_{jt} + \gamma_j + \delta_{st} + \epsilon_{jt}$$

$$\tag{4}$$

where Y_{jt} is the outcome of firm j in year t such as the number of patents, S_{jC} , S_{jG} and S_{jT} are firm-specific supply shocks in each of spaces respectively, and T_t is a dummy variable for the years 1917 and 1918. I include the quartic of years of experience of firm j in year t, X_{jt} , firm fixed effects, γ_j , and state-by-year fixed effects, δ_{st} , that thus control for state-specific yearly shocks. I define years of experience of firm j as years after firm j had a first patent application. The coefficients of interest are β_1 , β_3 and β_3 , indicating how supply shocks of inventors in collaboration space, geographic space, and idea space affect firm outcomes, respectively.

In the next subsection, I account for the possible endogeneity issues when unobserved characteristics would affect innovation rates of firms, thereby biasing the OLS coefficients.

4.2 Accounting for endogeneity

Firms that lost many inventors to WWI military service may differ in various characteristics from firms that lost few inventors. The innovation rates of firms that lost many inventors and firms that lost few inventors are assumed to not differ in the absence of supply shocks. On the condition that men registered, more than 90 percent of the population who served were drafted. Therefore, the majority of inventors who were registered and served were involuntarily drafted. However, inventors were not random subsamples of the total population

of draft-eligible males and thus it was not evident that this percentage held for inventors. Further, those who were drafted could evade the draft, fail physical examination, or claim exemption during appearance before the local board.

The above characteristics of firms and drafts could have biased the OLS coefficients. The assumption that the majority of inventors who registered and served were involuntarily drafted and inducted into the military due to the WWI draft could be weakened. For example, if inventors who voluntarily served were more productive and motivated than other inventors, this could differentially impact the innovation rates within firms. To address these concerns, I propose the draftable age group to construct an instrument for each of the supply shocks.

The instruments rely on the age profile of the inventors. Only the third draft registration required registration by all men aged 18-45. Specifically, 95 percent of inductions came from the first and second registrations. The third registration took place on September 12, 1918, and numbers were drawn on September 30, 1918, just before WWI ended on 11 November, 1918. Because WWI ended two months after the third registration, the majority of inductions thus occurred in the first and second registration, where young men were disproportionately likely to register. Further, men aged 21-30 were less likely to fail physical examination when they were drafted and were less likely to file claims of dependents such as spouses or children. I find that inventors aged 21 to 30 were more than 5 times as likely to serve than inventors aged 31 or older (4 percent versus 0.7 percent). Thus, using variation in the proportion of inventors who were between the ages of 21 and 30 within given firms, and given geographic locations and idea spaces, I am able to calculate powerful instruments for the proportion of such inventors who were drafted and served.

I first construct the instrument for the service rate in collaboration space. Let P_{jCs}^* be the number of patent applications per year between the years 1910 and 1916 by firm j and inventors aged 21-30 at risk of being drafted and let P_{jC}^* be pre-WWI patents by all inventors of firm j. I can then define the instrument for the supply shock in collaboration space as

$$S_{jC}^* = \frac{P_{jCs}^*}{P_{jC}} \tag{5}$$

The instrumental variable S_{jC}^* measures the proportion of firm j's team members who are at risk of being drafted and inducted at the time of the WWI draft.

Similarly, I construct the instrument for the supply shock in geographic space. Let P_{Gs}^* be the number of pre-WWI patents annually by inventors between the ages of 21 and 30 who lived in county G, and let P_G^* be the number of pre-WWI patents by all inventors in county G. The instrument for the supply shock in geographic space is then defined as

$$S_{jG}^* = \frac{P_{Gs}^*}{P_G} \tag{6}$$

The instrumental variable S_{jG}^* measures the proportion of inventors who were at risk of being drafted and inducted in county G where firm j was located during the WWI draft.

Finally, I define the instrument for the supply shock in idea space using the field composition of firm j. Let $patent_{jf}$ be the number of pre-WWI patent in field f by firm j, and $patent_{j}$ be the total number of pre-WWI patents by firm j. Then, let P_{fs}^{*} be the number of pre-WWI patents in field f by inventors aged 21-30, and let P_{f} be the number of pre-WWI patents in field f by all inventors. The instrument is then constructed as

$$S_{jF}^* = \sum_{f} \frac{patent_{jf}}{patent_i} \frac{P_{fs}^*}{P_f} \tag{7}$$

The instrumental variable S_{jF}^* measures the proportion of inventors at risk of being served in a similar field mix of firm j.

I use these variables constructed by draftable age groups to develop the instrument to measure the supply shocks of inventors who served in the military. Figure 2 represents how many inventors served in the military within firms that contained many inventors of draftable age and within control firms that contained few inventors of draftable age in each of the spaces. I identify two groups of firms using each of the instruments that measure supply shocks. Firms with many peers at risk of being served have values that are above the median, while firms with few peers have values that are below the median. This demonstrates that firms with many inventors of draftable age had higher service rates of inventors than control firms, thus leading to a large decline in the pool of inventors who could affect the innovation

activities within firms.

In the next subsection, I explain how the dynamics of the effects are measured.

4.2.1 Effect relative to the year before the WWI draft

I complement my empirical analysis with a difference-in-differences specification relative to the base year prior to the WWI draft. I use the regression model

$$Y_{jt} = \sum_{t=1910}^{1918} \left[\beta_{1t} (S_{jC}^* \times D_t) + \beta_{2t} (S_{jG}^* \times D_t) + \beta_{3t} (S_{jF}^* \times D_t) \right] + \theta X_{jt} + \gamma_j + \delta_{st} + \epsilon_{jt}$$
 (8)

where D_t is a year dummy except the base year 1916, the year before the WWI draft. Instruments for supply shocks, S_{jC}^* , S_{jG}^* , S_{jF}^* , interacted with a set of indicator variables corresponding to a particular year, D_t , provide the reduced form estimates relative to the base year. I also include state-by-year fixed effects, δ_{st} . The parameters of interest, β_t , therefore measure the effect of the supply shocks on innovation rates of firms at the year t relative to the omitted base year 1916, the year before the WWI draft registrations and lotteries began in 1917.

In the next section, I will report the results of this analysis.

${f V} \quad {f Results}$

5.1 Effect of supply shocks on firm's invention

The supply shocks of inventors as a result of the WWI draft affected the innovation rates of firms as measured by the number of patents. Figure 3 illustrates the raw data on the number of patents by treated and control firms. Figure 3 clearly shows a decrease in the number of patents by firms that lost their team members. The supply shock in geographic space did not significantly impact a firm's innovation rate. Firms exposed to the supply shock in idea space experienced an increase in innovation rates when they lost inventors outside the firms working on the same topics.

These results provide graphical evidence of the effects of supply shocks on firms' in-

novation rates, but do not consider the presence of unobservable firm-specific factors and state-specific economic trends that could have impacted firm outcomes. Furthermore, treated firms and control firms in Figure 3 are defined by a median value of supply shocks that is a continuous variable. To address this issue, I show the dynamics of effects relative to the omitted year 1916, which is before the WWI draft. Specifically, Figure 4 shows the estimates of the regression model in equation (8) in each of the supply shocks demonstrating that most of the estimated coefficients are insignificant before the omitted base year 1916. This result indicates that there are no differential pre-trends between treatment and control groups. Consistent with the results of Figure 3, estimates shown in Figure 4 move in the same direction. The supply shock of team members decreased firms' innovation rates, but the supply shock of inventors in the space of ideas increased firms' innovation rates.

I use the regression model to investigate the effect of supply shocks on firm outcomes. In Table 2, I report the coefficients from the first-stage regressions. 10 The relationship between the instrument and the supply shock respectively is presented in the first three columns. The last three columns show the coefficients when three instruments are included at the same time in the regression. Each instrument for the supply shock in its space has a significantly positive effect on the proportion of inventors who served. The instrument for the supply shock in idea space is correlated with supply shock in collaboration space, but correlation coefficient is relatively small. The pairwise correlations between the three instruments range from 0.003 to 0.015. The multivariate F test of excluded instruments produces high F statistics, indicating p values that are close to zero. Firms with many inventors at risk decreased the number of inventors in all three spaces during 1917 and 1918. This indicates that firms with a large portion of draftable inventors were more likely to lose inventors at the time of the WWI draft. For example, the estimated coefficient in collaboration space suggests that a 10 percent supply shock of inventors of draftable age group working with the firm, weighted by their pre-WWI patents, leads to a 1.1 percent supply shock of inventors

$$\boldsymbol{S_{j}} \times T_{t} = \alpha_{1}(S_{jC}^{*} \times T_{t}) + \alpha_{2}(S_{jG}^{*} \times T_{t}) + \alpha_{3}(S_{jF}^{*} \times T_{t}) + \theta X_{jt} + \gamma_{j} + \delta_{st} + \epsilon_{jt}$$

where S_j is a vector of firm-specific supply shocks (S_{jC}, S_{jG}, S_{jF}) measured by inventors who served in the military, and S_{jC}^*, S_{jG}^* and S_{jF}^* are firm-specific instruments measured by draftable inventors.

 $^{^{10}}$ The first-stage regression equation is

who served, weighted by their pre-WWI productivity. i.e., an increase in the instrument in collaboration space by 0.1 increases the supply shock in collaboration space by 0.01065 (column 4).

Table 3 reports estimated coefficients from IV specifications of the regression model¹¹ and Table A.2 reports OLS estimates in equation (4). The analysis sample includes firms which had at least one patent application prior to the WWI draft and no patent applications within fields that include arms industries such as weapons, ammunition, and explosives. The dependent variable is the number of patent applications not belonging to the arms industries.

I find that the supply shock in collaboration space has a negative effect on innovation rates of firms while the supply shock in idea space has a positive effect. I also find a negative, though insignificant, effect of the supply shock in geographic space. In particular, a supply shock that 10 percent of inventors who worked with a firm and served in the military, i.e., an increase in the supply shock in collaboration space by 0.1, decreases the number of patent applications per year by 0.01946 (column 4). Given the average number of patents per year by firms (0.1618), a 10 percent of team members who served reduces patent applications by a 12 percent. A 10 percent increase in the supply shock in idea space increases the number of patents annually by 0.01262 (column 4) associated with a 8 percent increase in the number of patents. This suggests that a firm increases its innovation rates if the firm loses inventors outside the firm working on the same topics in the space of ideas.

I identify two groups of firms based on the average number of pre-WWI patent applications per year. The results for highly innovative firms that had the number of pre-WWI patents above the median are consistent with the main findings. Specifically, a 10 percent increase in the supply shock in collaboration space leads to a 10 percent decrease given the average number of patents. I also find that a 10 percent increase in the supply shock in idea space increases patent applications by 8 percent.

Less innovative firms are defined as firms that had pre-WWI patent applications per year below the median. A 10 percent supply shock in collaboration space results in a 3 percent

$$Y_{jt} = \beta_1(\widehat{S_{jC}} \times T_t) + \beta_2(\widehat{S_{jG}} \times T_t) + \beta_3(\widehat{S_{jF}} \times T_t) + \theta X_{jt} + \gamma_j + \delta_{st} + \epsilon_{jt}$$

¹¹The second-stage regression equation is

decrease in patents, but it's insignificant. Interestingly, the supply shock in idea space has a significantly negative effect on innovation rates of less innovation firms. In particular, a 10 percent supply shock in idea space decreases the number of patents per year by 0.00649 (column 4) associated with a 8 percent decrease in patent applications. Highly innovative firms benefits from losing inventors in the space of ideas, but less innovative firms suffers from such supply shock. This provides evidence that less innovative firms depends heavily on knowledge spillovers in the space of ideas.

I report regression results of the reduced form in Table 4. Specifically, I estimate the effect using the regression model in equation (4) in which the supply shocks (S_{jC}, S_{jG}, S_{jF}) are replaced with instruments $(S_{jC}^*, S_{jG}^*, S_{jF}^*)$, respectively.¹² The parameters of interest measure the effect of losing draftable inventors on patenting by firms. The results are consistent with the IV estimates. The supply shock of inventors working with the firm decreases the firm's innovation rates, but the supply shock of inventors outside the firm working on the same topics increases the firm's innovation rates. These effects are driven by more innovative firms and less innovative firms experience a decline in invention in response to the loss of inventors in the space of ideas.

I estimate the long-run effect of supply shocks on innovation rates of firms. I investigate whether the effect of the supply shocks of losing inventors is persistent using the reduced form. The empirical analysis above uses years between 1910 and 1918, thus defining the pretreatment period as the years between 1910 and 1916 and the post-treatment period as the years between 1917 and 1918. To estimate the long-run effect, I examine several modifications of the pre- and post-treatment period. In Table A.3, the reduced form estimates are reported relying on the different pre- and post-treatment years. I also report the subsamples of more innovative firms and less innovative firms in Table B.1 and Table B.2, respectively. The supply shocks in geographic space and idea space have negative effects on innovation rates of firms in the long term, as result that is not consistent with the main findings between 1910 and 1918. Table B.1 and Table B.2 show that the results are driven by less innovative

$$Y_{jt} = \beta_1(S_{jC}^* \times T_t) + \beta_2(S_{jG}^* \times T_t) + \beta_3(S_{jF}^* \times T_t) + \theta X_{jt} + \gamma_j + \delta_{st} + \epsilon_{jt}$$

¹²The reduced form regression equation is

firms. However, a positive effect of the supply shock in idea space is still persistent for highly innovative firms reported in Table B.3. It should be noted that the results in the long run are less precisely measured because of the possibility that firms might change the company name or location over time.

In the next subsection, I estimate a number of robustness checks to provide the validity of the instrument and the sensitivity of the results.

5.2 Robustness of results

5.2.1 Validity of the instrument

The validity of the IV estimates rely on the consistency of innovation rates in the absence of the WWI draft between firms with and without draftable inventors. To address validity, I implement a placebo test using inventors in different age groups. Placebo supply shocks are created using inventors older than 30 years old. Inventors aged 30 or below were more likely have been drafted or enlisted in the military from the first and second draft registrations. Inventors aged 31-45 registered in the third draft registration but rarely served. Further, inventors aged 46 or above were not required to register, thus they were not draftable inventors. This would suggest that there should be no reduced-form relationship between placebo supply shocks of undraftable inventors and firm's innovation rates, since firms did not lose inventors older than 30 years old who rarely served.

I first investigate which age groups are more likely to serve in the military. I report correlations between placebo supply shocks and actual supply shocks in Table B.4. Panel A presents the correlation between supply shocks of draftable inventors, the instruments, and supply shocks of served inventors. The correlation between each supply shock of different age groups corresponding to the actual supply shock of inventors who served in each space shows that these two variables are substantially correlated. However, placebo supply shocks within panel B and C are not significantly correlated with actual supply shocks. Table B.5 shows first-stage regressions using placebo age groups. The estimated coefficients were quite small compared to the estimates reported in Table 2. Further, the low F statistics confirm that placebo supply shocks do not have a strong first stage. This provides evidence that

inventors older than 30 years old did not serve in the military and firms did not lose those inventors in three spaces in the regression analysis.

Table 5 reports reduced-form OLS coefficients using placebo supply shocks. The coefficients are small and insignificant in all specifications, indicating that there is no evidence of any reduced-form relationship between placebo supply shocks and innovation rates of firms. This result supports the validity of the instrument using the age profile of inventors.

Additionally, I implement another test using a placebo supply shock in geographic space. The instrument measuring the effect of supply shock within geographic space on innovation rates would be invalid if a supply shock of all men in geographic space inconsistently changed firm innovation rates, even in the absence of the WWI draft. I find that there is no evidence of relationships among a placebo supply shock and inventions, as reported in Table B.6, Table B.7, and Table B.8.

Another threat to the validity of the instruments involves the possibility that the WWI shock before the WWI draft affects innovation rates of firms differently. I examine placebo tests treating each of the years prior to the WWI draft of 1917-1918 as a placebo draft year. Figure B.5 and Table A.4 present the results of placebo draft year. I do not observe any evidence of significant impacts of supply shocks on inventions in each prior year, demonstrating that pre-trends captured by the WWI shock prior to the WWI draft do not alter the results of this study.

5.2.2 Controlling for the government's effect

Any labor shock that occurred during WWI but not captured by the WWI draft and military service may have also differentially impacted firms. For this reason, I only consider patents that are unrelated to the arms industry. Another possibility is that firms which closely worked with the government were differently affected by such supply shocks. The firms would increase inventions if the government supported or invested in these firms when the U.S. entered the war and invested heavily in war-related efforts. Furthermore, inventors working with firms that were close to the government or the arms industry might have been unlikely to be drafted, even if those inventors were 21-30 years old. This leads to biased results.

To address these concerns, I identify which firms and which patents are related to the government. I then use the sample of firms that had no patent applications joint with the government prior to the WWI draft. I also use the outcome of patents not assigned to the government. I find that the results are similar to main findings in every respect (Table A.5). This provides evidence that the measured effects of supply shocks on firms' innovation rates are valid after controlling for the government's effect.

5.2.3 Alternative specifications

I consider a number of alternative specifications to show that the results are robust to several modifications to the main estimation equation. I first use all firms and all patents regardless of the relationship to the arms industry. I find that the results are robust reported in Table B.9 using the reduced form.

I directly measure the differential effects of supply shocks on innovation rates of firms depending on their pre-WWI innovation rates. I include an interaction term for two groups of firms (more innovative firms and less innovative firms) rather than restrict the full sample to two subsamples. Table B.10 reports the reduced form estimates with the interaction term. Consistent with the results using the subsamples, more innovative firms increased their innovation rates when they lost inventors in idea space, while less innovative firms decreased innovation rates in response to such supply shock.

The counts of patents include a large proportion of zeros in the panel data. I thus consider the Poisson regression of the reduced form to address the nature of the data. I find that the results are similar, but a fixed effect for firms is not included.¹³

I also consider a slightly modified form of supply shocks. I modify the supply shock in collaboration space given the same weight (pre-WWI patents) of the same inventor. In the previous specification, I defined P_{jC} as the number of patents between the years 1910 and 1916 by firm j and inventors who worked with firm j. Therefore, considering the supply shock for firm j depends on how closely inventors worked with firm j before the shock. In

¹³Standard regression packages such as *poisson* and *xtpoisson* lead to convergence problems for the maximum likelihood because of a fixed effect for firms that had few patents (Santos Silva and Tenreyro, 2010, 2011). Even though I use the *ppml* command to address this problem, as suggested by Santos Silva and Tenreyro (2011), the regression does not produce the results due to a large number of observations with a fixed effect.

the modified supply shock, I only consider the productivity of inventors. Specifically, P_{jC} represents the pre-WWI patents by inventors who worked with firm j. The results are very similar (Table B.11).

In another modified form of supply shocks, I replace supply shocks in geographic space and idea space with supply shocks measured by inventors who were not a team member. For example, suppose an inventor in county c worked with firm j in f industry. This inventor was used to calculate the supply shocks in three spaces. i.e., supply shock in collaboration space for firm j, supply shock in geographic space for firm j located in county c, and supply shock in idea space for firm j in f industry. In the modified form of supply shocks, an inventor used to measure the supply shock in collaboration space was not used to measure supply shocks in geographic space and idea space. Specifically, supply shock in geographic space for firm jin county c was measured by inventors in county c excluding team members for firm j and, in the same fashion, supply shock in idea space for firm j in f industry was measured by inventors in f industry excluding team members for firm j. Although the regression analysis that includes three shocks simultaneously provides the marginal effect of supply shock while holding other supply shocks fixed, this modified form of supply shocks explicitly excludes inventors inside the firm when supply shocks in geographic space and idea space indirectly affect firm outcome. I report the results in Table B.12 using the reduced form and find the results to be consistent with other models.

In the next section, I discuss possible drivers and channels of innovation spillover.

VI Mechanism

6.1 The effect on invention of individuals

Individual inventors as well as firms play a crucial role in creating new ideas and inventions. Because individual inventors may depend heavily on knowledge spillovers, or may find it difficult to obtain patents for the first time, I consider the effects of supply shocks on patenting by individual inventors. Aggregated data on patents by individual inventors do not represent patents by firms, and individual inventors could work as an individual or in a team, or work

for a single firm or with several firms. Thus, this analysis seeks to understand the channels through which individual inventors generate knowledge spillovers and which individuals depend heavily on spillovers in response to the loss of their peers. I focus on inventors who did not serve in the military when they lost peers due to the WWI draft. The impact of peers on the level of innovation of inventors who did not serve in the military could be a key driver of patenting by firms in response to the supply shocks of losing peers.

Table B.13 shows summary statistics for the samples of native inventors who had at least one patent application between 1910 and 1916. Inventors who served in the military were more productive in the number of patents prior to their service even though their inventions substantially decreased during the WWI draft. However, it should be noted that served inventors are used to measure the supply shocks and are therefore not included in the analysis sample. Since men who were drafted and served were more likely to be registered during the first and second registration, inventors who served in the military were young than inventors who did not serve. The age profile has a strong positive impact on innovation rates Bell et al. (2019b). Further, I find there is little difference between the number of patents by inventors who served in the military before the WWI draft and patents of inventors who did not serve but registered between the ages of 21 and 30 during the war.

The supply shock in collaboration space is measured in a similar fashion. Individual i had collaborators who had one or more patent applications with inventor i for the pre-WWI period, 1910-1916. Let P_{iC1} be the number of pre-WWI patent applications per year by inventors who collaborated with individual i and served in the military, and let P_{iC0} be the number of pre-WWI patents by collaborators of individual i who did not serve. The collaboration-specific service rate at the individual level is then given as

$$S_{iC} = \frac{P_{iC1}}{P_{iC0} + P_{iC1}} \tag{9}$$

The variable S_{iC} measures the supply shock inventor i faced when they lost their network of collaborators.

I measure the supply shock in geographic space using the information on the 1920 Census merged into the patent data. A person observed in the 1920 Census is assumed to have lived

in the same place during the WWI draft. Let G be the county where inventor i lived, then let P_{G1} be the number of pre-WWI patents by inventors who registered and served in county G geographically close to inventor i, and let P_{G0} be the number of pre-WWI patents by inventors who did not serve in county G. The geographic-specific service rate is then given by

$$S_{iG} = \frac{P_{G1}}{P_{G0} + P_{G1}} \tag{10}$$

The variable S_{iG} measures the size of the supply shock where inventor i who lived in county G encountered due to the WWI draft.

Finally, I calculate the supply shock in idea space at the firm level using the field composition of individual i. Let $patent_{if}$ be the number of pre-WWI patent applications that inventor i had in field f, and $patent_i$ be the total number of pre-WWI patents by inventor i. Then, let P_{f1} be the number of pre-WWI patents in field f by inventors who registered and served, and let P_{f0} be the number of pre-WWI patents in field f by inventors who did not serve. The field-specific service rate, calculated by the field composition of inventor i, is defined as

$$S_{iF} = \sum_{f} \frac{patent_{if}}{patent_{i}} \frac{P_{f1}}{P_{f0} + P_{f1}}$$

$$\tag{11}$$

The variable S_{iF} measures the supply shock experienced by inventor i when they lost their peers who had a similar field.

I employ analogous difference-in-differences specifications to examine the impact of the supply shocks on innovation rates of inventors who did not serve. Specifically, I use the regression model

$$Y_{it} = \beta_1(S_{iC} \times T_t) + \beta_2(S_{iG} \times T_t) + \beta_3(S_{iF} \times T_t) + \theta X_{it} + \gamma_i + \delta_{st} + \epsilon_{it}$$
 (12)

where Y_{it} is the number of patents of inventor i in year t, S_{iC} , S_{iG} , and S_{iF} are personspecific supply shocks in collaboration space, geographic space and idea space, and T_t is a dummy variable indicating WWI draft years 1917 and 1918. I include the quartic of age of inventor i in year t, X_{it} , individual fixed effects, γ_i , and state-by-year fixed effects, δ_{st} .

Using this regression model, I find large declines in the number of patent applications per

year by inventors who did not serve when those inventors lost their collaborators (Table B.14). The supply shocks in idea space have a positive effect on innovation rates of inventors who did not serve. Specifically, a supply shock that 10% of collaborators disappear, which is a 0.1 increase in the WWI service rate in collaborator space, weighted by their pre-WWI patents, decreases the number of patent applications per year by 0.003. Conversely, a 0.1 increase in the service rate in idea space increases the number of patents annually by 0.08. Given the dependent variable mean, when inventors lost 10% of their peers, there was a corresponding 2.2% in innovation rates within collaboration space and a 5% increase in idea space.

I find that the supply shocks affect some inventors who registered but did not serve and other inventors who neither registered nor served differently. For relatively young inventors aged 18 and 45 who registered, the supply shock in a network of collaborators had an insignificant and small effect. Conversely, the supply shocks in idea space had substantial positive effects on innovation rates. In particular, a 10% increase in the service rate in idea space resulted in a 9.5% increase in the number of patent applications per year by inventors who registered but did not serve. Inventors who neither registered nor served did not benefit from the supply shocks in idea space. Most of these inventors were aged 46 or older, and experienced a large decline in innovation rates when their collaborators served in the military. Specifically, a 10% increase in the service rate in a network of collaboration decreased the number of patents by inventors who did not register or serve by 3.1%.

To address the possible endogeneity issues, I employ the IV strategy using the instruments that rely on the age profile of inventors. Specifically, I create the analogous supply shocks at the individual level in each of three spaces measured by pre-WWI patents by draftable inventors.

Table B.15 reports the coefficients from the analogous first-stage regressions at the individual level. The regressions support the validity of the instruments that have a strong first-stage. The estimated coefficients on the diagonal demonstrate that the instruments are substantially correlated with the supply shocks measured by inventors who served. The pairwise correlations between the three instruments range from 0.002 to 0.079. The multivariate F test of excluded instruments demonstrated that p-values were close to zero.

The IV estimates suggest that losing peers in collaboration spaces had a negative effect

on patenting by inventors who did not serve, while the supply shocks in idea space had a positive but less significant effect compared to the OSL estimates (Table B.16). Specifically, losing 10% of collaborators decreased the number of patent applications per year by 0.007, or 4.5% given the average number of patents.

The supply shock in collaboration space had a negative effect on innovation rates of inventors aged 18 and 45 who registered but did not serve. However, the supply shock in idea space had a strong positive impact on innovation rates. Specifically, a 10% supply shock in idea space increased patenting by inventors who registered but did not serve by 6.9%. The supply shock in collaboration space had a significantly negative effect on innovation rates of inventors who neither registered nor served and were aged 46 or above. A 10% supply shock in collaboration space decreases the number of patents by neither registered nor served inventors by 8.4%. I also consider the reduced form specifications and find that the results are robust to all outcomes reported in Table B.17.

Overall, the results here demonstrate that innovation rates of individual inventors who did not serve were negatively impacted when they lost peers in their collaboration spaces. Individual inventors and entire firms experienced a large decline in their inventions when they lost their team members. I find that individual inventors benefited from the supply shock in idea space less than firms. Firms substantially increased their innovation rates when they lost competitors in idea space, but individual inventors gained less benefits from losing peers in the space of ideas. I also find that the supply shock in idea space had a strong positive effect on innovation rates of both inventors who registered but did not serve, thus being relatively young and more active before the shock, and more innovative firms, which had more innovation activities before the shock. These results provide evidence that the position of knowledge producers relative to the frontier in its field plays a role in explaining peer effects when they lose peers.

In next subsection, I measure the effect of losing inventors on innovation rates of highly innovative firms.

6.2 Highly innovative firm

I investigate how the effect of the supply shocks varies with the productivity of firms. The main results are driven by more innovative firms, but firms could be categorized more specifically other than pre-WWI patents above the median. To understand clearly how the results are driven, I use the different subsamples of highly innovative firms. This analysis contributes to a growing literature that examines how the careers of knowledge producers, the quality of their outputs, and the quality of their peers affect their productivity and outcomes Azoulay et al. (2010); Waldinger (2010, 2012); Iaria et al. (2018); Bell et al. (2019a). Specifically, I determine whether the supply shocks have a large impact on highly innovative firms rather than on other firms.

I reestimate innovation rates within subsamples. Table A.6 reports the reduced form estimates using the subsamples of top innovative firms. I find that firms in the top 5th percentile of inventions decrease their patent applications per year by 0.5824, translating into a 3% decrease when those firms faced a 10% supply shock of team members. Instead, those firms substantially increase innovation rates when they lose their competitors in idea space. Specifically, patent applications increase 7.8% in response to a 10% supply shock in idea space. The results for firms in the top 10th and 25th percentiles are similar to those for firms in the top 5th percentile. The reduced form estimates using the full sample in the same specification reported in Table 4 show that losing 10% of inventors in collaboration space and idea space affect inventions of firms by a 1.3% decrease and 3.9% increase, respectively. These findings indicate that highly innovative firms are most affected by the supply shocks.

I additionally measure the differential effects of supply shocks on innovation rates of firms depending on their pre-WWI patents. I include an interaction term for presenting the percentile in their pre-wwi patents. Table B.18 reports reduced form estimates with the interaction term. The first three rows show the results for firms at the bottom percentile in pre-WWI patents without any interaction term and the next three rows show the differential effects of the supply shocks when firms increase their pre-WWI patent by one percentile. The results here suggest that the closer the firms get to knowledge frontiers, the more increase in their patent applications in response to the loss of inventors in the space of ideas.

In the next subsection, I investigate whether the supply shocks affect new ideas and originality of firm's invention.

6.3 New ideas

I determined whether the supply shocks affect new ideas introduced by firms using an alternative dependent variable: original patent application. I construct an indicator from the new words identified from patent titles by first defining any words contained in patent titles in 1900 as new words. Then, words that already appeared in previous patent titles are defined as non-novel. I then defined a dependent variable of the number of original patent applications containing at least one new word that had not already appeared in previous patent titles.

Table A.7 reports the reduced form estimates using original patent applications as a dependent variable. There are no significant effects of supply shocks on the number of original patents. However, highly innovative firms create more original ideas when they lose inventors in the space of ideas (Table B.19). This supports the hypothesis that highly innovative firms are a knowledge frontier during this time in history.

In the next subsection, I estimate the effect of the supply shocks on another measure of innovation: citation-weighted patents.

6.4 Citations

It could be assumed that firms would have invested most heavily in the most successful inventions rather than the marginal inventions. To investigate whether the supply shocks affect patents weighted by the later influence of the invention, I reestimate the results in Table 4 with patent citations as the outcome variable. The results are similar in response to the supply shocks shown in Table A.8, but the impact of the supply shock in idea space varies more heavily with firms' previous innovation rates. More innovative firms largely increase citation-weighted patents in the space of ideas, but less innovative firms significantly decrease them. Thus, these results indicate that highly innovative firms act as knowledge frontiers by investing more in useful inventions, while less innovative firms depend more on spillovers.

In the next subsection, I estimate the effect of inventor quality on innovation rates of firms during supply shocks.

6.5 Quality of inventors

Firms aim to attract high-skilled workers and productive inventors to encourage innovation and increase productivity. Inventor quality is considered to be one of the key drivers for firm innovation. It is not evident, however, that this positive relationship between inventor quality and firm innovation leads to a causal relationship. To identify the causal effect of inventor quality on a firm's innovation rate, I measure the effect of the supply shocks depending on the distribution of inventor quality on innovation rates of firms.

Table 6 reports the reduced form estimates relying on the supply shocks of different quality inventors in place of all inventors shown in Table 4. The results are similar if firms lose high-quality inventors who worked for them or collaborated with them. The supply shock of losing high-quality or low-quality inventors in the space of ideas increases firms' innovation rates. Interestingly, I find that the supply shock of losing high-quality inventors in geographic space decreases innovation rates of firms, but the supply shock of low-quality inventors increases innovation rates. Thus, the overall effect of losing inventors in geographic space is cancelled out (Table 4). This provides empirical support for previous findings from existing studies of the dynamics of knowledge spillovers in geographic space. High-quality knowledge producers has large spillovers on geographically close others, but low-quality knowledge producers had a less than proportionate impact. Losing low-quality inventors actually increases a firm's innovation in the presence of diminishing marginal returns.

I also investigate whether supply shocks have differential effects on a firm's invention depending on a firm's previous innovation rates, as shown in Table B.21 and Table B.22 for more innovative firms and less innovative firms, respectively. More innovative firms benefit more from losing more productive inventors in similar topics, but less innovative firms experience a larger decline in invention when they lose more innovative inventors. This supports the hypothesis that highly innovative firms act as knowledge frontiers that fill a gap immediately according to the law of diminishing returns, but less innovative firms depend on knowledge spillovers in the space of ideas.

In sum, I find evidence of the causal relationship between inventor quality and firm's innovation rates. Losing very high-quality inventors have larger effects on innovation rates of firms than losing high-quality or low-quality inventors. Highly innovative knowledge producers have large spillovers, thus largely affecting firm innovation.

VII Conclusion

In this paper, I explore the effect of the supply shocks on firm innovation rates. I use a novel approach to this question by using the new natural experiment of the WWI drafting and subsequent military service of inventors. I distinguish separate effects of the decreasing labor supply of inventors on a firm's innovation rate. A firm could lose inventor team members due to the WWI military service who worked directly for or with the firm. The firm also could lose inventors close to the firm's location in geographic space (working in the same county) and idea space (working on similar topics).

My analysis revealed three major findings. First, losing inventors who work for a firm itself decreases the firm's innovation rate. Consistent with results of previous empirical studies and the theory of human capital externalities, the loss of team members has a significant negative impact on the productivity of knowledge producers.

Second, I find that losing inventors working on the same topics in which a firm engages increases the firm's innovation rate. This increase suggests that the law of diminishing returns prevails over knowledge spillovers in the space of ideas. There exists intense competition in the space of ideas, and inventors who are not a team member of a firm in the same industry have a negative impact on the firm's innovation rate. This result provides evidence that new ideas grow up from knowledge producers when other knowledge producers within a field leave, consistent with Planck's Principle.

Third, the effects of supply shock on firm innovation rate depend on how close the firm is to the knowledge frontier. The overall results are driven by frontier firms. Firms that are highly innovative prior to the WWI draft benefit from the loss of inventors outside the firms working on similar topics, but less innovative firms decrease innovation rates in response to such supply shock in the space of ideas. This result provides evidence that less innovative

firms depend heavily on knowledge spillovers. As innovators are removed to serve in the military during WWI, the frontier knowledge producers fill the empty space of ideas.

Taken together, these results provide a deeper understanding of the dynamics of knowledge spillovers and how innovation rates depend on the distribution of inventors across firms and the space of ideas. However, all of the evidence presented here pertains to firms' inventions during the early 20th century, a time during which the system of mass production was introduced in the U.S. It is unclear how the findings here might extend to current industries or other areas of knowledge production. In particular, an increase in the mobility of skilled labor may generate large spillovers in geographic space as they move from one location to another more frequently, but at the same time a rise in working remotely or from home is unlikely to generate peer effects. Furthermore, the importance of capital equipment and collaborative work in inventive activity differs across the industry. For example, industrial research in some fields could not be conducted without capital equipment and collaborative work, and thus is likely to evolve by frontier firms that already possess specialized equipment and rich collaborative network. In contrast, other fields in which researchers tend to work alone and capital equipment is less required may provide more opportunities for new comers to produce new innovation. Future research should address where new innovations tend to originate.

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Figure 1: WORLD WAR I DRAFT LOTTERY

(a) The first draft number in the lottery

```
First Hundred Drawn.
258, 2522, 9613, 4532, 10218, 458, 3403,
10015, 9899, 8934, 1436, 2624, 4762, 854,
6985, 7183, 6597, 5977, 1894, 4614, 4501,
9922, 1878, 4142, 4083, 10425, 9018, 8251,
6423, 9736, 3257, 5799, 10240, 6767, 1095,
8666, 2022, 3383, 6551, 6952, 9420, 3382,
9258, 4306, 4320, 7103, 9852, 4881, 1455,
3679, 6183, 3755, 783, 1813, 8462,
1858, 8239, 2389, 10385, 5034, 7269, 8904,
5706, 3567, 3637, 9938, 5227, 1752, 5497,
8830 8596, 4520, 2494, 6453, 4137, 5885,
3674, 5939, 5769, 3200, 3082, 6132, 6809,
3505, 1117, 8343, 1572, 5897, 2762, 9594,
1748, 5938, 7952, 9316, 5019, 2195, 4487,
8159, 837,
```

(b) The order of the first hundred numbers drawn

Notes: Secretary of War Newton D. Baker drew the first draft number in the lottery shown in the first figure. The order of the first hundred numbers drawn from the first registration was published to the public (Pittsburgh Post on July 21, 1917).

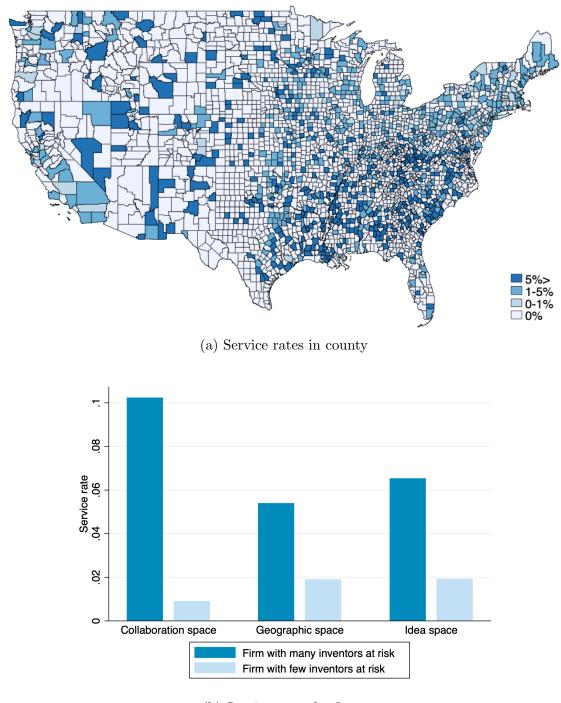
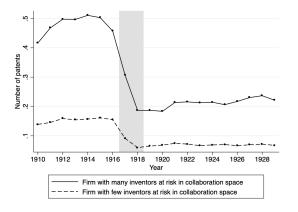


Figure 2: SUPPLY SHOCK OF INVENTORS

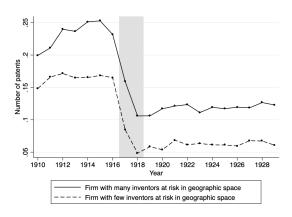
(b) Service rates for firms

Notes: The figures show a supply shock of inventors caused by the WWI draft and military service. The first figure presents inventors who served in the military as a pecentage of total inventors in each U.S. county. The second figure shows the percentage of inventors who served in the military for firms with many inventors at risk which have a portion of inventors in draftable age groups above the median and firms with few inventors at risk which have a portion of inventors in draftable age groups equal to or below the median in each space, respectively.

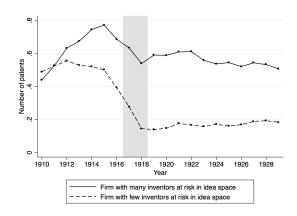
Figure 3: IMPACT OF SUPPLY SHOCK ON INNOVATION RATES



(a) Collaboration space



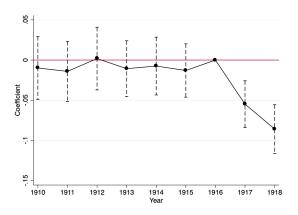
(b) Geographic space



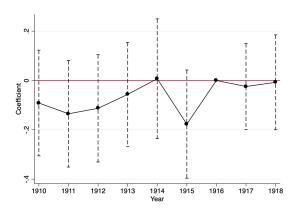
(c) Idea space

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and no patent application belonging to the arms industry such as weapon, ammunition, and explosives. Firms with many peers at risk have a portion of inventors in draftable age groups above the median, while firms with few peers have a portion of inventors in draftable age groups equal to or below the median in each space, respectively. The outcome variable is the number of patent applications per year not relevant for the arms industry. The number of patent applications is winsorized at 10.

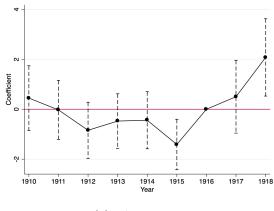
Figure 4: IMPACT OF SUPPLY SHOCK ON INNOVATION RATES RELATIVE TO A YEAR BEFORE THE WWI DRAFT



(a) Collaboration space



(b) Geographic space



(c) Idea space

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and no patent application belonging to the arms industry such as weapon, ammunition, and explosives. The outcome variable is the number of patent applications per year not relevant for the arms industry. The number of patent applications is winsorized at 10.

Table 1: SUMMARY STATISTICS

	Mean (1)	Median (2)	SD (3)	Min. (4)	Max. (5)
Patent applications per year					
1910-1918	0.1414	0	0.5311	0	10
1910-1916 (before the draft)	0.1644	0	0.5676	0	10
1917-1918 (during the draft)	0.0608	0	0.3650	0	10
Number of inventors					
In collabroation space	3	2	2	1	81
In geographic space	556	352	626	0	2,145
In idea space	1,134	36	3,214	0	36,758
Number of firms					
	29,031				

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. Patent applications per year is not relevant for the arms industry, winsorized at 10. The number of inventors represents the number of inventors per firm in each space between the years 1910 and 1918.

Table 2: FIRST-STAGE REGRESSIONS

	Dependent variable						
Instrument	Collaboration (1)	Geographic (2)	Idea (3)	Collaboration (4)	Geographic (5)	Idea (6)	
Collaboration (S_{iC}^*)	0.1064*** (0.0051)	-	_	0.1065*** (0.0051)	0.0002* (0.0001)	-0.0000 (0.0004)	
Geographic (S_{iG}^*)	_	0.5693*** (0.0120)	_	-0.0010 (0.0025)	0.5692*** (0.0120)	0.0163 (0.0172)	
$Idea~(S_{iF}^*)$	_	_	0.6690*** (0.0092)	0.0238*** (0.0090)	0.0004 (0.0003)	0.6690*** (0.0092)	
F-test of excluded instruments	438.10	2,236.66	5,261.10	147.10	795.51	1,810.59	
Number of observations Number of firms	$222,795 \\ 29,031$						

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. Standard errors are clustered by firms.

Table 3: IMPACT OF SUPPLY SHOCK ON INNOVATION RATES, IV COEFFICIENTS

		Speci	fication	
	(1)	(2)	(3)	(4)
Dependent variable: Patent	applications per	r year		
A. Firms				
Supply shock in Collaboration space β_1	-0.2000*** (0.0685)	-	_	-0.1946*** (0.0683)
Geographic space β_2	_	0.0630 (0.0781)	_	0.0639 (0.0777)
Idea space β_3	_	_	$0.1201^{**} $ (0.0549)	$0.1262^{**} $ (0.0549)
Dependent variable mean	0.1618			
Number of observations	222,795			
Number of firms	29,031			
B. More innovative firms				
Supply shock in Collaboration space β_1	-0.3596* (0.2008)	_	-	-0.3334^* (0.1997)
Geographic space β_2	_	0.4072 (0.2676)	_	$0.4136 \\ (0.2673)$
Idea space β_3	_	_	0.2646** (0.1233)	0.2729^{**} (0.1232)
Dependent variable mean	0.3481			
Number of observations	76,338			
Number of firms	9,093			
C. Less innovative firms				
Supply shock in Collaboration space β_1	-0.0175 (0.0260)	_	_	-0.0192 (0.0259)
Geographic space β_2	_	-0.0435 (0.0344)	_	-0.0434 (0.0343)
Idea space β_3	_	-	-0.0657*** (0.0207)	-0.0649*** (0.0207)
Dependent variable mean	0.0768			
Number of observations	146,457			
Number of firms	19,938			

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. More innovative firms had pre-WWI patents above the median and less innovative firms had pre-WWI patents equal to or below the median. The outcome variable is the number of patent applications per year not relevant for the arms industry. The number of patent applications is winsorized at 10. Standard errors are clustered by firms. State-year fixed effects are included.

Table 4: IMPACT OF SUPPLY SHOCK ON INNOVATION RATES, REDUCED FORM

		Speci	fication	
	(1)	(2)	(3)	(4)
Dependent variable: Patent	applications per	r year		
A. Firms				
Supply shock in Collaboration space β_1	-0.0213*** (0.0077)	_	_	-0.0208*** (0.0077)
Geographic space β_2	_	0.0366 (0.0481)	_	0.0367 (0.0481)
Idea space β_3	_	_	0.0806** (0.0390)	0.0793^{**} (0.0390)
Dependent variable mean	0.1618			
Number of observations	222,795			
Number of firms	29,031			
B. More innovative firms				
Supply shock in Collaboration space β_1	-0.0370^* (0.0217)	_	_	-0.0340 (0.0217)
Geographic space β_2	-	0.2322 (0.1627)	_	0.2303 (0.1624)
Idea space β_3	-	_	0.1827** (0.0901)	$0.1795^{**} (0.0901)$
Dependent variable mean	0.3481			
Number of observations	76,338			
Number of firms	9,093			
C. Less innovative firms				
Supply shock in Collaboration space β_1	-0.0019 (0.0030)	_	_	-0.0021 (0.0030)
Geographic space β_2	_	-0.0254 (0.0212)	_	-0.0252 (0.0212)
Idea space β_3	_	_	-0.0432*** (0.0144)	-0.0433*** (0.0144)
Dependent variable mean	0.0768			
Number of observations	$146,\!457$			
Number of firms	19,938			

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. More innovative firms had pre-WWI patents above the median and less innovative firms had pre-WWI patents equal to or below the median. The outcome variable is the number of patent applications per year not relevant for the arms industry. The number of patent applications is winsorized at 10. Standard errors are clustered by firms. State-year fixed effects are included.

Table 5: IMPACT OF SUPPLY SHOCK ON INNOVATION RATES, PLACEBO AGE GROUPS OF INVENTOR

	Specification				
	(1)	(2)	(3)	(4)	
Dependent variable: Patent	applications pe	r year			
A. Supply shock of less likely	y draftable inve	ntors (age 31-4	5)		
Supply shock in Collaboration space	0.0012 (0.0039)	_	_	0.0013 (0.0040)	
Geographic space	-	-0.0098 (0.0148)	_	-0.0111 (0.0152)	
Idea space	_	_	-0.0965*** (0.0224)	-0.0964^{***} (0.0224)	
Dependent variable mean	0.1323				
Number of observations	$255,\!861$				
Number of firms	29,031				
B. Supply shock of not draft	able inventors	(age 46 or abov	e)		
Supply shock in Collaboration space	0.0055 (0.0036)	_	_	0.0041 (0.0037)	
Geographic space	_	0.0148 (0.0154)	_	0.0117 (0.0156)	
Idea space	_	_	-0.1037*** (0.0241)	-0.1034*** (0.0241)	
Dependent variable mean	0.1323				
Number of observations	255,861				
Number of firms	29,031				

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. The outcome variable is the number of patent applications per year not relevant for the arms industry. The number of patent applications is winsorized at 10. Standard errors are clustered by firms. State-year fixed effects are included.

Table 6: QUALITY OF INVENTORS AND IMPACT OF SUPPLY SHOCK ON INNOVATION RATES

	Specification				
	(1)	(2)	(3)	(4)	
Dependent variable: Patent	applications pe	er year			
A. Supply shock of very high	h-quality invent	tors			
Supply shock in Collaboration space	-0.0530*** (0.0135)	_	_	-0.0479*** (0.0135)	
Geographic space	_	-0.0700*** (0.0109)	_	-0.0667*** (0.0108)	
Idea space	_	_	0.0619^* (0.0359)	0.0649^* (0.0359)	
B. Supply shock of high-qua	lity inventors				
Supply shock in Collaboration space	-0.0512*** (0.0102)	_	-	-0.0467*** (0.0101)	
Geographic space	_	-0.0683*** (0.0105)	_	-0.0641*** (0.0104)	
Idea space	_	_	0.0545^* (0.0317)	0.0564^* (0.0317)	
C. Supply shock of low-qual	ity inventors				
Supply shock in Collaboration space	$0.0001 \\ (0.0102)$	_	-	-0.0030 (0.0102)	
Geographic space	_	0.0705^{***} (0.0195)	_	0.0733^{***} (0.0196)	
Idea space	_	_	0.0354^* (0.0209)	0.0361^* (0.0209)	
Dependent variable mean	0.1618				
Number of observations	$261,\!279$				
Number of firms	29,031				

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. Very high-quality inventors are those with pre-WWI patents in the top 10 percentile of invention. High-quality inventors are those with pre-WWI patents above the median and low-quality inventors are those with pre-WWI patents below the median. The outcome variable is the number of patent applications per year not relevant for the arms industry. The number of patent applications is winsorized at 10. Standard errors are clustered by firms. State-year fixed effects are included.

Appendix

A.1 Conceptual Framework

The theories of human capital externalities explain how innovations and new ideas generated by knowledge producers or highly-skilled workers influence the productivity of firms. To show these spillovers can coexist with the law of diminishing returns, I develop a modified version of the model following Jones and Romer (2010) and Borjas and Doran (2015a). The production function for invention Y is modeled as an increasing function of the stock of ideas A, the stock of resources K such as buildings, land, and machinery, and the stock of inventors L. Suppose that the Cobb-Douglas production functions is given by

$$Y = A^{\phi} K^{\alpha} L^{1-\alpha} \tag{13}$$

where ϕ is the externalities elasticity for all firms that presents a percent increase in the stock of ideas is associated with a percent increase in invention. The stock of ideas is assumed to be proportional to the number of inventors and dependent on firms $A = L^{\gamma}$ where γ measures the firm-specific degree of increasing returns. A marginal product is then given by

$$MPL = (\gamma \phi + 1 - \alpha) K^{\alpha} L^{\gamma \phi - \alpha}$$
(14)

To analyze how the productivity responses to a supply shock, I show that 14

$$d\log MPL = \alpha d\log K + (\gamma \phi - \alpha) d\log L$$
 (15)

In the short run, it is assumed that resources K are fixed, $d\log K = 0$, and let $m = d\log L$. The change in innovation rates following a supply shock of inventors is then given by

$$d\log MPL = (\gamma \phi - \alpha)m \tag{16}$$

The equation (16) shows that spillover effects can coexist with competitive effects that are caused by the law of diminishing returns. This model of innovation rates demonstrates that spillovers will increase a firm's innovation rates as a result of increased innovations

¹⁴See the mathematical appendix for more details.

when the externality's elasticity ϕ is large, which leads to $\gamma \phi > \alpha$. On the other hand, the law of diminishing returns will act to decrease firms' innovations arising from the supply shock if the spillover effects are small $\gamma \phi < \alpha$. In the context of the WWI draft, losing inventors could either decrease innovation rates of firms when spillover effects come into play or increase innovation rates when competitors disappear following the law of diminishing returns. This model can also be used to explain the differential effects of the supply shock on invention among firms. In response to losing inventors, a firm would reduce productivity if the firm's degree of increasing returns to the stock of ideas is large γ , while they would not decrease inventions if a small degree of returns γ makes spillovers non-existent.

The degree to which human capital externalities come into play depends on the supply shock in three different spaces. Local externalities within a network of each space allows a generalizable model of the human capital externalities. The production function in equation (13) implies that the supply shock alters the productivity by the same proportion for all firms. It seems plausible, however, that a firm's productivity depends on different externalities in all three spaces. Specifically, the externalities may have a large influence on the firm's innovation rates when the firm loses their inventors who worked for the firm prior to the supply shock. For example, a supply shock of inventors in the same collaboration network would significantly reduce inventions of firms when those inventors lose that network. It is also possible that a supply shock that affects a particular area would have a strong effect on the productivity of firms at that location. Moreover, because patent applications can draw from many fields. For example, a supply shock of inventors in a field of chemistry would have a strong impact on inventions to firms in the chemistry field, but could also impact inventors within related fields.

To illustrate differential effects, I introduce local externalities as the number of efficiency units provided by the total invention workforce. Suppose there are N distinct networks and the generalized workforce is written as

$$L = (A_1^{\theta} L_1^{\beta} + \dots + A_N^{\theta} L_N^{\beta})^{\frac{1}{\beta}}$$
 (17)

where A_n is the stock of ideas in network n, L_n is the stock of invention workforce in network n, and θ is the local externality elasticity. Suppose that the stock of ideas

is proportional to the size of invention workforce depending on firms and networks. It is then defined by $A_n = L_n^{\gamma\sigma}$ where γ is a firm-specific parameter and σ is a network-specific parameter that can vary for each of three spaces. These parameters are used to measure the network-specific spillovers for each firm. The model shows that the change in marginal product for a firm in network n as a result of a supply shock in network n is

$$d \log MPL_n = (\gamma \phi + 1 - \alpha - \beta)m + (\gamma \sigma \theta + \beta - 1)m_n$$
(18)

where $d\log L = m$, $d\log L_n = m_n$, and resources K are fixed in the short run.

The supply shock is therefore measured by the percent change in the total supply of inventors and the percent change in the supply of inventors in network n. In the generalized model, the relative strengths of the total externalities elasticity ϕ , the local externalities elasticity θ in network n, and measures of the firm-specific γ and network-specific σ degree of returns to the stock of ideas will determine the impact of the supply shock in network n.

The model described above implies that the short-run impact of the supply shock on invention can be either positive of negative. Further, the model suggests that the supply shock of inventors differentially affects innovation rates of firms depending on the firm's characteristics and on which of the three distinct spaces is considered. The mixed evidence on the spillover effects in empirical studies can therefore be caused by the fact that the studies measure the conceptually different spillovers arising from distinct supply shocks.

To address this issue and test whether the theory is consistent with empirical evidence, I specifically measure the short-run impact of the supply shock in collaboration space, geographic space, and idea space on firm innovation rates. In this case, supply shocks are caused by the WWI draft during 1917-1918 that enlisted inventors into military service. I provide evidence that team members in collaboration space generate knowledge spillovers supporting the theory of human capital externalities, but other inventors outside the firm working on the same topics have a negative impact on firm's innovation rates supporting the law of diminishing returns in the space of ideas.

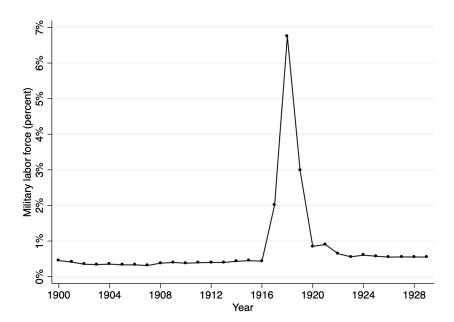
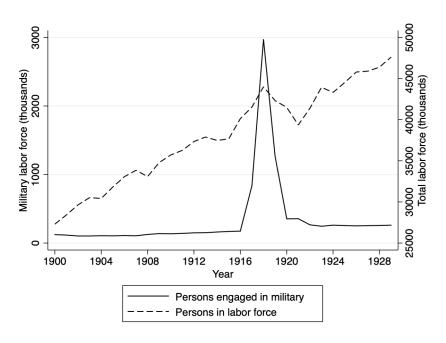


Figure A.1: WORLD WAR I LABOR FORCE

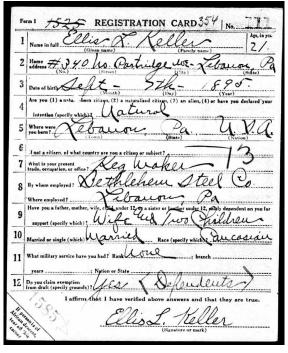
(a) Persons engaged in military, percentage of total labor force



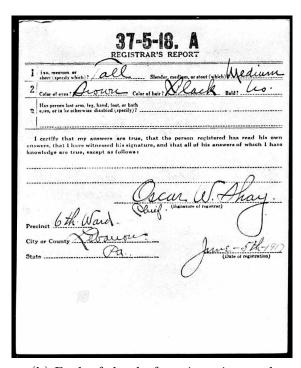
(b) Persons engaged in military (thousands)

Notes: The figures show persons engaged in military across the years from administrative data (Kendrick, 1961).

Figure A.2: EXAMPLE OF DRAFT REGISTRATION CARD



(a) Front of the draft registration card



(b) Back of the draft registration card

Notes: This figure shows an example of a draft registration card that was registered in the first registration on June 5, 1917.

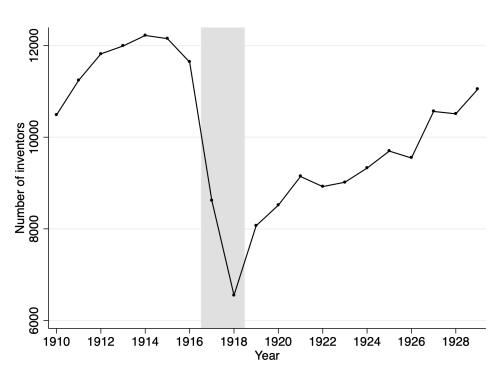


Figure A.3: ACTIVE INVENTORS

Notes: The figure shows the number of inventors who had at least one patent application given a year.

Table A.1: WORLD WAR I DRAFT

Register	Draft	Serve	Number		Percent		Patent data	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
Y	Y	Y	2,566,748	58.66	11.6	91.33	Served inventor	
Y	N	Y	$243,\!548$	50.00	11.0	8.67	(1,783)	
Y	Y	N	367,200	-	88.4	1.52	Registered, but not served	
Y	N	N	$21,\!423,\!725$	-		98.48	(67,342)	
N	N	Y	1,980,876	41.34	-	-	-	
N	N	N	25,314,807	-	-	-	Not registered, not served (51,902)	

Notes: Administrative data from U.S. Provost Marshal General (1919) provides the numbers in column (4). The numbers in parenthesis in column (8) present the number of inventors who had at least one patent application prior to the WWI draft.

Table A.2: IMPACT OF SUPPLY SHOCK ON INNOVATION RATES, OLS COEFFICIENTS

		Speci	fication	
	(1)	(2)	(3)	(4)
Dependent variable: Patent	applications pe	r year		
A. Firms				
Supply shock in Collaboration space	-0.0271 (0.0175)	-	_	-0.0282 (0.0176)
Geographic space	_	0.0449 (0.0745)	_	0.0448 (0.0745)
Idea space	_	_	$0.0953^* \ (0.0574)$	$0.0963^* \ (0.0575)$
Dependent variable mean	0.1618			
Number of observations	222,795			
Number of firms	29,031			
B. More innovative firms				
Supply shock in Collaboration space	-0.0433 (0.0487)	_	_	-0.0463 (0.0491)
Geographic space	_	0.3347 (0.2687)	_	0.3380 (0.2689)
Idea space	_	_	0.2065^* (0.1246)	0.2084^* (0.1247)
Dependent variable mean	0.3481			
Number of observations	$76,\!338$			
Number of firms	9,093			
C. Less innovative firms				
Supply shock in Collaboration space	-0.0111 (0.0071)	_	_	-0.0105 (0.0071)
Geographic space	_	-0.0372 (0.0328)	_	-0.0366 (0.0327)
Idea space	_	_	-0.0661*** (0.0200)	-0.0658*** (0.0200)
Dependent variable mean	0.0768			
Number of observations	$146,\!457$			
Number of firms	19,938			

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. More innovative firms had pre-WWI patents above the median and less innovative firms had pre-WWI patents equal to or below the median. The outcome variable is the number of patent applications per year not relevant for the arms industry. The number of patent applications is winsorized at 10. Standard errors are clustered by firms. State-year fixed effects are included.

Table A.3: LONG-RUN IMPACT OF SUPPLY SHOCK ON INNOVATION RATES

		Specif	ication	
	(1)	(2)	(3)	(4)
Dependent variable: Patent	applications pe	r year		
A. Year: 1910-1929, Post-to	reatment years:	1917-1929		
Supply shock in Collaboration space	-0.0257^{***} (0.0051)	_	_	-0.0254^{***} (0.0051)
Geographic space	_	-0.1548*** (0.0340)	_	-0.1532^{***} (0.0339)
Idea space	_	_	0.0078 (0.0178)	0.0063 (0.0178)
Dependent variable mean	0.1618			
Number of observations	580,620			
Number of firms	29,031			
B. Year: 1910-1950, Post-tr	reatment years:	1917-1950		
Supply shock in Collaboration space	-0.0311*** (0.0050)	_	_	-0.0316*** (0.0050)
Geographic space	_	-0.1857^{***} (0.0347)	_	-0.1827*** (0.0346)
Idea space	_	_	-0.0910*** (0.0134)	-0.0929*** (0.0134)
Dependent variable mean	0.1618			
Number of observations	1,190,271			
Number of firms	29,031			
C. Year: 1900-1950, Post-tr	reatment years:	1917-1950		
Supply shock in Collaboration space	-0.0265*** (0.0033)	_	_	-0.0266*** (0.0033)
Geographic space	_	-0.1375*** (0.0232)	_	-0.1354*** (0.0231)
Idea space	-	_	-0.0315*** (0.0087)	-0.0330*** (0.0087)
Dependent variable mean	0.1444			
Number of observations	1,480,581			
Number of firms	29,031			

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. The outcome variable is the number of patent applications per year not relevant for the arms industry. The number of patent applications is winsorized at 10. Standard errors are clustered by firms.

Table A.4: IMPACT OF SUPPLY SHOCK ON INNOVATION RATES, PLACEBO WWI DRAFT YEARS

	Specification				
	(1)	(2)	(3)	(4)	
Dependent variable: Paten	t applications pe	r year			
A. Placebo pre-treatment y	ears (1910) and	post-treatment	years (1911-191	6)	
Supply shock in Collaboration space	-0.0279 (0.0284)	_	_	-0.0259 (0.0288)	
Geographic space	_	-0.0703 (0.1174)	_	-0.0505 (0.1187)	
Idea space	_	-	0.0073 (0.0648)	0.0064 (0.0649)	
B. Placebo pre-treatment y	ears (1910-1911 ₎) and post-treatr	nent years (1912	2-1916)	
Supply shock in Collaboration space	-0.0071 (0.0241)	_	_	-0.0007 (0.0244)	
Geographic space	_	-0.1393 (0.1038)	_	-0.1384 (0.1054)	
Idea space	_	_	0.0754 (0.0608)	0.0753 (0.0608)	
C. Placebo pre-treatment y	ears (1910-1912)) and post-treatr	nent years (191	3-1916)	
Supply shock in Collaboration space	-0.0120 (0.0223)	_	_	-0.0089 (0.0225)	
Geographic space	_	-0.0644 (0.0983)	_	-0.0573 (0.0993)	
Idea space	_	-	$0.0666 \\ (0.0601)$	0.0662 (0.0601)	
D. Placebo pre-treatment y	ears (1910-1913 ₎) and post-treatr	nent years (191.	4-1916)	
Supply shock in Collaboration space	0.0278 (0.0220)	_	_	0.0313 (0.0222)	
Geographic space	_	-0.0357 (0.0910)	-	-0.0590 (0.0920)	
Idea space	-	-	0.0888 (0.0599)	0.0899 (0.0599)	

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft. The outcome variable is the number of patent applications per year. The number of patent applications is winsorized at 10. Standard errors are clustered by firms.

Table A.5: IMPACT OF SUPPLY SHOCK ON INNOVATION RATES, CONTROLLING FOR THE GOVERNMENT'S EFFECT

	Specification				
	(1)	(2)	(3)	(4)	
Dependent variable: Patent	applications per	r year			
A. Firms					
Supply shock in Collaboration space	-0.0219*** (0.0067)	_	_	-0.0215*** (0.0067)	
Geographic space	_	-0.0611 (0.0430)	_	-0.0602 (0.0429)	
Idea space	_	_	0.0590^* (0.0343)	0.0579^* (0.0343)	
Dependent variable mean	0.1617				
Number of observations	260,901				
Number of firms	28,989				
B. More innovative firms					
Supply shock in Collaboration space	-0.0372^* (0.0205)	_	_	-0.0351^* (0.0205)	
Geographic space	_	0.1413 (0.1444)	_	0.1383 (0.1444)	
Idea space	_	_	$0.1502^* \ (0.0869)$	$0.1468* \\ (0.0870)$	
Dependent variable mean	0.3484				
Number of observations	81,567				
Number of firms	9,063				
C. Less innovative firms					
Supply shock in Collaboration space	-0.0029 (0.0027)	_	_	-0.0030 (0.0027)	
Geographic space	-	-0.0315^* (0.0190)	-	-0.0310 (0.0190)	
Idea space	-	-	-0.0423*** (0.0127)	-0.0424*** (0.0127)	
Dependent variable mean	0.0768				
Number of observations	179,334				
Number of firms	19,926				

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives and for which firms collaborated with the government. More innovative firms are those with pre-WWI patents above the median and less innovative firms are those with pre-WWI patents below the median. The outcome variable is the number of patent applications per year not relevant for the arms industry and the government. The number of patent applications is winsorized at 10. Standard errors are clustered by firms.

Table A.6: IMPACT OF SUPPLY SHOCK ON INNOVATION RATES, HIGHLY INNOVATIVE FIRM

		Specif	ication	
	(1)	(2)	(3)	(4)
Dependent variable: Patent	applications per	r year		
A. Firms in the top 5 perce	ntile of inventio	n		
Supply shock in Collaboration space	-0.6461*** (0.1839)	_	_	-0.5824*** (0.1816)
Geographic space	_	$ \begin{array}{c} 1.6473 \\ (2.0079) \end{array} $	_	$ \begin{array}{c} 1.1058 \\ (1.9774) \end{array} $
Idea space	-	_	1.5834*** (0.3360)	$1.4985^{***} \\ (0.3362)$
Dependent variable mean	1.9227			
Number of observations	12,447			
Number of firms	1,383			
B. Firms in the top 10 perc	entile of inventi	on		
Supply shock in Collaboration space	-0.4037*** (0.1043)	_	_	-0.3666*** (0.1032)
Geographic space	_	0.5951 (0.9502)	_	0.4793 (0.9336)
Idea space	-	_	$1.3563^{***} \\ (0.2374)$	$1.3143^{***} \\ (0.2374)$
Dependent variable mean	1.2970			
Number of observations	26,397			
Number of firms	2,933			
C. Firms in the top 25 perc	entile of inventi	ion		
Supply shock in Collaboration space	-0.2066*** (0.0414)	_	_	-0.1909*** (0.0412)
Geographic space	-	$0.3260 \\ (0.3088)$	-	0.2528 (0.3066)
Idea space	_	_	0.8679*** (0.1446)	0.8469*** (0.1445)
Dependent variable mean	0.6663			
Number of observations	73,314			
Number of firms	8,146			

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. The outcome variable is the number of patent applications per year not relevant for the arms industry. The number of patent applications is winsorized at 10. Standard errors are clustered by firms.

Table A.7: IMPACT OF SUPPLY SHOCK ON ORIGINAL PATENT APPLICATIONS

	Specification					
	(1)	(2)	(3)	(4)		
Dependent variable: Origina	al patent applica	ations per year				
A. Firms						
Supply shock in Collaboration space	$0.0020 \\ (0.0020)$	_	_	0.0021 (0.0019)		
Geographic space	_	0.0007 (0.0121)	_	0.0004 (0.0121)		
Idea space	-	-	0.0112 (0.0089)	0.0113 (0.0089)		
Dependent variable mean	0.0190					
Number of observations	261,279					
Number of firms	29,031					
B. More innovative firms						
Supply shock in Collaboration space	0.0037 (0.0059)	_	_	$0.0040 \\ (0.0059)$		
Geographic space	_	0.0309 (0.0376)	_	0.0313 (0.0377)		
Idea space	_	_	$0.0200 \ (0.0227)$	0.0205 (0.0227)		
Dependent variable mean	0.0403					
Number of observations	81,837					
Number of firms	9,093					
C. Less innovative firms						
Supply shock in Collaboration space	0.0025** (0.0011)	_	_	0.0025** (0.0011)		
Geographic space	-	$0.0005 \\ (0.0097)$	_	0.0003 (0.0096)		
Idea space	-	_	-0.0002 (0.0051)	-0.0001 (0.0051)		
Dependent variable mean	0.0093					
Number of observations	$179,\!442$					
Number of firms	19,938					

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. More innovative firms are those with pre-WWI patents above the median and less innovative firms are those with pre-WWI patents below the median. The outcome variable is the number of original patent applications per year defined as those with new one/two/three word phrases in title that did not exist in patent applications in previous years. The number of patent applications is winsorized at 10. Standard errors are clustered by firms.

Table A.8: IMPACT OF SUPPLY SHOCK ON PATENT CITATIONS

	Specification					
	(1)	(2)	(3)	(4)		
Dependent variable: Patent	citations per y	ear				
A. Firms						
Supply shock in Collaboration space	-0.0360** (0.0157)	_	_	-0.0352^{**} (0.0157)		
Geographic space	_	-0.1424 (0.1068)	_	-0.1412 (0.1067)		
Idea space	_	_	0.1035 (0.0861)	0.1017 (0.0860)		
Dependent variable mean	0.2674					
Number of observations	261,279					
Number of firms	29,031					
B. More innovative firms						
Supply shock in Collaboration space	-0.0686 (0.0466)	_	_	-0.0635 (0.0465)		
Geographic space	_	0.0979 (0.3720)	_	0.0921 (0.3720)		
Idea space	_	_	0.3829^* (0.2137)	0.3765^* (0.2135)		
Dependent variable mean	0.5610					
Number of observations	81,837					
Number of firms	9,093					
C. Less innovative firms						
Supply shock in Collaboration space	-0.0076 (0.0091)	_	_	-0.0082 (0.0091)		
Geographic space	-	-0.1224** (0.0605)	_	-0.1208** (0.0601)		
Idea space	-	_	-0.1713*** (0.0430)	-0.1716*** (0.0430)		
Dependent variable mean	0.1335					
Number of observations	179,442					
Number of firms	19,938					

Notes: The sample consists of firms which had at least one patent application prior to the WWI draft and had no patent application relevant for the arms industry such as weapons, ammunition, and explosives. More innovative firms are those with pre-WWI patents above the median and less innovative firms are those with pre-WWI patents below the median. The outcome variable is the number of patent citations. The number of patent citations is winsorized at 20. Standard errors are clustered by firms.

Mathematical Appendix: Conceptual Framework

$$Y = A^{\phi} K^{\alpha} L^{1-\alpha}$$

Assume $A = L^{\gamma}$ where γ is a firm-specific parameter.

$$Y = L^{\gamma\phi} K^{\alpha} L^{1-\alpha} = K^{\alpha} L^{\gamma\phi+1-\alpha}$$

$$MPL = (\gamma \phi + 1 - \alpha) K^{\alpha} L^{\gamma \phi - \alpha}$$

$$\log MPL = \log(\gamma \phi + 1 - \alpha) + \alpha \log K + (\gamma \phi - \alpha) \log L$$

$$d\log MPL = \alpha d\log K + (\gamma \phi - \alpha) d\log L$$

Assume $d\log K = 0$ in the short run and let $d\log L = m$

$$\Rightarrow d \log MPL = (\gamma \phi - \alpha)m$$

Now, assume L is given by

$$L = (A_1^{\theta} L_1^{\beta} + \dots + A_N^{\theta} L_N^{\beta})^{\frac{1}{\beta}}$$

Assume $A_n = L_n^{\gamma \sigma}$ where σ is a network-specific parameter.

$$L = (L_1^{\gamma \sigma \theta + \beta} + \dots + L_N^{\gamma \sigma \theta + \beta})^{\frac{1}{\beta}}$$

$$Y = A^{\phi} K^{\alpha} L^{1-\alpha} = K^{\alpha} L^{\gamma \phi + 1 - \alpha}$$

$$=K^{\alpha}(L_1^{\gamma\sigma\theta+\beta}+\cdots+L_N^{\gamma\sigma\theta+\beta})^{\frac{1}{\beta}(\gamma\phi+1-\alpha)}$$

$$MPL_n = \frac{1}{\beta} (\gamma \phi + 1 - \alpha) K^{\alpha} (L_1^{\gamma \sigma \theta + \beta} + \dots + L_N^{\gamma \sigma \theta + \beta})^{\frac{1}{\beta} (\gamma \phi + 1 - \alpha) - 1} (\gamma \sigma \theta + \beta) L_n^{\gamma \sigma \theta + \beta - 1}$$

$$\log MPL_n = \log \frac{1}{\beta} (\gamma \phi + 1 - \alpha) + \alpha \log K + \left(\frac{1}{\beta} (\gamma \phi + 1 - \alpha) - 1 \right) \log L^{\beta} + \log(\gamma \sigma \theta + \beta) + (\gamma \sigma \theta + \beta - 1) \log L_n$$

where
$$L^{\beta} = L_1^{\gamma\sigma\theta+\beta} + \cdots + L_N^{\gamma\sigma\theta+\beta}$$

$$d\log MPL_n = \alpha d\log K + (\gamma \phi + 1 - \alpha - \beta) d\log L + (\gamma \sigma \theta + \beta - 1) d\log L_n$$

Assume $d\log K = 0$ in the short run and let $d\log L = m$, $d\log L_n = m_n$

$$\Rightarrow d \log MPL_n = (\gamma \phi + 1 - \alpha - \beta)m + (\gamma \sigma \theta + \beta - 1)m_n$$