HW7 數值方法 E94114023 工科系 115 鐘俐欣

$$\vec{A}=\begin{bmatrix}4&-1&0&-1&0&0\\-1&4&-1&0&-1&0\\0&-1&4&0&1&-1\\-1&0&0&4&-1&-1\\0&-1&0&-1&4&-1\\0&0&-1&0&-1&4\end{bmatrix}\quad \vec{b}=\begin{bmatrix}0\\-1\\9\\4\\8\\6\end{bmatrix}$$

(a) Jacobi method

$$U=\begin{bmatrix}0&-1&0&-1&0&0\\0&0&-1&0&-1&0\\0&0&0&0&1&-1\\0&0&0&0&-1&-1\\0&0&0&0&0&-1\\0&0&0&0&0&0\end{bmatrix}\quad L=\begin{bmatrix}0&0&0&0&0&0\\-1&0&0&0&0&0\\0&-1&0&0&0&0\\-1&0&0&0&0&0\\0&-1&0&-1&0&0\\0&0&-1&0&-1&0\end{bmatrix}\quad D=\begin{bmatrix}4&0&0&0&0&0\\0&4&0&0&0&0\\0&0&4&0&0&0\\0&0&0&4&0&0\\0&0&0&0&4&0\\0&0&0&0&0&4\end{bmatrix}$$

$$\vec{x}=-D^{-1}(L+U)\vec{x}+D^{-1}\vec{b}\equiv T\vec{x}+\vec{c}\quad \vec{x}^{(0)}=\begin{bmatrix}\\\\\\\\\\\end{bmatrix}$$

$$\Rightarrow \vec{x}=\begin{bmatrix}1.1744519\\1.6400099\\2.4480686\\3.0556619\\3.949207\\3.0993981\end{bmatrix}\#$$

(b) Gauss - Seidel method

$$\vec{x}=-(D+L)^{-1}U\vec{x}+(D+L)^{-1}\vec{b}\equiv T_g\vec{x}+\vec{c}\quad \vec{x}^{(0)}=\begin{bmatrix}\\\\\\\\\\\end{bmatrix}$$

$$\vec{x}=\begin{bmatrix}1.1747334\\1.6431547\\2.4482573\\3.0559503\\3.9496441\\3.0994753\end{bmatrix}\#$$

(c) SOR method $w = 0.5$

$$\vec{x} = (D+wL)^{-1}[(1-w)D - wU]\vec{x} + w(D+wL)^{-1}\vec{b} \equiv T_w\vec{x} + \vec{c} \qquad \vec{x}^{(0)} = \begin{bmatrix} | \\ | \\ | \\ | \\ | \\ | \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 1.27740673 \\ 1.77746127 \\ 2.52444282 \\ 3.33506718 \\ 4.31021642 \\ 3.75535492 \end{bmatrix} \#$$

(d) The conjugate gradient method

$$t_k = \frac{\vec{v}_k^T \vec{v}_k}{\vec{v}_k^T A \vec{v}_k} \qquad \vec{x}^{(k)} = \vec{x}^{(k-1)} + t_k \vec{v}_k \qquad \vec{v}_{k+1} = b - \vec{A}\vec{x}^{(k)} \qquad \vec{x}^{(0)} = \begin{bmatrix} | \\ | \\ | \\ | \\ | \\ | \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} 1.17447536 \\ 1.64296141 \\ 2.44811446 \\ 3.05565523 \\ 3.94924477 \\ 3.09936634 \end{bmatrix}$$

輸出結果截圖

```
(a)Jacobi method:
X0:
[[1]
 [1]
 [1]
 [1]
 [1]
 [1]]
X1:
[[0.5 ]
 [0.5 ]
 [2.5 ]
 [1.75]
 [2.75]
 [2.  ]]
X2:
[[0.5625]
 [1.1875]
 [2.1875]
 [2.3125]
 [3.0625]
 [2.8125]]
X3:
[[0.875   ]
 [1.203125]
 [2.484375]
 [2.609375]
 [3.578125]
 [2.8125  ]]
X4:
[[0.953125  ]
 [1.484375  ]
 [2.359375  ]
 [2.81640625]
 [3.65625   ]
 [3.015625  ]]
X5:
[[1.07519531]
 [1.4921875 ]
 [2.4609375 ]
 [2.90625   ]
 [3.82910156]
 [3.00390625]]
X6:
[[1.09960938]
 [1.59130859]
 [2.41674805]
 [2.97705078]
 [3.85058594]
 [3.07250977]]
X7:
[[1.14208984]
 [1.59173584]
 [2.45330811]
 [3.00567627]
 [3.91021729]
 [3.0668335 ]]
X8:
[[1.14935303]
 [1.62640381]
 [2.43708801]
 [3.02978516]
 [3.9160614 ]
 [3.09088135]]
X9:
[[1.16404724]
 [1.62562561]
 [2.45030594]
 [3.03907394]
 [3.93676758]
 [3.08828735]]
X10:
[[1.16617489]
 [1.63778019]
 [2.44428635]
 [3.04727554]
 [3.93824673]
 [3.09676838]]
X11:
[[1.17126393]
 [1.63717699]
 [2.44907546]
 [3.0502975 ]
 [3.94545603]
 [3.09563327]]
X12:
[[1.17186862]
 [1.64144886]
 [2.44683856]
 [3.05308831]
 [3.94577694]
 [3.09863287]]
X13:
[[1.17363429]
 [1.64112103]
 [2.4485762 ]
 [3.05406961]
 [3.94829251]
 [3.09815387]]
X14:
[[1.17379766]
 [1.64262575]
 [2.4477456 ]
 [3.05502017]
 [3.94833613]
 [3.09921718]]
X15:
[[1.17441148]
 [1.64246985]
 [2.4483767 ]
 [3.05533774]
 [3.94921577]
 [3.09902043]]
X16:
[[1.1744519 ]
 [1.64300099]
 [2.44806863]
 [3.05566192]
 [3.949207  ]
 [3.09939812]]
```

```
(b)Gauss_seidel method:
X0:
[[1]
 [1]
 [1]
 [1]
 [1]
 [1]]
X1:
[[0.5      ]
 [0.375    ]
 [2.34375  ]
 [1.625    ]
 [2.75     ]
 [2.7734375]]
X2:
[[0.5       ]
 [1.1484375 ]
 [2.54296875]
 [2.50585938]
 [3.60693359]
 [3.03747559]]
X3:
[[0.91357422]
 [1.51586914]
 [2.48660278]
 [2.88949585]
 [3.86071014]
 [3.08682823]]
X4:
[[1.10134125]
 [1.61216354]
 [2.45957041]
 [3.01221991]
 [3.92780292]
 [3.09684333]]
X5:
[[1.15609586]
 [1.6358673 ]
 [2.45122693]
 [3.04518553]
 [3.94447404]
 [3.09892524]]
X6:
[[1.17026321]
 [1.64149104]
 [2.44898556]
 [3.05341562]
 [3.94845798]
 [3.09936088]]
X7:
[[1.17372667]
 [1.64279255]
 [2.44842386]
 [3.05538638]
 [3.94938495]
 [3.0994522 ]]
X8:
[[1.17454473]
 [1.64308839]
 [2.44828891]
 [3.05584547]
 [3.94959652]
 [3.09947136]]
X9:
[[1.17473347]
 [1.64315472]
 [2.44825739]
 [3.05595033]
 [3.9496441 ]
 [3.09947537]]
```

```
(c)SOC method:
X0:
[[1]
 [1]
 [1]
 [1]
 [1]
 [1]]
X1:
[[0.75      ]
 [0.71875   ]
 [1.69921875]
 [1.34375   ]
 [1.9296875 ]
 [1.97705078]]
X2:
[[0.6328125 ]
 [0.76708984]
 [2.06079102]
 [1.73931885]
 [2.57215118]
 [2.59108067]]
X3:
[[0.62970734]
 [0.91637611]
 [2.25668371]
 [2.09377682]
 [3.03310479]
 [2.9802014 ]]
X4:
[[0.69112279]
 [1.08080197]
 [2.36620417]
 [2.38494203]
 [3.36917057]
 [3.23046004]]
X5:
[[0.77877939]
 [1.22967025]
 [2.42884705]
 [2.61477227]
 [3.61582311]
 [3.39425129]]
X6:
[[0.86994501]
 [1.35416202]
 [2.4653723 ]
 [2.79238856]
 [3.79738679]
 [3.50340803]]
X7:
[[0.95329133]
 [1.45408731]
 [2.48707472]
 [2.92795505]
 [3.93124969]
 [3.57743207]]
X8:
[[1.02440096]
 [1.53238433]
 [2.5002332 ]
 [3.03061286]
 [4.0300535 ]
 [3.62843937]]
X9:
[[1.08257513]
 [1.59279989]
 [2.50838982]
 [3.10793993]
 [4.10304915]
 [3.66408706]]
X10:
[[1.12888004]
 [1.63893982]
 [2.51356713]
 [3.165972  ]
 [4.15702444]
 [3.68930497]]
X11:
[[1.165054  ]
 [1.67392561]
 [2.51693433]
 [3.20940892]
 [4.19696716]
 [3.70732767]]
X12:
[[1.19294382]
 [1.70031847]
 [2.51917704]
 [3.24185929]
 [4.22654676]
 [3.72031681]]
X13:
[[1.21424413]
 [1.72015522]
 [2.52070418]
 [3.26606811]
 [4.2484659 ]
 [3.72974216]]
X14:
[[1.23039998]
 [1.73502387]
 [2.52176461]
 [3.28411006]
 [4.26471746]
 [3.73661884]]
X15:
[[1.24259173]
 [1.74614616]
 [2.52251325]
 [3.29754603]
 [4.27677261]
 [3.74165765]]
X16:
[[1.25175739]
 [1.75445349]
 [2.52304894]
 [3.30754647]
 [4.28571851]
 [3.74536226]]
X17:
[[1.25862869]
 [1.76065126]
 [2.52343635]
 [3.31498692]
 [4.29235931]
 [3.74809308]]
X19:
[[1.26760861]
 [1.76871283]
 [2.52392598]
 [3.32463671]
 [4.30095263]
 [3.75160242]]
X20:
[[1.270473  ]
 [1.77127537]
 [2.52407864]
 [3.32769686]
 [4.30367315]
 [3.75270768]]
X21:
[[1.27260803]
 [1.77318266]
 [2.52419147]
 [3.32997204]
 [4.30569437]
 [3.75352707]]
X22:
[[1.27419835]
 [1.77460185]
 [2.52427505]
 [3.33166349]
 [4.30719624]
 [3.75413495]]
X23:
[[1.27538234]
 [1.77565763]
 [2.52433707]
 [3.33292094]
 [4.30831231]
 [3.75458615]]
X24:
[[1.27626349]
 [1.77644292]
 [2.52438313]
 [3.33385571]
 [4.30914175]
 [3.75492118]]
```

```
X25:
[[1.27691908]
 [1.77702696]
 [2.52441736]
 [3.33455061]
 [4.30975822]
 [3.75517004]]
X26:
[[1.27740673]
 [1.77746127]
 [2.52444282]
 [3.33506718]
 [4.31021642]
 [3.75535492]]
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(d)The conjugate gradient method:
X0:
[[1]
 [1]
 [1]
 [1]
 [1]
 [1]]
X1:
[[0.42718447]
 [0.42718447]
 [2.7184466 ]
 [1.8592233 ]
 [3.00485437]
 [2.14563107]]
X2:
[[0.56211407]
 [1.23109279]
 [2.17986204]
 [2.35925654]
 [3.10123266]
 [2.87923993]]
X3:
[[0.92007309]
 [1.20944218]
 [2.52388538]
 [2.65417226]
 [3.65199427]
 [2.81632135]]
X4:
[[0.96232489]
 [1.4994266 ]
 [2.35753236]
 [2.83203247]
 [3.66857921]
 [3.02619379]]
X5:
[[1.09076258]
 [1.49695727]
 [2.47125314]
 [2.91966299]
 [3.85060631]
 [3.00523938]]
X6:
[[1.10307331]
 [1.5945775 ]
 [2.41761117]
 [2.98124112]
 [3.85507246]
 [3.07438863]]
X7:
[[1.14663676]
 [1.59389736]
 [2.45582624]
 [3.00989794]
 [3.91632287]
 [3.06776298]]
X8:
[[1.1505979 ]
 [1.62678307]
 [2.43792065]
 [3.03082655]
 [3.91776207]
 [3.09098041]]
X9:
[[1.16530763]
 [1.62655619]
 [2.45079247]
 [3.04042583]
 [3.9384187 ]
 [3.08878561]]
X10:
[[1.16662826]
 [1.63764526]
 [2.4447658 ]
 [3.04749997]
 [3.93889925]
 [3.09660833]]
X11:
[[1.17159164]
 [1.63756861]
 [2.44910555]
 [3.05073284]
 [3.94586703]
 [3.0958709 ]]
X12:
[[1.1720359 ]
 [1.64130884]
 [2.44707361]
 [3.05312033]
 [3.94602872]
 [3.09850884]]
X13:
[[1.17371028]
 [1.64128297]
 [2.44853727]
 [3.05421042]
 [3.94837907]
 [3.09826029]]
X14:
[[1.17386004]
 [1.64254458]
 [2.44785193]
 [3.05501586]
 [3.94843358]
 [3.09915004]]
X15:
[[1.17442485]
 [1.64253585]
 [2.44834563]
 [3.05538353]
 [3.94922638]
 [3.09906622]]
X16:
[[1.17447536]
 [1.64296141]
 [2.44811446]
 [3.05565523]
 [3.94924477]
 [3.09936634]]
```