1-D Simulation of Ekman Layers

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1 Ekman layer

The Ekman layer is the layer in a fluid where there is a force balance between pressure gradient force, Coriolis force and lateral friction. The mathematical formation is

$$-f \cdot v = \frac{1}{\rho}\frac{\partial}{\partial x}p + \frac{\partial}{\partial z}(A_z \frac{\partial}{\partial z}u) \tag{1}$$

$$f \cdot u = \frac{1}{\rho}\frac{\partial}{\partial y}p + \frac{\partial}{\partial z}(A_z \frac{\partial}{\partial z}v) \tag{2}$$

where u and v are velocities in the x and y directions, respectively, f is the Coriolis parameter, ρ is water density, p is pressure, A_z is the eddy viscosity. To simplify the equations, rewrite Eq 1 and 2 using complex notation

$$i \cdot f\vec{u} = -\frac{1}{\rho}\nabla p + \frac{\partial}{\partial z}(A_z \frac{\partial}{\partial z}\vec{u}) \tag{3}$$

where $\vec{u} = u + iv$ is the complex velocity. Since time stepping is used to achieve Ekman balance, a time dependence term is also added

$$\frac{\partial}{\partial t}\vec{u} + i \cdot f\vec{u} = -\frac{1}{\rho}\nabla p + \frac{\partial}{\partial z}(A_z \frac{\partial}{\partial z}\vec{u}). \tag{4}$$

For simplicity we assume $\rho = \rho_0$ is a constant, ∇p is dependent on time t (tidal pressure gradient) and depth z (baroclinicity), and A_z is only a function of z.

2 Model setup

Formulation of finite difference and discretization are described in the previous report thus not repeated here. In this model the major difference from the 1-D Western Boundary Current case is the grid setup. As shown in Fig 1, the whole water column is divided into n cells, and the weight of each cell is denoted by asterisks. Sea surface is located $0.5\Delta z$ above the uppermost grid point. For velocity components and body forces (Coriolis force, pressure gradient force), values are calculated at weight of each cell; for surface forces (viscous friction/diffusion), values are calculate at boundary of each grid cell. Thus, the discretized form of friction term becomes

$$[\frac{\partial}{\partial z}(A_z \frac{\partial}{\partial z}\vec{u})]_i^n = \frac{A_z^{i-1/2}(\vec{u}_{i-1}^n - \vec{u}_i^n) - A_z^{i+1/2}(\vec{u}_i^n - \vec{u}_{i+1}^n)}{\Delta z^2} \tag{5}$$

where $A_z^{i-1/2} = 0.5(A_{z,i-1} + A_{z,i})$ and $A_z^{i+1/2} = 0.5(A_{z,i} + A_{z,i+1})$ are eddy viscosity at upper and lower surface of each cell. For other terms, discretization formulation is similar to that of numerical exercise 1.

2.1 Boundary condition

The surface ocean is forced by wind stress near the air-water interface

$$\vec{\tau}_{wind} = \vec{\tau}_{-1/2} = \frac{A_{z,-1/2}(\vec{u}_{-1} - \vec{u}_0)}{\Delta z} \tag{6}$$

and the bottom is non-slip

$$\vec{u}_{bottom} = \vec{u}_{n+1/2} = 0.5(\vec{u}_n + \vec{u}_{n+1}) = 0. \tag{7}$$

Note that at both surface and bottom the condition is applied at the surface instead of the weight of a grid cell, which is the major difference from numerical exercise 1.

3 Results

3.1 Case 1

Firstly, we test the model setup with a simpler case - constant pressure gradient and eddy viscosity. Model parameters are chosen as $f = 1 \times 10^{-4}$ s^{-1}, $A_z = 1 \times 10^{-2}$ m^{2}s^{-1}, $\frac{1}{\rho}\nabla p = -3 \times 10^{-6}$ m^{2}s^{-1}, and $\vec{\tau}_{wind} = 1 \times 10^{-2}$ m^{2}s^{-1}. The model has 2000 vertical grid points, and it is run with $dt = 0.2$ hr for 1000 time steps.

The fully developed structure of Ekman spiral is shown in Fig 2. Temporal evolution of velocity profiles are shown in Fig 3 and 4. Both surface and bottom Ekman layers are developed.Since the governing equation is time dependent, signals of inertial oscillation also appears in the final solution; to filter out this signal, the vector plot in Fig 2 is averaged over one inertial period.

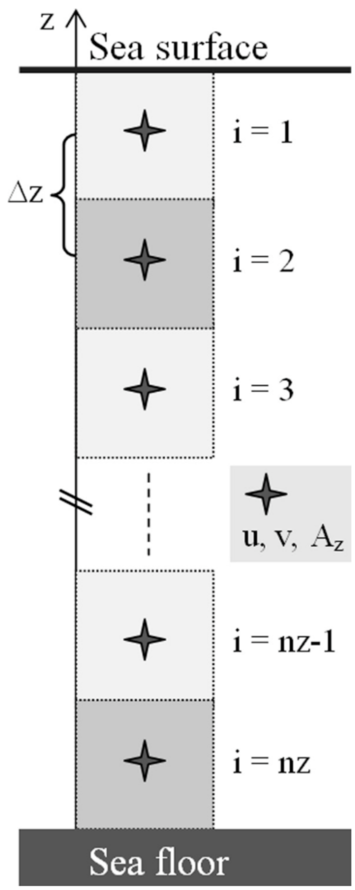

Figure 1: Grid cell setup in this numerical excercise. (Kämpf, 2010)

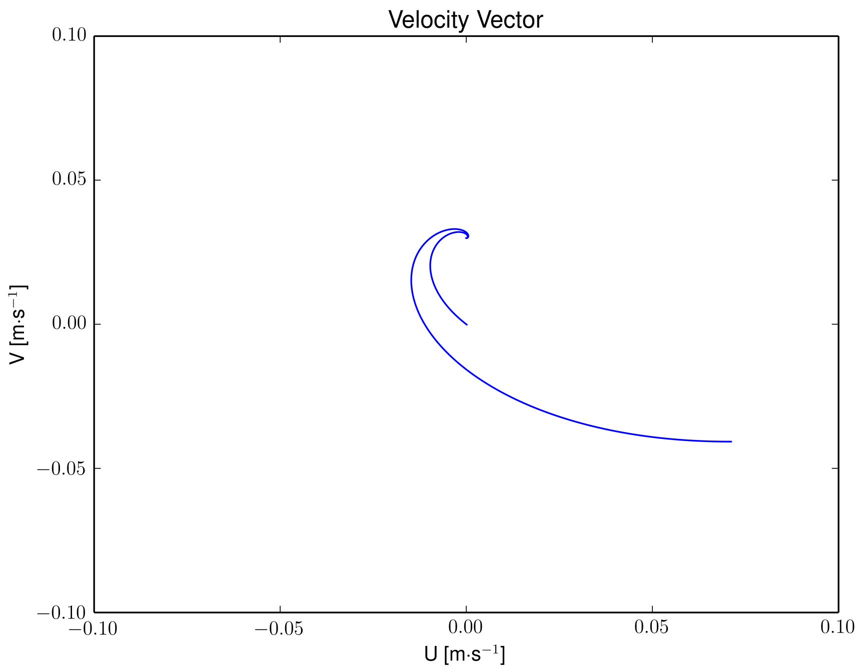

Figure 2: Velocity vector of the last inertial period.

Velocity is maximum near the surface, 45 degrees right to surface wind stress. It then decreases and turns right to form the surface spiral. Below the bottom Ekman depth (dash line in Fig 3 and 4) and above the bottom Ekman depth is geostrophic flow, which is mixed with inertial oscillation signal. Below the bottom Ekman depth is the bottom Ekman layer, a reverse spiral (Fig 2), the velocity of which decreases towards zero due to non-slip bottom boundary condition.

4 Case 2

For the second testing case, we use a slightly complicated set of parameters. Firstly, eddy viscosity A_z increases linearly from bottom (10^{-3} m^{2}s^{-1}) to surface (10^{-2} m^{2}s^{-1}). This has very little influence on the surface, however, bottom Ekman layer is nearly 10 times thinner due to decreased A_z. Secondly, pressure gradient force is characterized with a fortnightly tidal cycle (with a period of 14 days). Besides, tidal forcing is also baroclinic, which increases from bottom (0) to surface ($-3.0 \times 10^{-3}\hat{i} + 3.0 \times 10^{-3}\hat{j}$ m^{2}s^{-1}). Lastly, we use the first tidal cycle to spin-up the model, thus wind stress ($-10^{-2}\hat{j}$ m^{2}s^{-1}) only kicks in after the 14th day.

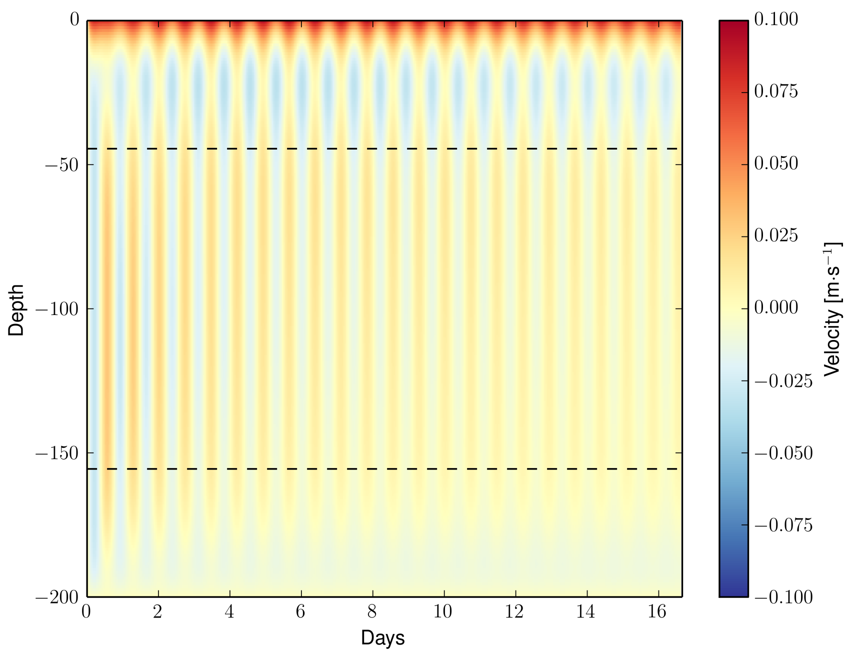

Figure 3: Hovmöller diagram of U velocity. Dash lines denote surface and bottom Ekman depth.

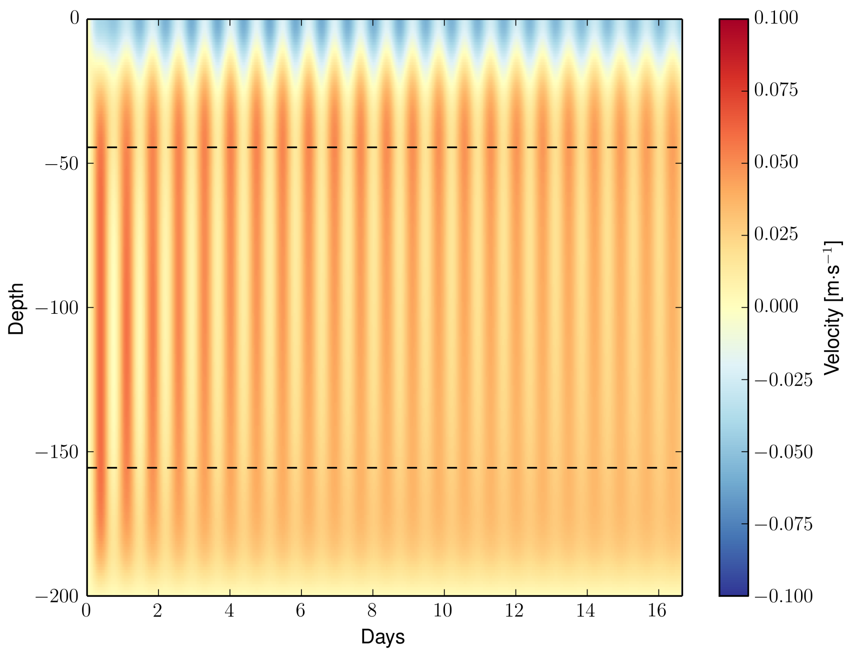

Figure 4: Hovmöller diagram of V velocity.

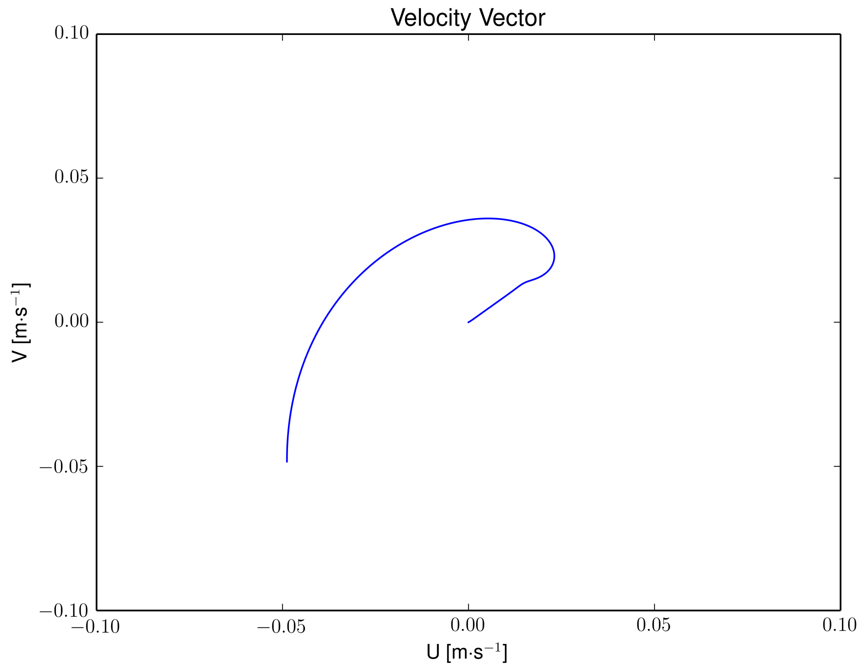

Figure 5: Velocity vector of the last inertial period.

Results are shown in Fig 5, 6 and 7. Overall it behaves as expected. With this set of parameters, modeled velocity is a combination of pressure gradient driven tidal flow and surface Ekman flow.

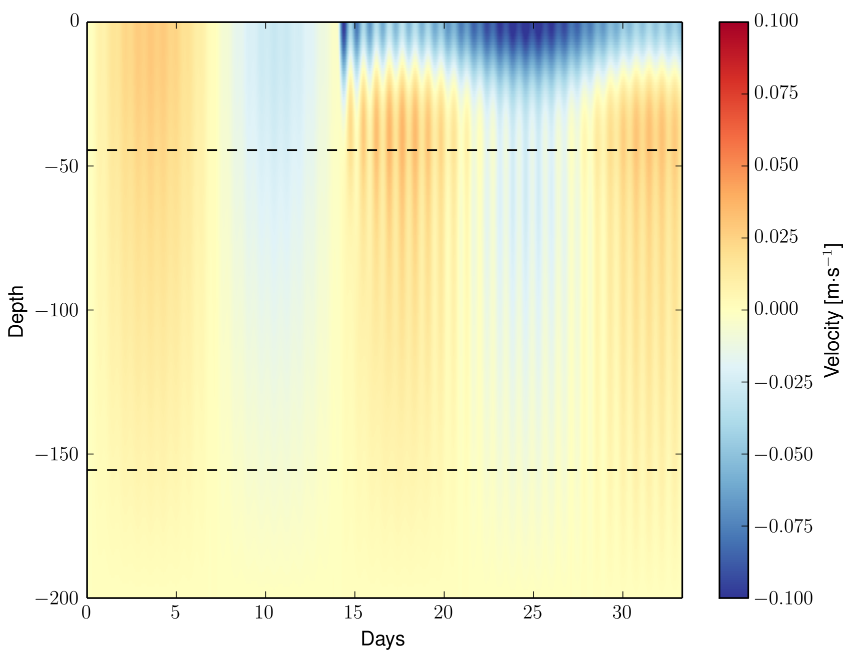

Figure 6: Hovmöller diagram of U velocity.

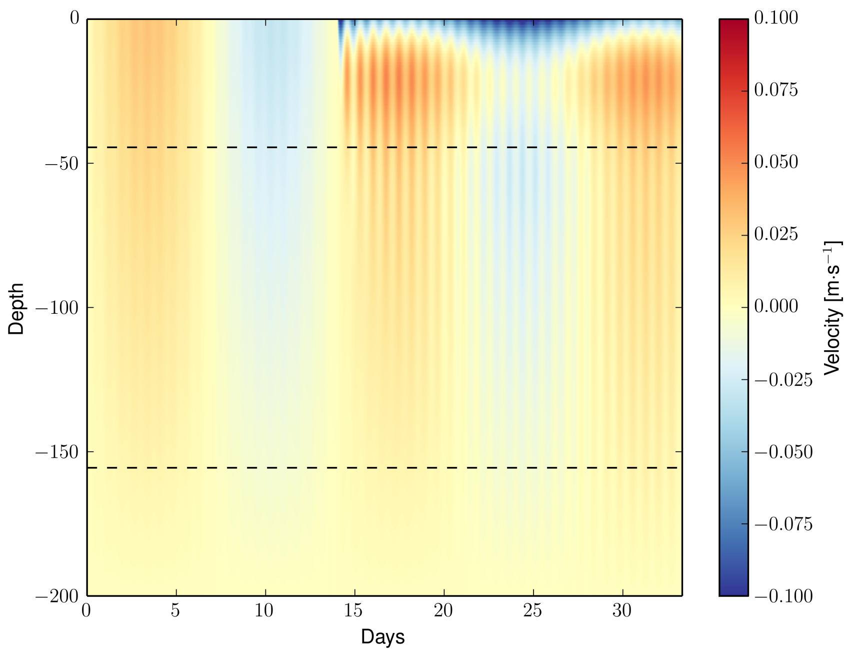

Figure 7: Hovmöller diagram of V velocity.

References

Kämpf, J. (2010). *Advanced Ocean Modelling: Using Open-source Software*. Springer Science & Business Media.