1-D Simulation of Western Boundary Current

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1 Model setup

The numerical model solved in this report is based on the 1-D vorticity equation

$$\frac{\partial}{\partial t}\phi_{xx} + \beta\phi_x = WC - \gamma\phi_{xx} \tag{1}$$

where the terms on the left side are the 'storage term' and 'β effect term', while terms on the right side are wind curl and friction, respectively. To solve this equation numerically, the spatial derivation terms must be discretized with finite difference method:

$$(\phi_x)_i^n = \frac{\phi_{i+1}^n - \phi_{i-1}^n}{2\Delta x} \tag{2}$$

$$(\phi_{xx})_i^n = \frac{\phi_{i+1}^n + \phi_{i-1}^n - \phi_i^n}{\Delta x^2} \tag{3}$$

and temporal derivation can discretized with either forward scheme

$$\frac{\partial}{\partial t}(\phi_{xx})_i = \frac{(\phi_{xx})_i^{n+1} - (\phi_{xx})_i^n}{\Delta t} \tag{4}$$

or leap frog scheme

$$\frac{\partial}{\partial t}(\phi_{xx})_i = \frac{(\phi_{xx})_i^{n+1} - (\phi_{xx})_i^{n-1}}{2\Delta t} \tag{5}$$

The superscript n and subscript i are indices for time steps and spatial grid points, respectively. Substitute Eq 4 into Eq 1 and rearrange terms, we have

$$(\phi_{xx})_i^{n+1} = (\phi_{xx})_i^n + \Delta t(-\beta(\phi_x)_i^n + WC - \gamma(\phi_{xx})_i^n) \tag{6}$$