1-D Simulation of Ekman Layers

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1 Ekman layer

The Ekman layer is the layer in a fluid where there is a force balance between pressure gradient force, Coriolis force and lateral friction. The mathematical formation is

$$-f \cdot v = \frac{1}{\rho}\frac{\partial}{\partial x}p + \frac{\partial}{\partial z}(A_z\frac{\partial}{\partial z}u) \tag{1}$$

$$f \cdot u = \frac{1}{\rho}\frac{\partial}{\partial y}p + \frac{\partial}{\partial z}(A_z\frac{\partial}{\partial z}v) \tag{2}$$

where u and v are velocities in the x and y directions, respectively, f is the Coriolis parameter, ρ is water density, p is pressure, A_z is the eddy viscosity. To simplify the equations, rewrite Eq 1 and 2 using complex notation

$$i \cdot f\vec{u} = -\frac{1}{\rho}\nabla p + \frac{\partial}{\partial z}(A_z\frac{\partial}{\partial z}\vec{u}) \tag{3}$$

where $\vec{u} = u + iv$ is the complex velocity. Since time stepping is used to achieve Ekman balance, a time dependence term is also added

$$\frac{\partial}{\partial t}\vec{u} + i \cdot f\vec{u} = -\frac{1}{\rho}\nabla p + \frac{\partial}{\partial z}(A_z\frac{\partial}{\partial z}\vec{u}). \tag{4}$$

For simplicity we assume $\rho = \rho_0$ is a constant, ∇p is dependent on time t (tidal pressure gradient) and depth z (baroclinicity), and A_z is only a function of z.

2 Model setup

Formulation of finite difference and discretization are described in the previous report thus not repeated here. In this model the major difference from the 1-D Western Boundary Current case is the grid setup. As shown in Fig 1, the whole water column is divided into n cells, and the weight of each cell is denoted by asterisks. Sea surface is located $0.5\Delta z$ above the uppermost grid point. For velocity components and body forces (Coriolis force, pressure gradient force), values are calculated at weight of each cell; for surface forces (viscous friction/diffusion), values are calculate at boundary of each grid cell. Thus, the discretized form of friction term becomes

$$[\frac{\partial}{\partial z}(A_z\frac{\partial}{\partial z}\vec{u})]_i = \frac{A_z^{i-1/2}(\vec{u}_{i-1}^n - \vec{u}_i^n) - A_z^{i+1/2}(\vec{u}_i^n - \vec{u}_{i+1}^n)}{\Delta z^2} \tag{5}$$

where $A_z^{i-1/2} = 0.5(A_{z,i-1} + A_{z,i})$ and $A_z^{i+1/2} = 0.5(A_{z,i} + A_{z,i+1})$ are eddy viscosity at upper and lower surface of each cell. For other terms, discretization formulation is similar to that of numerical exercise 1.

2.1 Boundary condition

The surface ocean is forced by wind stress near the air-water interface

$$\vec{\tau}_{wind} = \vec{\tau}_{-1/2} = \frac{A_{z,-1/2}(\vec{u}_{-1} - \vec{u}_0)}{\Delta z} \tag{6}$$

and the bottom is non-slip

$$\vec{u}_{bottom} = \vec{u}_{n+1/2} = 0.5(\vec{u}_n + \vec{u}_{n+1}) = 0. \tag{7}$$

Note that at both surface and bottom the condition is applied at the surface instead of the weight of a grid cell, which is the major difference from numerical exercise 1.

3 Results

3.1 Case 1

Firstly, we test the model setup with a simpler case - constant pressure gradient and eddy viscosity. Model parameters are chosen as $f = 1 \times 10^{-4}$ s^{-1}, $A_z = 1 \times 10^{-2}$ m^{2}s^{-1}, $\frac{1}{\rho}\nabla p = -3 \times 10^{-6}$ m^{2}s^{-1}, and $\vec{\tau}_{wind} = 1 \times 10^{-2}$ m^{2}s^{-1}. The model has 2000 vertical grid points, and it is run with $dt = 0.2$ hr for 1000 time steps. The structure of Ekman spiral is shown in Fig 2. Temporal evolution of velocity profiles are shown in Fig 4 and **??**.

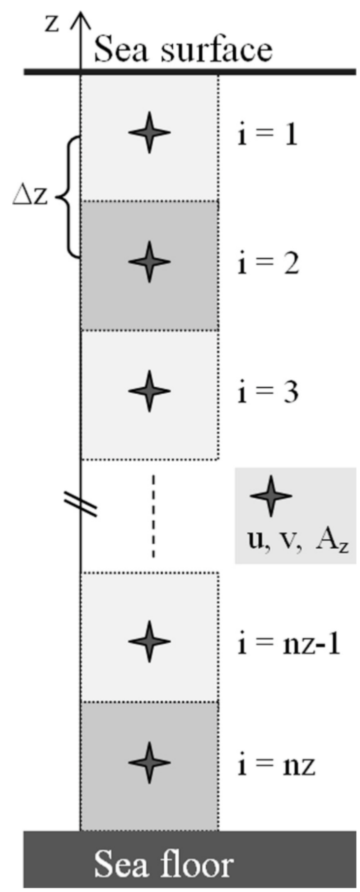

Figure 1: Grid cell setup in this numerical excercise. (Kämpf, 2010)

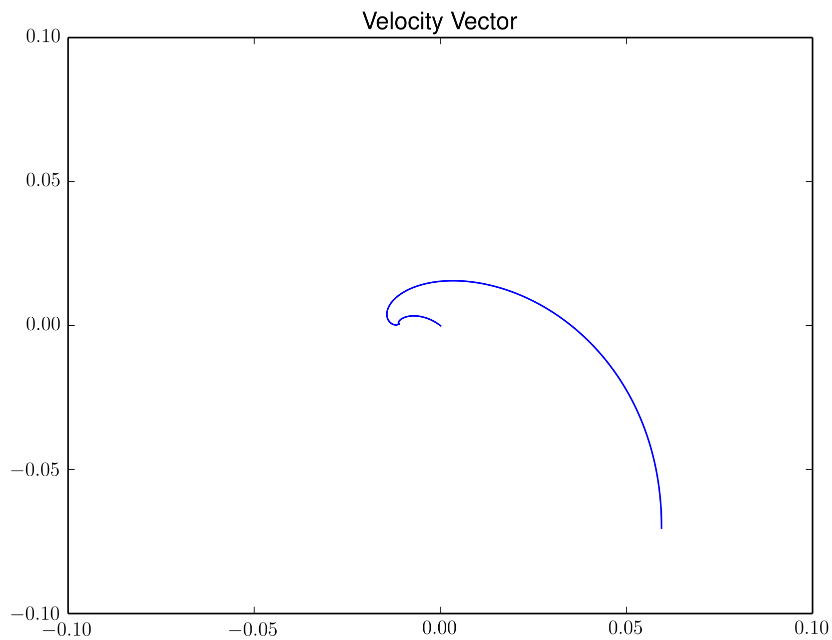

Figure 2: Velocity vector of the last time step. Both surface and bottom Ekman layers are shown.

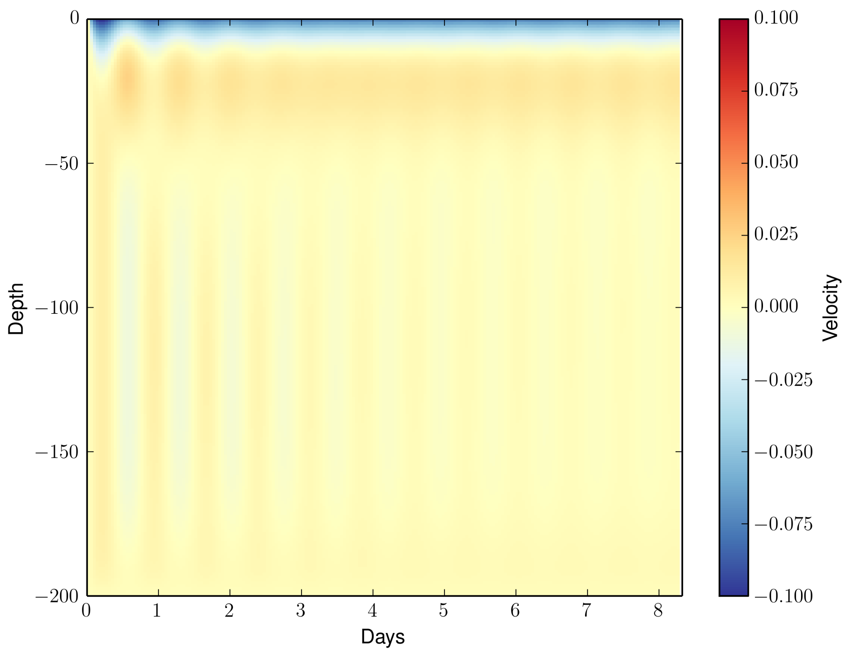

Figure 3: Hovmöller diagram of U velocity.

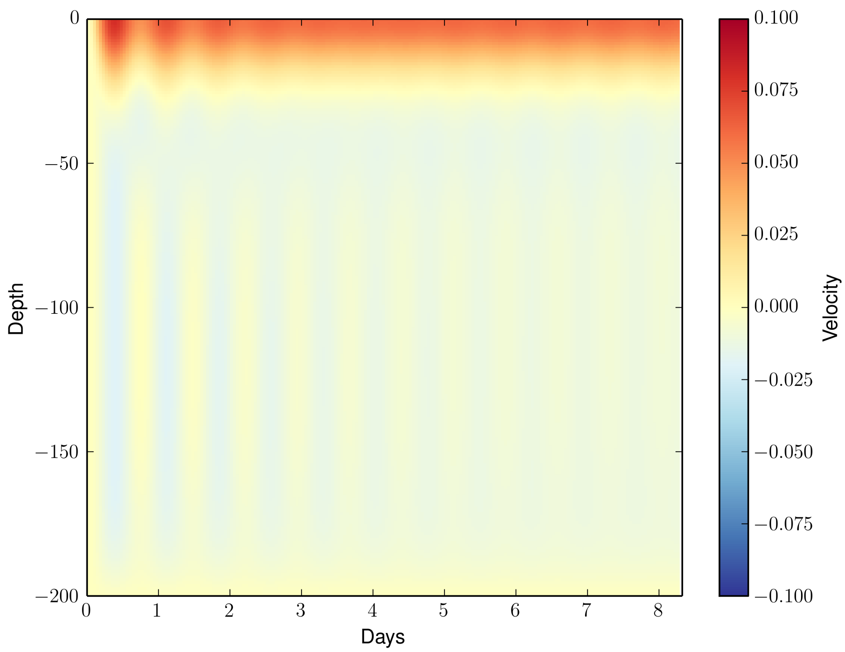

Figure 4: Hovmöller diagram of V velocity.

References

Kämpf, J. (2010). *Advanced Ocean Modelling: Using Open-source Software*. Springer Science & Business Media.