We consider a game-playing population and a third-party population, each of which consists of N=100 agents. The contribution c to the public pool of each game player is randomly drew from the normal distribution c~N(1, 0.5) and c~N(1, 0.8). Agents update their strategies according to the Fermi process (a standard imitative process for simulating replicator dynamics in finite populations) such that at every time step, an agent explores a strategy with probability 0.01 or otherwise imitates the strategy of another randomly selected agent with the probability following the Fermi function. Under each condition of \delta_C and \delta_D-\alpha (the condition predicted by our theory), we consider several different initialization of agent strategies. For each setting, we conduct 50 independent simulation runs and the presented results are the averaged results.

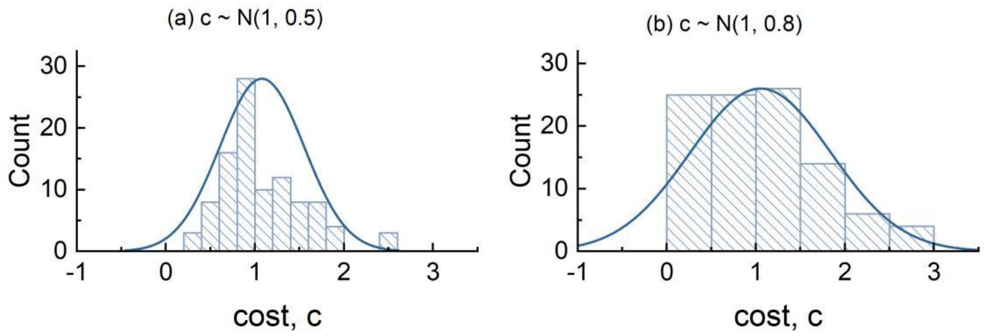

Fig 1. Distributions of the cost, c. The curve represents the probability distribution, and the histogram represents the frequency distribution. To ensure cooperators contribute positively to the public pool, we limit the c<0 to 0 when generating random values.

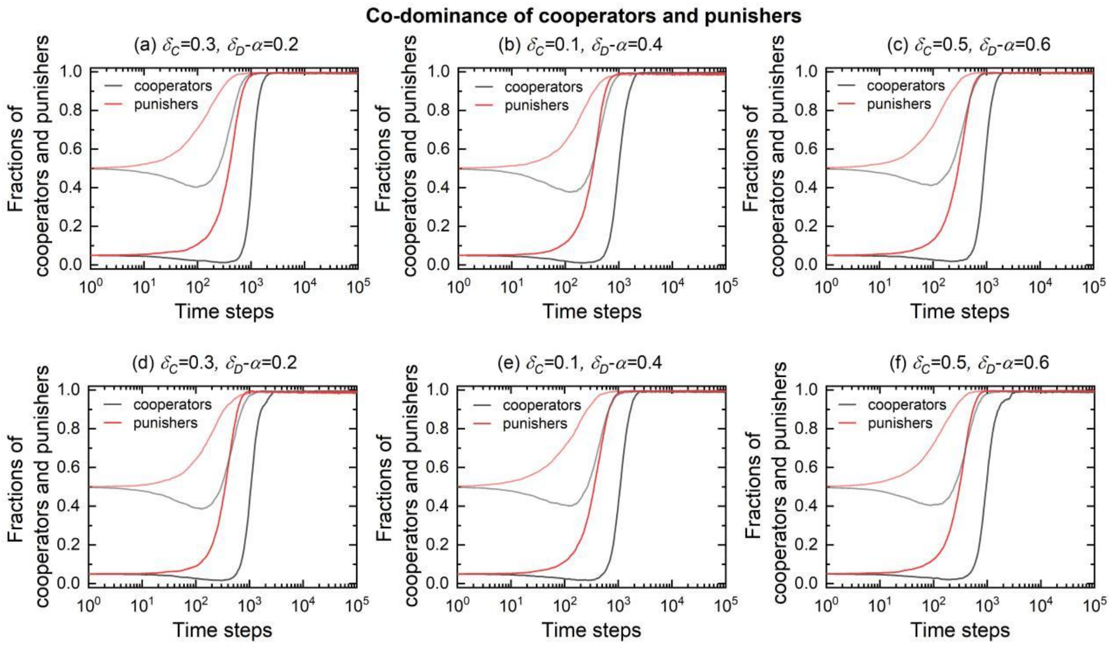

Fig 2. Co-dominance of cooperators and punishers in small, finite, heterogeneous populations, given the conditions of Theorem 1 are met. c~N(1, 0.5) in the top row, c~N(1, 0.8) in the bottom row. Light and dark curves represent different initial states, respectively.

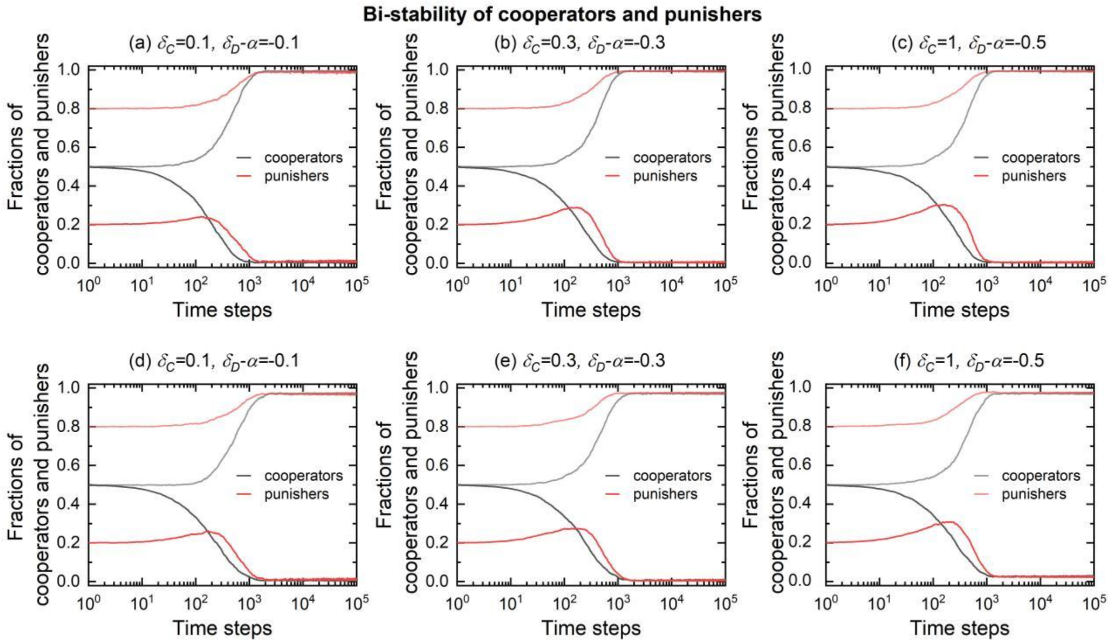

Fig 3. Bi-stability of cooperators and punishers in small, finite, heterogeneous populations, given the conditions of Theorem 5 are met. c~N(1, 0.5) in the top row, c~N(1, 0.8) in the bottom row. Light and dark curves represent different initial states, respectively.

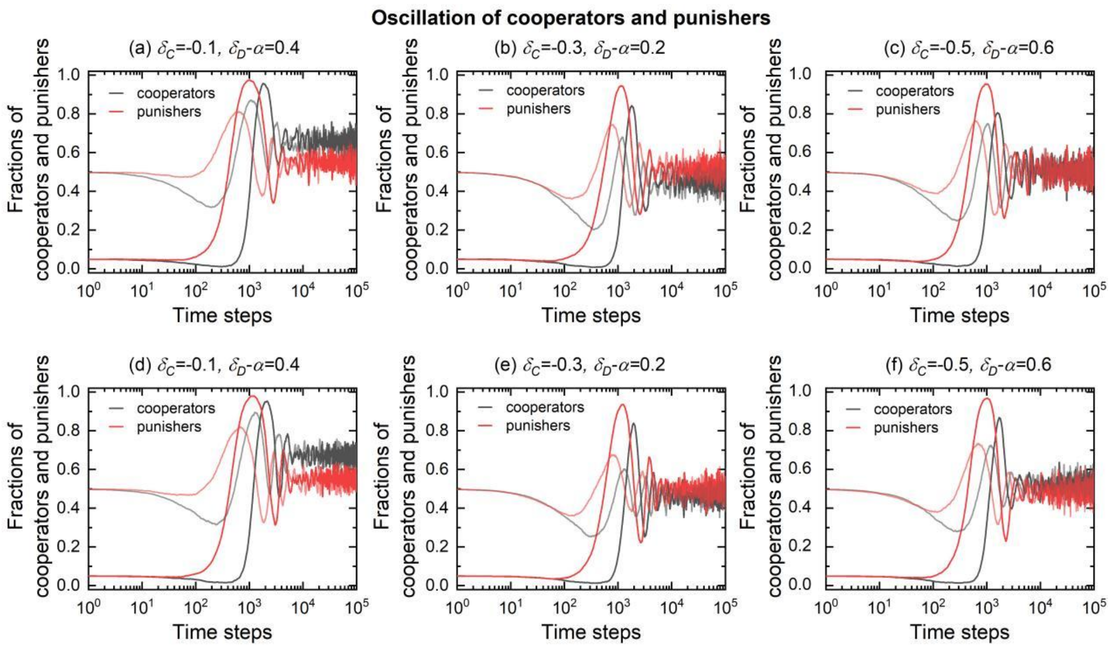

Fig 4. Oscillation of cooperators and punishers in small, finite, heterogeneous populations, given the conditions of Theorem 6 are met. c~N(1, 0.5) in the top row, c~N(1, 0.8) in the bottom row. Light and dark curves represent different initial states, respectively.