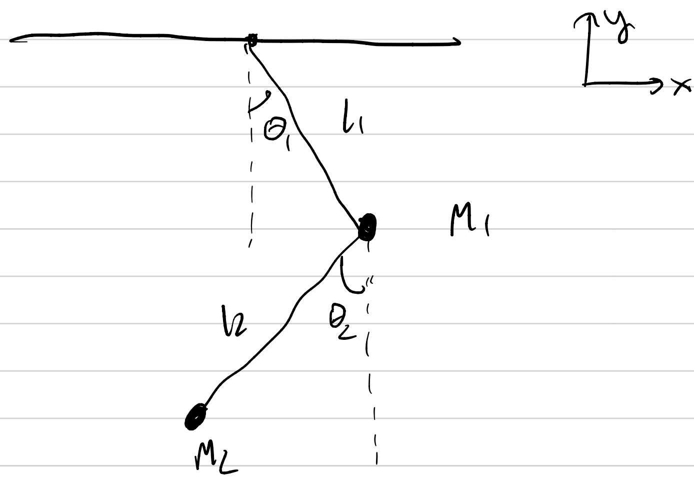

$x_1 = l_1 \sin(\theta_1)$ $\qquad$ $y_1 = -l_1 \cos(\theta_1)$

$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_1$ $\qquad$ $y_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2$

$\dot{x}_1 = \dot{\theta}_1 l_1 \cos\theta_1$ $\qquad$ $\dot{y}_1 = \dot{\theta}_1 l_1 \sin\theta_1$

$\dot{x}_2 = \dot{\theta}_1 l_1 \cos\theta_1 + \dot{\theta}_2 l_2 \cos\theta_2$

$\dot{y}_2 = \dot{\theta}_1 l_1 \sin\theta_1 + \dot{\theta}_2 l_2 \sin\theta_2$

$L = T - V$

$T = \frac{1}{2} m(\dot{x} + \dot{y})^2$ $\qquad$ $U = -mgy$

$\Rightarrow$)

$$T = \frac{1}{2} m_1 \underbrace{(\dot{x}_1 + \dot{y}_1)^2} + \frac{1}{2} m_2 \underbrace{(\dot{x}_2 + \dot{y}_2)^2}$$

$$l_1^2 \sin^2(\theta_1)\, \dot{\theta}_1^2 + l_1^2 \cos^2(\theta_1)\, \dot{\theta}_1^2$$
$$= (l_1 \dot{\theta}_1)^2$$

$$l_1^2 \cos^2(\theta_1)\, \dot{\theta}_1^2 + 2 l_1 l_2 \cos(\theta_1) \cos(\theta_2)\, \dot{\theta}_1 \dot{\theta}_2$$
$$+ l_2^2 \cos^2(\theta_2)\, \dot{\theta}_2^2 + l_1^2 \sin^2(\theta_1)\, \dot{\theta}_1^2 + 2 l_1 l_2 \sin(\theta_1)$$
$$\cdot \sin(\theta_2)\, \dot{\theta}_1 \dot{\theta}_2 + l_2^2 \sin^2\theta_2\, \dot{\theta}_2$$
$$= (l_1 \dot{\theta}_1)^2 + (l_2 \dot{\theta}_2)^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \left[\sin(\theta_1)\sin\theta_2) + \cos(\theta_1)\cos(\theta_2)\right]$$

trig identity

$$= (l_1 \dot{\theta}_1)^2 + (l_2 \dot{\theta}_2)^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)$$

$$\Rightarrow T = \frac{1}{2} m_1 (l_1 \dot{\theta}_1)^2 + \frac{1}{2} m_2 \left[(l_1 \dot{\theta}_1)^2 + (l_2 \dot{\theta}_2)^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cdot \cos(\theta_1 - \theta_2)\right]$$

$$U = -g l_1 (m_1+m_2)\cos(\theta_1) - m_2 g l_2 \cos\theta_2$$

$$L = T - U$$

$$= \frac{1}{2} m_1 (l_1\dot{\theta}_1)^2 + \frac{1}{2} m_2 \left[(l_1\dot{\theta}_1)^2 + (l_2\dot{\theta}_2)^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1-\theta_2) \right]$$

$$+ g l_1 (m_1+m_2)\cos(\theta_1) - m_2 g l_2 \cos(\theta_2)$$

For ease of solving with computer, use Hamiltonian Formalism:

$$H(p_i\, q_i) = \sum_i p_i \dot{q}_i - L$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i} \qquad p_1 = \frac{\partial L}{\partial \dot{\theta}_1} \qquad p_2 = \frac{\partial L}{\partial \dot{\theta}_2}$$

$\Rightarrow$

$$p_1 = (m_1+m_2) l_1^2 \dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_1-\theta_2)$$

$$p_2 = m_2 l_2^2 \dot{\theta}_2 + m_2 l_2 l_1 \dot{\theta}_1 \cos(\theta_1-\theta_2)$$

$\hookrightarrow$ Solve for $\dot{\theta}_2$:

$$m_2 l_2^2 \dot{\theta}_2 = p_2 - m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_1-\theta_2)$$

$$\dot{\theta}_2 = \frac{p_2}{m_2 l_2^2} - \frac{l_1}{l_2}\dot{\theta}_1 \cos(\theta_1-\theta_2)$$

use $\dot{\theta}_2$ to solve for $\dot{\theta}_1$ and plug into $\dot{\theta}_2$ (ugly af, used Maple LOL):

$$\boxed{\dot{\theta}_1 = \frac{l_2 p_2 - l_1 p_2 \cos(\theta_1 - \theta_2)}{l_1 l_2 (m_1 + m_2 \sin^2(\theta_1 - \theta_2))}}$$

$$\boxed{\dot{\theta}_2 = \frac{l_1 (m_1 + m_2) p_2 - l_2 m_2 p_1 \cos(\theta_1 - \theta_2)}{l_1 l_2^2 m_2 [m_1 + m_2 \sin^2(\theta_1 - \theta_2)]}}$$

Plug into Hamiltonian:

$$\to H = \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 l_2^2 \dot{\theta}_2^2 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) - g l_1 (m_1 + m_2) \cos(\theta_1) - m_2 g l_2 \cos\theta_2$$

Plug into Hamiltonian and use Hamilton's equation (used Maple for this gross crap).

$$\dot{q}_i = \frac{\partial H}{\partial p_i}, \quad \dot{p}_i = \frac{\partial H}{\partial q_i}$$

These are very gross, assume lengths are equal and masses are equal to 1 for simplicity.

$$\boxed{\dot{\theta}_1 = \frac{p_1 - p_2 \cos(\theta_1 - \theta_2)}{1 + \sin^2(\theta_1 - \theta_2)}}$$

$$\dot{\theta}_2 = \frac{2p_2 - p_1 \cos(\theta_1 - \theta_2)}{1 + \sin^2(\theta_1 - \theta_2)}$$

$$\dot{p}_1 = -2g\sin(\theta_1) - \frac{p_1 p_2 \sin(\theta_1 - \theta_2)}{1 + \sin^2(\theta_1 - \theta_2)} + \frac{p_1^2 + 2p_2^2 - p_1 p_2 \cos(\theta_1 - \theta_2)}{2[1 + \sin^2(\theta_1 - \theta_2)]^2} \cdot \sin[2(\theta_1 - \theta_2)]$$

$$\dot{p}_2 = -g\sin(\theta_2) + \frac{p_1 p_2 \sin(\theta_1 - \theta_2)}{1 + \sin^2(\theta_1 - \theta_2)} - \frac{p_1^2 + 2p_2^2 - p_1 p_2 \cos(\theta_1 - \theta_2)}{2[1 + \sin^2(\theta_1 - \theta_2)]^2} \cdot \sin(2(\theta_1 - \theta_2))$$

YAY!!!