

Off-policy Learning For Prediction

Objectives

- Understand how off-policy learning can help deal with the exploration problem
- Understand importance sampling
- Use importance sampling to estimate the expected value of a target distribution using samples from a different distribution.

Off-Policy Learning

- What is it ?
 - Off-policy learning is a reinforcement learning technique where the agent learns to estimate value functions, state-value function $V(s)$ or action-value function $Q(s,a)$ from data generated by following a different behavior policy than the one being evaluated.
 - In off-policy learning, the behavior policy determines the agent's actions, while the target policy is the one whose value function the agent aims to estimate.

Off-Policy Learning

- On-Policy: improve and evaluate the policy being used to select actions
- Off-policy: improve and evaluate a different policy from the one used to select actions
- Example:
 - Learning the optimal policy involves following a completely random policy, termed the target policy, as it serves as the objective for the agent's learning process

Off-Policy Learning

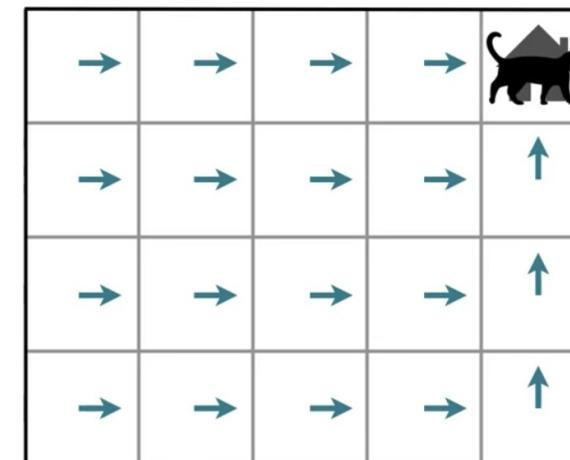
- The value function that the agent is learning is based on the target policy. One example of a

Target policy is the optimal policy we call the policy that the agent is using to select actions the behavior policy because it defines our

agents π

Target Policy
 $\pi(a | s)$

- Learn values for this policy

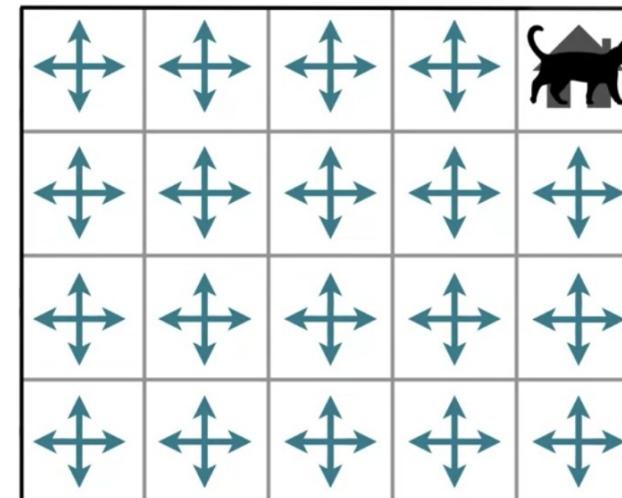


Off-Policy Learning

- The behavior policy is usually denoted by B .
- The behavior policy is in charge of selecting actions for the agent.
- The behavior policies shown here is the uniform random policy

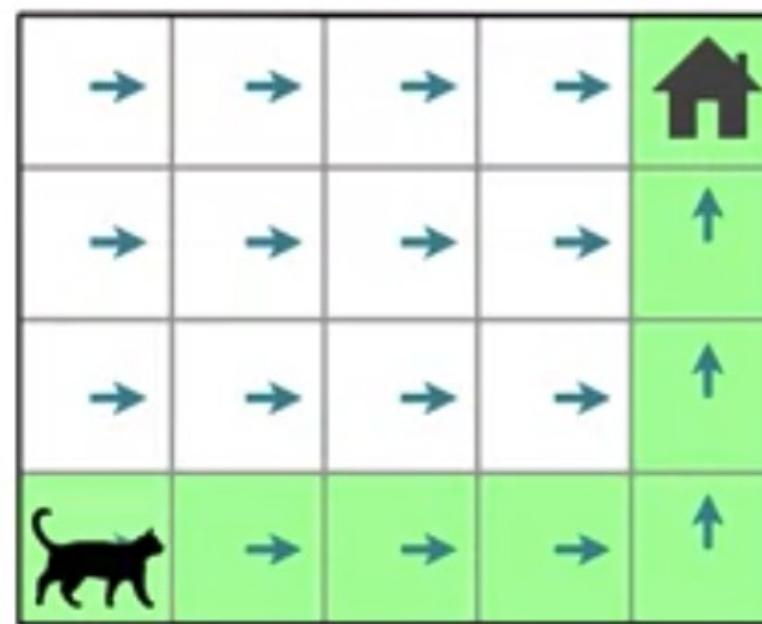
Behavior Policy
 $b(a|s)$

- Select **actions** from this policy
- Generally an **exploratory policy**



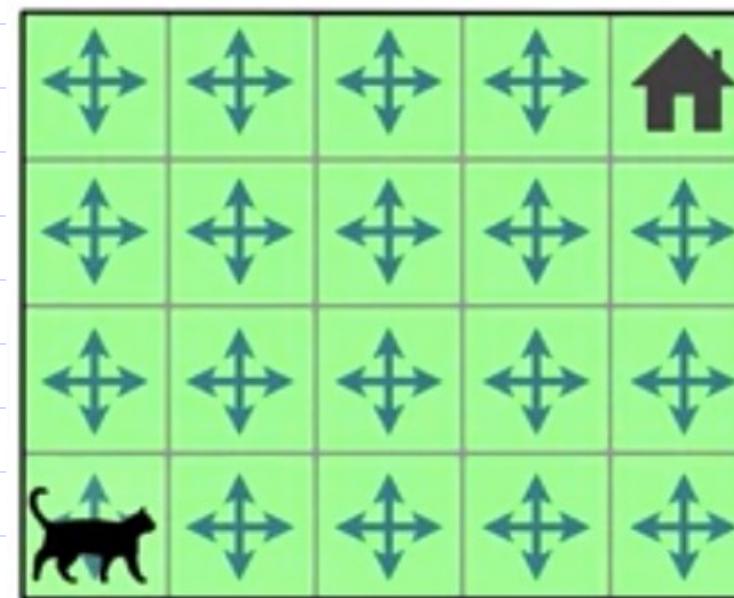
Off-Policy Learning

- If our agent behaves according to the Target policy it might only experience a small number of states.



Off-Policy Learning

- If our aging can behave according to a policy that favors exploration.
- It can experience a much larger number of states.

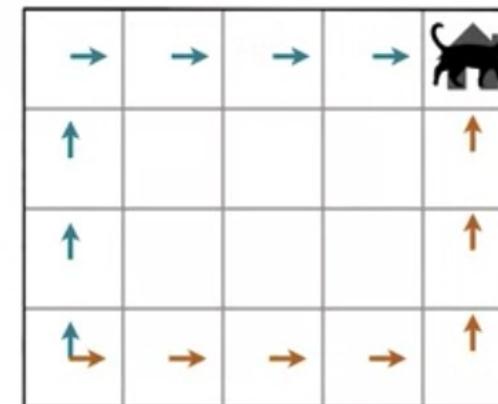


Off-Policy Learning

Off-Policy Learning

- One key rule of off policy learning is that the behavior policy must cover the target policy.
 - If the target policy says the probability of selecting an action a given State s is greater than zero then the behavior policy must say the probability of selecting that action

$\pi(a | s) > 0$ where $b(a | s) > 0$

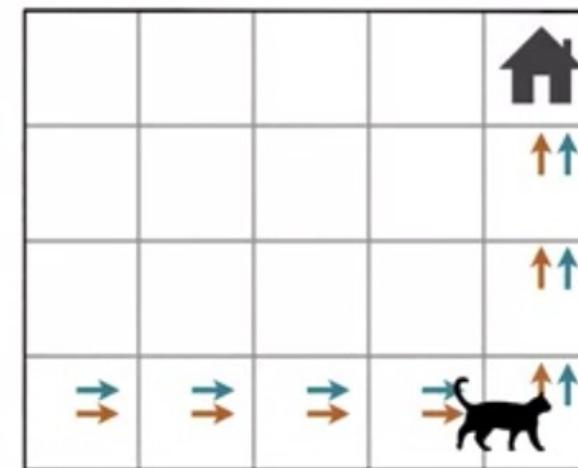


Off-Policy Learning

- It's worth noting that off policy learning is a strict generalization of on policy learning on policies the specific case where the target policy is equal to the behavior policy.

$\pi(a|s) > 0$ where $b(a|s) > 0$

On-Policy: $\pi(a|s) = b(a|s)$



Importance Sampling

- We have some random variable x that's being sampled from a probability distribution b .
- We want to estimate the expected value of x but with respect to the target distribution π .
- Because x is drawn from b , we cannot simply use the sample average to compute the expectation under π .
- This sample average will give **Sample: $x \sim b$** **value** under b instead.

Estimate: $\mathbb{E}_\pi[X]$

Importance Sampling

□ Importance sampling ratio

$$\begin{aligned}\mathbb{E}_{\pi}[X] &\doteq \sum_{x \in X} x\pi(x) \\&= \sum_{x \in X} x\pi(x) \frac{b(x)}{b(x)} \\&= \sum_{x \in X} x \frac{\pi(x)}{b(x)} b(x)\end{aligned}$$

Importance sampling ratio

□ We can write the importance sampling ratio as Rho of x.

$$\sum_{x \in X} x\rho(x)b(x)$$

Importance Sampling

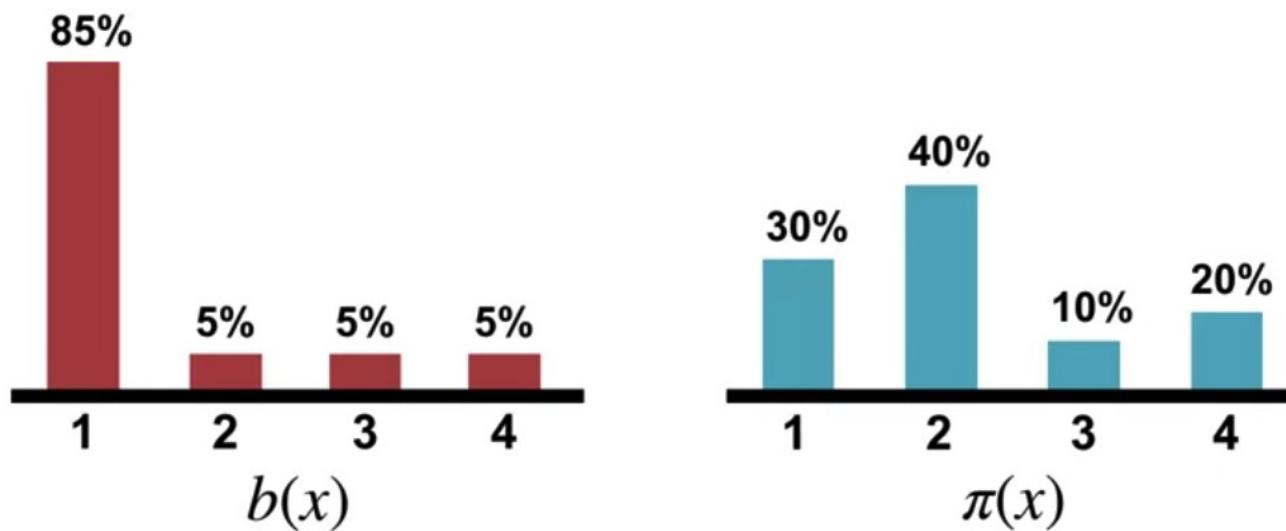
- How do we use it to estimate the expectation from data?
 - We just need to compute a weighted sample average with the importance sampling ratio as the weightings
 - We can now estimate the expected value of x under distribution drawn from π as the samples

$$\mathbb{E}_{\pi}[X] \approx \frac{1}{n} \sum_{i=1}^n x_i \rho(x_i)$$

$$x_i \sim b$$

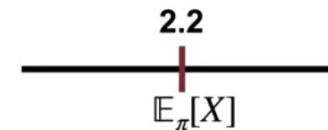
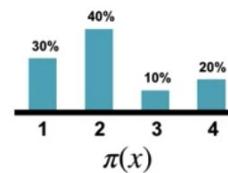
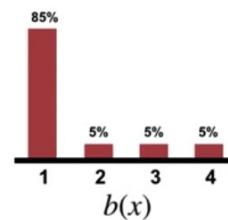
Importance Sampling

- We have two rather different distributions: b and π . We'll draw samples according to b and try to estimate the expected value under π .



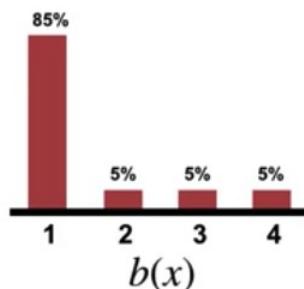
Importance Sampling

- Draw samples according to b and try to estimate the expected value under π .
 - On the right: track of our current estimate for the expected value under π .
 - For reference, The true expected value is in the middle of the

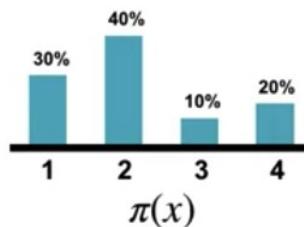


Importance Sampling

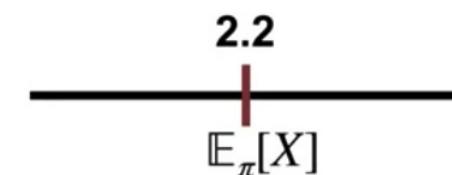
- Sample from $b(x=1)$, we get estimate of 0.35



$$\xrightarrow{x=1} \quad b(x) = .85 \quad \xrightarrow{} \quad x = [1]$$



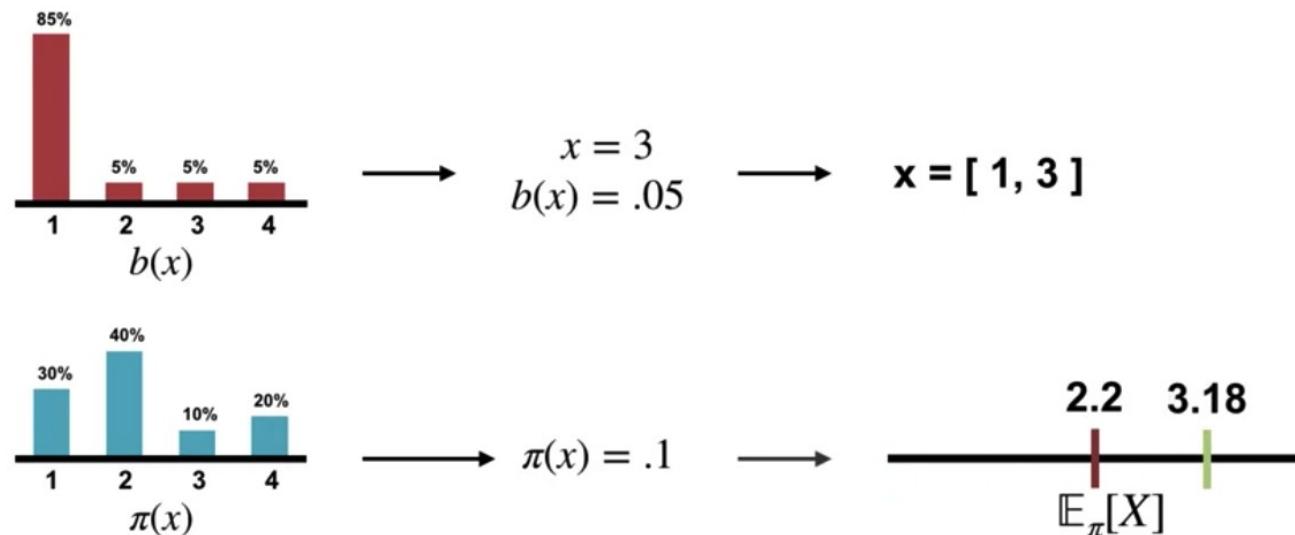
$$\xrightarrow{} \quad \pi(x) = .3$$



$$\frac{1}{n} \sum_1^n x \rho(x) \longrightarrow 1 \times \frac{.3}{.85} = 0.35$$

Importance Sampling

- Sample from $b(x=3)$, we get estimate of 3.18



$$\frac{1}{n} \sum_1^n x \rho(x) \rightarrow \frac{(1 \times \frac{.3}{.85}) + (3 \times \frac{.1}{.05})}{2} = 3.18$$

Summary

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- Understand importance sampling
- Use importance sampling to estimate the expected value of a target distribution using samples from a different distribution.

Q & A