1. $P(x_j|w_i) = \frac{1}{\Gamma(P_i)} \lambda_j^{P_i} x_j^{P_i-1} e^{-\lambda_j x_j}$ $\quad P_i, \lambda_j > 0$.

(a) $P(w_i | x_1 = x_1, \cdots x_k = x_k) = \frac{P(x_1 = x_1, \cdots x_k = x_k | w_i) P(w_i)}{P(x_1, x_2 \cdots x_k)}$

$$\propto P(x_1|w_i) \cdot P(x_2|w_i) \cdots P(x_k|w_i) P(w_i).$$

$$= \prod_{j=1}^{k} \frac{1}{\Gamma(P_i)} \lambda_j^{P_i} \cdot x_j^{P_i-1} e^{-\lambda_j x_j} P(w_i)$$

$$= \frac{P(w_i)}{\Gamma(P_i)} \cdot \prod_{j=1}^{k} \lambda_j^{P_i} x_j^{P_i-1} e^{-\lambda_j x_j}$$

Decide w_i if.

$$\frac{P(w_1)}{\Gamma(P_i)} \cdot \prod_{j=1}^{k} (\lambda_j)^{P_1} (x_j)^{P_1-1} e^{-\lambda_j x_j} \overset{w_1}{>} \frac{P(w_2)}{\Gamma(P_2)} \prod_{j=1}^{k} (\lambda_j)^{P_2} (x_j)^{P_2-1} e^{-\lambda_j x_j}$$

$$\Rightarrow \frac{P(w_1)}{\Gamma(P_i)} \prod_{j=1}^{k} (\lambda_j)^{P_1} (x_j)^{P_1-1} \overset{w_1}{>} \frac{P(w_2)}{\Gamma(P_2)} \prod_{j=1}^{k} (\lambda_j)^{P_2} (x_j)^{P_2-1}$$

(b) when $|P_2 - P_1| = 1$, it's linear.

(c) when $P_1 = 4$, $P_2 = 2$, $c = 2$, $k = 4$. $\lambda_1 = \lambda_3 = 1$, $\lambda_2 = \lambda_4 = 2$.

$$\frac{P(w_1)}{\Gamma(P_1)} \cdot \prod_{j=1}^{4} (\lambda_j)^{P_1} (x_j)^{P_1-1} > \frac{P(w_2)}{\Gamma(P_2)} \prod_{j=1}^{4} (\lambda_j)^{P_2} (x_j)^{P_2-1}$$

$$\Rightarrow \frac{1}{3 \times 2 \times 1} \cdot \prod_{j=1}^{4} (\lambda_j)^4 (x_j)^3 > \frac{1}{1} \prod_{j=1}^{4} (\lambda_j)^2 (x_j)^1$$

$$\Rightarrow \frac{1}{6} \cdot 4^2 \cdot (0.1 \times 0.2 \times 0.3 \times 4)^3 > (0.1) \times 0.2 \times 0.3 \times 0.4.$$

$$\Rightarrow \frac{8}{3} \times (0.024)^2 > 1 \quad \Rightarrow \quad 0.015 < 1$$

$\Rightarrow$ W_2 is the decision $C(x) = 2$.

(d) $P_1 = 3.2 \quad P_2 = 8, \quad c = 2, \lambda_1 = 1 \quad k = 1.$

$$\frac{P(w_1)}{P(P_1)} \cdot \prod_{j=1}^{k} (\lambda_j)^{P_1} (x_j)^{P_1 - 1} > \frac{P(w_2)}{P(P_2)} \prod_{j=1}^{k} (\lambda_j)^{P_2} (x_j)^{P_2 - 1}$$

$$\Rightarrow \frac{1}{P(3.2)} \times 1 \times x^{2.2} > \frac{1}{P(8)} \times 1 \times x^{7}.$$

$$\Rightarrow \quad x^{4.8} = \frac{P(8)}{P(3.2)}$$

$$\Rightarrow \quad x = \sqrt[4.8]{\frac{5040}{2.423}} = 4.912 .$$

$$\Rightarrow x^* = 4.912.$$

Type-1 error. (false positive.

$$\Rightarrow T_1 = 1 - P(w_1|X).$$

$$= 1 - \frac{P(x_1|w_i) - P(x_k|w_i) P(w_1)}{P(X)}.$$

$$= 1 - \frac{\frac{P(w_1)}{P(3.2)} x^{2.2}}{\frac{P(w_1)}{P(3.2)} x^{2.2} + \frac{P(w_2)}{P(8)} x^{7}}$$

$$= 1 - \frac{1}{1 + \frac{P(3.2)}{P(8)} x^{4.8}} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \text{Type-1 error} = \frac{1}{2}$$

$$T_2 = 1 - P(W_2|x)$$

$$= 1 - \frac{\frac{P(W_2)}{P(8)} x^7}{\frac{P(W_1)}{P(3.2)} x^{2.2} + \frac{P(W_2)}{P(8)} x^7}$$

$$= 1 - \frac{1}{\frac{P(8)}{P(3.2)} x^{-4.8}}$$

$$= 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

(e)

$P_1 = P_2 = 4 \quad C = 2, \; k = 2 \quad \lambda_1 = 8 \quad \lambda_2 = 0.3.$

$P(W_1) = \frac{1}{4}, \quad P(W_2) = \frac{3}{4}.$

$$\frac{P(W_1)}{P(P_1)} \prod_{j=1}^{2} (\lambda_j)^4 (x_j)^{P_1 - 1} = \frac{P(W_2)}{P(P_2)} \cdot \prod_{j=1}^{k} (\lambda_j)^{P_2} (x_j)^{P_2 - 1}$$

$$\Rightarrow \frac{1}{4} \cdot (x_1 x_2)^3 = \frac{3}{4} (x_1 x_2)$$

$$\Rightarrow 2(x_1 x_2)^3 = 0.$$

$\therefore f(x_1, x_2) = 0$ when $x_1 x_2 = 0.$

2. As we know $x_i | w_j \sim Lap(m_{ij}, \lambda_i)$.

$$P_{x_i|w_j}(x_i|w_j) = \frac{\lambda_i}{2} e^{-\lambda_i |x_i - m_{ij}|}$$

$$P(w_1) = P(w_2) = \cdots P(w_j) \quad (j \in (1, 2, \cdots c))$$

$$\Rightarrow P(w_i|x) > P(w_j|x) \quad \forall j \neq i.$$

$$\Rightarrow \frac{P(x|w_i)\ P(w_i)}{P(x)} > \frac{P(x|w_j)\, P(w_j)}{P(x)}$$

$$\Rightarrow P(w_i) \cdot \prod_{j=1}^{k} P(x_j|w_i) > P(w_j) \prod_{j=1}^{k} P(x_j|w_j)$$

$$\Rightarrow \prod_{j=1}^{k} \left(\frac{\lambda_j}{2} \cdot e^{-\lambda_j |x_i - m_{ij}|}\right) > \prod_{j=1}^{k} \frac{\lambda_i}{2} e^{-\lambda_j |x_j - m_{jn}|}$$

$$\Rightarrow \sum_{j=1}^{n} [\lambda_j (|x_j - m_{ji}| - (x_j - m_{jn})] > 0. \quad \forall n = i.$$

$\Rightarrow$ it's the weighted Manhattance Distance classifer.

the Min Manhattan Distance classifer when $M_{ji} = M_{jn}$.

(3)

(a) $R(a_i|x) = \sum_{j=1}^{4} \lambda(a_i|w_j) P(w_j|x)$

$$\Rightarrow R(a_1|x_1) = \sum_{j=1}^{4} \lambda(a_1|w_j) P(w_j|x_1).$$

$$= \sum_{j=1}^{4} \lambda(a_1|w_j) \frac{P(x_1|w_j) \cdot P(w_j)}{P(x_1)}.$$

$$\because P(x_1) = \sum_{j=1}^{4} P(x_1|w_j) \cdot P(w_j) = 0.2967.$$

$$\Rightarrow R(a_1|x_1) = \frac{1}{0.2967} \cdot \left(0 \times \frac{1}{3} \times \frac{1}{10} + 2 \times \frac{1}{2} \times \frac{1}{5} + 3 \times \frac{1}{6} \times \frac{1}{2} + 4 \times \frac{2}{3} \times \frac{1}{5}\right).$$

$$= \frac{0.77}{0.2967} = 2.5952.$$

$$R(a_2|x_1) = \frac{1}{P(x_1)} \cdot \sum_{j=1}^{4} (\lambda(a_2|w_j) \cdot P(x_1|w_j) P(w_j))$$

$$= \frac{1}{0.2917} \cdot \left(1 \times \frac{1}{10} \times \frac{1}{3} + 0 \times \frac{1}{2} \times \frac{1}{5} + 1 \times \frac{1}{6} \times \frac{1}{2} + 8 \times \frac{2}{5} \times \frac{1}{5}\right).$$

$$= 2.5506$$

$\Rightarrow R(a_3|x_1) = 1.5506$

$\Rightarrow R(a_4|x_1) = 1.8539$

$\therefore$ when $x = x_1$, the minum is $R(a_3|x_1) = 1.5506$.

$$P(x_2) = \sum_{j=1}^{4} P(x_2|w_j) \cdot P(w_j) = 0.33.$$

$$R(a_1|x_2) = \frac{1}{P(x_2)} \cdot \sum_{j=1}^{4} (\lambda(a_1|w_j) \cdot P(x_2|w_j) P(w_j))$$

$$= \frac{1}{0.33} \cdot \left(\frac{1}{10} \times \frac{1}{3} \times 0 + \frac{1}{5} \times \frac{1}{4} \times 2 + \frac{1}{2} \times \frac{1}{3} \times 3 + \frac{1}{5} \times \frac{2}{5} \times 10.\right)$$

$$= 2.2879.$$

$R(a_2|x_2) = 2.5455$, $R(a_3|x_2) = 1.0909$ $R(a_4|x_2) = 1.4646$

$\Rightarrow$ when $x = x_2$, the minum is $R(a_3|x_2) = 1.0909$.

$$P(x_3) = \sum_{j=1}^{4} P(x_3|w_j) \cdot P(w_j) = 0.3733.$$

$$R(a_1|x_3) = \frac{1}{P(x_3)} \cdot \sum_{j=1}^{4} (\lambda(a_1|w_j) \cdot P(x_3|w_j) P(w_j))$$

$$= \frac{1}{0.3733} \cdot \left(\frac{1}{10} \times \frac{1}{3} \times 0 + \frac{1}{5} \times \frac{1}{4} \times 2 + \frac{1}{2} \times \frac{1}{2} \times 3 + \frac{1}{5} \times \frac{1}{5} \times 4\right)$$

$= 2.2054$.

$R(a_2|x_3) = 1.6161$

$R(a_3|x_3) = 0.7500$

$R(a_4|x_3) = 1.5179$.

when $x = x_3$, the minum is $R(a_3|x_3) = 0.7500$

(b)

$$R = \sum_{i=1}^{3} R(\alpha(x_i)|x_i) \cdot P(x_i)$$

$$= R(\alpha(x_1)|x_1)P(x_1) + R(\alpha(x_2)|x_2)P(x_2) + R(\alpha(x_3)|x_3) \cdot P(x_3)$$

$$= 0.2967 \times 1.5506 + 0.33 \times 1.0909 + 0.3733 \times 0.75$$

$$\simeq 1.1$$

4.

(a) w_1 = evade taxes.

w_2 = x evade taxes.

$P(w_1) = \frac{3}{10}$ $\quad P(w_2) = \frac{7}{10}$

(b) conditional Gaussianality.

$$P(x|w_i) = N(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma_i} e^{-\frac{1}{2}\left(\frac{x-\mu_i}{\sigma_i}\right)^2}$$

$$\mu_1 = \frac{88+90+85}{3} = 87.67 \quad \sigma_1^2 = \frac{\sum(x-\mu_1)^2}{n-1} = 4.22 \quad \sigma = 2.5166$$

$$\mu_2 = \frac{122+77+106+210+72+117+60}{7} = 109.143$$

$\sigma_2^2 = 2539.5 \quad b_2 = 50.3932$

$$\Rightarrow P(x|w_1) = \frac{1}{6.3082} \cdot e^{-\frac{1}{2}\left(\frac{x-87.67}{2.5166}\right)^2}$$

$$P(x|w_2) = \frac{1}{126.317} e^{-\frac{1}{2}\left(\frac{x-109.143}{50.3932}\right)^2}$$

$x \rightarrow$ income.

(c) $x_1 = \{$ refund, No refund $\}$

$P(x_1 = \text{refund} | w_1) = 0$ $\quad$ $P(x_1 = \text{refund} | w_2) = \frac{3}{7}$

$P(x_1 = \text{No refund} | w_1) = 1$ $\quad$ $P(x_1 = \text{No refund} | w_2) = \frac{4}{7}$

$x_2 = \{$ single, married, divorced $\}$

$P(x_2 = \text{single} | w_1) = \frac{2}{3}$ $\quad$ $P(x_2 = \text{single} | w_2) = \frac{2}{7}$

$P(x_2 = \text{married} | w_1) = 0$ $\quad$ $P(x_2 = \text{married} | w_2) = \frac{4}{7}$

$P(x_2 = \text{Divorced} | w_1) = \frac{1}{3}$ $\quad$ $P(x_2 = \text{Divorced} | w_2) = \frac{1}{7}$

We know that a certain amount of samples is required to get an estimate distribution with confidence $1-\alpha$. But, it is the most accurate estimate we can make without upscaling the 'evade taxes' or downscaling the 'not evade taxes'

(d) No

As we know. Laplace correction corrects for this by not allowing each conditional probability $=0$

But

$$P(x|w_i) = P(x_1|w_i)\,P(x_2|w_i)\cdots P(x_k|w_i)\,P(w_i)$$

if one of this conditional probability $=0$, then the entire expression is 0.

it's districted.

(e) Minumn error Rate

$$P(w_1|x) > P(w_2|x)$$

$$\Rightarrow \frac{\prod_{i=1}^{k} P(x_i|w_1)\,P(w_1)}{P(x)} > \frac{\prod_{i=1}^{k} P(x_i|w_2)\,P(w_2)}{P(x)}$$

$$\Rightarrow P(w_1)\cdot\prod_{i=1}^{k} P(x_i|w_1) > P(w_2)\cdot\prod_{i=1}^{k} P(x_i|w_2)$$

$$\Rightarrow P(w_1)\,P(x_1|w_1)\cdot P(x_2|w_1)\cdot P(x_3|w_1) > P(w_2)\cdot P(x_1|w_2)\cdot P(x_2|w_2)\,P(x_3|w_2)$$

$P(x_1=\text{refund}\,|\,w_1) = \frac{1}{5}$	$P(x_1=\text{refund}\,|\,w_2) = \frac{4}{9}$
$P(x_1=\text{No refund}\,|\,w_1) = \frac{4}{5}$	$P(x_1=\text{No refund}\,|\,w_2) = \frac{5}{9}$
$P(x_2=\text{single}\,|\,w_1) = \frac{1}{2}$	$P(x_2=\text{single}\,|\,w_2) = \frac{3}{10}$
$P(x_2=\text{Married}\,|\,w_1) = \frac{1}{6}$	$P(x_2=\text{Married}\,|\,w_2) = \frac{1}{2}$
$P(x_2=\text{Divorced}\,|\,w_1) = \frac{1}{3}$	$P(x_2=\text{Divorced}\,|\,w_2) = \frac{1}{5}$

$$P(x_3|w_1) = \frac{1}{6.3082} \cdot e^{-\frac{1}{2}\left(\frac{x-87.67}{2.5166}\right)^2}$$

$$P(x_3|w_2) = \frac{1}{126.317} e^{-\frac{1}{2}\left(\frac{x-109.143}{50.3932}\right)^2}$$