Lab 3: Superposition

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Abstract

In this weeks lab we are inspecting a R-2R ladder circuit with three 5 volt sources. The goal is to analyze the circuit and show that $V_o = \frac{V_3}{2} + \frac{V_2}{4} + \frac{V_1}{8}$ using superposition. The analytical solution will be compared to values calculated from an LTSpice simulation of the same circuit.

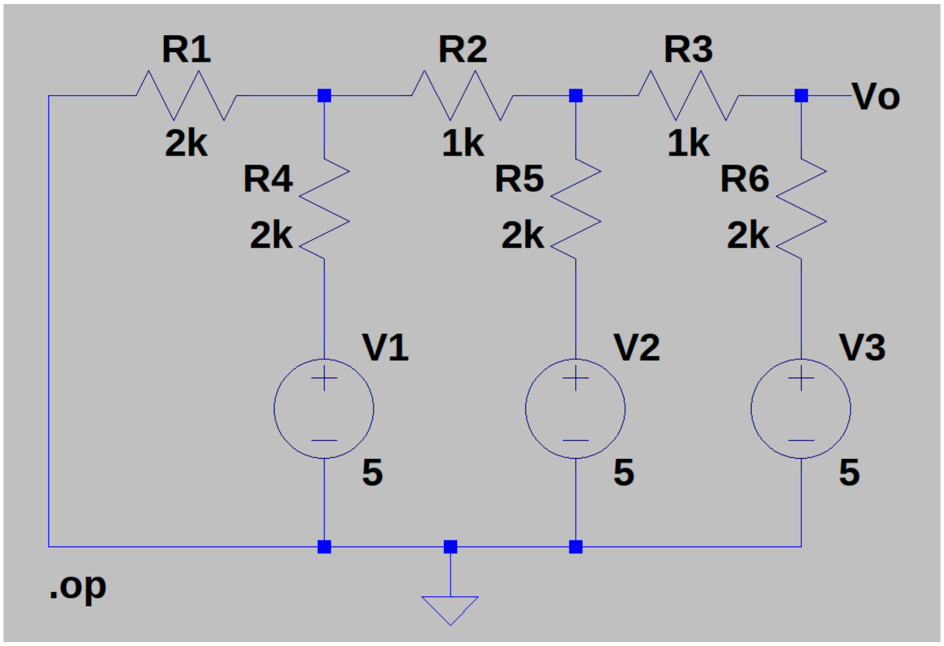
Equipment

- Acer Nitro 5 - OS: Ubuntu 22.04.1 LTS
- LTspice - Version: 17.0.35.0

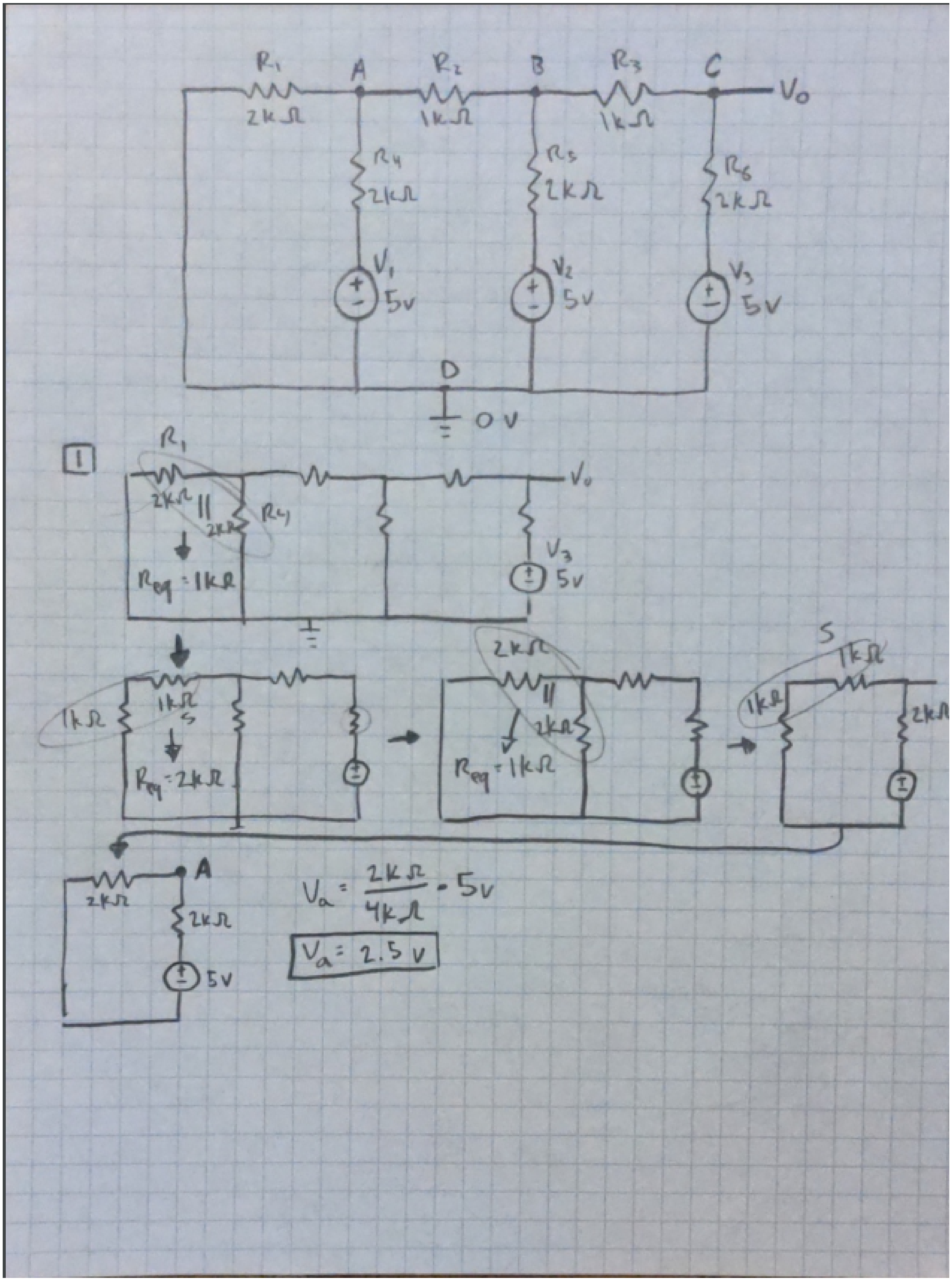
Procedure

1. Theoretically derive the voltage at V_o using superposition and confirm that the statement, $V_o = \frac{V_3}{2} + \frac{V_2}{4} + \frac{V_3}{8}$, is true.
2. Simulate the same circuit schematic using LTSpice.
3. Create a data table documenting the output voltage with respect to each possible state of input voltages.

Circuit Diagram



Calculations



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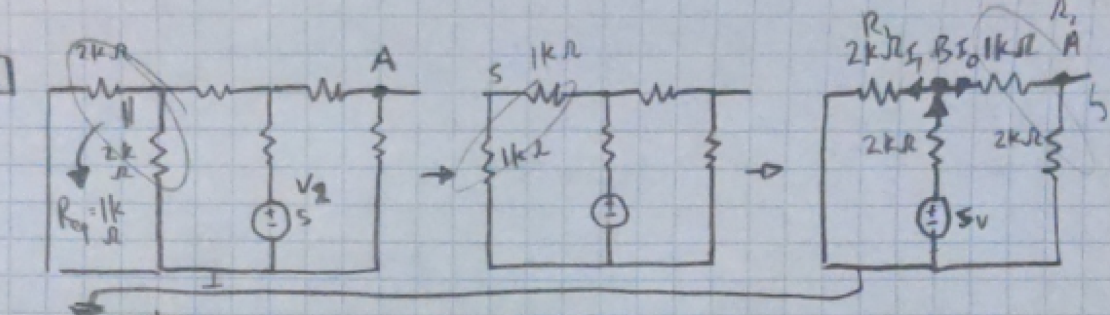
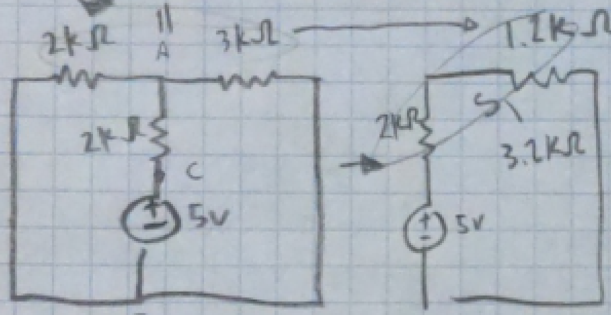




$I_s = 1.5625\ mA$

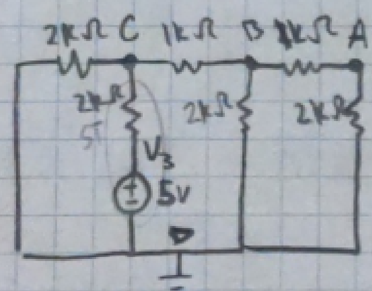
$I_o = I_s\left(\frac{2000\Omega}{5000\Omega}\right)$

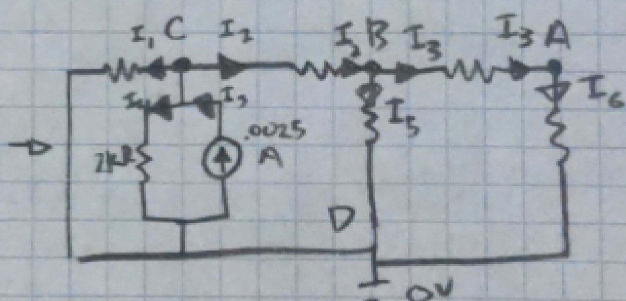
$I_o = .000625A$

$V_b = V_s - (2000 \cdot .0015625)$

$V_b = 1.875 \rightarrow V_a = V_b - I_s \cdot 1000 \rightarrow \boxed{V_a = 1.25v}$

3





C) $\left(.0025A = \frac{V_c}{2k\Omega} + \frac{V_c}{2k\Omega} + \frac{V_c - V_b}{1k\Omega}\right) 2k\Omega$

$5v = V_c + V_c + 2V_c - 2V_b$

$5 = -2V_b + 4V_c$

B) $\left(\frac{V_c - V_b}{1k} = \frac{V_b - V_a}{1k} + \frac{V_b}{2k}\right) 2k\Omega$

$2V_c - 2V_b = 2V_b - 2V_a + V_b$

$0 = -2V_a + 5V_b - 2V_c$

a) $\left(\frac{V_b - V_a}{1k} = \frac{V_a}{2k}\right) 2k \rightarrow 2V_b - 2V_a = V_a$

$0 = 3V_a - 2V_b$

$$\begin{bmatrix} 0 & -2 & 4 \\ -2 & 5 & -2 \\ 3 & -2 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 0.625 \\ 0.9375 \\ 1.71875 \end{bmatrix} \quad \boxed{V_a = 0.625v}$$

From these calculations we derive that:

$$V_{a1} = \frac{V_3}{2} = 2.5V \qquad V_{a2} = \frac{V_2}{4} = 1.25V \qquad V_{a3} = \frac{V_1}{8} = 0.625V$$

$$V_o = V_{a1} + V_{a2} + V_{a3} \Rightarrow V_o = 4.375V$$

Data

V_3, volts	V_2, volts	V_1, volts	V_o, volts
0	0	0	0
0	0	5	0.625
0	5	0	1.25
0	5	5	1.875
5	0	0	2.5
5	0	5	3.125
5	5	0	3.75
5	5	5	4.375

Conclusion

Our theoretical calculations matched the simulated results from LTSpice. From this we were able to see that in fact the statement $V_o = \frac{V_3}{2} + \frac{V_2}{4} + \frac{V_1}{8}$ holds true. This leads to further questions regarding the pattern of voltage superposition as more voltages are added to the ladder. The pattern we witness here is $V_o = \sum_{n=1}^{m} \frac{V_{in}}{2^n}$ where n begins on the ladder rung connected to the point of interest and m is the furthest rung from the point of interest.