

# Frequency Response of R-L and R-C Networks

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## Abstract

The purpose of this lab is to investigate how series R-L and R-C circuits respond with respect to frequency. Since we know that the impedance of capacitors and inductors are  $j\omega L$  and  $\frac{1}{j\omega C}$  respectively, we assume that circuits with these passive elements should contribute to a change of currents and voltages at different points in the circuit. We will build two and simulate four simple circuits containing these elements and measure the voltage out at a point in the circuit, plot the data and examine for patterns in the relationship between frequency and voltage.

## Equipment

- Function Generator - GWInstek GFG-8250A Serial: GCR906609
- Oscilloscope - Keysight DSOX1102G Serial: CN57336404
- LCR Meter - Keysight U1732C Serial: MY57300007
- DMM - Extech EX330
- Resistor  $R_1 = 984.1 \Omega$
- Capacitor  $C_1 = 47.69 nF$
- Inductor  $L_1 = 45.65 mH$

## Set-Up



Figure 1: RL Circuit

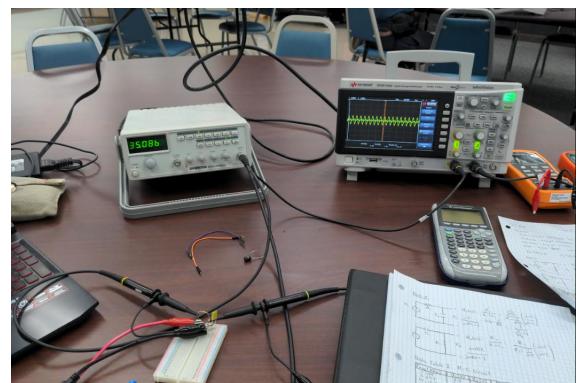
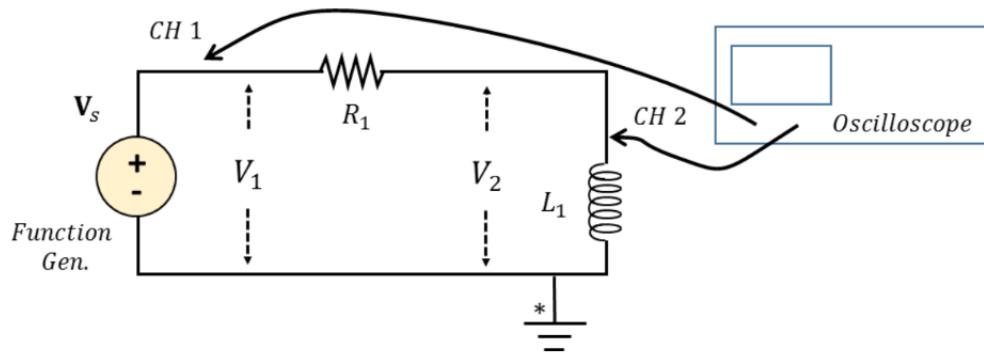


Figure 2: RC Circuit

### Part 1: The R-L Network



### Derivation of Transfer Function for R-L and L-R Circuits

**Part 1)**

$$H_V(\omega) = \frac{V_o}{V_s} = \frac{L_1}{R_1 + L_1} \rightarrow \frac{j\omega L}{R + j\omega L} \left( \frac{1}{j\omega L} \right)$$

$$H_V(\omega) = \frac{1}{\frac{-jR}{\omega L} + 1} \quad |H_V(\omega)| = \frac{1}{\sqrt{1 + (\frac{\omega_0}{\omega})^2}}$$

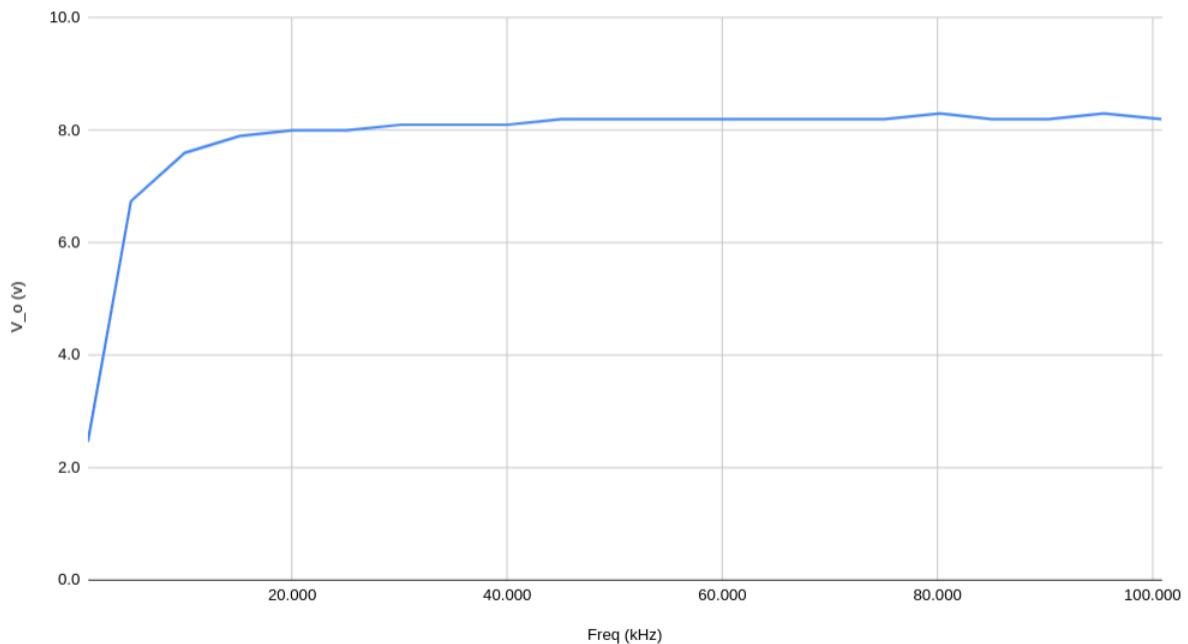
$$\phi(\omega) = \tan^{-1} \left( -\frac{\omega_0}{\omega} \right)$$
  

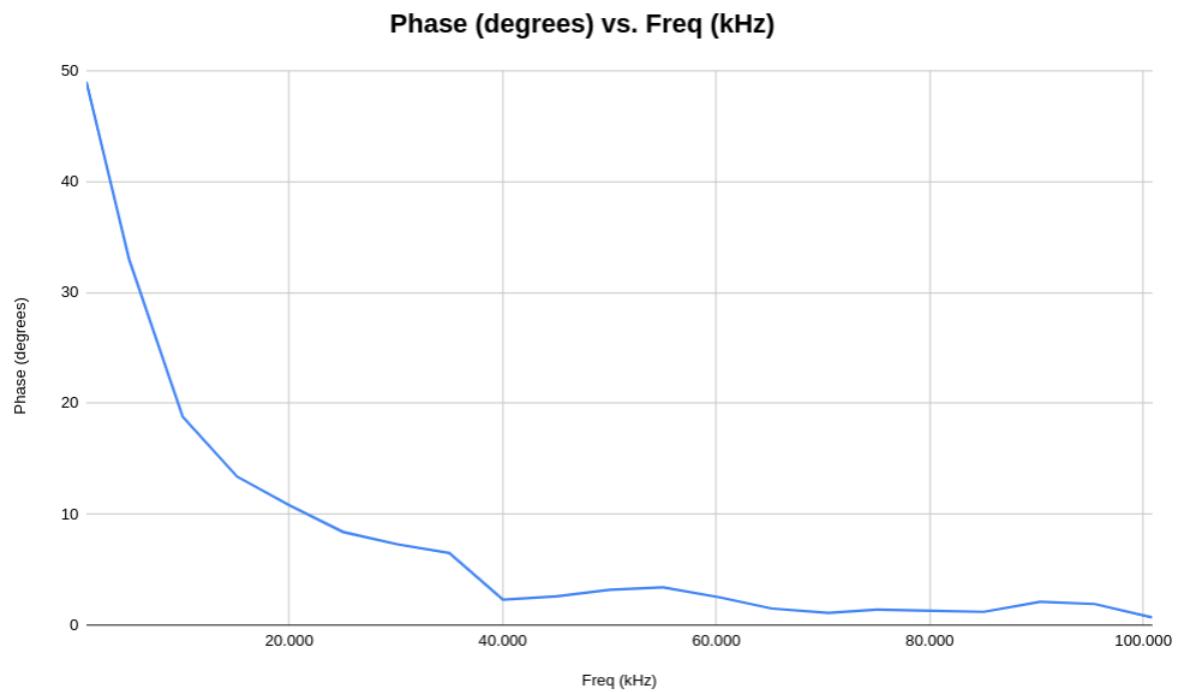
$$H_V(\omega) = \frac{V_o}{V_s} = \frac{R_1}{R_1 + L_1} \rightarrow \frac{R}{R + j\omega L} \left( \frac{1}{j\omega L} \right)$$

$$H_V(\omega) = \frac{R}{\frac{jR}{\omega L} + 1} \quad |H_V(\omega)| = \frac{\omega_0 / \omega}{\sqrt{1 + (\omega_0 / \omega)^2}}$$

**Constructed R-L Circuit: Data**

Freq (kHz)	V_s (v)	V_o (v)	Phase (degrees)
1.008	8.8	2.5	49
5.004	8.8	6.7	33
10.012	8.8	7.6	18.8
15.103	8.8	7.9	13.4
20.044	8.8	8.0	10.8
25.053	8.8	8.0	8.4
30.098	8.8	8.1	7.3
35.000	8.8	8.1	6.5
40.023	8.8	8.1	2.3
45.030	8.8	8.2	2.6
50.024	8.8	8.2	3.2
55.030	8.8	8.2	3.4
60.350	8.8	8.2	2.5
65.200	8.8	8.2	1.5
70.500	8.8	8.2	1.1
75.050	8.8	8.2	1.4
80.260	8.8	8.3	1.3
85.060	8.8	8.2	1.2
90.330	8.8	8.2	2.1
95.460	8.8	8.3	1.9
100.840	8.8	8.2	0.7

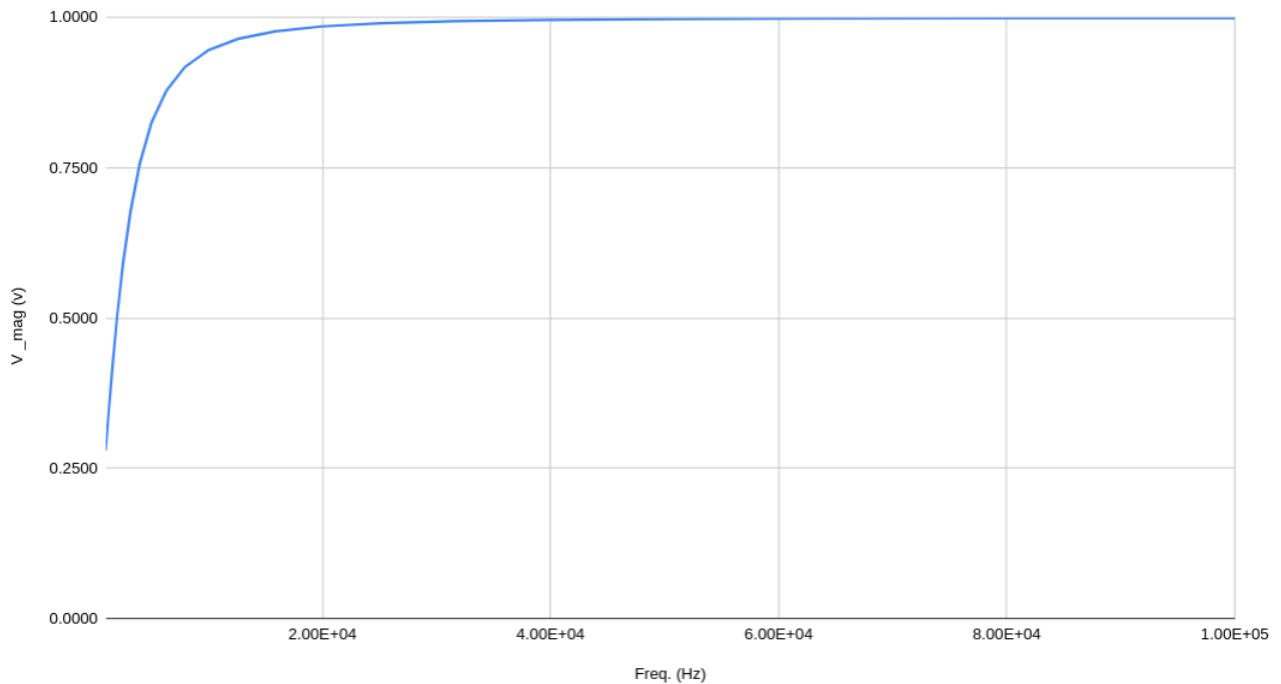
**V\_o (v) vs. Freq (kHz)**

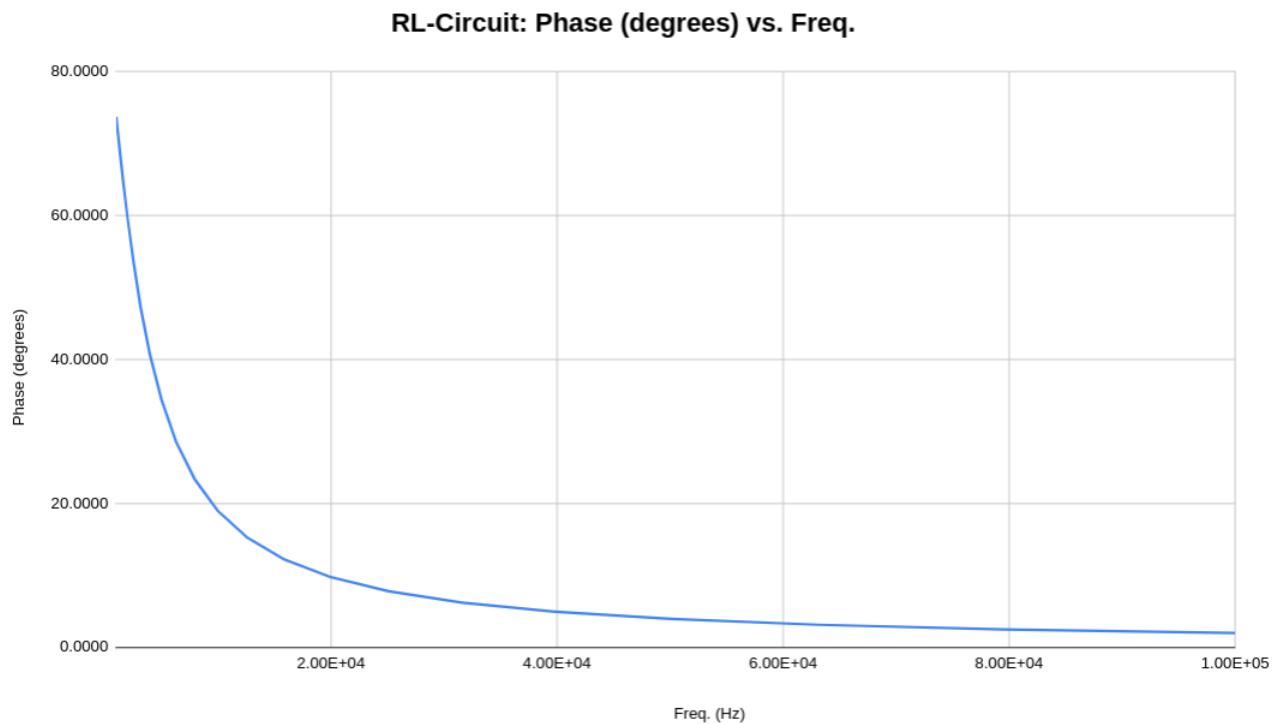


### Simulated R-L Circuit: Data

Freq.	V_o Real	Imaginary	V_mag (v)	Phase (degrees)
1.00E+03	7.83E-02	2.69E-01	0.2798	73.7504
1.26E+03	1.19E-01	3.23E-01	0.3445	69.8503
1.58E+03	1.76E-01	3.81E-01	0.4194	65.2060
2.00E+03	2.53E-01	4.35E-01	0.5027	59.8201
2.51E+03	3.49E-01	4.77E-01	0.5907	53.7914
3.16E+03	4.59E-01	4.98E-01	0.6777	47.3337
3.98E+03	5.74E-01	4.95E-01	0.7575	40.7555
5.01E+03	6.81E-01	4.66E-01	0.8252	34.3944
6.31E+03	7.72E-01	4.20E-01	0.8785	28.5362
7.94E+03	8.43E-01	3.64E-01	0.9180	23.3612
1.00E+04	8.95E-01	3.07E-01	0.9459	18.9370
1.26E+04	9.31E-01	2.54E-01	0.9648	15.2447
1.58E+04	9.55E-01	2.07E-01	0.9774	12.2149
2.00E+04	9.71E-01	1.67E-01	0.9855	9.7570
2.51E+04	9.82E-01	1.34E-01	0.9908	7.7779
3.16E+04	9.88E-01	1.07E-01	0.9942	6.1922
3.98E+04	9.93E-01	8.55E-02	0.9963	4.9257
5.01E+04	9.95E-01	6.81E-02	0.9977	3.9162
6.31E+04	9.97E-01	5.42E-02	0.9985	3.1125
7.94E+04	9.98E-01	4.31E-02	0.9991	2.4733
1.00E+05	9.99E-01	3.43E-02	0.9994	1.9650

RL-Circuit V\_mag (v) vs. Freq.

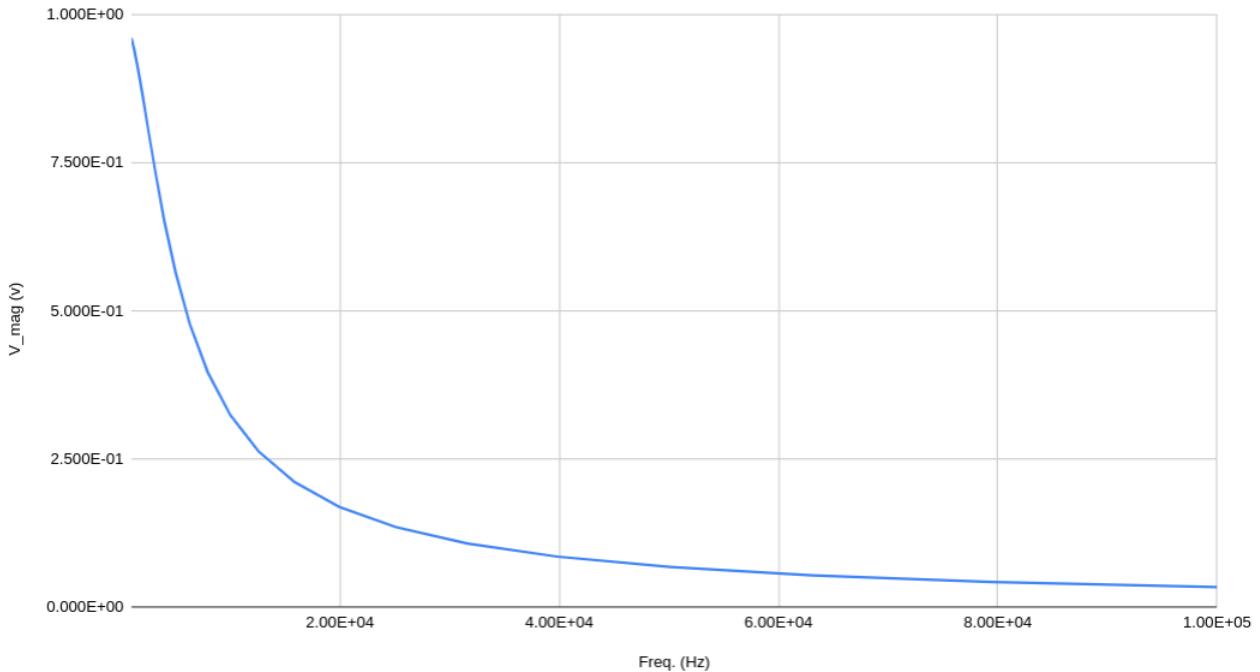


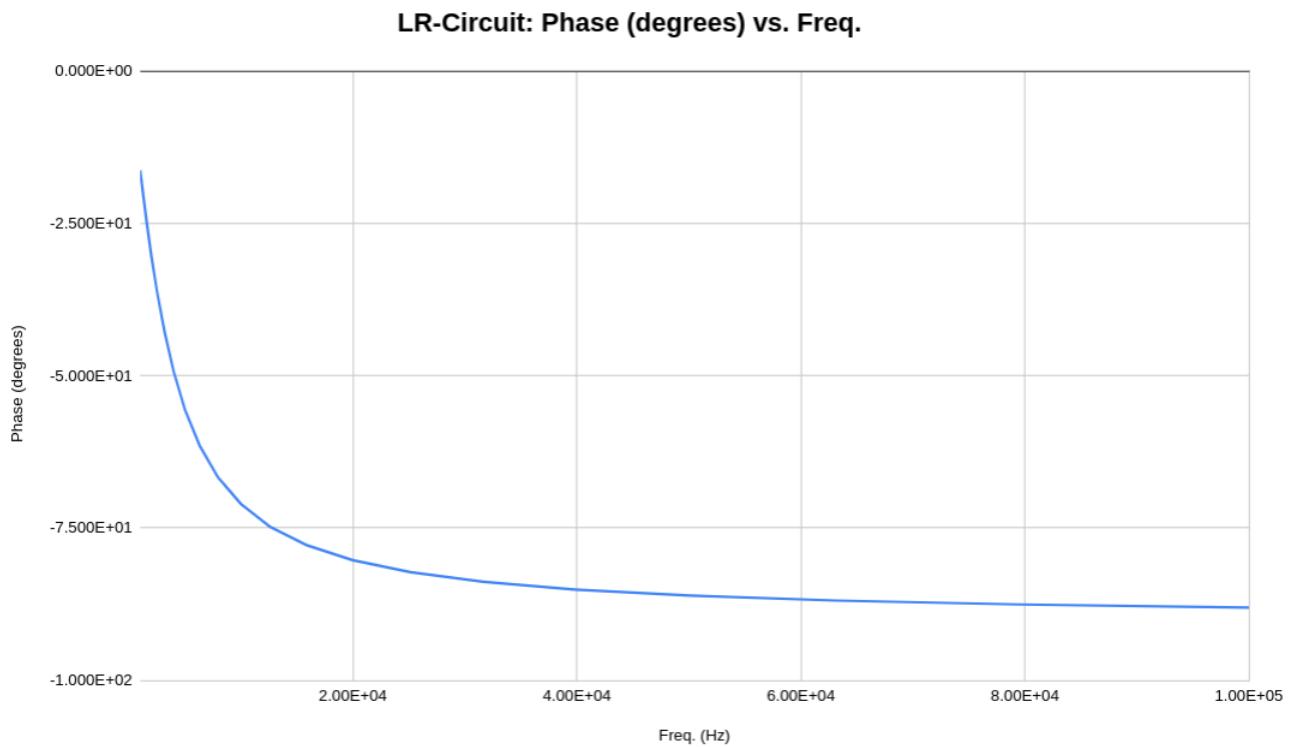


### Simulated L-R Circuit: Data

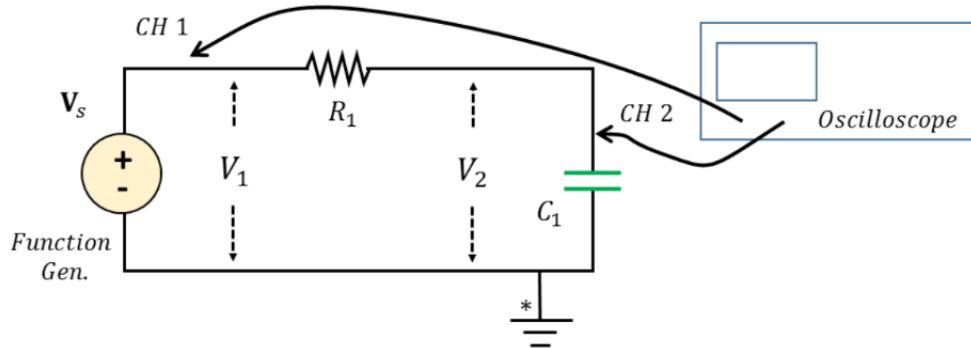
Freq.	V_o Real	Imaginary	V_mag (v)	Phase (degrees)
1.00E+03	9.22E-01	-2.69E-01	9.601E-01	-1.625E+01
1.26E+03	8.81E-01	-3.23E-01	9.388E-01	-2.015E+01
1.58E+03	8.24E-01	-3.81E-01	9.078E-01	-2.479E+01
2.00E+03	7.47E-01	-4.35E-01	8.645E-01	-3.018E+01
2.51E+03	6.51E-01	-4.77E-01	8.069E-01	-3.621E+01
3.16E+03	5.41E-01	-4.98E-01	7.353E-01	-4.267E+01
3.98E+03	4.26E-01	-4.95E-01	6.528E-01	-4.924E+01
5.01E+03	3.19E-01	-4.66E-01	5.649E-01	-5.561E+01
6.31E+03	2.28E-01	-4.20E-01	4.777E-01	-6.146E+01
7.94E+03	1.57E-01	-3.64E-01	3.965E-01	-6.664E+01
1.00E+04	1.05E-01	-3.07E-01	3.245E-01	-7.106E+01
1.26E+04	6.91E-02	-2.54E-01	2.629E-01	-7.476E+01
1.58E+04	4.48E-02	-2.07E-01	2.116E-01	-7.779E+01
2.00E+04	2.87E-02	-1.67E-01	1.695E-01	-8.024E+01
2.51E+04	1.83E-02	-1.34E-01	1.353E-01	-8.222E+01
3.16E+04	1.16E-02	-1.07E-01	1.079E-01	-8.381E+01
3.98E+04	7.37E-03	-8.55E-02	8.586E-02	-8.507E+01
5.01E+04	4.66E-03	-6.81E-02	6.830E-02	-8.608E+01
6.31E+04	2.95E-03	-5.42E-02	5.430E-02	-8.689E+01
7.94E+04	1.86E-03	-4.31E-02	4.315E-02	-8.753E+01
1.00E+05	1.18E-03	-3.43E-02	3.429E-02	-8.803E+01

LR-Circuit: V\_mag (v) vs. Freq.





## Part 2: The R-C Network



### Derivation of Transfer Function for R-C and C-R Circuits

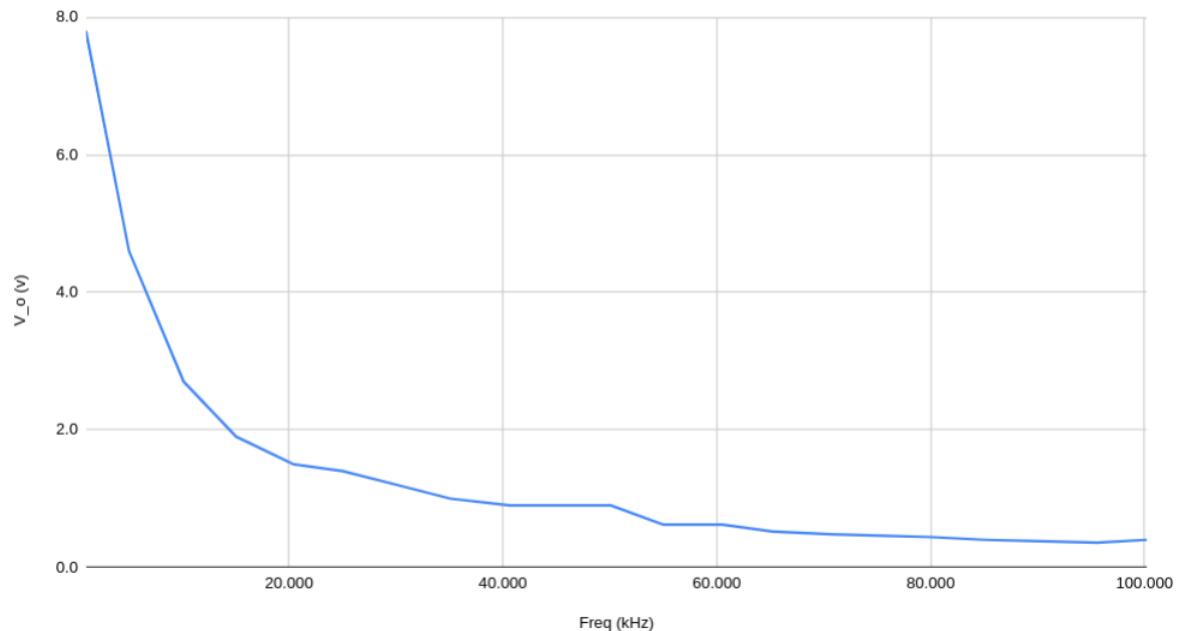
Part 2)

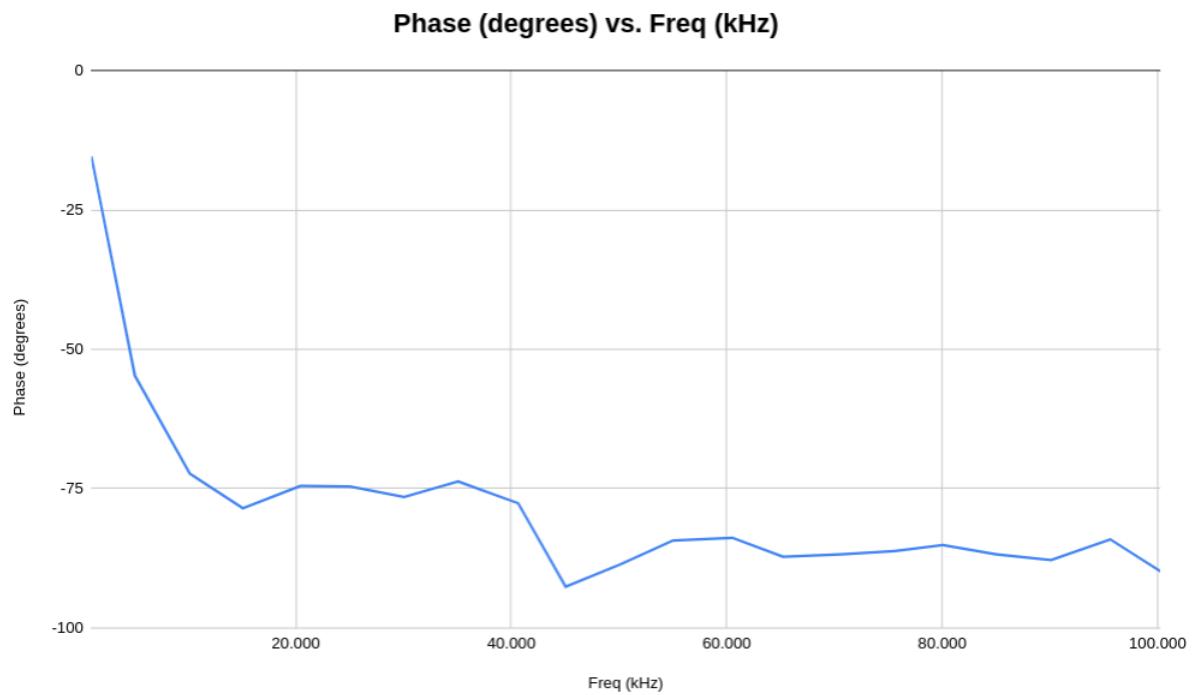
	$H_v(\omega) = \frac{V_o}{V_s} = \frac{C_1}{R_1 + C_1} \rightarrow \frac{1/j\omega C}{R + 1/j\omega C}$ $\omega_0 = \frac{1}{RC}$ $ H_v(\omega)  = \frac{1}{\sqrt{1 + (\omega/\omega_0)^2}}$ $\phi(\omega) = -\tan^{-1}(\omega/\omega_0)$
	$H_v(\omega) = \frac{V_o}{V_s} = \frac{R_1}{R_1 + C_1} \rightarrow \frac{R}{R + 1/j\omega C}$ $\omega_0 = \frac{1}{RC}$ $ H_v(\omega)  = \frac{\sqrt{(\omega/\omega_0)^2}}{\sqrt{1 + (\omega/\omega_0)^2}}$ $\phi(\omega) = -\tan^{-1}((\omega/\omega_0)^2)$

### Constructed R-C Circuit: Data

Freq (kHz)	V_s (v)	V_o (v)	Phase (degrees)
1.019	8.8	7.8	-15.4
5.047	8.8	4.6	-54.7
10.152	8.8	2.7	-72.3
15.077	8.8	1.9	-78.5
20.430	8.8	1.5	-74.5
25.022	8.8	1.4	-74.6
30.054	8.8	1.2	-76.5
35.076	8.8	1.0	-73.7
40.630	8.8	0.9	-77.6
45.056	8.8	0.9	-92.6
50.106	8.8	0.9	-88.6
55.010	8.8	0.6	-84.3
60.530	8.8	0.6	-83.8
65.230	8.8	0.5	-87.2
70.600	8.8	0.5	-86.8
75.600	8.8	0.5	-86.2
80.070	8.8	0.4	-85.1
85.020	8.8	0.4	-86.8
90.110	8.8	0.4	-87.8
95.620	8.8	0.4	-84.1
100.240	8.8	0.4	-89.8

**V\_o (v) vs. Freq (kHz)**

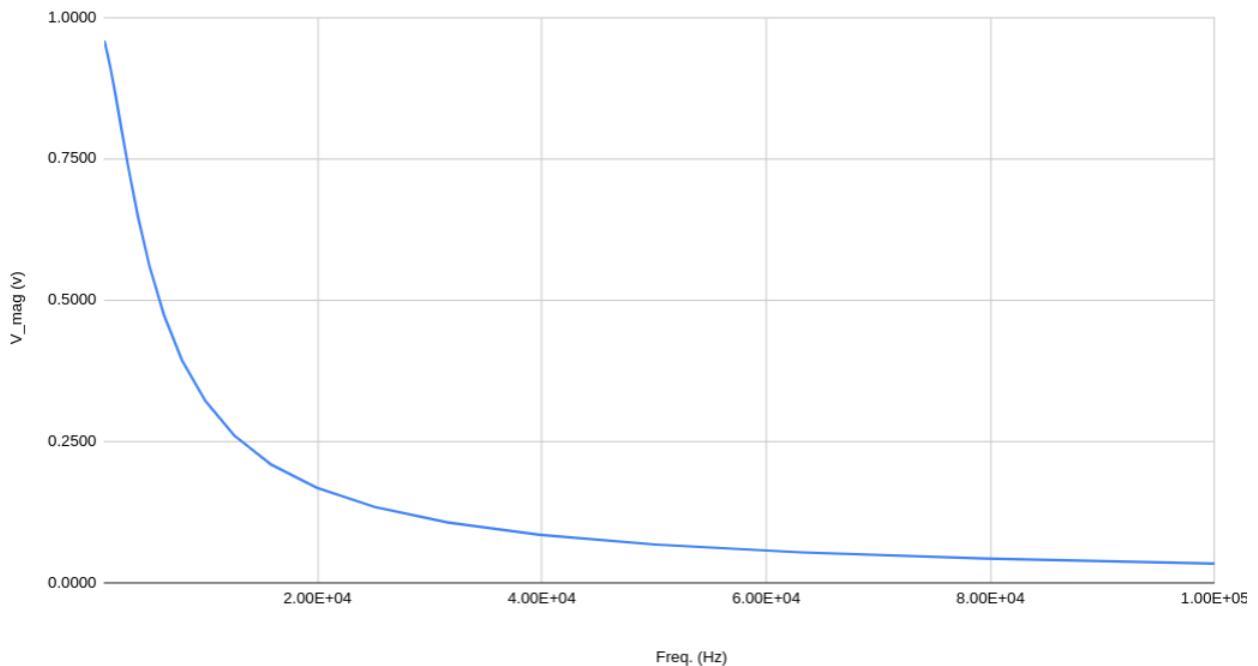


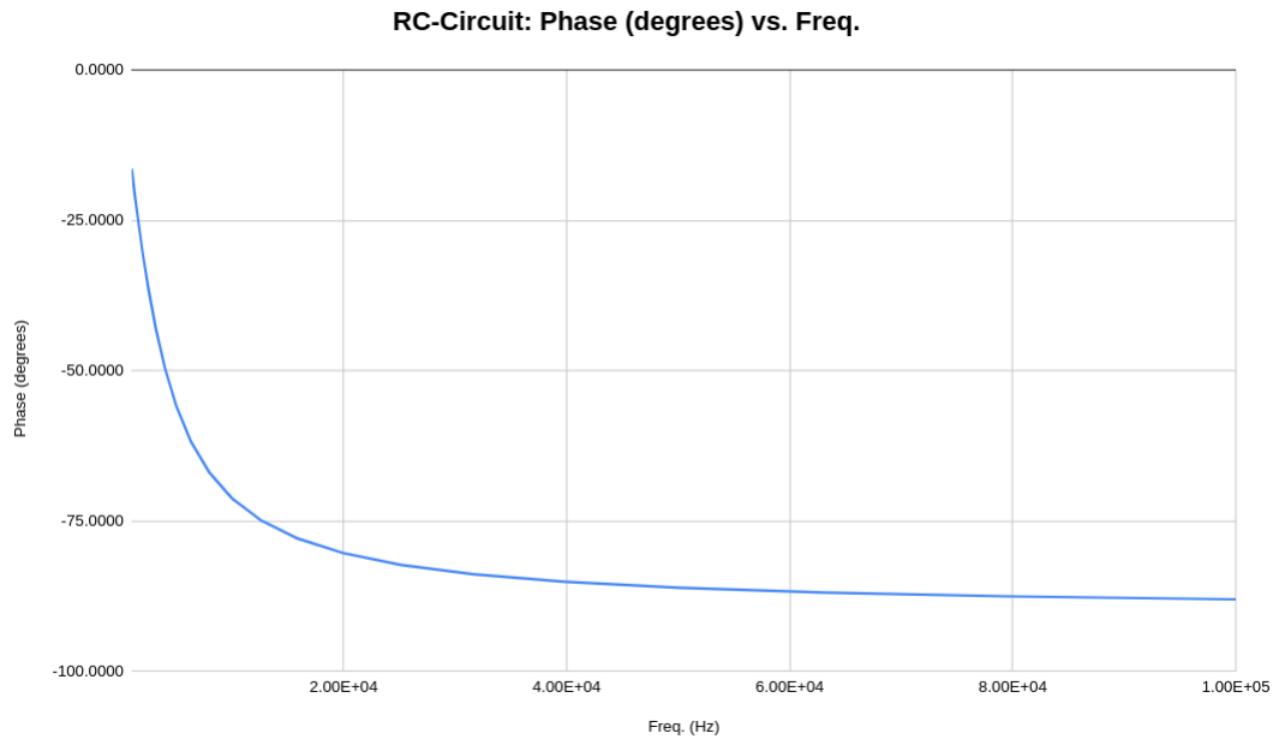


### Simulated R-C Circuit: Data

Freq.	V_o Real	Imaginary	V_mag (v)	Phase (degrees)
1.00E+03	9.20E-01	-2.71E-01	0.9592	-16.4298
1.26E+03	8.79E-01	-3.26E-01	0.9375	-20.3666
1.58E+03	8.21E-01	-3.84E-01	0.9059	-25.0492
2.00E+03	7.43E-01	-4.37E-01	0.8619	-30.4710
2.51E+03	6.46E-01	-4.78E-01	0.8036	-36.5276
3.16E+03	5.35E-01	-4.99E-01	0.7314	-42.9994
3.98E+03	4.20E-01	-4.94E-01	0.6485	-49.5746
5.01E+03	3.14E-01	-4.64E-01	0.5604	-55.9164
6.31E+03	2.24E-01	-4.17E-01	0.4734	-61.7433
7.94E+03	1.54E-01	-3.61E-01	0.3926	-66.8811
1.00E+04	1.03E-01	-3.04E-01	0.3212	-71.2672
1.26E+04	6.77E-02	-2.51E-01	0.2601	-74.9239
1.58E+04	4.38E-02	-2.05E-01	0.2092	-77.9225
2.00E+04	2.81E-02	-1.65E-01	0.1676	-80.3540
2.51E+04	1.79E-02	-1.33E-01	0.1338	-82.3112
3.16E+04	1.14E-02	-1.06E-01	0.1066	-83.8790
3.98E+04	7.20E-03	-8.46E-02	0.0849	-85.1311
5.01E+04	4.56E-03	-6.74E-02	0.0675	-86.1291
6.31E+04	2.88E-03	-5.36E-02	0.0537	-86.9235
7.94E+04	1.82E-03	-4.26E-02	0.0427	-87.5554
1.00E+05	1.15E-03	-3.39E-02	0.0339	-88.0577

RC-Circuit: V\_mag (v) vs. Freq.

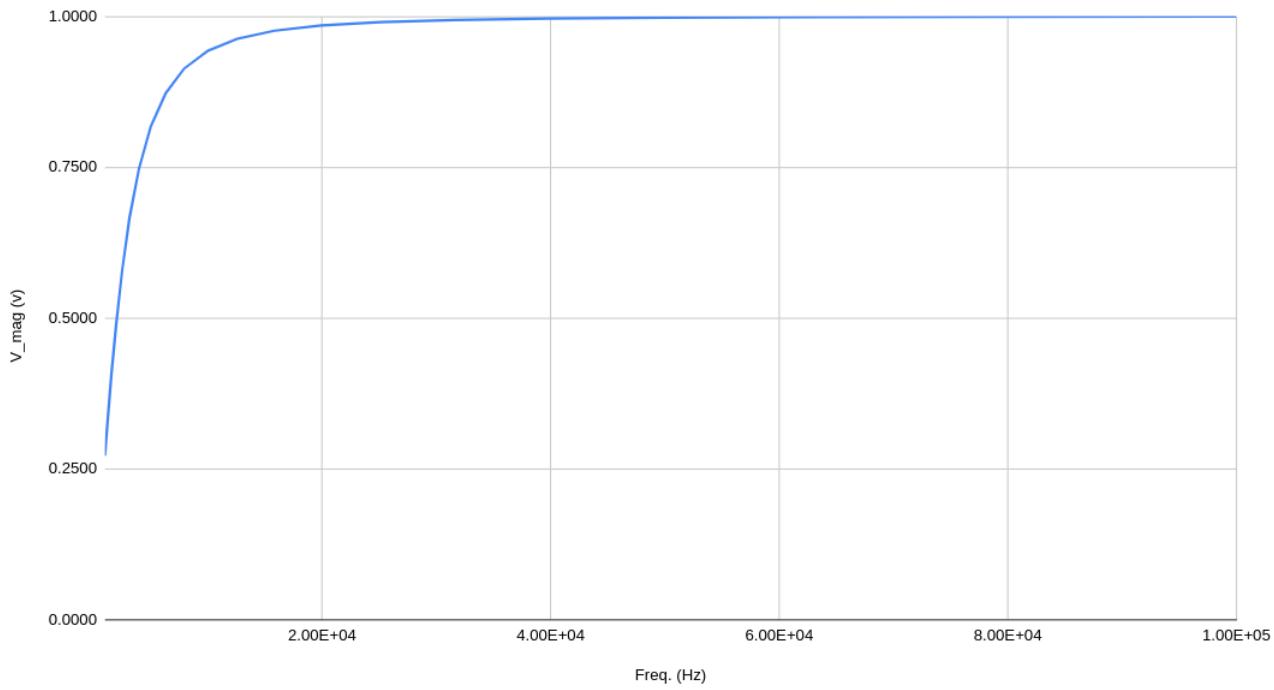


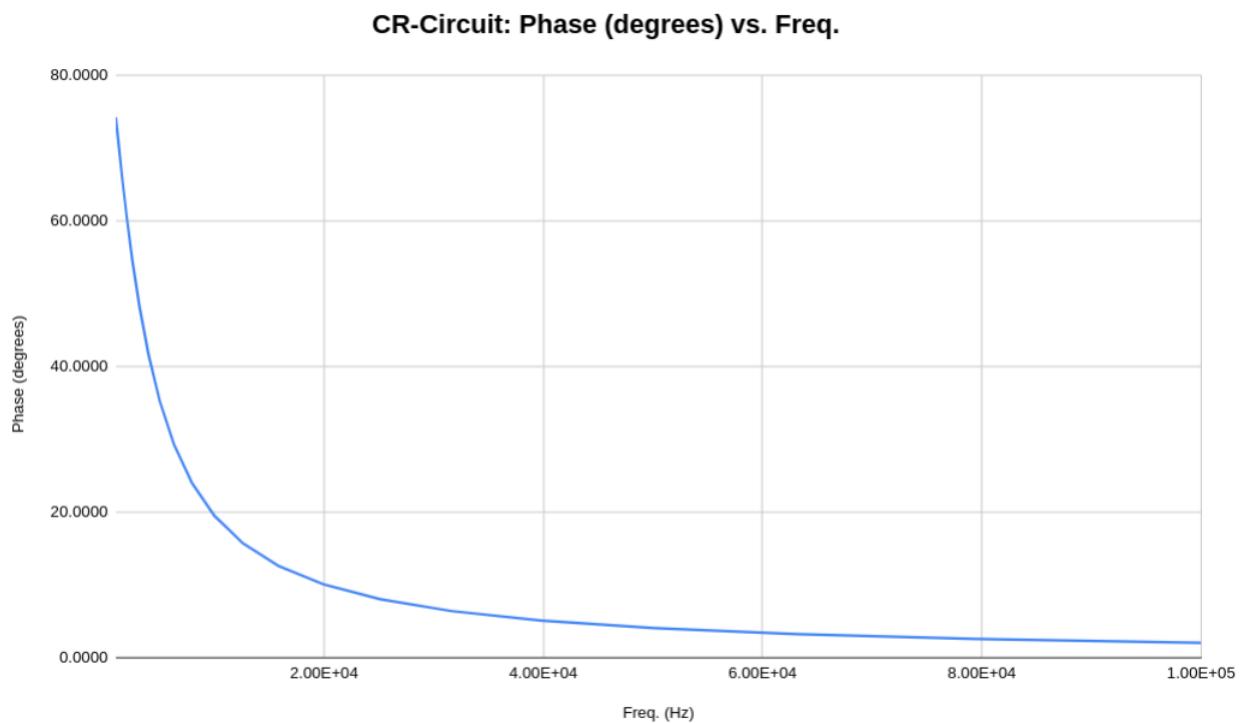


### Simulated C-R Circuit: Data

Freq.	V_o Real	Imaginary	V_mag (v)	Phase (degrees)
1.00E+03	7.39E-02	2.62E-01	0.2719	74.2243
1.26E+03	1.12E-01	3.16E-01	0.3351	70.4213
1.58E+03	1.67E-01	3.73E-01	0.4087	65.8793
2.00E+03	2.41E-01	4.28E-01	0.4910	60.5905
2.51E+03	3.35E-01	4.72E-01	0.5787	54.6388
3.16E+03	4.44E-01	4.97E-01	0.6662	48.2228
3.98E+03	5.58E-01	4.97E-01	0.7473	41.6409
5.01E+03	6.67E-01	4.71E-01	0.8168	35.2317
6.31E+03	7.61E-01	4.27E-01	0.8721	29.2922
7.94E+03	8.34E-01	3.72E-01	0.9134	24.0185
1.00E+04	8.89E-01	3.15E-01	0.9427	19.4922
1.26E+04	9.27E-01	2.61E-01	0.9627	15.7041
1.58E+04	9.52E-01	2.13E-01	0.9760	12.5896
2.00E+04	9.69E-01	1.72E-01	0.9846	10.0598
2.51E+04	9.81E-01	1.38E-01	0.9902	8.0211
3.16E+04	9.88E-01	1.11E-01	0.9938	6.3867
3.98E+04	9.92E-01	8.82E-02	0.9961	5.0809
5.01E+04	9.95E-01	7.03E-02	0.9975	4.0398
6.31E+04	9.97E-01	5.59E-02	0.9984	3.2109
7.94E+04	9.98E-01	4.45E-02	0.9990	2.5515
1.00E+05	9.99E-01	3.54E-02	0.9994	2.0272

CR-Circuit: V\_mag (v) vs. Freq.





## Conclusion

The aim of this lab was to construct two circuits, an RL and an RC circuit, and measure the output voltage between the two components as the frequency of the AC source is adjusted from 1 KHz to 100 KHz. Then compare this to the simulated data. When comparing the real world measurements to the simulated measurements they produce very similar response curves. For the RL circuit the voltage out with respect to frequency goes from near 25% the voltage source to nearly 100% the voltage source. This is because as the frequency increases the impedance across the inductor grows linearly with respect to the frequency. The phase goes from near 90 degrees out of phase to near 0 degrees out of phase. For the RC circuit the voltage out with respect to the frequency goes from close to 100% the source voltage to nearly 0% the source voltage. This is because as the frequency increases the impedance across the capacitor is inversely proportional to the frequency. The phase goes from near 0 degrees out of phase to near -90 degrees out of phase. When simulating the LR circuit, the voltage vs frequency curve is inverted but the phase vs frequency curve is the same. The same pattern can be found with the CR circuit.