Mutual Inductance

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Abstract

This lab's aim is to provide hands on experience with mutually coupled coils. The steps performed will demonstrate how to measure self and mutual impedance of the coils, how to interpret the mutually coupled coil data to then determine self-impedance and mutual impedance and finally how to use the maximum-power-transfer theorem in an AC-circuit containing inductively coupled coils.

Equipment

- Oscilloscope
- Function Generator - GwInstek GFG-8250A Serial: GCR906010
- 3 x Digital Multimeter - EXTECH Instruments
- LCR Meter
- Resistance Substitution Box
- Transformer: "Optimus"
- Current Limiting Resistor

Part 1: Dot Marking

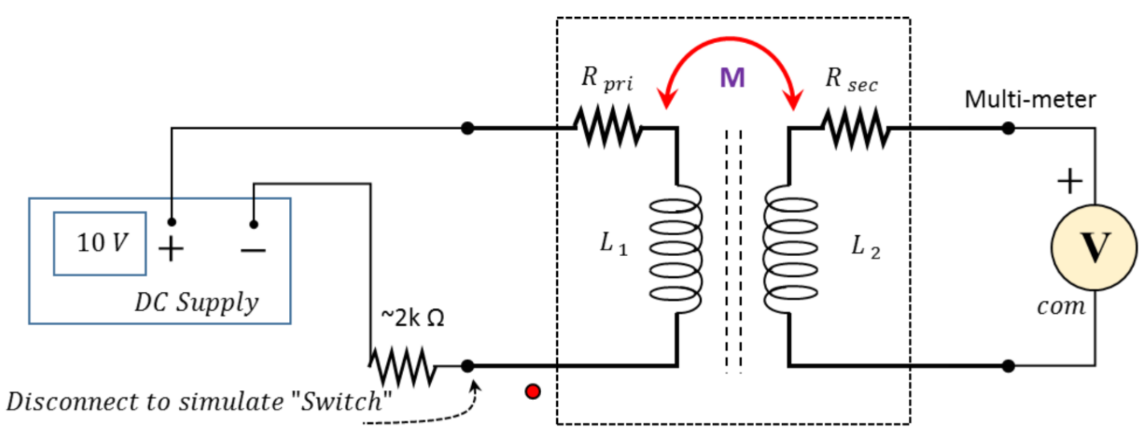


Fig 1.

1. Arbitrarily assign a dot to a terminal on the primary side of the transformer.

2. Attach the negative terminal of the dc power supply to this terminal through the current limiting resistor and switch (Since we don't have switches, we will quickly disconnect the leads to replicate effect).

3. Arbitrarily connect the positive lead (red) of the digital multi-meter to the secondary side of the transformer with the meter set in the dc voltage mode.

4. Open the "switch" and observe the digital multi-meter.

5. If the digital multi-meter "kicks" positive assign the dot on the secondary side terminal that connects to the positive lead (red) of the digital voltmeter. Otherwise assign the dot to the negative lead (black).

Part 2: Measurement of the Self-Impedance of Each Coil

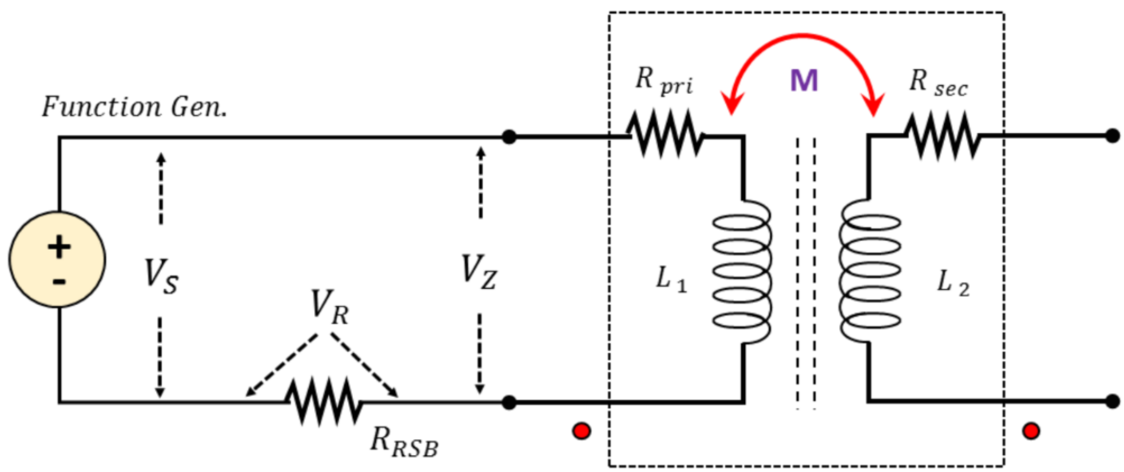


Fig 2.

To measure the self-impedance of each coil the function generator was set to these frequency and RMS settings:

Settings for Coil 1:

$$f = 99.81\ Hz \qquad V_S = 2.007\ v$$

For coil 1 the resistor R_{RSB1}, and the voltages V_{R1} and V_{Z1} were measured as follows:

$$R_{RSB1} = 1850\Omega \qquad V_{R1} = 1.239v \qquad V_{Z1} = 1.238v$$

Settings for Coil 2:

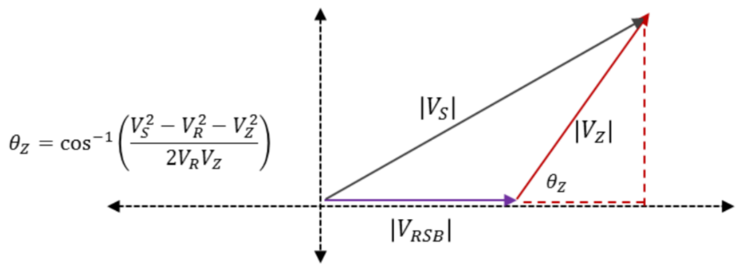
$$f = 99.68\ Hz \qquad V_S = 1.995\ v$$

For coil 2 the resistor R_{RSB2}, and the voltages V_{R2} and V_{Z2} were measured as follows:

$$R_{RSB2} = 185\Omega \qquad V_{R2} = 1.207v \qquad V_{Z2} = 1.208v$$

Calculations:

From these measurements it is possible to find the impedance of each coil. Using the phasor relationships below, we will find the phase angle of the voltage across the coil, the current through the coil by means of the voltage across the resistor and from that calculate the impedance of the coil.



Coil 1:

Find θ_{Z1}:

$$\theta_{Z1} = cos^{-1}\left(\frac{V_S^2 - V_{R1}^2 - V_{Z1}^2}{2V_{R1}V_{Z1}}\right) \rightarrow \theta_{Z1} = 71.76^o$$

Find $|I_1|$:

$$|I_1| = \frac{|V_{R1}|}{R_{RSB1}} \rightarrow |I_1| = 0.67\ mA$$

Find the complex impedance, Z_1, and use the reactance to find the inductance, L_1, of coil 1:

$$|Z_1| = \frac{|V_{Z1}|}{|I_1|} \rightarrow |Z_1| = 1848\ \Omega$$

$$X_1 = Z_1 sin(\theta_{Z1}) \rightarrow X_1 = j1754.9\ \Omega \qquad R_1 = Z_1 cos(\theta_{Z1}) \rightarrow R_1 = 578.3\ \Omega$$

$$L_1 = \frac{X_1}{j2\pi f} \rightarrow L_1 = 2.8\ H$$

Coil 2:

Find θ_{Z2}:

$$\theta_{Z2} = cos^{-1}\left(\frac{V_S^2 - V_{R2}^2 - V_{Z2}^2}{2V_{R2}V_{Z2}}\right) \rightarrow \theta_{Z2} = 68.69^o$$

Find $|I_2|$:

$$|I_2| = \frac{|V_{R2}|}{R_{RSB2}} \rightarrow |I_2| = 6.52\ mA$$

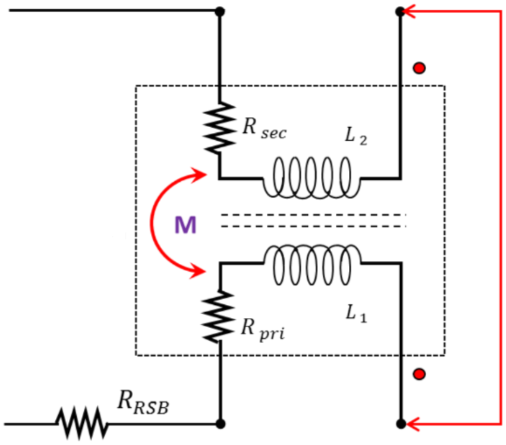
Find the complex impedance, Z_2, and use the reactance to find the inductance, L_2, of coil 2:

$$|Z_2| = \frac{|V_{Z2}|}{|I_2|} \rightarrow |Z_2| = 185.3\ \Omega$$

$$X_2 = Z_2 sin(\theta_{Z2}) \rightarrow X_2 = j172.64\ \Omega \qquad R_2 = Z_2 cos(\theta_{Z2}) \rightarrow R_2 = 67.34\ \Omega$$

$$L_2 = \frac{X_2}{j2\pi f} \rightarrow L_2 = 275.6\ mH$$

Part 3: Measurement of Mutual Inductance



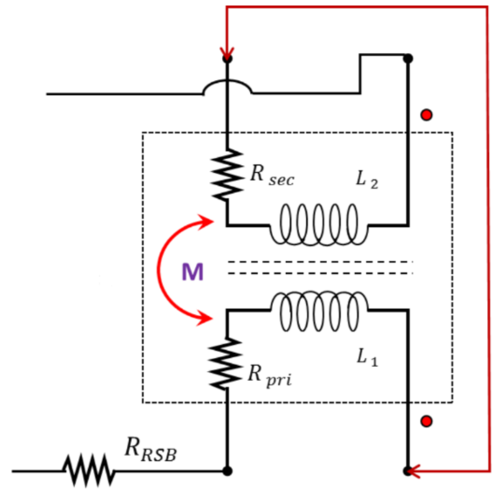


Fig 3. Fig 4.

1. Connect the primary and secondary transformer coils in series with the dotted leads connected together as in figure 4.4a. Use this combination as the test impedance.
2. Measure VS, VR, VZ, and IS.
3. Repeat this procedure crossing the terminals of the primary and secondary coils as shown in fig 4. Use these values to calculate the mutual inductance in the same manner as you calculated the self-inductance in Part II.

Calculations for Fig 3:

Function Generator settings:

$$f = 99.93\ Hz \qquad V_S = 2.007\ v$$

Measured voltages and resistance:

$$V_{Ztot} = 1.202\ v \qquad V_R = 1.204\ v \qquad R_{RSB} = 2940\ \Omega$$

Find θ_{Ztot}:

$$\theta_{Ztot} = cos^{-1}\left(\frac{V_S^2 - V_R^2 - V_{Ztot}^2}{2V_R V_{Ztot}}\right) \rightarrow \theta_{Ztot} = 66.94^o$$

Find $|I_tot|$:

$$|I_tot| = \frac{|V_R|}{R_{RSB}} \rightarrow |I_tot| = 0.41\ mA$$

Find the complex impedance, Z_{tot}, and use the reactance to find the inductance, L_{tot}:

$$|Z_{tot}| = \frac{|V_{Ztot}|}{|I_{tot}|} \rightarrow |Z_{tot}| = 2935.12\ \Omega$$

$$X_{tot} = Z_{tot}sin(\theta_{Ztot}) \rightarrow X_{tot} = j2700.63\ \Omega \qquad R_{tot} = Z_{tot}cos(\theta_{Ztot}) \rightarrow R_{tot} = 1149.6\ \Omega$$

$$L_{tot} = \frac{X_2}{j2\pi f} \rightarrow L_{tot} = 4.31\ H$$

Find Mutual Inductance, M_1:

$$M_1 = \frac{L_{tot} - L_1 - L_2}{2} \rightarrow M_1 = 0.6172\ H$$

Calculations for Fig 4:

Function Generator settings:

$$f = 99.97\ Hz \qquad V_S = 2.005\ v$$

Measured voltages and resistance:

$$V_{Ztot} = 1.192\ v \qquad V_R = 1.190\ v \qquad R_{RSB} = 1301\ \Omega$$

Find θ_{Ztot}:

$$\theta_{Ztot} = cos^{-1}\left(\frac{V_S^2 - V_R^2 - V_{Ztot}^2}{2V_R V_{Ztot}}\right) \rightarrow \theta_{Ztot} = 65.24^o$$

Find $|I_{tot}|$:

$$|I_{tot}| = \frac{|V_R|}{R_{RSB}} \rightarrow |I_{tot}| = 0.91\ mA$$

Find the complex impedance, Z_tot, and use the reactance to find the inductance, L_tot:

$$|Z_{tot}| = \frac{|V_{Ztot}|}{|I_{tot}|} \rightarrow |Z_{tot}| = 1303\ \Omega$$

$$X_{tot} = Z_{tot}sin(\theta_{Ztot}) \rightarrow X_{tot} = j1183.4\ \Omega \qquad R_{tot} = Z_{tot}cos(\theta_{Ztot}) \rightarrow R_{tot} = 545.75\ \Omega$$

$$L_{tot} = \frac{X_{tot}}{j2\pi f} \rightarrow L_{tot} = 1.88\ H$$

Find Mutual Inductance, M_2:

$$M_2 = \frac{L_{tot} - L_1 - L_2}{2} \rightarrow M_2 = 0.5978\ H$$

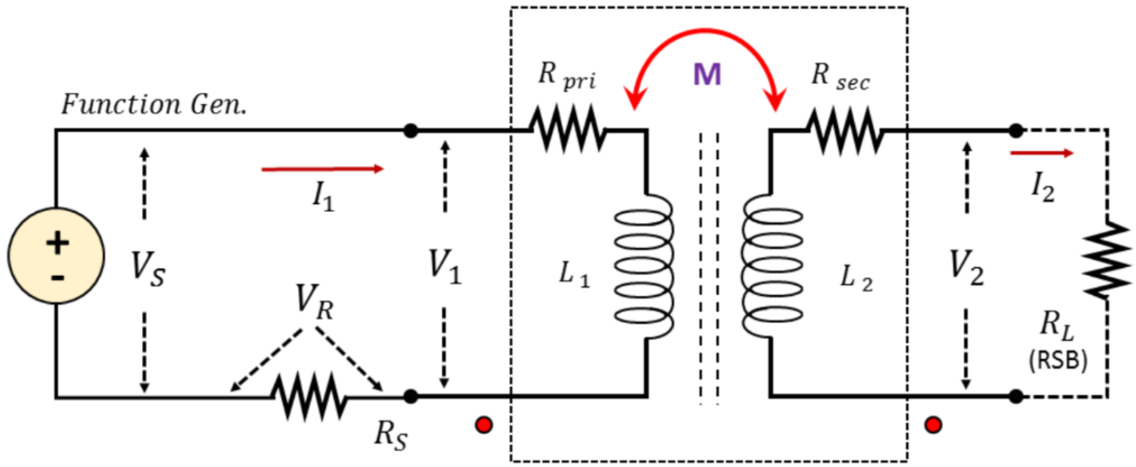
Find Mutual Inductance and Coupling Coefficient:

Using M_1 and M_2 we will calculate the average mutual inductance, M, and the coupling coefficient, k:

$$M = \frac{M_1 + M_2}{2} \rightarrow M = 0.6075\ H \qquad k = \frac{M}{\sqrt{L_1 * L_2}} \rightarrow k = 0.6916$$

Part 4: Maximum Power

Fig 5. Max Power Experimental Circuit Diagram

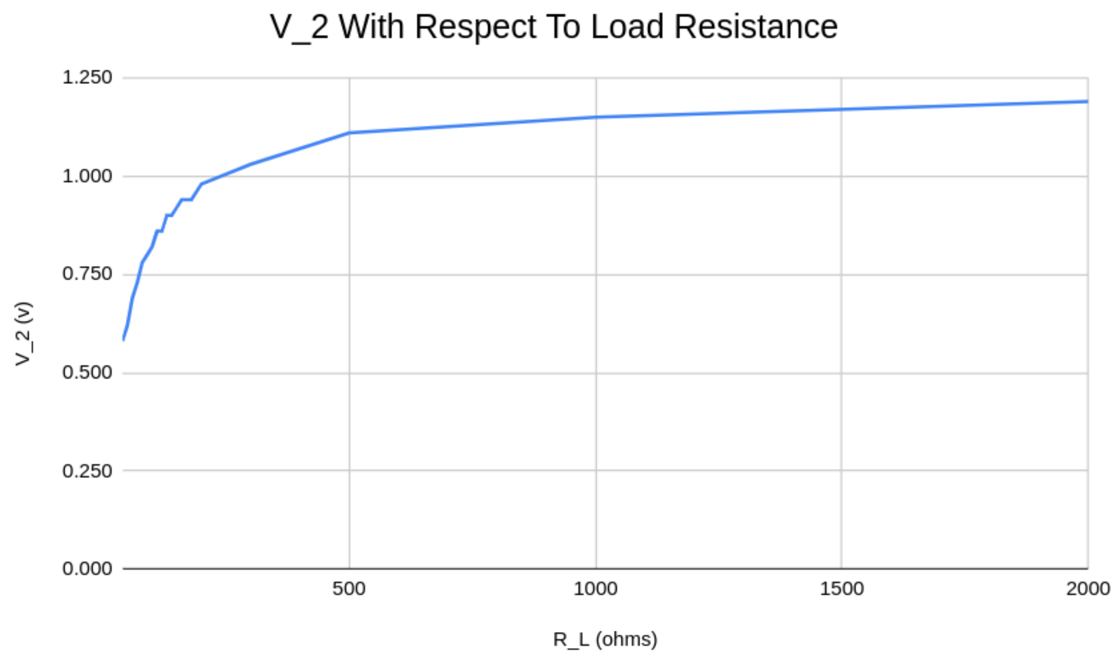


A variable load resistor will be used on the secondary side of the transformer. The load will begin at 40 Ω and increased until 2000 Ω. A 1k resistor will be placed in series with the primary transformer to allow for accurate voltage reading across V_1. After the test, the results will be compared to a Thevenin Equivalent circuit and we will use the Maximum-Power-Transfer Theorem to find the value R_L which will yield the maximum power.

Data

Voltage and Power with Respect to Resistance

R_L (ohms)	V_s (v)	V_1 (v)	V_2 (v)	P_L (W)	Turns Ratio (V_2/V_1)
40	2.005	2.410	0.580	0.00841	0.24
50	2.005	2.650	0.620	0.00769	0.23
60	2.006	2.850	0.690	0.00794	0.24
70	2.004	3.020	0.730	0.00761	0.24
80	2.005	3.180	0.780	0.00761	0.25
90	2.005	3.260	0.800	0.00711	0.25
100	2.005	3.420	0.820	0.00672	0.24
110	2.005	3.500	0.860	0.00672	0.25
120	2.004	3.580	0.860	0.00616	0.24
130	2.005	3.660	0.900	0.00623	0.25
140	2.006	3.660	0.900	0.00579	0.25
150	2.005	3.740	0.920	0.00564	0.25
160	2.005	3.820	0.940	0.00552	0.25
180	2.005	3.900	0.940	0.00491	0.24
200	2.005	3.980	0.980	0.00480	0.25
300	2.005	4.220	1.030	0.00354	0.24
400	2.006	4.300	1.070	0.00286	0.25
500	2.005	4.460	1.110	0.00246	0.25
1000	2.005	4.620	1.150	0.00132	0.25
2000	2.004	4.700	1.190	0.00071	0.25

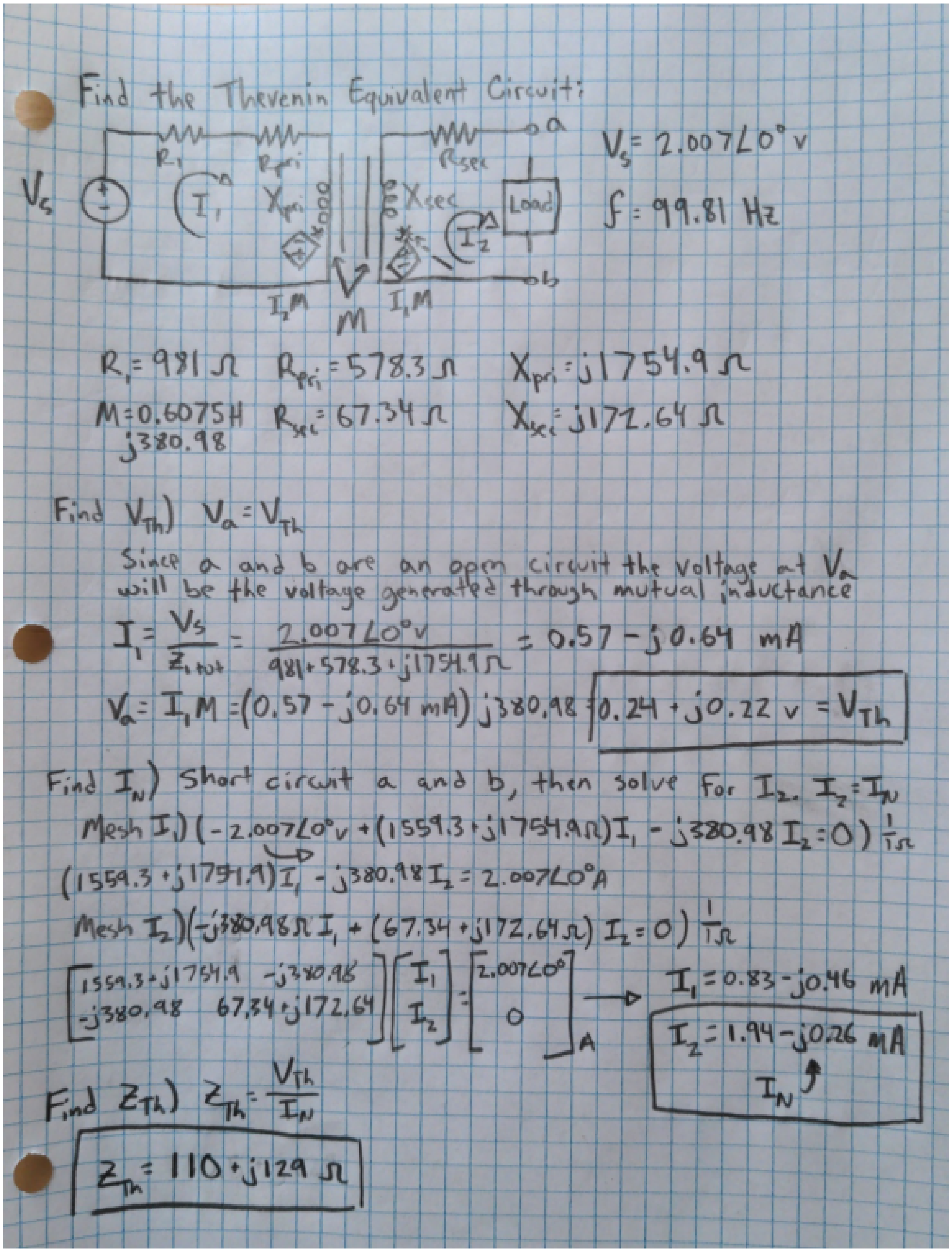


After the test, the Thevenin Equivalent circuit was calculated and we will use the Maximum-Power-Transfer Theorem to find the value R_L which will yield the maximum power.

$$V_{Th} = .33\angle 43.78^o \qquad Z_{Th} = 110 + j129\Omega$$

Max Power occurs when $Z_L = Z^*_{Th}$. This would mean that the point of max power across the Load is theoretically calculated to be when $R_L = 110\Omega$. Max Power can be found using the Max-Power-Transfer Theorem: $P_{max} = \frac{|V_{Th}|^2}{8R_{Th}}$.

$$P_{max} = \frac{|V_{Th}|^2}{8R_{Th}} \rightarrow P_{max} = .12mW$$



Calculations to find Thevenin Equivalent

Conclusion

The purpose of this lab was to gain hands on experience using and testing transformers. Parts 1-3 produced results that are within reasonable limits. The self-impedance for coils 1 and 2 were 2.8 H and 275.6 mH, respectively. Their mutual inductance was calculated to be 607.5 mH with a coupling coefficient of 0.6916 and an average turns ratio of 0.24.

In Part 4 the power that was calculated based on the voltage across the load resistor did not match the pattern that was estimated using a Thevenin Equivalent Circuit and the Max-Power-Theorem. From the Thevenin Equivalent Circuit, max power was estimated to occur when the load resistor was set to 110 Ω. When compared to the data collected, Max Power was reached when R_L was at it's lowest 40Ω. In addition to this, Max Power was estimated to be 0.12 mW via the Thevenin Equivalent, yet when testing in the lab, max power was recorded to be 8.41 mW, approximately 67 times more power than predicted.

These discrepancies between the theoretical calculations and the data collected during lab may have sprung from several sources. The two mostly likely sources of error are: user error in measurement or calculation and random variability in measurement tools or components of the circuit. More testing should be done to gain a more comprehensive investigation of max power and mutually inductive circuits.