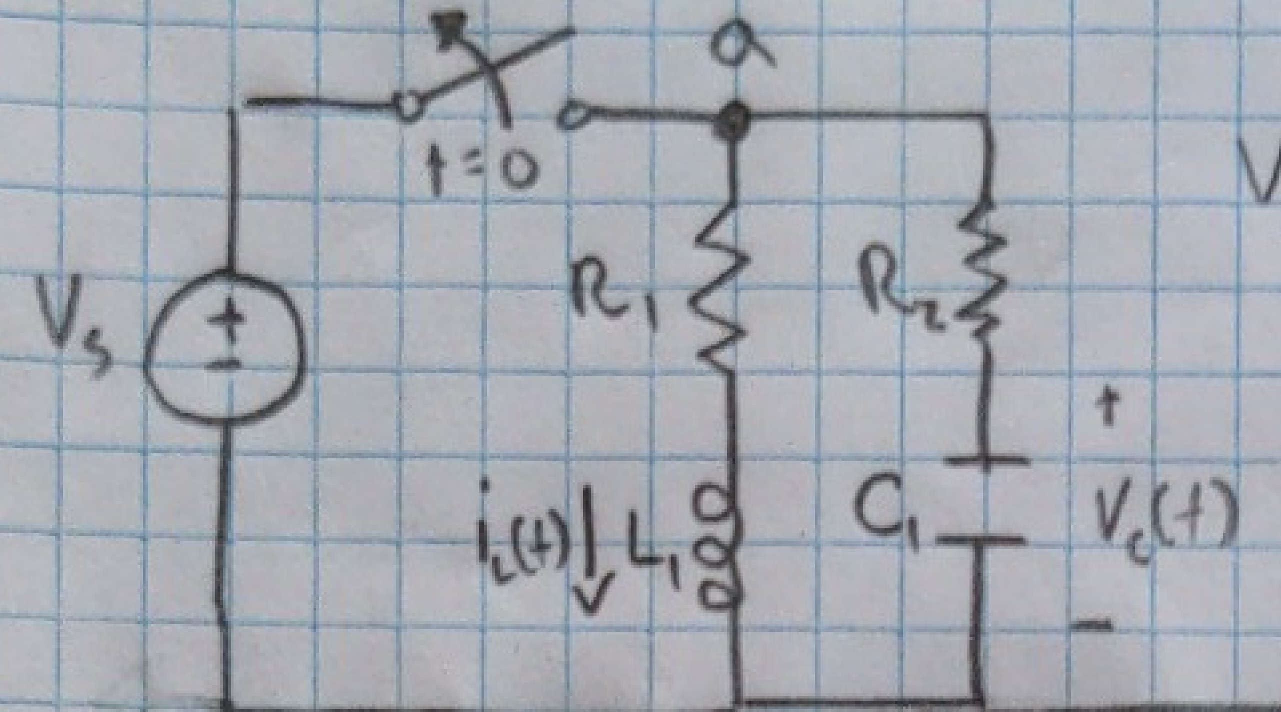
Chris Hunt HW4 ENGR 203

4.1) The switch has been closed for a long time, at $t=0$ the switch opens.

Find: a) $i_L(0^+)$, $V_c(0^+)$ b) $\frac{di_L(0^+)}{dt}$, $\frac{dV_c(0^+)}{dt}$ c) $i_L(\infty)$, $V_c(\infty)$



$V_s = 12V$ $R_1 = 6\Omega$ $R_2 = 4\Omega$

$C_1 = 400mF$ $L_1 = 2H$

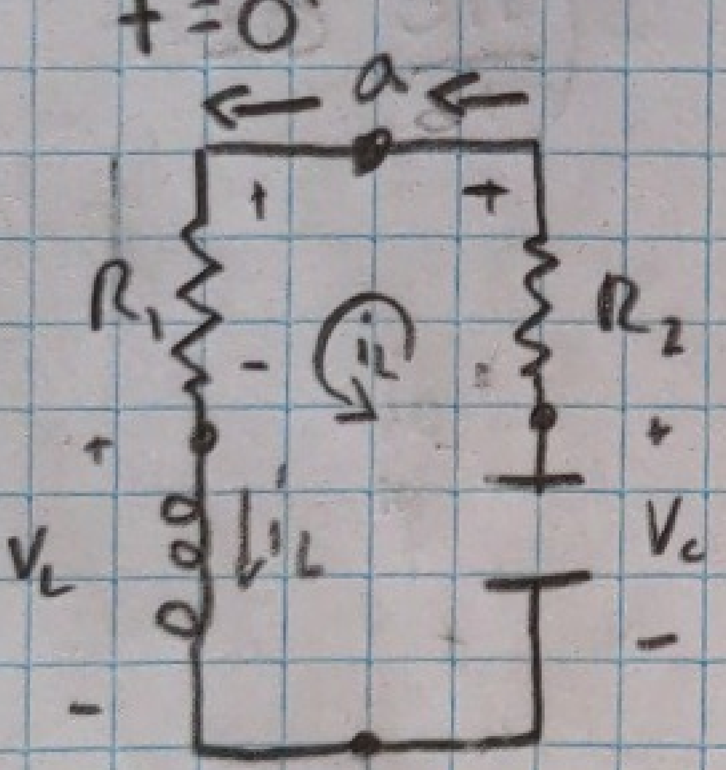
At $t=0^-$ the voltage across the capacitor with equal V_a and will have 0 current. While the current across the inductor will be the current through the branch, V_a/R_1

Answers:

t	$i_L(t)$	$V_c(t)$	$\frac{di_L}{dt}$	$\frac{dV_c}{dt}$
0^-	2A	12V		
0^+	2A	12V	-4A/s	5V/s
∞	0A	0V		

At $t=0^+$ since the capacitor cannot tolerate discontinuous voltages V_c will equal 12V. Likewise, the current across an inductor cannot change instantaneously, i_L will equal 2A

$t=0^+$



KCL @ a) $i_c = i_L = 2A$

KVL) $+V_L + (-V_c) + V_{R_2} + V_{R_1} = 0$

$+V_L - 12v + \underbrace{(2A \cdot 4\Omega)}_{8v} + \underbrace{(2A \cdot 6\Omega)}_{12v} = 0$

$V_L = -8v$

Find $\frac{di_L}{dt}$)

$\frac{di_L}{dt} = \frac{V_L(0^+)}{L} = \frac{-8v}{2H} = -4 A/s$

Find $\frac{dV_c}{dt}$)

$\frac{dV_c}{dt} = \frac{i_c(0^+)}{C} = \frac{2A}{400mF} = 5 V/s$

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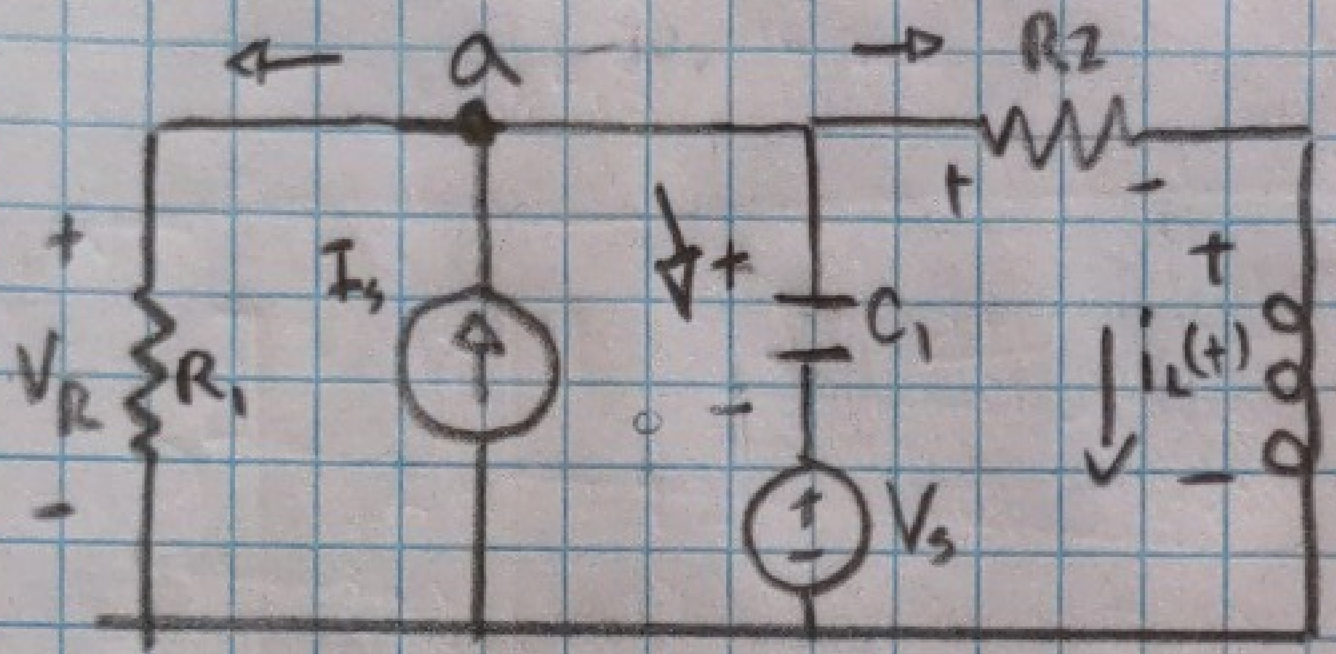
4.2) For the following circuit, Find:

t	i_L	V_c	V_R	$\frac{di_L}{dt}$	$\frac{dV_c}{dt}$	$\frac{dV_R}{dt}$
0^-	0A	-10v	0v			
0^+	0A	-10v	0v	0A/s	8 V/s	0V/s
∞	0.4A	6V	16v			



$V_s = 10v$ $I_s = 2u(t)\,A$

$R_1 = 10\Omega$ $R_2 = 40\Omega$

$C_1 = 250mF$ $L_1 = 125mH$

At $t=0^-$ the capacitor acts as an open circuit, so there is a voltage across C_1 but no current able to flow through any circuit branches, and no voltage difference across R_1

At $t=0^+$ since the capacitor cannot tolerate instantaneous voltage changes $V_c(0^+) = V_c(0^-)$ and since inductors cannot tolerate instantaneous changes in current $i_L(0^+) = i_L(0^-) = 0A$. Since there is no voltage change at node a the voltage across R_1 stays the same, 0 V.

Find $\frac{dV_c}{dt}$) KCL) $I_s = i_{R_1}' + i_c' + i_L'$

$2A = 0 + i_c + 0$

$i_c = 2A \longrightarrow \frac{dV_c(0^+)}{dt} = \frac{2A}{250mF} = 8\,V/s$

Find $\frac{di_L}{dt}$) KVL) $-V_s - V_c + V_{R_2} + V_L = 0$

$-10v - (-10v) + V_{R_2} + V_L = 0$

$V_L = 0v \longrightarrow \frac{di_L(0^+)}{dt} = \frac{0v}{125mH} = 0\,A/s$

Find $\frac{dV_R}{dt}$) $\frac{dV_R}{dt} = \frac{d(i_{R_1} \cdot R_1)}{dt} = 0\,V/s$

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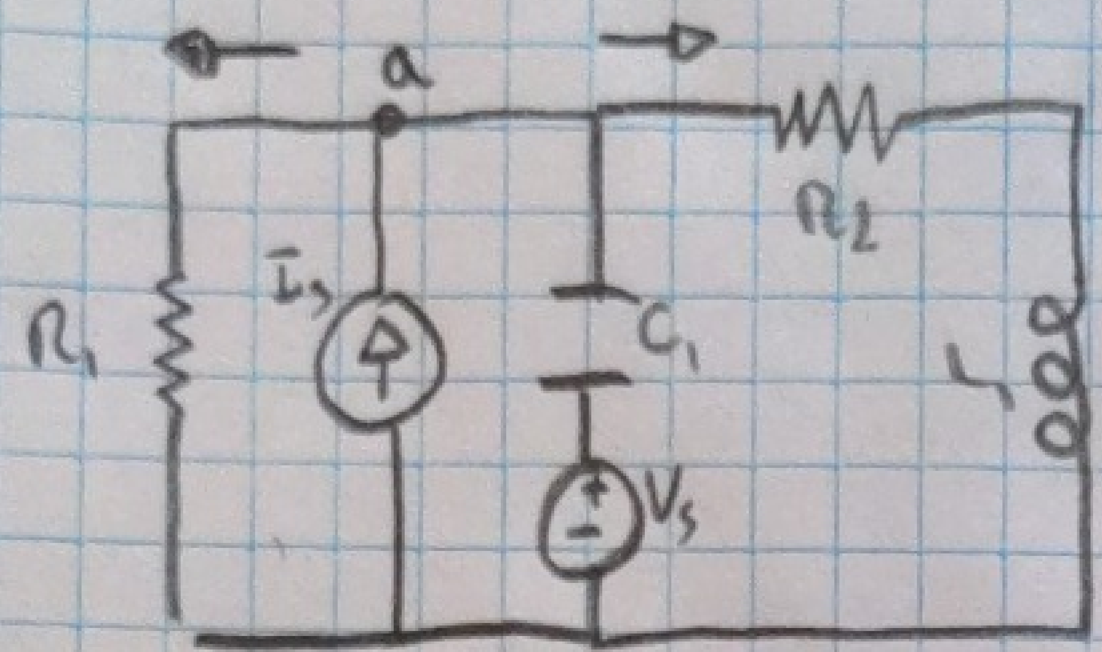
4.2) cont.

At $t = \infty$)



Find $i_{R_1}(\infty)$ and $i_{L_1}(\infty)$

Use a current divider.

$i_L = 2A\left(\frac{10\Omega}{50\Omega}\right) = 0.4A$

$i_{R_1} = 2A - 0.4A = 1.6A$

Find $V_R(\infty)$) $V_R(\infty) = i_{R_1} \cdot R_1 = 1.6A \cdot 10\Omega = 16V$

Find $V_c(\infty)$) $V_c(\infty) = V_R(\infty) - V_s = 16V - 10V = 6V$

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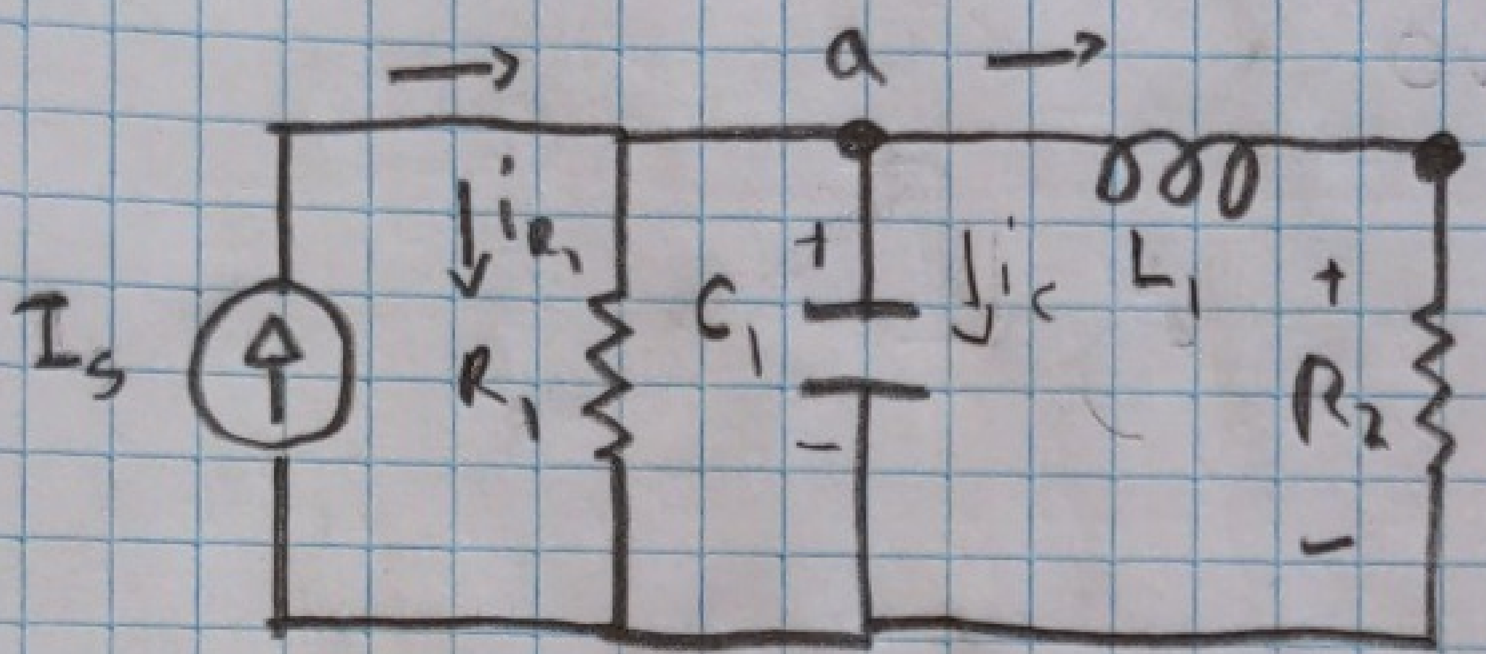
4.3) For the Following Circuit, Find:

t	i_L	V_c	i_{R_1}	V_{R_2}	$\frac{di_{R1}}{dt}$	$\frac{dV_{R2}}{dt}$	$\frac{dV_c}{dt}$
0^-	0A	0V	0A	0V			
0^+	0A	0V	0A	0V	0 A/s	0 V/s	16 V/s
∞	1.6A	9.6V	2.4A	9.6V			



$I_s = 4u(t)$ A

$R_1 = 4\,\Omega$ $R_2 = 6\,\Omega$

$C_1 = 250$mF $L_1 = 1$ H

At $t=0^-$ there is no source current or voltage. All values are zero

At $t=\infty$ the capacitor acts as an open circuit and the inductor acts as a short circuit.

Find $i_{R_1}(\infty)$) $i_{R_1}(\infty) = 4A\left(\frac{6\Omega}{10\Omega}\right) = 2.4A$

Find $V_{R_2}(\infty)$) $V_{R_2}(\infty) = V_a(\infty) = 2.4A \cdot 4\Omega = 9.6V$

At $t=0^+$ however, the current source initiates 4 A of current, changing the circuit dynamics. The capacitory maintains the voltage of 0V at node a and the inductor maintains 0 A of current down its branch. With no voltage across R_1 there is no current

Find $\frac{dV_c(0^+)}{dt}$) KCL @a) $I_s = i_{R1} + i_L + i_c$

$4A = 0 + 0 + i_c$

$i_c(0^+) = 4A \rightarrow \frac{dV_c(0^+)}{dt} = \frac{4A}{250mF} = 16 V/s$

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4.3) cont.

Find $\frac{di_L}{dt}$) KVL) $-V_c + V_L + V_{R_2} = 0$

$0 + V_L + i_L(0^+)\cdot 6\Omega = 0$

$V_L = 0 \rightarrow \frac{di_L(0^+)}{dt} = \frac{0V}{1H} = 0\,A/s$

Find $\frac{dV_R}{dt}$) $\frac{dV_{R_2}(0^+)}{dt} = \frac{d}{dt}(i_L R_2) = 0\,V/s$

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4.4) The current in an RLC series circuit is described by:

$$\frac{d^2i}{dt^2} + 10\frac{di}{dt} + 25i = 0 \qquad i(0) = 10A \quad \frac{di(0)}{dt} = 0 A/s$$

Find $i(t)$ for $t>0$:

$$\frac{R}{L} = 10 \qquad \frac{1}{LC} = 25$$

$$\alpha = \frac{R}{2L} = \frac{1}{2}\cdot 10 = 5 \ np/s$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \left(\frac{1}{LC}\right)^{.5} = (25)^{.5} = 5 \ rad/s$$

$\alpha = \omega_0$ Critically dampened

$i(t)$ for the critically damped circuit: $i(t) = (A_2 + A_1 t)e^{-\alpha t}$

$$i(t) = (A_2 + A_1 t)e^{-5t}$$

Find $\frac{di(t)}{dt}$) $\frac{di(t)}{dt} = A_1 e^{-5t} - 5(A_2 + A_1 t)e^{-5t}$

$i(0) = 10A = A_2$ $\qquad \frac{di(0)}{dt} = 0 A/s = A_1 - 5A_2$

$A_2 = 10A \rightarrow 0 A/s = A_1 - 5(10) \rightarrow A_1 = 50$

Therefore, $\boxed{i(t) = (10 + 50t)e^{-5t}}$