Chris Hunt HW17 ENGR 203

17.1) Find the Fourier Transform of $\delta(t+3) - \delta(t-3)$

$$F(\omega) = \mathcal{F}[\delta(t+3) - \delta(t-3)] = \int_{-\infty}^{\infty} \delta(t+3) e^{-j\omega t} dt - \int_{-\infty}^{\infty} \delta(t-3) e^{-j\omega t} dt$$

$$F_1(\omega) = \int_{-\infty}^{\infty} \delta(t+3) e^{-j\omega t} dt \qquad t_0 = -3$$

The only time this function won't be zero is at $t_0 = -3$

$$F_1(\omega) = e^{j3\omega}$$

$$F_2(\omega) = -\int_{-\infty}^{\infty} \delta(t-3) e^{-j\omega t} dt \qquad t_0 = 3$$

The only time this will be nonzero is at $t_0 = 3$

$$F_2(\omega) = -e^{-j3\omega}$$

$$F(\omega) = e^{j3\omega} - e^{-j3\omega}$$

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17.2) Find the Fourier Transform of $\cos(2t)u(t)$.

$$f(t) = \cos(2t)u(t) \text{ or } \begin{cases} 0, & t<0 \\ \cos(2t), & t \geq 0 \end{cases}$$

$$\cos(2t) = \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right)$$

$$F(\omega) = \int_0^\infty \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right) e^{-j\omega t}\, dt$$

$$= \frac{1}{2}\left(\int_0^\infty e^{-j(\omega-2)t}\, dt + \int_0^\infty e^{-j(2+\omega)t}\, dt\right)$$

$$= \frac{1}{2}\left(\frac{1}{-j(\omega-2)}\int_0^\infty -j(\omega-2)e^{-j(\omega-2)t}\, dt + \frac{1}{-j(2+\omega)}\int_0^\infty -j(2+\omega)e^{-j(2+\omega)t}\, dt\right)$$

$$= \frac{1}{2}\left(\frac{1}{-j(\omega-2)}\left(e^{-j(\omega-2)t}\Big|_0^\infty\right) + \frac{1}{-j(2+\omega)}\left(e^{-j(2+\omega)t}\Big|_0^\infty\right)\right)$$

$$= \frac{1}{2}\left(\frac{1}{j2-j\omega}(-1) + \frac{1}{-j2-j\omega}(-1)\right)$$

$$= \frac{1}{2}\left(\frac{-1}{j2-j\omega} + \frac{1}{j2+j\omega}\right)$$

$$= \frac{1}{2}\left(\frac{-(j2+j\omega)+(j2-j\omega)}{(j2-j\omega)(j2+j\omega)}\right)$$

$$= \frac{1}{2}\left(\frac{-j2\omega}{j^2 4 + j^2 2\omega - j^2 2\omega - j^2\omega^2}\right)$$

$$= \frac{1}{\cancel{2}}\left(\frac{-j\cancel{2}\omega}{-4+\omega^2}\right)$$

$$= \frac{-j\omega}{-4+\omega^2}$$

$$= \boxed{\frac{j\omega}{4-\omega^2} = F(\omega)}$$

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17.3) Find the Fourier Transform of $e^{-3t}\sin(10t)u(t)$

$$f(t) = e^{-3t}\sin(10t)u(t) \quad \text{or} \quad \begin{cases} 0, & t<0 \\ e^{-3t}\sin(10t), & t \geq 0 \end{cases}$$

$$\sin(10t) = \frac{1}{j2}\left(e^{j10t} - e^{-j10t}\right)$$

$$f(t) = \frac{1}{j2}\left(e^{(-3+j10)t} - e^{(-3-j10)t}\right)$$

$$F(\omega) = \int_0^\infty \frac{1}{j2}\left(e^{(-3+j10)t} - e^{(-3-j10)t}\right)e^{-j\omega t}\,dt$$

$$= \frac{1}{j2}\left(\int_0^\infty e^{-t(3-j10+j\omega)}\,dt - \int_0^\infty e^{-t(3+j10+j\omega)}\,dt\right)$$

$$= \frac{1}{j2}\left(\frac{-1}{3-j10+j\omega}\int_0^\infty -3+j10-j\omega \; e^{-t(3-j10+j\omega)}\,dt\right.$$

$$\left. + \frac{1}{3+j10+j\omega}\int_0^\infty -3-j10-j\omega \; e^{-t(3+j10+j\omega)}\,dt\right)$$

$$= \frac{1}{j2}\left(\frac{1}{3-j10+j\omega} - \frac{1}{3+j10+j\omega}\right)$$

$$= \frac{1}{j2}\left(\frac{3+j10+j\omega-3+j10-j\omega}{(3-j10+j\omega)(3+j10+j\omega)}\right)$$

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17.3) cont.

$$F(w) = \frac{1}{\cancel{j}2}\left(\frac{\cancel{3}+\cancel{j}10+\cancel{jw}-\cancel{3}-\cancel{j}10-\cancel{jw}}{(3+j10+jw)(3-j10+jw)}\right)$$

$$F(w) = \frac{10}{((3+jw)+j10)((3+jw)-j10)}$$

$$F(w) = \frac{10}{(3+jw)^2 - j^2 100}$$

$$\boxed{F(w) = \frac{10}{(3+jw)^2 + 10^2}}$$