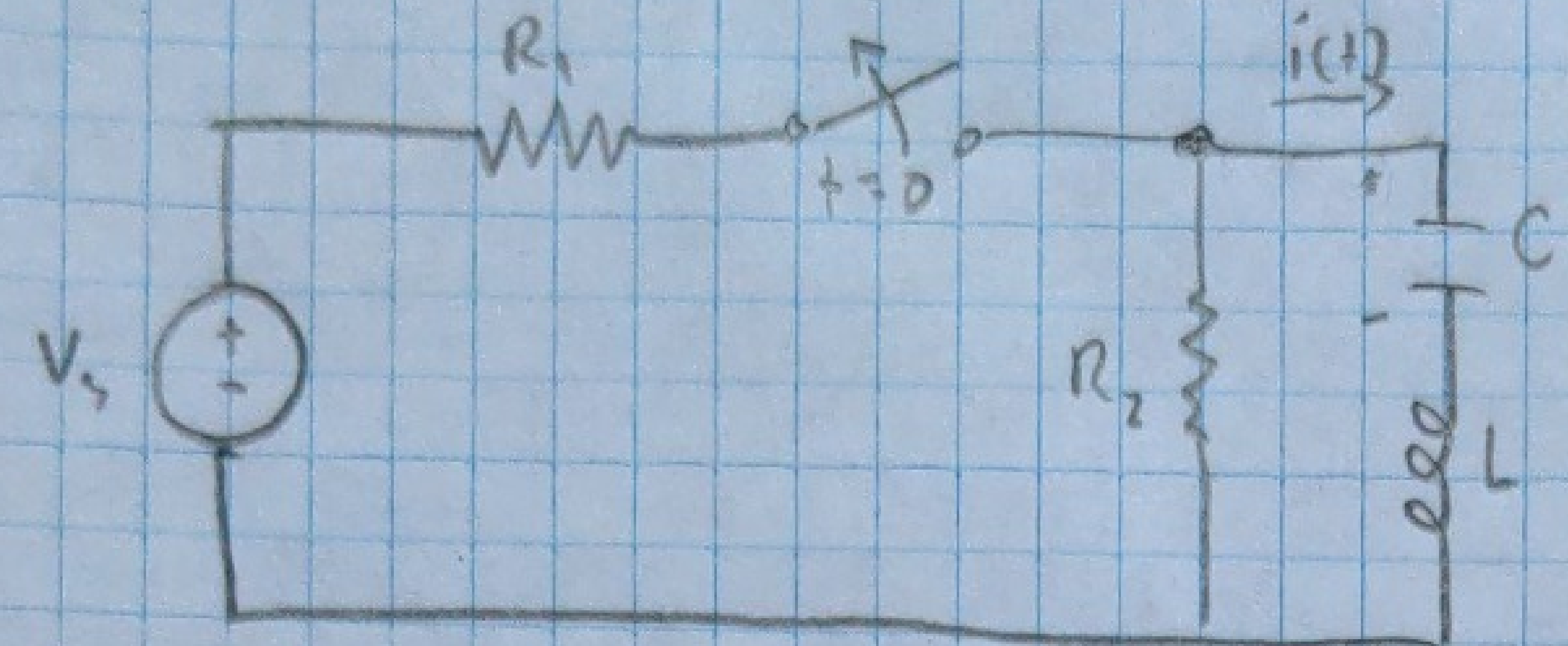
Chris Hunt HW 5 ENGR 203

5.1) The switch has been closed for a very long time and opens at $t=0$. Find $i(t)$ for $t \geq 0$.



$V_s = 20V$ $R_1 = 10\Omega$

$R_2 = 40\Omega$ $C = 1mF$

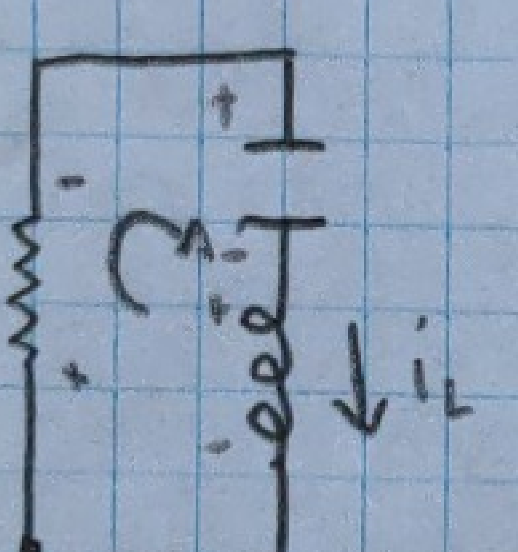
$L = 2.5H$

t	i_L	V_c	$\frac{di_L}{dt}$	$\frac{dV_c}{dt}$
0^-	0A	16V		
0^+	0A	16V	-6.4 A/s	0V/s
∞	0A	0V		

$t=0^-$) $i_L = 0A$ $V_c = V_s\left(\frac{R_2}{R_1+R_2}\right)$

$V_c = 16v$

$t=0^+$)



Find $\frac{di_L}{dt}$) $\frac{di_L}{dt} = \frac{V_L}{L}$

KVL) $V_{R_1} + V_c + V_L = 0$

$0 + 16v + V_L = 0 \rightarrow V_L = -16V$

$\frac{di_L}{dt} = \frac{-16V}{2.5H} = -6.4 A/s$

$\frac{dV_c}{dt} = \frac{i_c}{C} \rightarrow \frac{0}{C} = 0 V/s$

Find $i(t)$ for $t \geq 0$) $R = 40\Omega$ $C = 1mF$ $L = 2.5H$

$\alpha = \frac{R}{2L} = \frac{40\Omega}{5H} = 8\ np/s$ $\omega_0 = \frac{1}{\sqrt{LC}} = 20\ rad/s \rightarrow \alpha < \omega_0$ under damped complex roots

Find ω_d) $\omega_d = \sqrt{\omega_0^2 - \alpha^2} = \sqrt{336}\ rad/s$

$i(t) = e^{-8t}\left(B_1\cos(\sqrt{336}t) + B_2\sin(\sqrt{336}t)\right) A$

$\frac{di(t)}{dt} = -8e^{-8t}\left(B_1\cos(\sqrt{336}t) + B_2\sin(\sqrt{336}t)\right) + e^{-8t}\left(-\sqrt{336}B_1\sin(\sqrt{336}t) + \sqrt{336}B_2\cos(\sqrt{336}t)\right)$

cont. →

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5.1) cont.

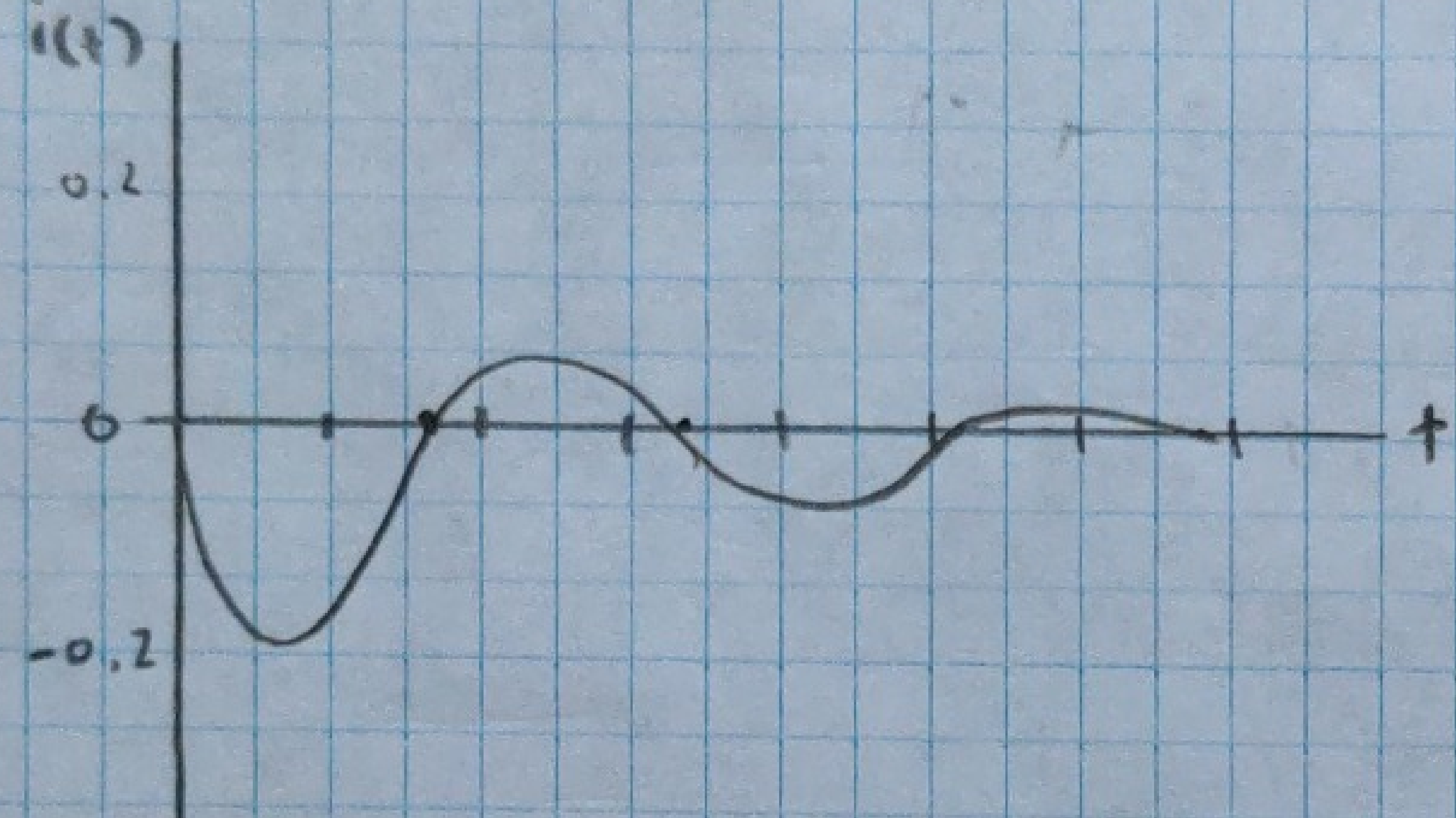
$$i(0) = 0\,A = B_1$$

$$\frac{di(0)}{dt} = -6.4\,A/s = -8B_1 + \sqrt{336}\,B_2$$

$$\frac{di(0)}{dt} = -6.4\,A/s = \sqrt{336}\,B_2$$

$$B_2 = \frac{-6.4}{\sqrt{336}} \approx -0.349$$

$$i(t) = -0.349e^{-8t}\sin(\sqrt{336}\,t)\ A$$



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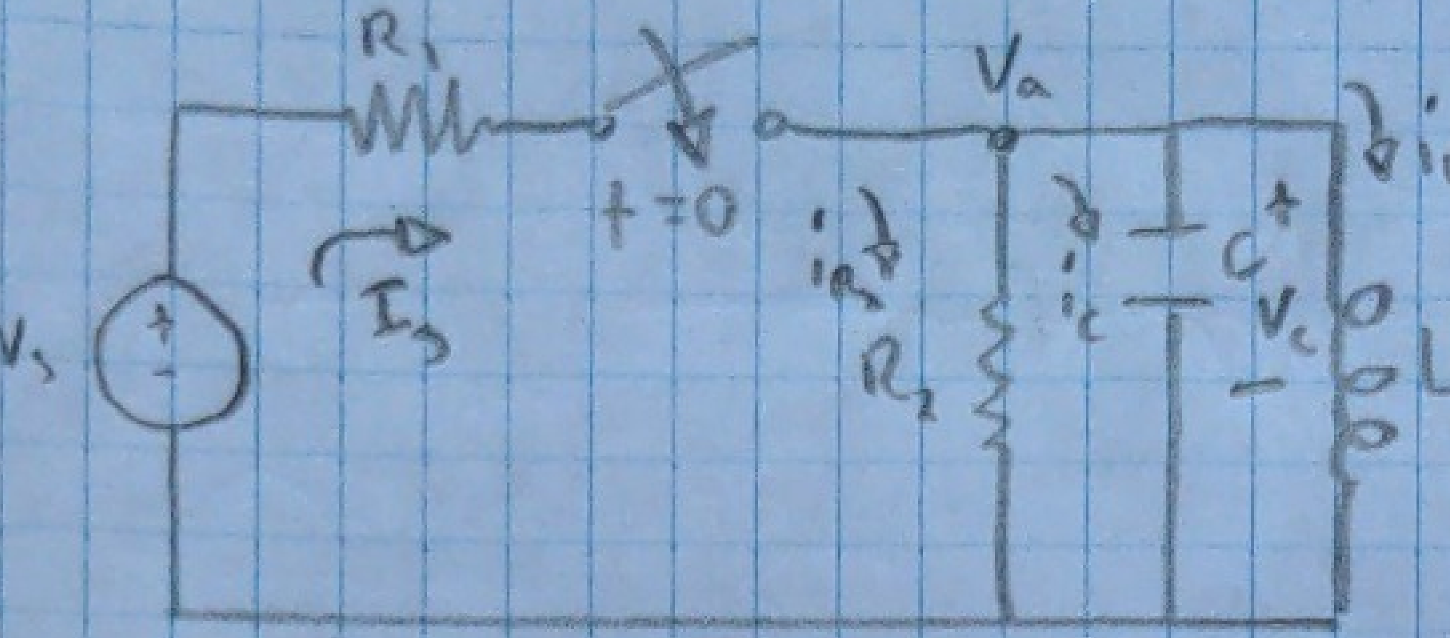
5.2) Find $V_c(t)$ for $t>0$. Assume initial conditions are 0.



$V_s = 20V$ $R_1 = 5\Omega$ $R_2 = 1\Omega$

$C = 1F$ $L = 250mH$

$V_c(0^-) = 0V$ $i_c(0^-) = 0A$

$i_L(0^-) = 0A$

t	i_L	V_c	$\frac{di_L}{dt}$	$\frac{dV_c}{dt}$
0^-	0A	0V		
0^+	0A	0V	0 A/s	4 V/s
∞	0A	0V		

At $t=0^+$) Since the capacitor maintains the voltage at V_a at 0V, the voltage across L is 0V.

$$V_L = 0V \rightarrow \frac{di_L}{dt} = \frac{V_L}{L} = \frac{0V}{250mH} = 0 A/s$$

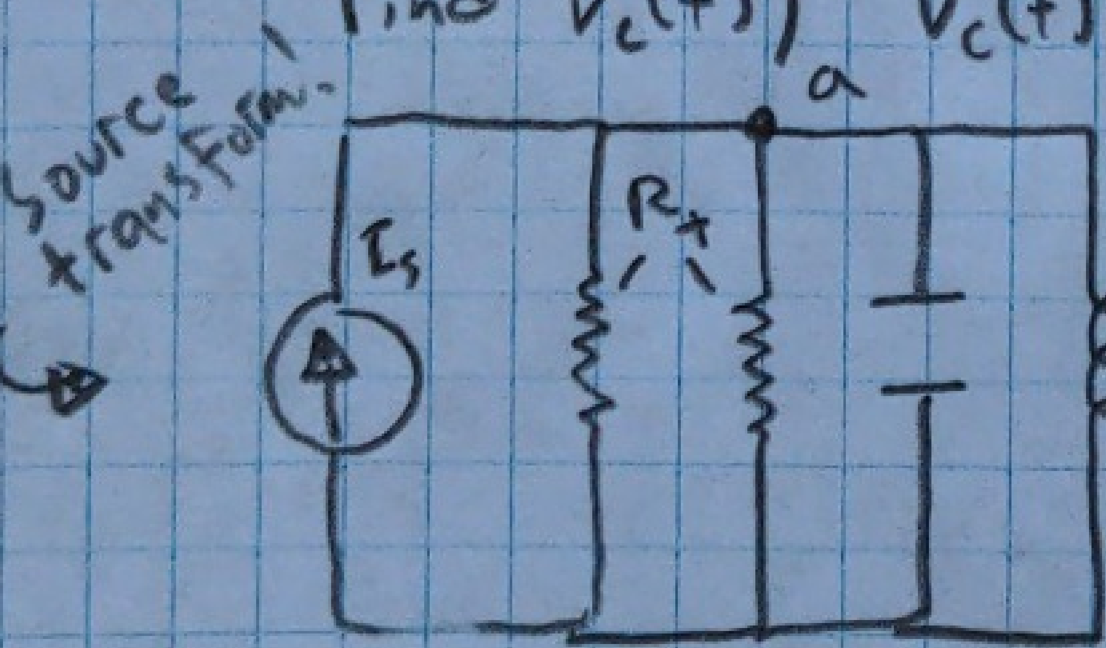
Find dV_c/dt) $\frac{dV_c}{dt} = \frac{i_c}{C}$

KCL @ a) $I_s = i_R + i_c + i_L \rightarrow \frac{20V}{5\Omega} = \frac{0V}{1\Omega} + i_c + 0A$

$$4A = i_c \rightarrow \frac{dV_c}{dt} = \frac{4A}{1F} = 4 V/s$$

Find $V_c(t)$) $V_c(t) = V_t + V_{ss}$ $V_{ss} = V_c(\infty) = 0V$

Source transform!



$$I_s = \frac{20V}{5\Omega} = 4A \qquad R_t = \frac{5}{6}\Omega$$

$C = 1F$ $L = 250mH$

$$\alpha = \frac{1}{2RC} = \frac{1}{10/6} = 3/5 \text{ np/s} \qquad w_0 = \frac{1}{\sqrt{.25}} = 2 \text{ rad/s}$$

$\alpha < w_0 \rightarrow$ under damped

$$w_d = \sqrt{2^2 - (3/5)^2} = 3.64 \text{ rad/s}$$

$$V(t) = e^{-3/5 t}(B_1 \cos(3.64t) + B_2 \sin(3.64t))$$

$$\frac{dV(t)}{dt} = -\frac{3}{5}e^{-3/5t}(B_1\cos(3.64t) + B_2\sin(3.64t)) + e^{-3/5t}(-3.64B_1\sin(3.64t) + 3.64B_2\cos(3.64t))$$

$\longrightarrow$

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5.2) Cont.

$$V(0) = 0\,V = e^{0}(B_1\cos(0) + B_2\sin(0)) = B_1 = 0$$

$$\frac{dV(0)}{dt} = -\frac{3}{5}e^{0}(B_1\cos(0) + B_2\sin(0)) + e^{0}(-3.64B_1\sin(0) + 3.64B_2\cos(0))$$

$$\frac{dV(0)}{dt} = 4\,V/s = 3.64B_2 \Rightarrow B_2 = 1.1\,A$$

$$\boxed{V(t) = e^{-3/5t}(1.1\sin(3.64t))\ V}$$

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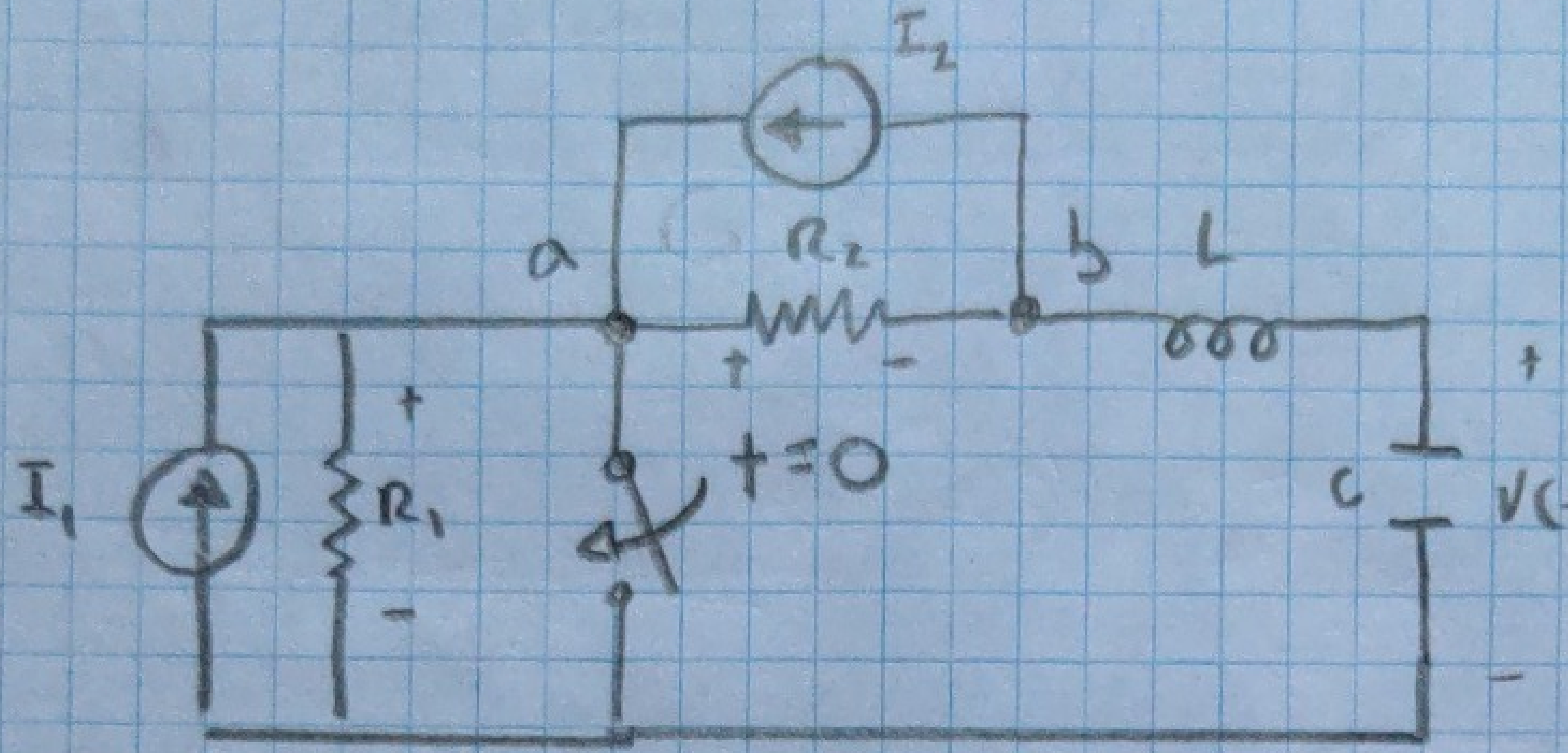
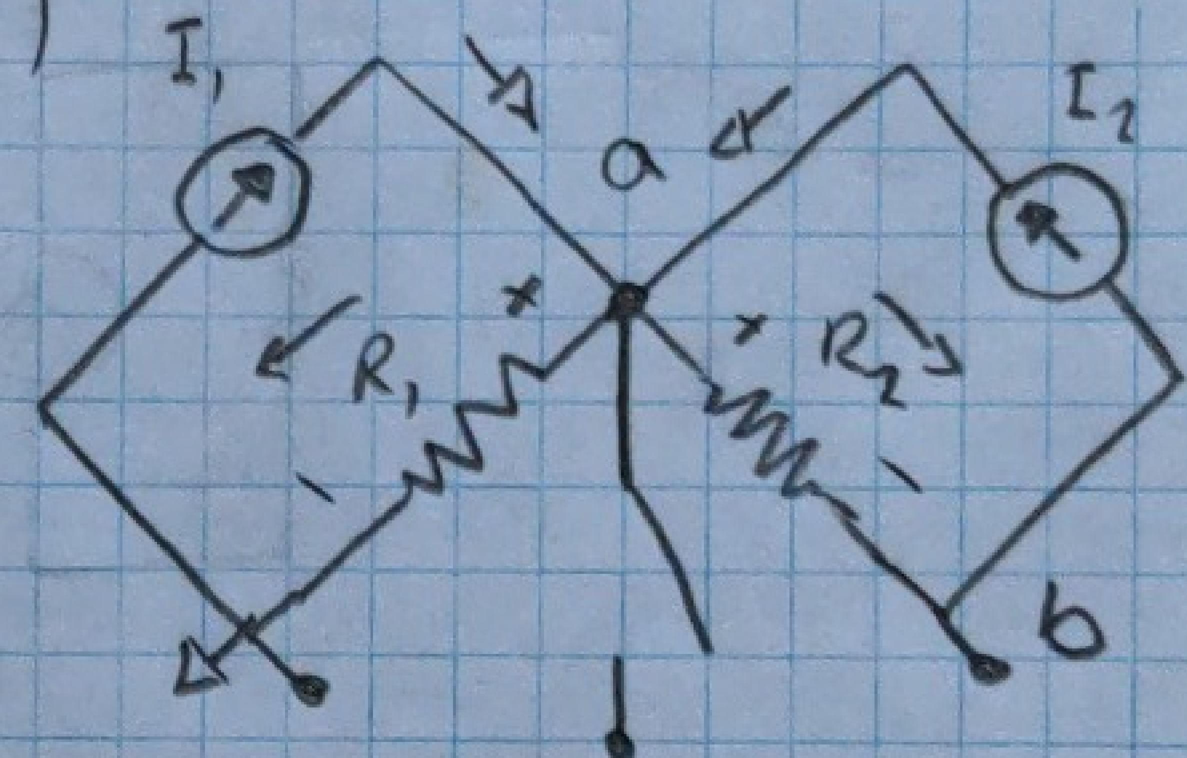
5.3) Find v(t) for t>0. The switch has been open for a long time and closes at t=0.



$I_1 = 4A$ $I_2 = 2A$ $R_1 = 1\Omega$ $R_2 = 6\Omega$ $L = 1H$ $C = 40mF$

t	i_L	V_c	$\frac{di_L}{dt}$	$\frac{dV_c}{dt}$
0^-	0A	-8V		
0^+	0A	-8V		0V/s
∞	0A			

$t = 0^-$)



KCL @ a) $I_1 + I_2 = i_{R_1} + i_{R_2}$

$4A + 2A = i_{R_1} + i_{R_2}$

i_{R_2} must equal I_2 and i_{R_1} must equal I_1

$V_a = I_1 \cdot R_1 = 4A \cdot 1\Omega = 4V$

$V_{R_2} = I_2 \cdot R_2 = 12V$

$V_b = V_a - V_{R_2} = 4V - 12V = \boxed{-8V = V_c}$

→

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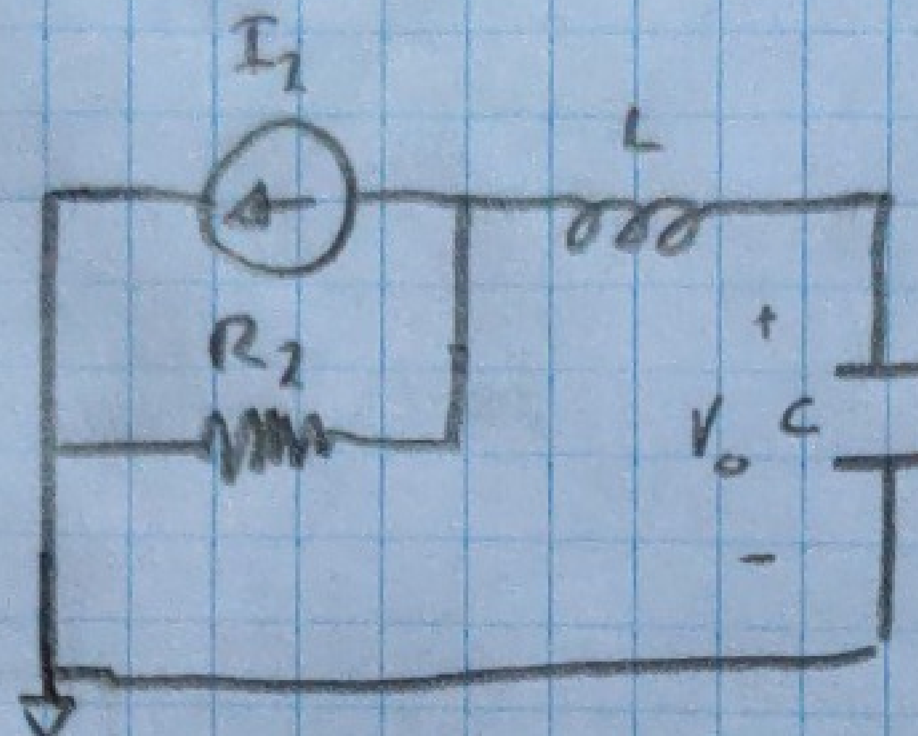
5.3) cont.

$t=0^+)$ $\frac{dV_c}{dt} = \frac{i_c}{C}$ Since there is no current through the inductor at $t=0^+$ there is no current through the capacitor

$\frac{dV_c}{dt} = \frac{0A}{40mF} = 0 V/s$



$\alpha = \frac{6\Omega}{2(1H)} = 3\ Np/s$

$\omega_0 = \frac{1}{\sqrt{1H \cdot 40mF}} = 5\ rad/s$

$\alpha < \omega_0$ underdamped

$\omega_d = \sqrt{25-9} = 4\ rad/s$

$v(t) = e^{-3t}(B_1\cos(4t) + B_2\sin(4t))$

$\frac{dv(t)}{dt} = -3e^{-3t}(B_1\cos(4t) + B_2\sin(4t)) + e^{-3t}(-4B_1\sin(4t) + 4B_2\cos(4t))$

$v(0^+) = -8v = \overset{1}{e}(B_1\overset{1}{\cos}(0) + B_2\overset{0}{\sin}(0))$

$-8v = B_1$

$\frac{dV(0^+)}{dt} = 0 V/s = -3B_1 + 4B_2$

$0 V/s = 24 + 4B_2$

$-24 = 4B_2$

$B_2 = 6v$

$$\boxed{v(t) = e^{-3t}(-8\cos(4t) + 6\sin(4t))\ v}$$

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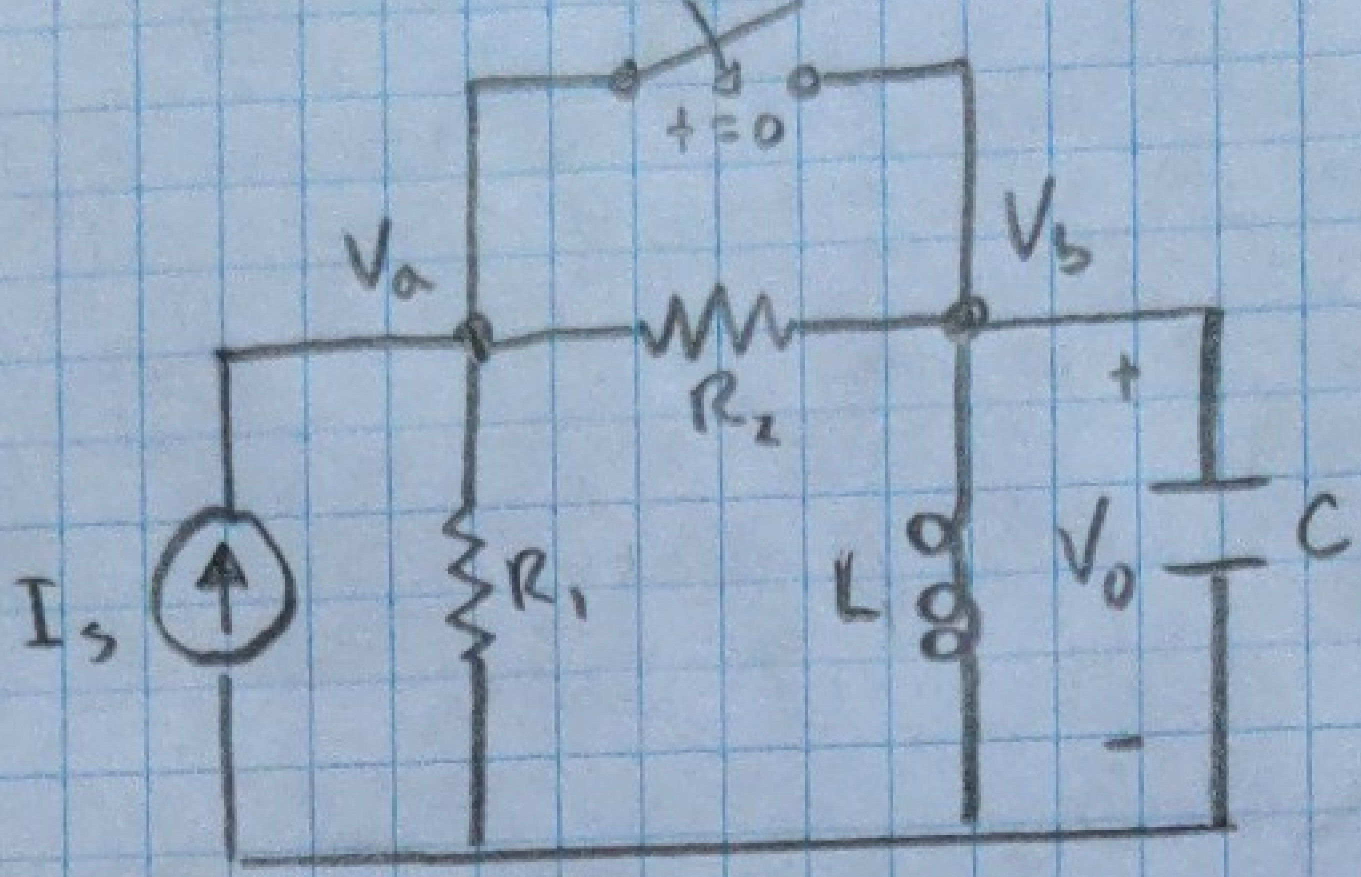
5.4) Find the output voltage $V_o(t)$. The switch has been open for a long time and closes at $t=0$



$I_s = 3A$ $R_1 = 5\Omega$

$R_2 = 10\Omega$ $L = 1H$

$C = 10mF$

At $t=0^-$) L acts as a short circuit to ground. Therefore $V_b(0^-) = 0v = V_o(0^-)$.

The current through L will be the same as the current through R_2

$$i_L(0) = I_s\left(\frac{R_1}{R_1+R_2}\right) = 3A\left(\frac{5\Omega}{15\Omega}\right) = 1A$$

t	i_L	V_c	$\frac{dV_c}{dt}$	$\frac{di_L}{dt}$
0^-	1A	0v		
0^+	1A	0v	200 V/s	0 A/s
∞	3A	0v		

At $t=0^+$) Since the capacitor maintains the voltage at V_b and now $V_a = V_b$ there is no current across R_1 or R_2

Find i_c) $I_s = i_L + i_c$

$3A = 1A + i_c \rightarrow i_c = 2A$

$$\frac{dV_c}{dt} = \frac{2A}{10mF} = 200 V/s \qquad \frac{di_L}{dt} = \frac{0v}{1H} = 0 A/s$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2(5\Omega)10mF} = 10\ np/s \qquad \omega_0 = \frac{1}{\sqrt{LC}} = 10\ rad/s$$

$\alpha = \omega_0 \rightarrow$ critically damped.

$$v(t) = (A_1 + A_2 t)e^{-\alpha t}$$

$$\frac{dv(t)}{dt} = A_2 e^{-\alpha t} + (A_1 + A_2 t) - \alpha e^{-\alpha t}$$

$\longrightarrow$

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5.4) cont.

$V(0) = 0v = A_1$

$\frac{dV(0)}{dt} = 200 V/s = A_2 e^0 - 10A_1$

$200 V/s = -10 A_1 \rightarrow A_1 = -20 A$

$\boxed{V_o(t) = -20te^{-10t} \; V}$