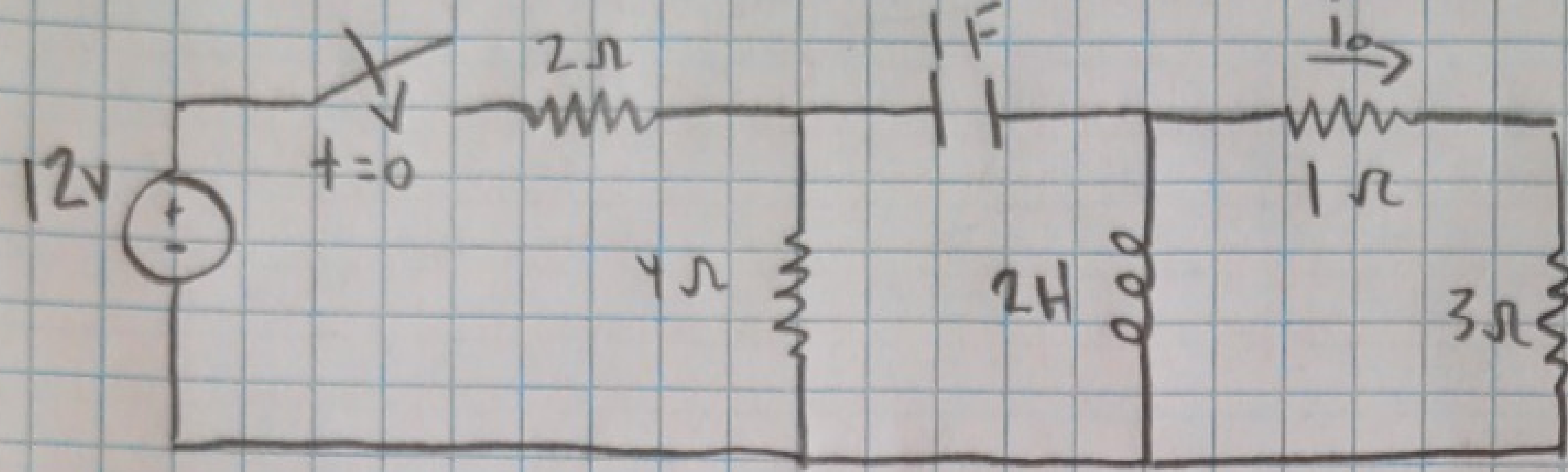
Chris Hunt HW 11 ENGR 203

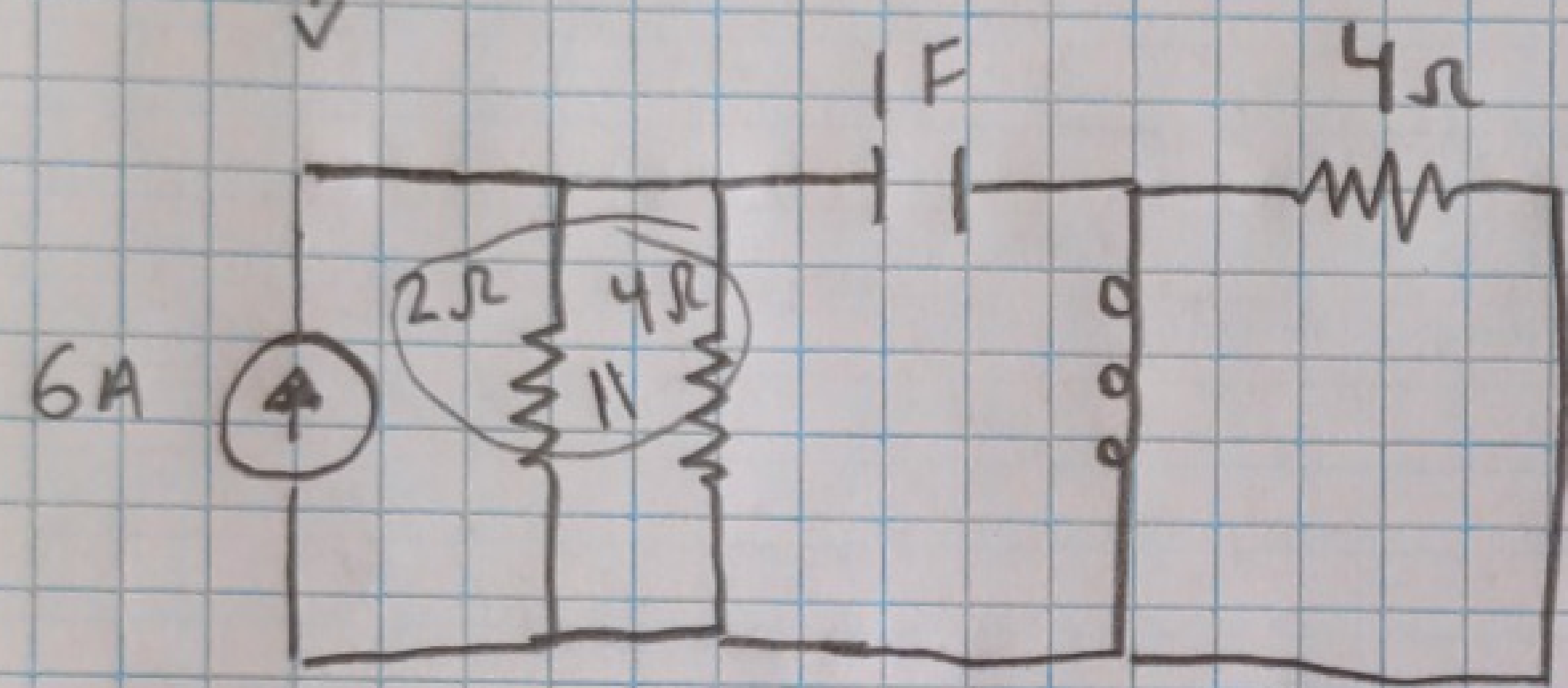
11.1) Find $i_o(t)$, $t>0$. The switch closes at $t=0$



$V_c(0^-)=0\,V$ $I_c(0^-)=0\,A$

$V_L(0^-)=0\,V$ $I_L(0^-)=0\,A$

↓ ST!



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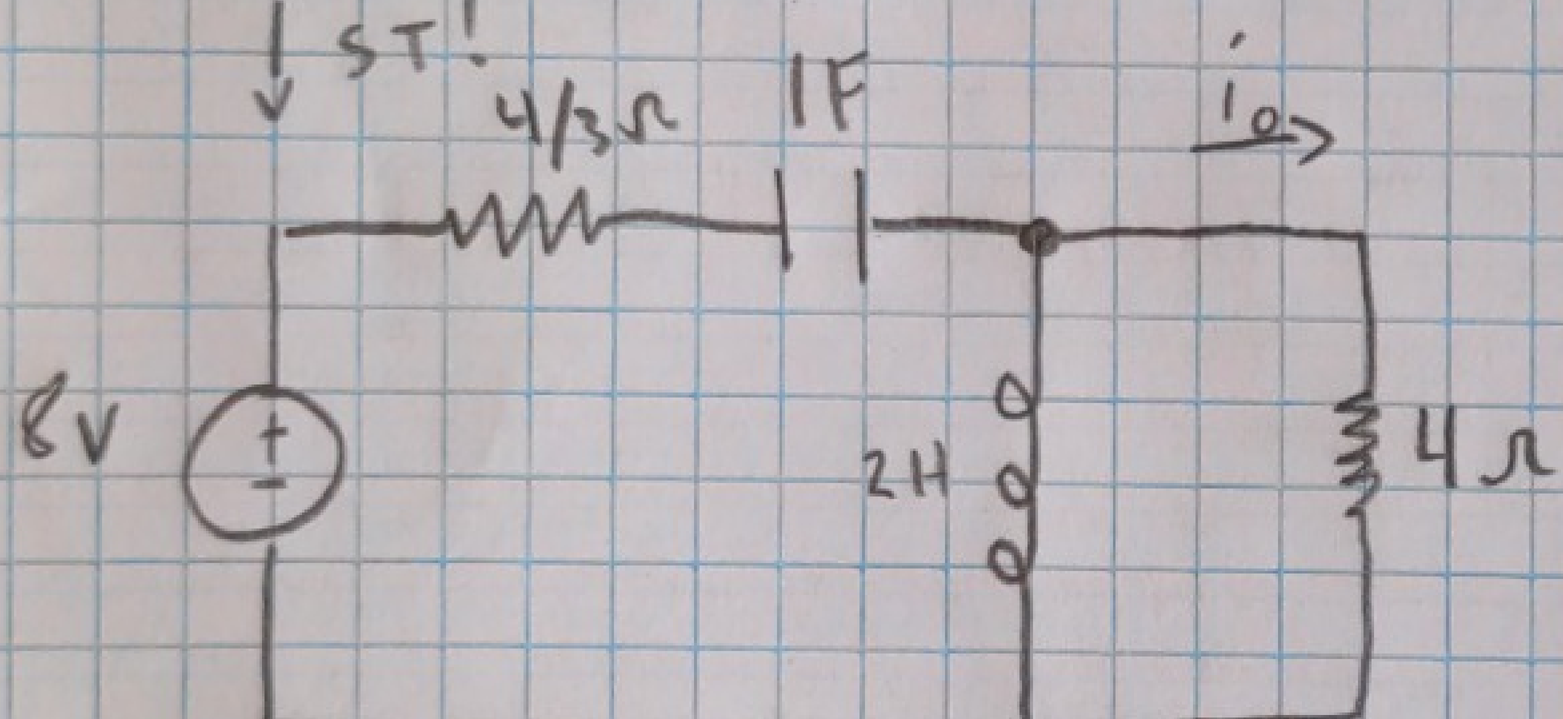
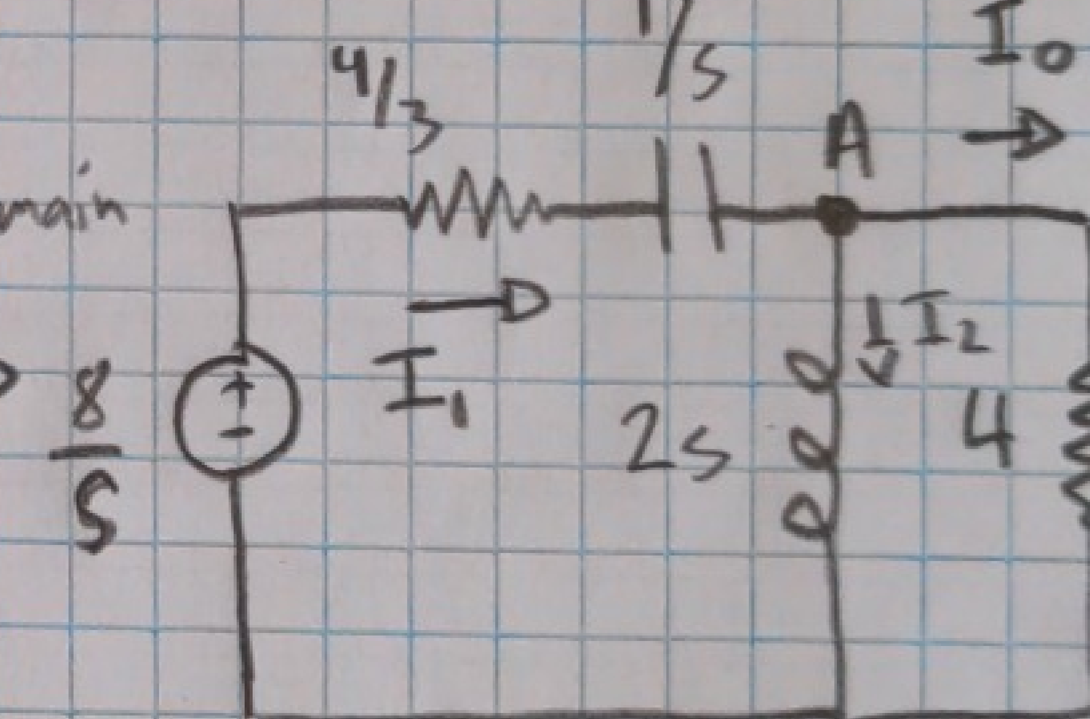


s-Domain →



Find I_o) @ A) $I_1 = I_2 + I_o$

$$\frac{\frac{8}{s} - V_a}{\frac{4}{3}+\frac{1}{s}} = \frac{V_a}{2s} + \frac{V_a}{4}$$

$$\frac{8}{s} - V_a = V_a\left(\frac{1}{2s}+\frac{1}{4}\right)\left(\frac{4}{3}+\frac{1}{s}\right)$$

$$\left(\frac{8}{s} - V_a = V_a\left(\frac{2}{3s}+\frac{1}{2s^2}+\frac{1}{3}+\frac{1}{4s}\right)\right)12s^2$$

$$96s - 12s^2V_a = V_a\left(8s+6+4s^2+3s\right)$$

$+12s^2V_a$

$$96s = V_a\left(16s^2+11s+6\right)$$

$$V_a = \frac{96s}{16s^2+9s+6} \quad \frac{1/16}{1/16}$$

$$V_a = \frac{6s}{s^2+\frac{11}{16}s+\frac{6}{16}}$$

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11.1) Cont.

$$V_a = \frac{6s}{s^2 + \frac{11}{16}s + \frac{6}{16}}$$

$$s^2 + \frac{11}{16}s + \frac{6}{16} = s^2 + 2as + a^2 + b^2$$

$$\frac{11}{16} = 2a \rightarrow a = \frac{11}{32}$$

$$\frac{6}{16} = a^2 + b^2 \rightarrow \frac{6}{16} = \left(\frac{11}{32}\right)^2 + b^2$$

$$b = \sqrt{\frac{6}{16} - \left(\frac{11}{32}\right)^2} = \sqrt{\frac{6}{16} - \frac{121}{1024}}$$

$$b = \sqrt{\frac{384}{1024} - \frac{121}{1024}}$$

$$b = \sqrt{\frac{263}{1024}} \rightarrow b = \frac{\sqrt{263}}{32}$$

$$V_a = \frac{6s}{(s+\frac{11}{32})^2 + \left(\frac{\sqrt{263}}{32}\right)^2}$$

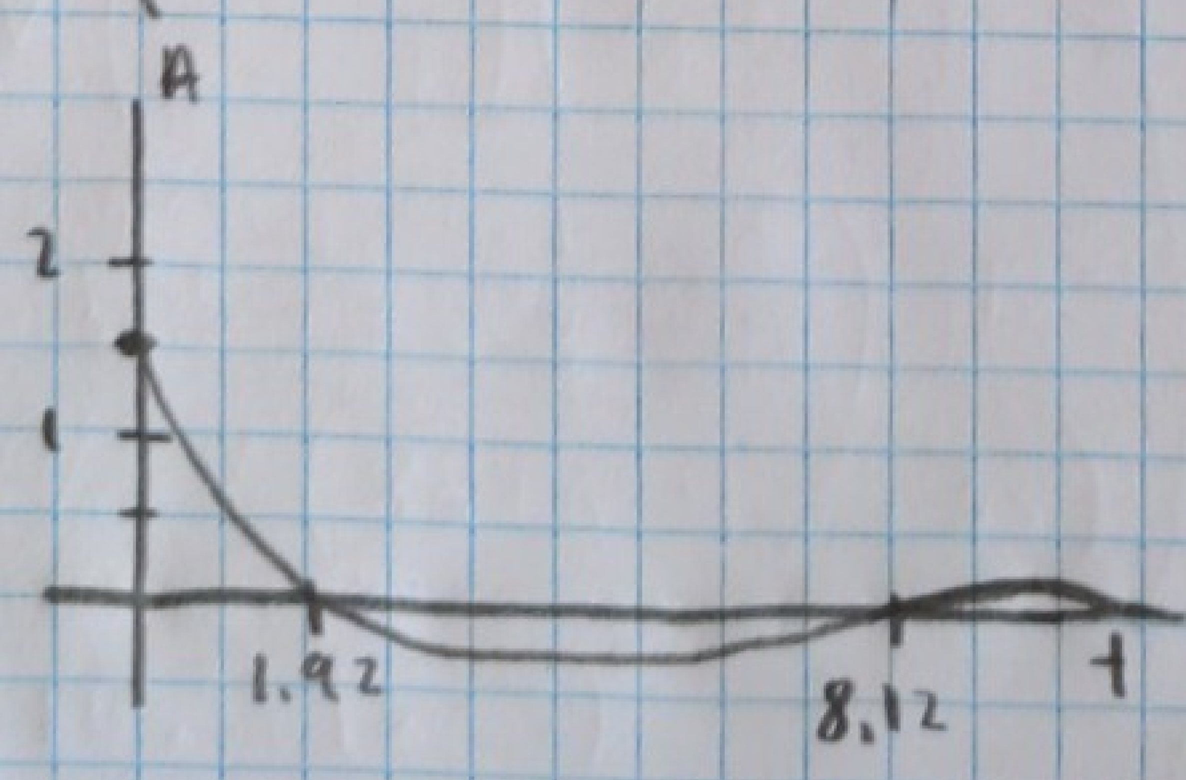
$$V_a = 6\left(\frac{s + \frac{11}{32} - \frac{11}{32}}{(s+\frac{11}{32})^2 + \left(\frac{\sqrt{263}}{32}\right)^2}\right)$$

$$V_a = 6\left(\frac{s + \frac{11}{32}}{(s+\frac{11}{32})^2 + \left(\frac{\sqrt{263}}{32}\right)^2} - \frac{\frac{11}{32}}{(s+\frac{11}{32})^2 + \left(\frac{\sqrt{263}}{32}\right)^2}\right)$$

$$V_a = 6\left(\frac{s + \frac{11}{32}}{(s+\frac{11}{32})^2 + \left(\frac{\sqrt{263}}{32}\right)^2} - \frac{11}{32} \cdot \frac{32}{\sqrt{263}} \cdot \frac{\sqrt{263}/32}{(s+\frac{11}{32})^2 + \left(\frac{\sqrt{263}}{32}\right)^2}\right)$$

$$I_o = \frac{V_a}{4\Omega} = \frac{3}{2}\left(\frac{s + \frac{11}{32}}{(s+\frac{11}{32})^2 + \left(\frac{\sqrt{263}}{32}\right)^2} - \frac{11}{\sqrt{263}} \frac{\sqrt{263}/32}{(s+\frac{11}{32})^2 + \left(\frac{\sqrt{263}}{32}\right)^2}\right)$$

$$\mathcal{L}^{-1}[I_o] = \frac{3}{2}\left(e^{-\frac{11}{32}t} \cos\left(\frac{\sqrt{263}}{32}t\right) - \frac{11}{\sqrt{263}} e^{-\frac{11}{32}t} \sin\left(\frac{\sqrt{263}}{32}t\right)\right) u(t) \text{ A}$$



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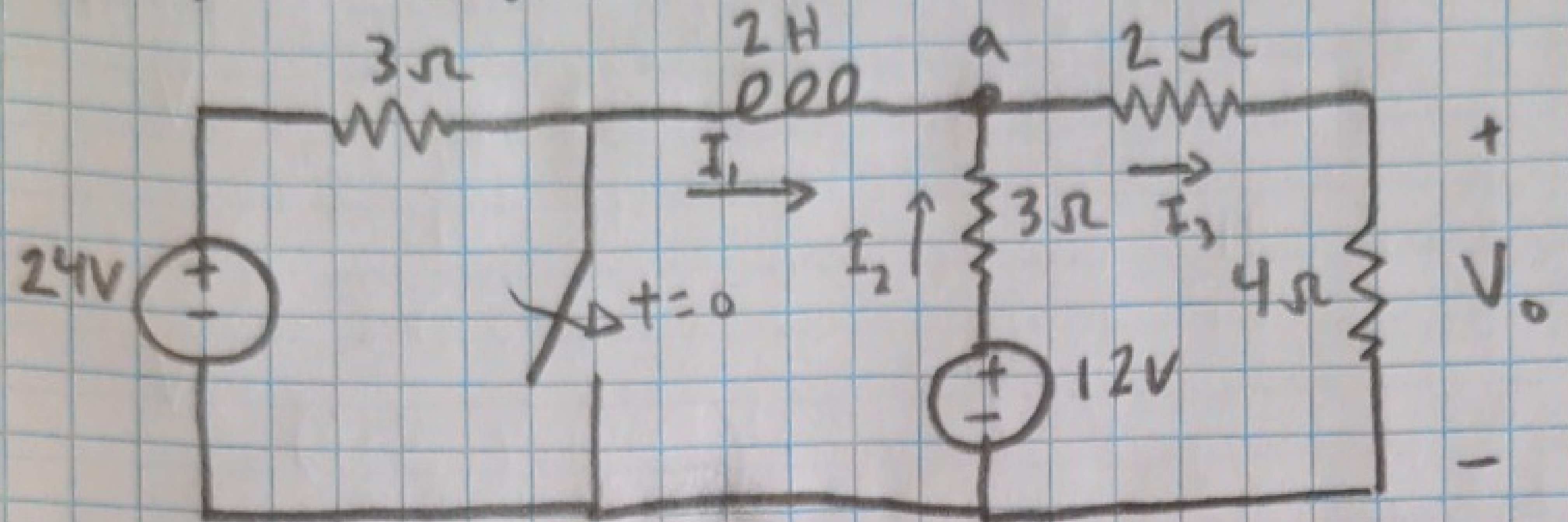
11.2) Find $V_o(t)$ for $t > 0$, the switch closes at $t = 0$



Find $i_L(0^-)$:

@a) $I_1 + I_2 = I_3$

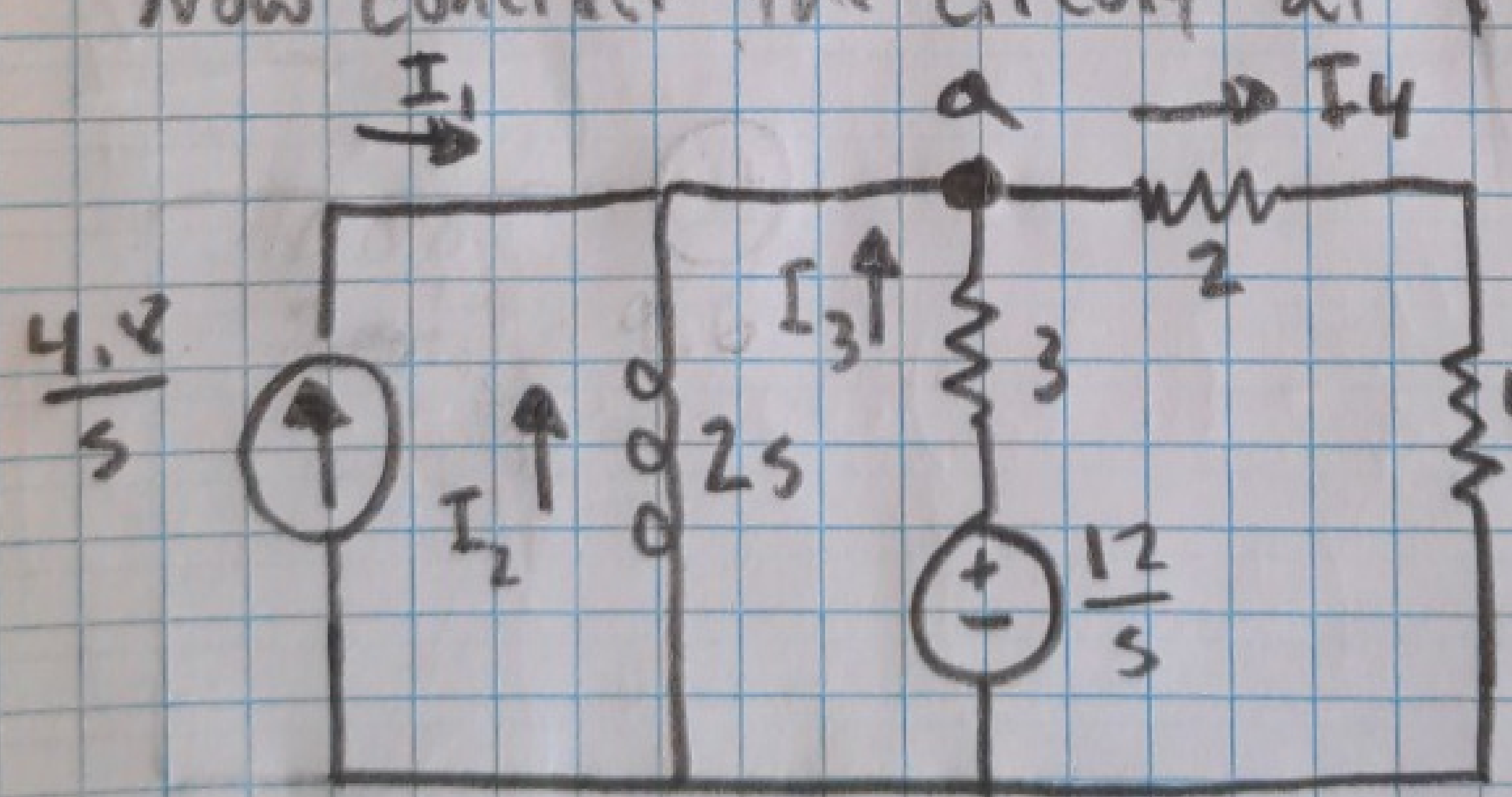
$$\left(\frac{24_V - V_a}{3\Omega} + \frac{12V - V_a}{3\Omega} = \frac{V_a}{6\Omega} \right) 6\Omega$$

$$48 - 2V_a + 24 - 2V_a = V_a \rightarrow 72 = 5V_a$$

$$V_a = \frac{72}{5} = 14.4$$

$$i_L(0^-) = \frac{14.4}{3} = 4.8\text{ A}$$

Now concider the circuit at $t > 0$, converted to s domain



@a) $I_1 + I_2 + I_3 = I_4$

$$\frac{4.8}{s} + \frac{0 - V_a}{2s} + \frac{12/s - V_a}{3} = \frac{V_a}{6}$$

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11.2) cont.

$$\frac{4.8}{s} + \frac{0 - V_a}{2s} + \frac{12/s - V_a}{3} = \frac{V_a}{6}$$

$$\left(\frac{4.8}{s} - \frac{V_a}{2s} + \frac{12}{3s} - \frac{V_a}{3} = \frac{V_a}{6}\right) 6s$$

$$28.8 - 3V_a + 24 - 2sV_a = sV_a$$

$$\left(52.8 = V_a(3s + 3)\right)\frac{1}{3}$$

$$17.6 = V_a(s+1) \longrightarrow V_a = \frac{17.6}{s+1}$$

Find $V_o(t)$

$$V_o(s) = V_a\left(\frac{4}{6}\right)$$

$$V_o(s) = \frac{2}{3}\left(\frac{17.6}{s+1}\right)$$

$$V_o(s) = \frac{11.73}{s+1}$$

$$\boxed{\mathcal{L}^{-1}\left[V_o(s)\right] = 11.73e^{-t}u(t)\,\text{V}}$$