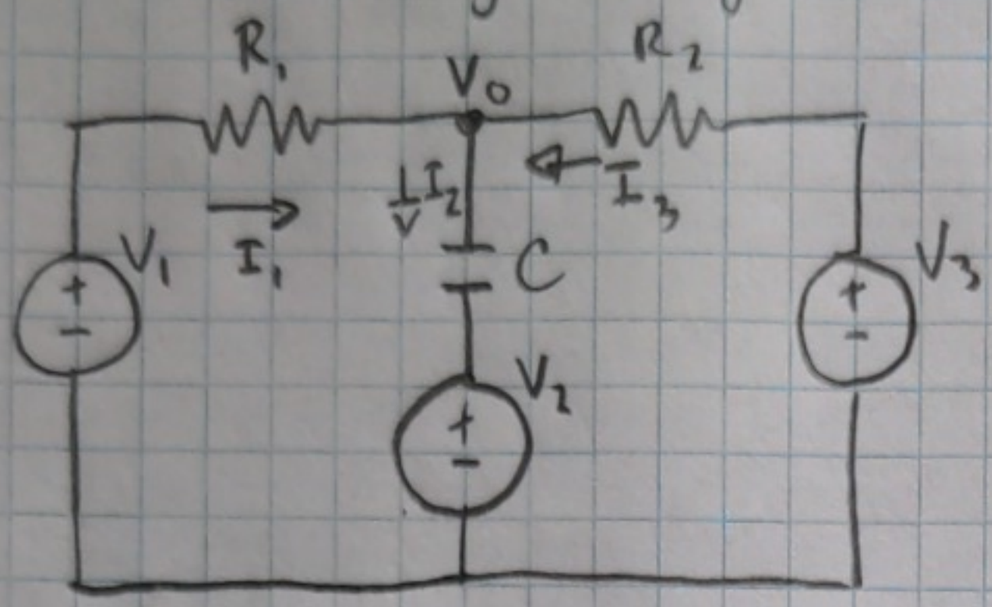
Chris Hunt HW10 ENGR 203

10.1) Find the s-domain solution, $V_o(s)$ of the circuit using node-voltage analysis.



$V_1 = 3/s$ V $V_2 = 1/s$ V $V_3 = 4/s$

$R_1 = R_2 = 1$ $C = 2/s$

@ V_o) $I_2 = I_1 + I_3$

$$\frac{V_o - 1/s}{2/s} = \frac{3/s - V_o}{1} + \frac{4/s - V_o}{1}$$

$$\frac{sV_o - 1}{2} = \frac{7}{s} - 2V_o$$

$$V_o\left(\frac{s}{2} + 2\right) = \frac{7}{s} + \frac{1}{2}$$

$$V_o\left(\frac{s}{2} + 2\right) = \frac{s + 14}{2s}$$

$$V_o = \frac{s+14}{2s}\left(\frac{1}{\frac{s}{2}+2}\right)$$

$$\boxed{V_o(s) = \frac{s+14}{s(s+4)}} = \frac{A}{s} + \frac{B}{s+4}$$

$$s + 14 = A(s+4) + Bs$$

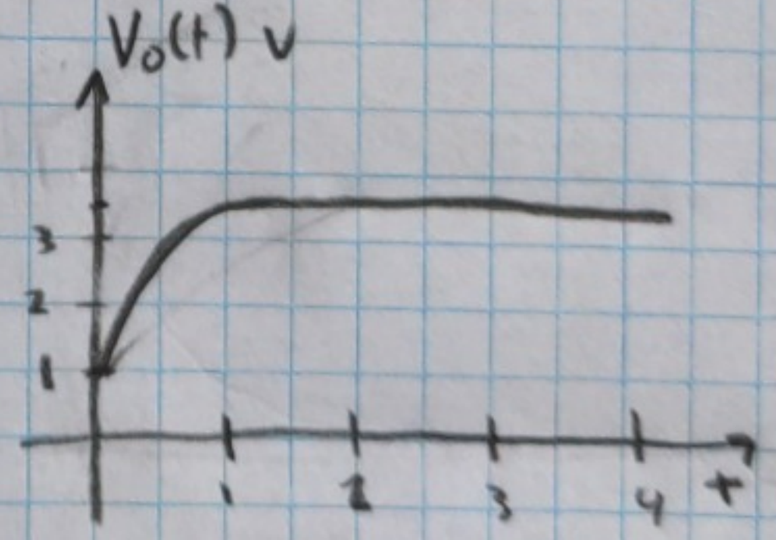
$$s + 14 = As + 4A + Bs$$

$14 = 4A \rightarrow A = \frac{7}{2}$

$1 = A + B \rightarrow 1 = \frac{7}{2} + B \rightarrow B = -\frac{5}{2}$

$$V_o(s) = \frac{7}{2}\left(\frac{1}{s}\right) - \frac{5}{2}\left(\frac{1}{s+4}\right)$$

$$\mathcal{L}^{-1}[V_o(s)] = \frac{7}{2}u(t) - \frac{5}{2}e^{-4t}\ V$$



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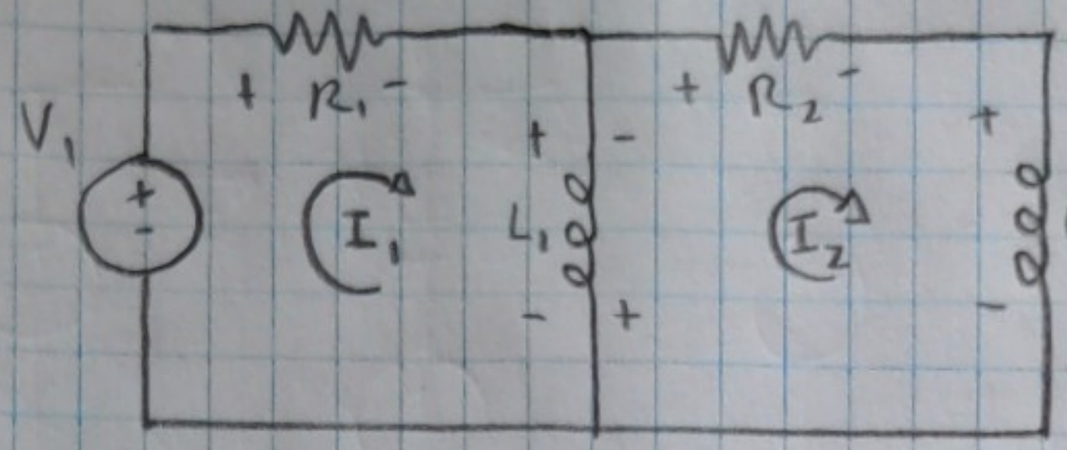
10.2) Find the s-domain solution of the circuit using mesh-current analysis.



$V_1 = \frac{10}{s}$ $R_1 = 1$

$R_2 = 4$ $L_1 = \frac{s}{4}$

$L_2 = s$

Mesh I_1)

$$-\frac{10}{s} + I_1 + \frac{s}{4}(I_1 - I_2) = 0$$

$$\left(1 + \frac{s}{4}\right) I_1 - \frac{s}{4} I_2 = \frac{10}{s}$$

Mesh I_2)

$$4 I_2 + s I_2 + \frac{s}{4}(I_2 - I_1) = 0$$

$$I_2\left(4 + \frac{5}{4}s\right) - \frac{s}{4} I_1 = 0$$

$$\frac{4}{s}\left(\frac{s}{4} I_1 = \left(4 + \frac{5}{4}s\right) I_2\right)$$

$$I_1 = \left(\frac{16}{s} + 5\right) I_2$$

Find I_2

$$s\left(\left(1 + \frac{s}{4}\right)\left(\frac{16}{s} + 5\right) I_2 - \frac{s}{4} I_2 = \frac{10}{s}\right)$$

$$\left(s + \frac{s^2}{4}\right)\left(\frac{16}{s} + 5\right) I_2 - \frac{s^2}{4} I_2 = 10$$

$$\left(16 + 5s + 4s + \frac{5}{4}s^2\right) I_2 - \frac{s^2}{4} I_2 = 10$$

$$I_2\left(s^2 + 9s + 16\right) = 10$$

$$\boxed{I_2 = \frac{10}{s^2 + 9s + 16}}$$

Find I_1)

$$I_1 = \left(\frac{16}{s} + 5\right)\left(\frac{10}{s^2 + 9s + 16}\right)$$

$$I_1 = \frac{160}{s(s^2 + 9s + 16)} + \frac{50}{s^2 + 9s + 16}$$

$$\boxed{I_1 = \frac{160}{s(s^2 + 9s + 16)} + \frac{50}{s^2 + 9s + 16}}$$

$$\boxed{I_2 = \frac{10}{s^2 + 9s + 16}}$$

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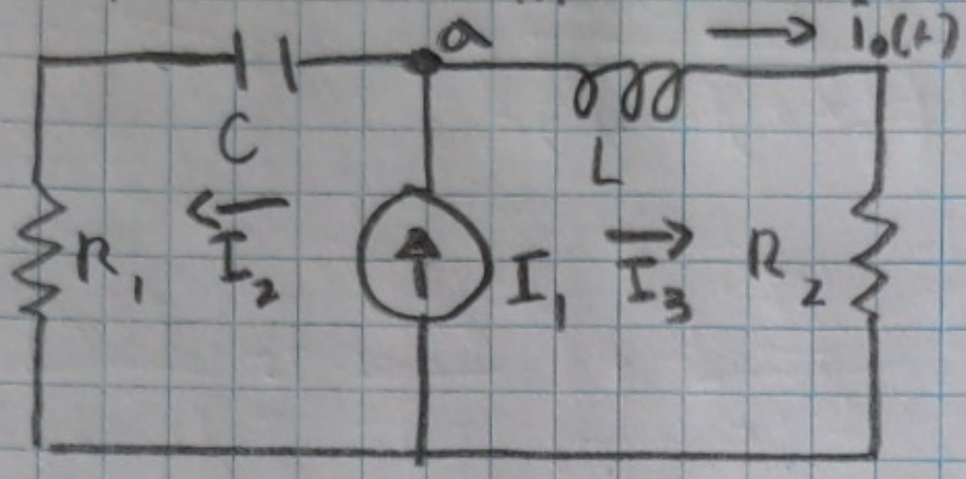
10.3) Find the time-domain current $I_o(t)$ given the s-domain circuit below.



$I_1 = \frac{2}{s+2}$ $R_1 = 2$

$R_2 = 1$ $C = \frac{1}{s}$

$L = 2s$ $I_o(s) = ?$

a) $I_1 = I_3 + I_2 \Rightarrow \frac{2}{s+2} = \frac{V_a}{2s+1} + \frac{V_a}{2+\frac{1}{s}}$

$\frac{2}{s+2} = \left(\frac{1}{(2s+1)(2+\frac{1}{s})}\right) V_a$

$V_a = \frac{(4s+2)(2+\frac{1}{s})}{s+2}$

$V_a = \frac{8s+4+4+\frac{2}{s}}{s+2}$

$V_a = \left(\frac{8s+8+\frac{2}{s}}{s+2}\right)\frac{s}{s}$

$V_a = \frac{8s^2+8s+2}{s^2+2s}$

Find $I_o(s)$ → Find $V_a(s)$

$I_o(s) = \frac{V_a}{2s+1}$

$I_o(s) = \left(\frac{8s^2+8s+2}{s^2+2s}\right)\left(\frac{1}{2s+1}\right)$

$I_o(s) = \frac{8s^2+8s+2}{2s^3+s^2+4s^2+2s}$

$I_o(s) = \frac{8s^2+8s+2}{s(2s^2+5s+2)}$

$I_o(s) = \frac{2}{2}\left(\frac{4s^2+4s+1}{s(s^2+\frac{5}{2}s+1)}\right) \rightarrow I_o(s) = \frac{4s^2+4s+1}{s(s^2+\frac{5}{2}s+1)}$

$\frac{4s^2+4s+1}{s(s^2+\frac{5}{2}s+1)} = \frac{A}{s} + \frac{B}{s^2+\frac{5}{2}s+1}$

→

10.3) Cont.

$$\frac{4s^2+4s+1}{s(s^2+\frac{5}{2}s+1)} = \frac{4(s^2+s+\frac{1}{4})}{s(s+2)(s+\frac{1}{2})} = \frac{4(s+\frac{1}{2})^2}{s(s+2)(s+\frac{1}{2})}$$

$$\frac{4(s+\frac{1}{2})}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

$$4s+2 = A(s+2) + Bs$$

$4 = A + B \quad B = 3$

$2 = 2A \rightarrow A = 1$

$$I_o(s) = \frac{1}{s} + \frac{3}{s+2}$$

$$\mathcal{L}^{-1}[I_o(s)] = u(t) + 3e^{-2t}\ A = i_o(t)$$