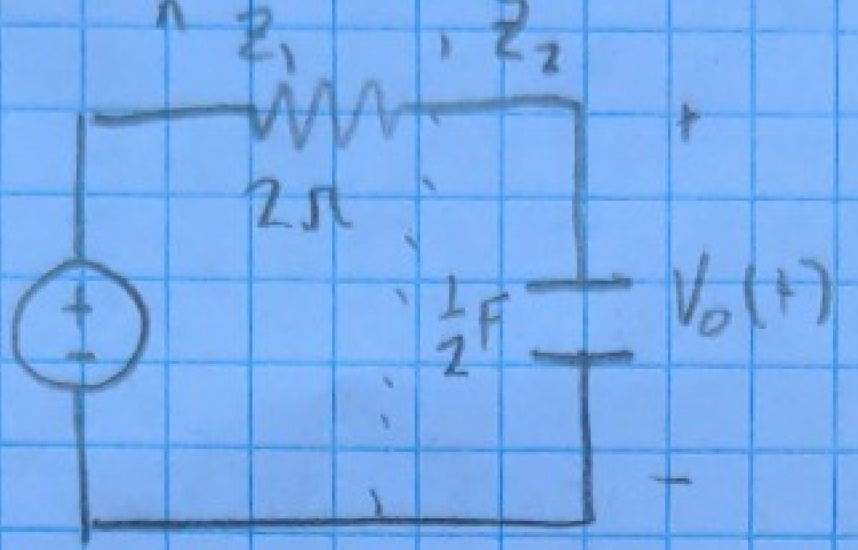
Chris Hunt HW15 ENGR 203

15.1) The voltage source $v_s(t)$ in the circuit is described by the Fourier Series.

$$V_s(t) = \frac{A}{2} - \frac{A}{\pi}\sum_{k=1}^{\infty}\frac{\sin(n\omega_0 t)}{n} \qquad A = 5V \qquad \omega_0 = \pi \frac{rad}{s} \rightarrow \omega_n = n\pi \frac{rad}{s}$$

$k = 2n-1$



$$C = \frac{1}{2}F \qquad Z_2 = \frac{2}{j\pi n}$$

$$\frac{Z_2}{Z_1+Z_2} = \frac{2/j\pi n}{2+2/j\pi n} = \frac{2}{2+j2\pi n} = \frac{1}{1+j\pi n}$$

Find $V_0(t)$) Begin by finding the DC component

Since at DC the capacitor acts like an open circuit, $V_{0,DC}$ is equal to the DC component to our excitation voltage V_s.

$$V_{0,DC} = \frac{5}{2}V$$

Let's rewrite the ac component in phasor form.

$$V_s = \frac{-5}{\pi n}\sin(n\omega_0 t)_V = \frac{5}{\pi n}\cos(n\omega_0 t + 90°)_V \qquad \omega_n = n\omega_0$$

$$V_s = \frac{5}{\pi}\angle 90 \ V$$

Now rewrite $\frac{Z_2}{Z_1+Z_2}$ in phasor form:

$$\frac{Z_2}{Z_1+Z_2} = \frac{1}{1+j\pi n} = \frac{1\angle 0}{\sqrt{1+(\pi n)^2}\angle \tan^{-1}\pi n}$$

Find V_0 using a voltage divider

$$V_0 = \frac{\frac{5}{\pi n}\angle 90}{\sqrt{1+(\pi n)^2}\angle \tan^{-1}\pi n}$$

$$V_0 = \frac{5}{\pi n\sqrt{1+\pi^2 n^2}}\angle 90 - \tan^{-1}\pi n$$

$$V_0(t) = \frac{5}{2} + \sum_{n=1}^{\infty}\frac{5}{n\pi\sqrt{1+\pi^2 n^2}}\cos(n\pi t + 90 - \tan^{-1}\pi n)\ V$$

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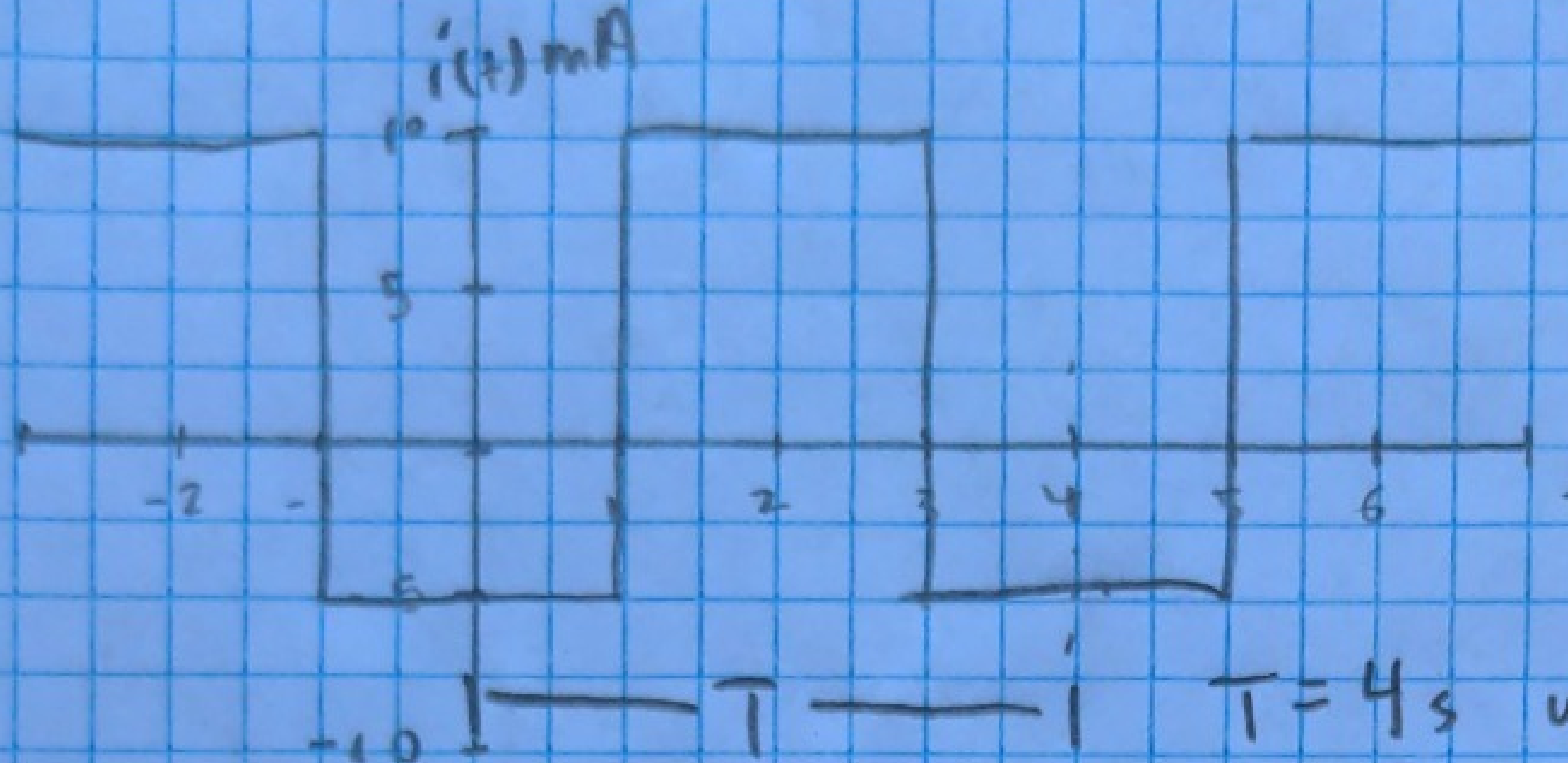
15.2) The Following waveform describes the current being fed into a 1.5kΩ resistor.



Even symmetry!

$$a_0 = \frac{2}{T}\int_0^{T/2} f(t)\,dt$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_0 t)\,dt$$

$T = 4s$ $\quad \omega_0 = \frac{\pi}{2}$ $\quad b_n = 0$

a) Find the power dissipated in the resistor using Fourier Series

$$a_0 = \frac{2}{4}\left(\int_0^1 -5\,dt + \int_1^2 10\,dt\right)$$

$$a_0 = \frac{1}{2}(-5+10)$$

$$a_0 = \frac{5}{2}\text{ mA}$$

$$a_n = \frac{4}{4}\left(\int_0^1 -5\cos(n\omega_0 t)\,dt + \int_1^2 10\cos(n\omega_0 t)\,dt\right)$$

$$a_n = \frac{-5\cdot 2}{\pi n}\left(\sin\left(n\frac{\pi}{2}t\right)\Big|_0^1\right) + \frac{10\cdot 2}{\pi n}\left(\sin\left(n\frac{\pi}{2}t\right)\Big|_1^2\right)$$

$$a_n = \frac{-10}{\pi n}\sin\left(\frac{n\pi}{2}\right) + \frac{20}{\pi n}\left(\cancel{\sin(n\pi)}^{\,0} - \sin\left(n\frac{\pi}{2}\right)\right)$$

$$a_n = \frac{-30}{\pi n}\sin\left(n\frac{\pi}{2}\right) \Longrightarrow i(t) = \frac{5}{2} - \frac{30}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{\frac{n-1}{2}}}{n}\cos\left(\frac{\pi}{2}nt\right)\text{ mA}$$

$$n = 2k-1$$

$$P_{1\Omega} = I_{rms}^2 = a_0^2 + \frac{1}{2}\sum_{k=1}^{\infty} a_n^2 + b_n^2$$

$$P_{1\Omega} = \frac{25}{4} + \frac{1}{2}\left(\left(\frac{-30}{\pi}\right)^2 + \left(\frac{10}{\pi}\right)^2 + \left(\frac{-6}{\pi}\right)^2\right)$$

$$P_{1\Omega} = \frac{25}{4} + \frac{518}{\pi^2} = 58.73\text{ mW} \longrightarrow \boxed{1000k\Omega\cdot P_{1\Omega} = 58.73\text{ W}}$$

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15.2) b) What fractions of the total power dissipated in the resistor are due to the DC component and 1st harmonic?

$$P_{1\Omega} = \frac{25}{4} + \frac{1}{2}\left(\frac{400}{\pi^2}\right) = 51.84 \text{ mW}$$

$$1000 k\Omega \cdot P_{1\Omega} = 51.84 \text{ W}$$

$$\frac{51.84 \text{ W}}{58.73 \text{ W}} \cdot 100 = 88.27\%$$

The DC and 1st Harmonic comprise 88.27% of the power dissipated through the resistor.