Chris Hunt HW 8 ENGR 203

8.1) Find the Laplace transform of $(t-4)u(t-2)$

For this proplem we will use the time shift properties of the Laplace transform

$$\mathcal{L}[f(t-a)u(t-a)] = e^{-as}F(s)$$

Find $F(s)$: $f(t) = (t-2)u(t)$

$$\mathcal{L}[f(t)] = \int_0^\infty (t-2)e^{-st}\,dt$$

$$= \int_0^\infty t e^{-st}\,dt - 2\int_0^\infty e^{-st}\,dt$$

integrate by parts!

$u = t$

$du = 1$

$dv = e^{-st}\,dt$

$v = -\frac{e^{-st}}{s}$

$$\left(-\frac{te^{-st}}{s} - \int_0^\infty -\frac{e^{-st}}{s}\,dt\right) - 2\left(-\frac{e^{-st}}{s}\Big|_0^\infty\right)$$

$$\left(-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\Big|_0^\infty\right) - 2\left(0 + \frac{1}{s}\right)$$

$$\left(0 - \left(0 - \frac{1}{s^2}\right)\right) - \frac{2}{s}$$

$$= \frac{1}{s^2} - \frac{2}{s}\left(\frac{s}{s}\right)$$

$$F(s) = \frac{1-2s}{s^2}$$

Time shift!

$$\boxed{\mathcal{L}[f(t-2)] = \frac{e^{-2s}(1-2s)}{s^2}}$$

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8.2) Find $F(t)$ given $F(s) = \frac{4}{s+3} - \frac{2.5s}{s^2+16}$

Using Laplace Transform pairs:

$$\frac{4}{s+3} = 4\left(\frac{1}{s+3}\right) \rightarrow a = 3 \rightarrow 4e^{-3t}$$

$$-\frac{2.5s}{s^2+16} = -2.5\left(\frac{s}{s^2+4^2}\right) \rightarrow \omega = 4 \rightarrow -2.5\cos(4t)$$

$$\boxed{\mathcal{L}^{-1}[F(s)] = 4e^{-3t} - 2.5\cos(4t)}$$

8.3) Find $F(t)$ given $F(s) = \frac{6s+12}{(s+1)(s+3)(s+4)}$

Use Partial Fraction Decomposition:

$$\frac{6s+12}{(s+1)(s+3)(s+4)} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s+4}$$

$$A = (s+1)F(s)\Big|_{s=-1} = \frac{6(-1)+12}{(-1+3)(-1+4)} = \frac{6}{6} = 1$$

$$B = (s+3)F(s)\Big|_{s=-3} = \frac{6(-3)+12}{(-3+1)(-3+4)} = \frac{-6}{-2} = 3$$

$$C = (s+4)F(s)\Big|_{s=-4} = \frac{6(-4)+12}{\underset{-3}{(-4+1)}\underset{-1}{(-4+3)}} = \frac{-12}{3} = -4$$

$$F(s) = \frac{1}{s+1} + \frac{3}{s+3} - \frac{4}{s+4}$$

$$\boxed{\mathcal{L}^{-1}[F(s)] = e^{-t} + 3e^{-3t} - 4e^{-4t}}$$

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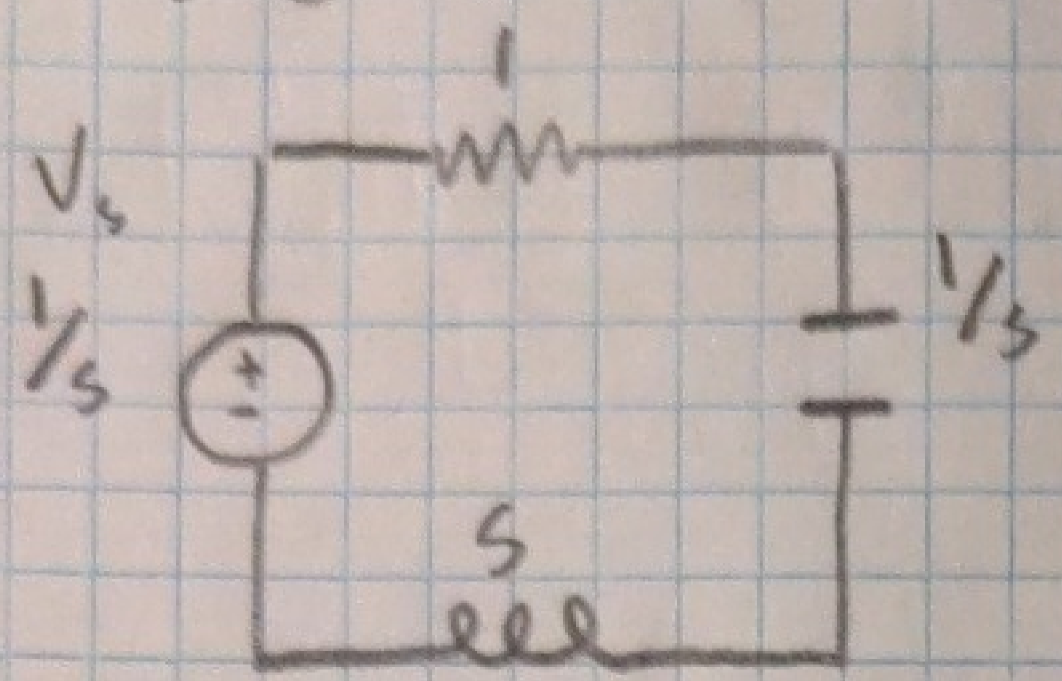
8.4) Find the time-domain current $i(t)$ for the given s-domain circuit



$$V_s(s) = \frac{1}{s}$$

$$Z_{tot} = \left(1 + \frac{1}{s} + s\right)\frac{s}{s} = s^2 + s + 1$$

$$I(s) = \frac{V_s}{Z_{tot}} = \frac{\frac{1}{s}}{s^2+s+1} = \frac{1}{s(s^2+s+1)}$$

$$I(s) = \frac{1}{s(s^2+s+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+s+1}$$

$$A = s\,F(s)\Big|_{s=0} = \frac{1}{1} = 1 = A$$

$$1 = A(s^2+s+1) + (Bs+C)s$$

$$1 = A(s^2+s+1) + B(s^2) + C(s)$$

$$0 = A + B \qquad B = -1$$

$$0 = A + C \qquad C = -1$$

$$1 = A$$

$$I(s) = \frac{1}{s} - \frac{s+1}{s^2+s+1} = \frac{1}{s} - 1\left(\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1}{\sqrt{3}}\frac{\frac{\sqrt{3}}{2}}{(s+\frac{1}{2})^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$u(t) \qquad e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right) \qquad \frac{e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)}{\sqrt{3}}$$

$$\boxed{\mathcal{L}^{-1}[i(t)] = u(t) - e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right) - \frac{e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)}{\sqrt{3}} \quad A}$$