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Set: a collection of things ("elements")

Two sets are "equal" when they contain exactly the same elements no more no less.

{} when listing set elements

$\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ $5 \in \mathbb{N}$

$\{5\}$ is not the same as 5.

subset: If every element of set A is also an element of set B, then set A is called a subset of set B

Notation $A \subseteq B$ "subset" underline is equal to

If $A \neq B$ A is a proper subset

$A \subset B$

Empty set = $\emptyset$ "null set" $\emptyset = \{\}$ $\{\emptyset\} = \{\{\}\}$

$\mathcal{U}$ = universal set "everything" based on the domain

$\mathcal{P}(A)$ = Power set of a set A
A set containing all subsets of A

$\cap$ intersection → "and"

$\cup$ union → "or"

$\times$ ordered pairs

Complement = $\overline{A}$ or A^c or A' or A^* or $\sim A$
everything that is not in A but in the domain

$x \in A^c \leftrightarrow x \notin A$
$\leftrightarrow \neg(x \in A)$

Size of set A is it's cardinality : $|A|$

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<u>Claim</u>: Any set A is a subset of itself

<u>Proof</u>: Let A be a set. We want to show $A \subseteq A$. To do this, let x be an arbitrary selected element of set A.

If $x \in A$, then $x \in A$

Since this applies to every element in A, every element of A is in set A; therefore

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$A \subseteq A$

0

$\uparrow$

$\emptyset$ vs. $\{\emptyset\}$

$\{ \; \{\} \} \; \{$

$P(\emptyset) = \{\emptyset\}$ is true!

$A = \{1\}$

$B = \{2\}$

$= \emptyset$

$A \cup B = \{1,2\}$

$A \cup \emptyset = \{1, \emptyset\}$

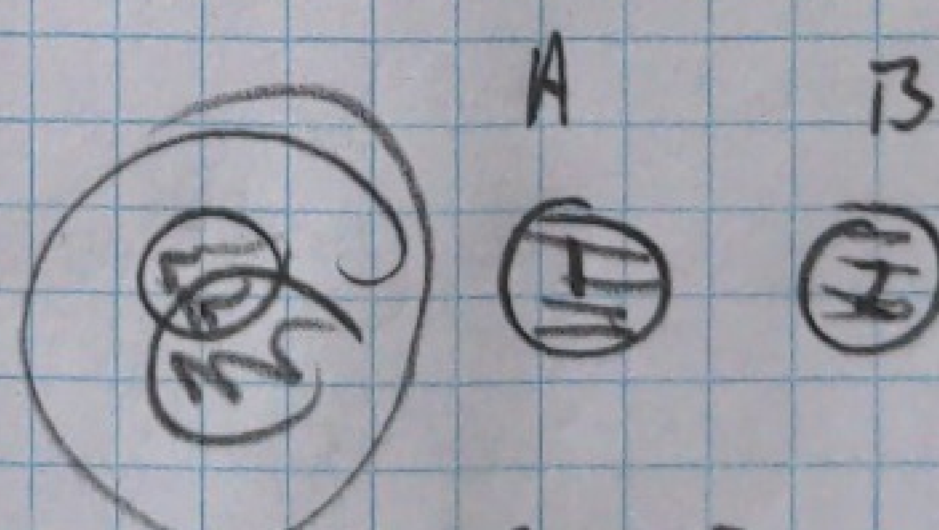
$A \cap \emptyset = \emptyset$,

$A \cup \emptyset = \cancel{\{A, \emptyset\}}$ A $\{A, \emptyset$

$A \setminus \emptyset = A$

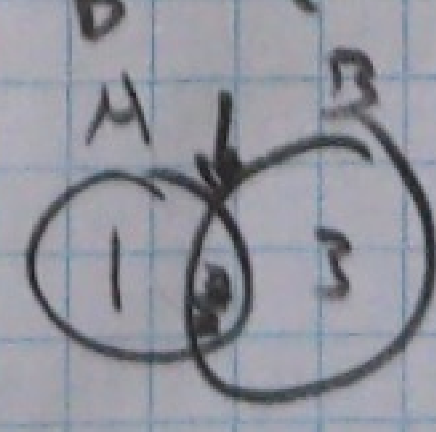
$\emptyset \setminus A = \emptyset$

A $\emptyset$



$A = \{1,2\}$

$B = \{2,3\}$



$A \cap B = \{2\}$

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11. $A \cap B$ $A \cap \bar{B} = A \setminus B$ $(B \cap C) \cup (C \cap \bar{A})$

16. 

$\rightarrow (A \cap \bar{B}) \cup (B \cap \bar{A})$

$(A \setminus B) \cup (B \setminus A)$

$A = \{1, 2, 3, 4\}$ We want $C = \{1, 2, 5, 6\}$

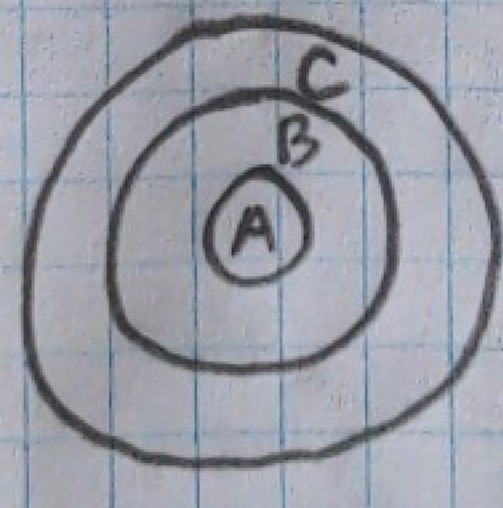
$B = \{3, 4, 5, 6\}$

Begin by removing the elements that are similar to both A and B, then union the two sets.

25. a. Suppose that $A \subseteq B$ and $B \subseteq C$. Does this mean $A \subseteq C$

$\forall x (x \in A \rightarrow x \in C)$ If x is in A, then x is in C

Proof: By the definition of a subset ($\subseteq$) we know that ~~if~~ x is in A, then x is in B. Likewise if x is in B, then x is in C.



By the definition of $\subseteq$ $A \subseteq C$ and the statement is true.

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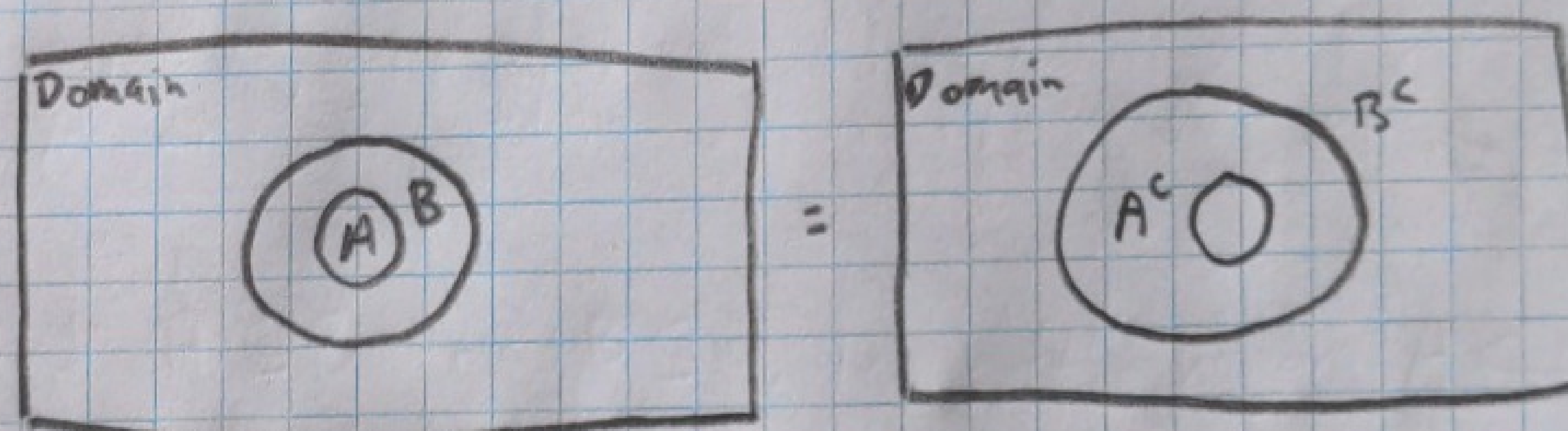
29. Explain why there is no set A which satisfies
$A = \{2, |A|\}$

This is an impossible set because sets do not contain duplicate elements.

The cardinality of the set A as defined should be 2, but 2 is already an element of the set and can not be represented twice. This would then create a contradiction.

SQ-4. For all sets A and B

a) Prove that $A \subseteq B$ if and only if $B^c \subseteq A^c$



$$A \subseteq B \rightarrow B^c \subseteq A^c \wedge B^c \subseteq A^c \rightarrow A \subseteq B$$

Proof:

Assume $A \subseteq B$, that is, every element of A is also an element of B.

Now consider all elements not in B and all the elements not in A. Since we know that $|A| \leq |B|$ then we know that $|A^c| \geq |B^c|$ and since every element of A is in B then every element of B^c is in A^c. Therefore the statement $A \subseteq B \rightarrow B^c \subseteq A^c$ is true.

Now assume $B^c \subseteq A^c$, that is every element not in B is also not in A. By the same logic as above we see that $|B^c| \leq |A^c|$ so that means $|A| \leq |B|$ and $A \subseteq B$ is therefore true

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SQ-4. b. Prove that if $A \subseteq B \wedge B \subseteq A$, then $A = B$

Suppose not, that is $A \subseteq B \wedge B \subseteq A \wedge A \neq B$

By the definition of subsets we see that for every x in A, x is in B and for every x in B, x is in A. That is there is no value x that is in A but not B and vice versa, so A must equal B. This is a contradiction. Therefore the original statement must be true