Chris Hunt DP7 MTH231

2.1) 1. a) 2, 5, 10, 17, 26, ...

$a_1 = 2 = 1^2 + 1$

$a_2 = 5 = 2^2 + 1$ $a_n = n^2 + 1$

$a_3 = 10 = 3^2 + 1$

$a_4 = 17 = 4^2 + 1$

1 2 3 4 5 6

b) 0, 2, 5, 9, 14, 20

~~$a_1 = 0 = 2^1 - 2$~~ X $a_1 = 0 = 2^n - 2(n) = 2^1 - 2 = 0$

~~$a_2 = 2 = 2^2 - 2$~~ X $a_2 = 2 = 2^2 - 2(2)$ X

~~$a_3 = 5 = 2^3 - 3$~~ X

~~$a_4 = 9 = 2^4 - 4$~~ X

2. a) $4 - 0 = 4$ $a_0 = 0$

$5 - 4 = 1$ 2^0

$7 - 5 = 2$ 2^1 $a_1 = a_{n-1} + 2^{n-1} = 4 + 2^0 = 4 + 1 = 5$

$11 - 7 = 4$ 2^2

$19 - 11 = 8$ 2^3 $a_2 = 5 + 2^{2-1} = 5 + 2^1 = 5 + 2 = 7$

$35 - 19 = 16$ 2^4

$a_3 = 7 + 2^2 = 7 + 4 = 11$

$a_n = a_{n-1} + 2^{n-1}$ $a_0 = 4$

How do I get the closed formula?

b) 0, 3, 8, 15, 24, 35, ... $a_0 = 0$

$0 - 0 = 0$

$3 - 0 = 3$ $a_n = a_{n-1} + (2n+1)$

$8 - 3 = 5$ $a_1 = 0 + (2 + 1) = 3$ ✓

$15 - 8 = 7$ $a_2 = 3 + (4 + 1) = 8$ ✓

$24 - 15 = 9$ $a_3 = 8 + (6 + 1) = 15$ ✓

$$\boxed{a_n = a_{n-1} + (2n+1)}$$

c) 6, 12, 20, 30, 42 $a_0 = 6$ $a_n = a_{n-1} + 2(n+2)$

$6 - 0 = 6$ $20 - 12 = 8$ $a_1 = 6 + 6 = 12$ $a_4 = 30 + 2(6) = 42$

$12 - 6 = 6$ $30 - 20 = 10$ $a_2 = 12 + 2(4) = 20$ $\boxed{a_n = a_{n-1} + 2(n+2)}$

$a_3 = 20 + 2(5) = 30$

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2.1) Describing sequences

- Sequence: an ordered list of numbers

$$a_0, a_1, a_2, a_3, \ldots$$

$$(a_n)\ n \in \mathbb{N}$$

index

1. 7	3. 4	5. 49	7. 28	9. -2
2. -3	4. 64	6. 37	8. 17	

Closed Formula: For a sequence $(a_n)\ n \in \mathbb{N}$ is a formula for a_n using a fixed finite number of operations n

Recursive Definition: An equation relating a term of the sequence to previous terms and an initial condition

Find a_6) $a_n = 2a_{n-1} - a_{n-2}$ $\quad a_0 = 3 \quad a_1 = 4$

I need a_3 and a_4

$a_2 = 2(4) - 3 = 5$

$a_3 = 2(5) - 4 = 6$

$a_4 = 2(6) - 5 = 7$ $\qquad a_n = n + 3$

$a_5 = 2(7) - 6 = 8$

$a_6 = 9$

1	2	3	4	5	6	7
3	8	14	24	33	33	45

→ Sequence of partial sums

$$\sum_{k=1}^{n} a_k$$

1. $\sum_{k=1}^{n=100} a_k$ 2. $\sum_{k=1}^{n=50} 2^k$

3. $\sum_{k=2}^{n} (4k-2)$

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11. Find a closed formula for the sequence with recursive definition:

$$a_n = 2a_{n-1} - a_{n-2} \qquad a_1 = 1 \quad a_2 = 2$$

$$a_3 = 2a_2 - a_1 = 2(2) - 1 = 3$$

$$a_4 = 2a_3 - a_2 = 2(3) - 2 = 4$$

$$a_5 = 2a_4 - a_3 = 2(4) - 3 = 5$$

? $a_n = n$

Looks good!

$a_6 = 6$	or	$a_6 = 2(a_5) - a_4 = 2(5) - 4 = 6$
$a_7 = 7$	or	$a_7 = 2(a_6) - a_5 = 2(6) - 5 = 7$
$a_8 = 8$	or	$a_8 = 2(a_7) - a_6 = 2(7) - 6 = 8$

12. Give 2 different recursive definitions for the sequence with closed formula $a_n = 3 + 2n$. Prove you are correct. At least one of the recursive definitions should makes use of use of two previous terms and no constants.

$$a_n = 3 + 2n$$
$$a_0 = 3 + 2(0) = 3$$
$$a_1 = 3 + 2(1) = 5$$
$$a_2 = 3 + 2(2) = 7$$
$$a_3 = 3 + 2(3) = 9$$

$$\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ \end{array}$$
$$(3, 5, 7, 9, 11, \ldots)$$

$a_n = 2(a_{n-1}) - a_{n-2}$? ✓

$$a_2 = 2(5) - 3 = 7$$
$$a_3 = 2(7) - 5 = 9$$
$$a_4 = 2(9) - 7 = 11$$
$$a_5 = 2(11) - 9 = 13$$

$a_n = a_{n-1} + a_{n-2} - a_{n-3}$? ✓

$$a_3 = 7 + 5 - 3 = 9$$
$$a_4 = 9 + 7 - 5 = 11$$
$$a_5 = 11 + 9 - 7 = 13$$

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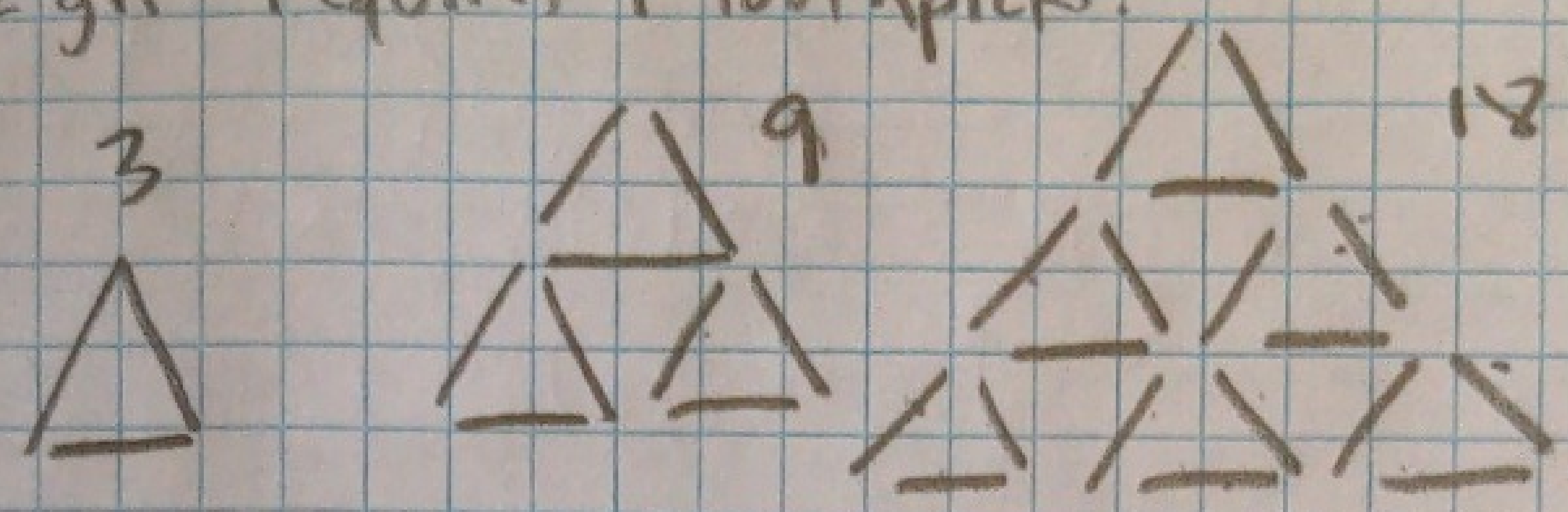
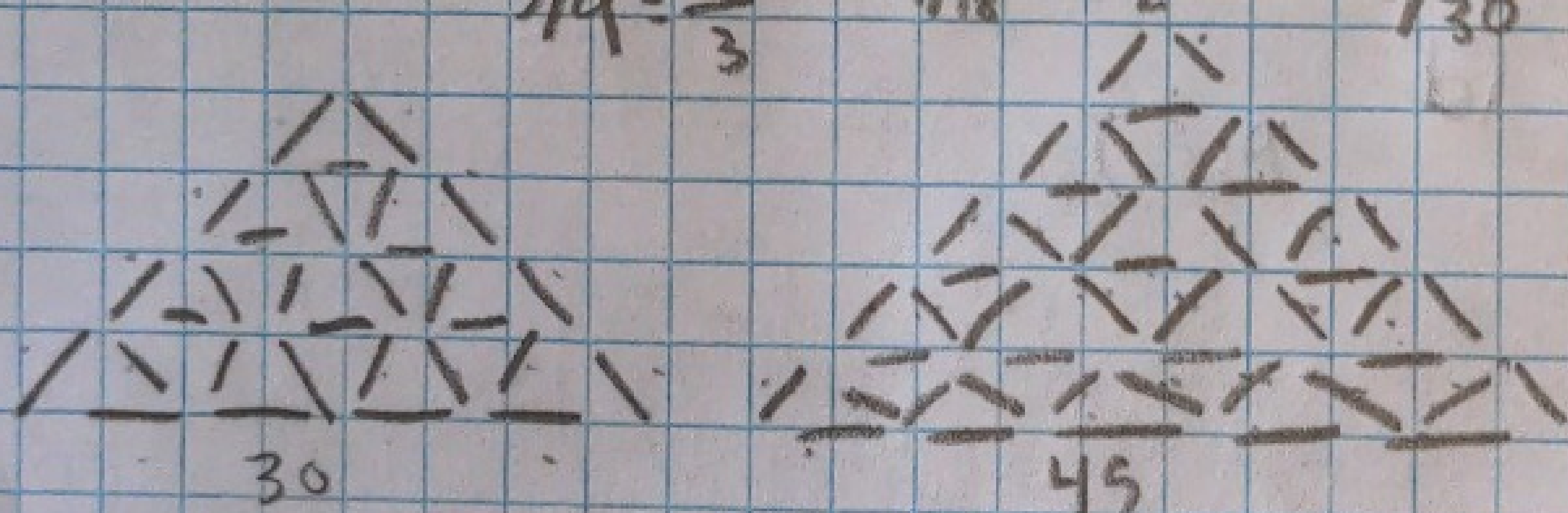
13. If you have enough toothpicks, you can make a large triangular grid. Below, are the triangular grids of size 1 and of size 2. The size 1 grid requires 3 toothpicks the size 2 grid requires 9 toothpicks.



a) Let t_n be the number of toothpicks required to make a size n triangular grid. Write out the first 5 terms of the sequence $t_1, t_2, \ldots$

$t_1 = 3$ $\xrightarrow{+6}$ $t_2 = 9$ $\xrightarrow{+9}$ $t_3 = 18$ $\xrightarrow{+12}$ $t_4 = 30$ $\xrightarrow{+15}$ $t_5 = 45$

$3/9 = \frac{1}{3}$ $9/18 = 1/2$ $18/30 = 3/5$ $30/45 = 2/3$



b) $t_n = t_{n-1} + 3n$ $t_1 = 3$

c) This sequence is not arithmetic (the difference between each is not a constant) and it is not geometric (the ratio between successive terms is not a constant). But the difference between the terms do form an arithmetic sequence → $\underset{1}{6}, \underset{2}{9}, \underset{3}{12}, \underset{4}{15}$ $a_n = 3n+3$

So this sequence is a sequence of partial sums

d) $a_n =$

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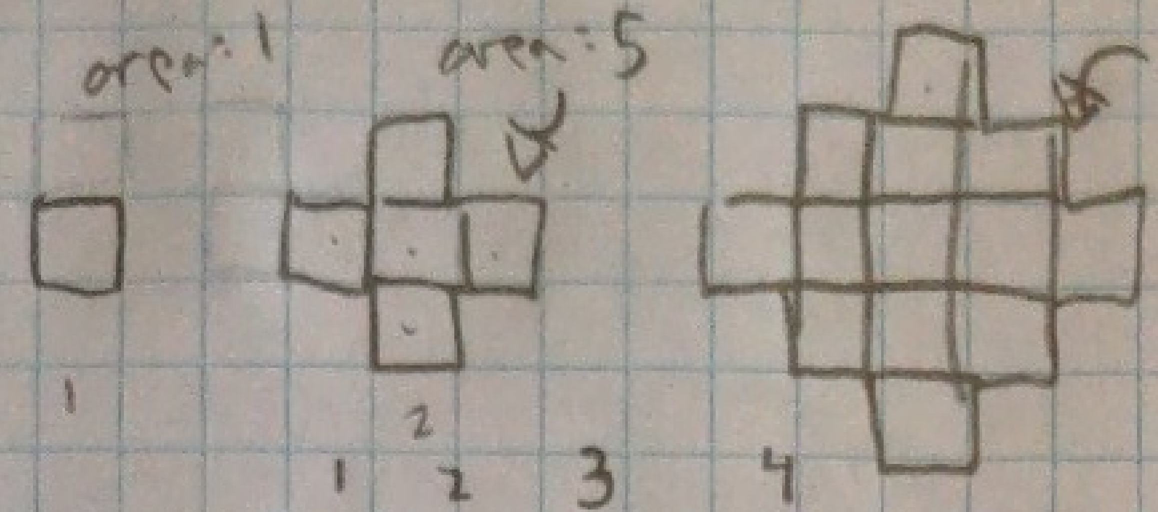
14. If you were to shade in a $n \times n$ square on graph paper you could do it the boring way or the interesting way



a) $a_n = (1, 5, 13, 25, 41?, \ldots$

$+4 \quad +8 \quad +12 \quad +16?$

$+4 \quad +4$

This is a partial sum of an arithmetic sequence

For each new square we add a middle row that is $2n-1$ squares and add the previous middle row again

$a_1 = 1$

$a_n = a_{n-1} + (2n-1 + 2(n-1)-1)$

$a_n = a_{n-1} + (4n-4)$

$a_2 = 1 + (4(2)-4) = 5$

$a_3 = 5 + (4(3)-4) = 13$

b) 1, 5, 13, 25, 41,

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15. How many license plates consist of 6 symbols using only #'s {1, 2, 3} and letters {a, b, c, d} no numeral appears after a letter

a) Case 1) No numerals

$\underline{4}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{4} = 4^6 = 4096$

Case 2) One digit

$\underline{3}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{4} = 3 \cdot 4^5 = 3072$

Case 3) Two digit

$\underline{3}\ \underline{3}\ \underline{4}\ \underline{4}\ \underline{4}\ \underline{4} = 3^2 \cdot 4^4 = 2304$

$3^x \cdot 4^y$

$3^3 \cdot 4^3 = 1728$

$3^4 \cdot 4^2 = 1296$

$3^5 \cdot 4^1 = 972$

$3^6 \cdot 4^0 = 729$ +

14197

Partial sum of geometric sequence

$n = 7$

b) $4^7 - 3^7 = 14197$

729, 972, 1296, 1728, 2304, 3072, 4096

+243 +324 +432 +576 +768 +1024

$324/243 = \frac{4}{3}$ $432/324 = \frac{4}{3}$ $\frac{576}{432} = \frac{4}{3}$

partial sum of a geometric sequence

common ratio is 4/3