Chris Hunt DP 2 MTH231

3.1 Propositional Logic

A proposition is a statement

Propositional Logic - studies the way statements can interact with each other

Truth Tables - method for determining the truth value of a statement

Tautology - a statement which is true on the basis of its logical form alone

Logical Equivalence - Two statements, P and Q, that have identical truth tables

De Morgan's Laws - $\neg(P \wedge Q) \equiv \neg P \vee \neg Q$

$\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

Implications are Disjunctions - $P \rightarrow Q \equiv \neg P \vee Q$

Negation of an implication - $\neg(P \rightarrow Q) \equiv P \wedge \neg Q$

Double negation - $\neg\neg P \equiv P$

Deduction Rule - argument form which is always valid

modus ponens:

$$\begin{array}{l} P \rightarrow Q \\ P \\ \hline \therefore Q \end{array}$$

P	Q	P→Q
T	T	T
T	F	F
F	T	T
F	F	T

Predicate Logic - Like propositional logic but with predicates allows for finer grain logical analysis

- No analog to truth tables

$\neg\exists x \forall y (x \leq y) \equiv$ "it is not true that there is a number x such that for all numbers y, x is less than or equal to y"

$\updownarrow$

$\forall x \exists y (y < x) \equiv$ "For every number x there is a number y which is smaller than x"

Law of Logic - A statement in predicate logic that is necessarily true.

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5) "I want either pepperoni or sausage. Also if I have sausage, then I must also include quail. Oh, and if I have pepperoni or quail then I must have ricotta cheese."

a) P = pepperoni Q = sausage R = quail S = ricotta

$P \vee Q$. $Q \rightarrow R$. $(P \vee R) \rightarrow S$.

P	Q	R	S	$P \vee Q$	$Q \rightarrow R$	$(P \vee R) \rightarrow S$
T	T	T	T	T	T	T
T	T	T	F	T	T	F
T	T	F	T	T	F	T
T	T	F	F	T	F	F
T	F	T	T	T	T	T
T	F	T	F	T	T	F
T	F	F	T	T	T	T
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	T	F	T	T	F
F	T	F	T	T	F	T
F	T	F	F	T	F	T
F	F	T	T	F	T	T
F	F	T	F	F	T	F
F	F	F	T	F	T	T
F	F	F	F	F	T	T

b) Looking at the truth tables there is no scenario in which every statement is false, therefore, Geoff must be a truth teller

c) The waiter can conclude that there will be ricotta cheese in the calzone.

18. Negate each statement as simply as possible

a) Every number is either even or odd

$P(x) = x \bmod 2 = 0$ $Q(x) = x \bmod 2 = 1$

$\neg(\forall x \in N (P(x) \vee Q(x))) = \exists x \in N (\neg P(x) \wedge \neg Q(x))$

"There is some number that is not even and not odd."

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18) b) There is a sequence that is both arithmetic and geometric".

X = some numeric sequence

$P(x)$ = X is arithmetic $Q(x)$ = X is geometric

$$\neg(\exists x(P(x) \wedge Q(x)) \equiv \forall x(\neg P(x) \vee \neg Q(x))$$

Every sequence is either not arithmetic or it's not geometric.

c) For all numbers n, if n is prime, then $n+3$ is not prime.

$P(n)$ = n is prime $Q(n)$ = $n+3$ is not prime

$$\neg(\forall n \in \mathbb{N}(P(n) \rightarrow Q(n)) \equiv \exists n \in \mathbb{N}(P(n) \wedge \neg Q(n))$$

There is some number n such that n is prime and $n+3$ is prime.

SQ-1) The notation $\exists!$ means "there exists a unique." Suppose that $P(x)$ is a predicate and D is the domain of discourse for x.

1. Rewrite the statement "$\exists! x \in D$ such that $P(x)$" without using $\exists!$

   Let: $Q(x)$ = X is unique

   "$\exists x \in D$ S.T. $(P(x) \wedge Q(x))$"

2. $\neg(\exists! x \in D \text{ s.T. } P(x)) = \forall! x \in D \text{ s.T. } \neg P(x)$

   "For all unique x in D not $P(x)$"

   As long as X is unique in D it is not $P(x)$