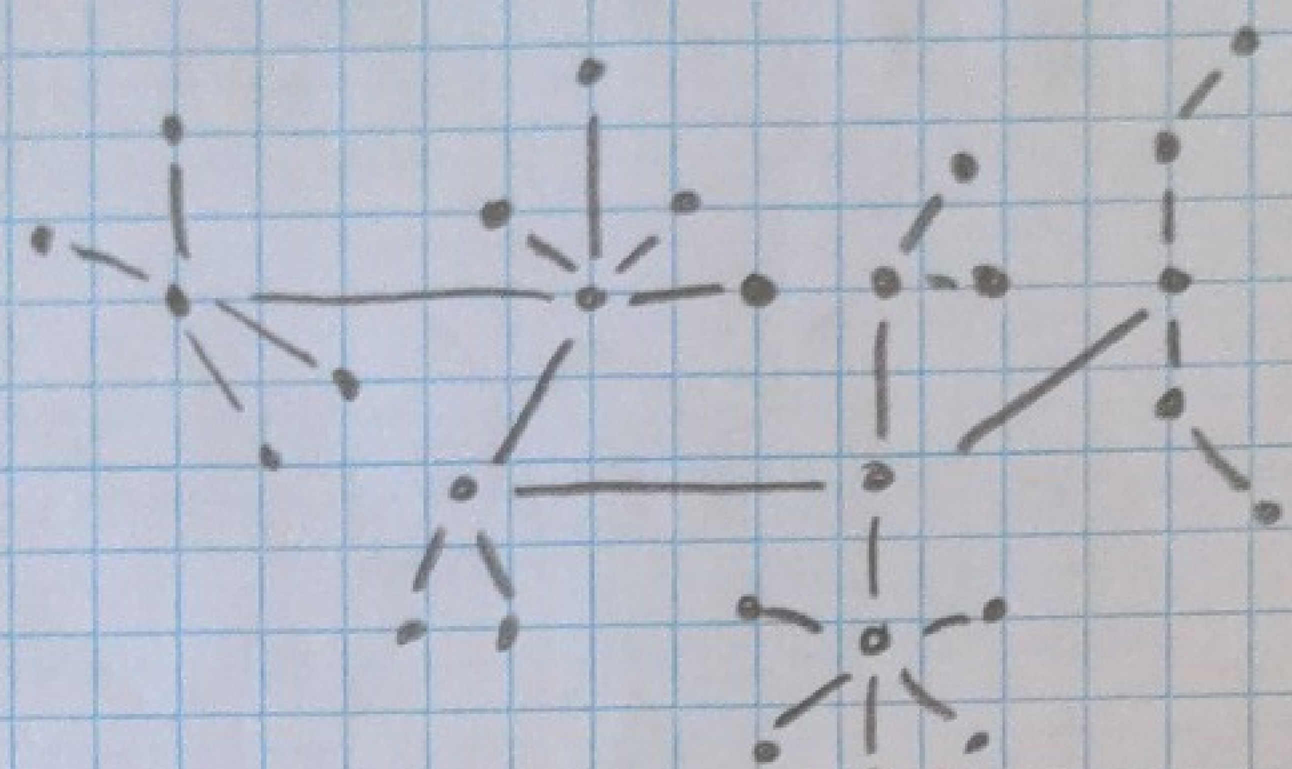
Chris Hunt DP9 MTH 231

6) Prove the chromatic number of any tree is two

- A tree is a connected graph with no cycles

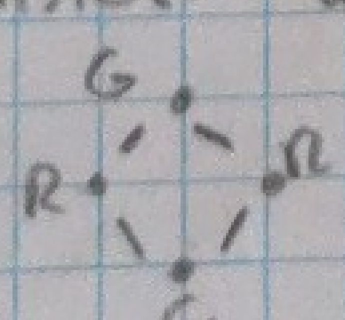
a) Describe a procedure to color the tree below.



Place a vertex and assign it a color, let's say black. Place a second vertex and make it adjacent to any vertex, in this instance V_1. Since V_1 is black V_2 must be another color, let's say red. For every other added vertex, V_n, connect it to an existing vertex and assign it either red or black, which ever the vertex being connected to isn't. Since each addition vertex is only being connected to one other vertex at a time the tree can be colored with only 2 colors.

b) ~~In a cycle graph with odd number of vertices~~

Consider a cycle graph with even vertices



When placing vertices and edges, each new vertex connected to the previous with the final vertex connected to the initial it is possible to alternate between colors. Since there is an even amount of vertices

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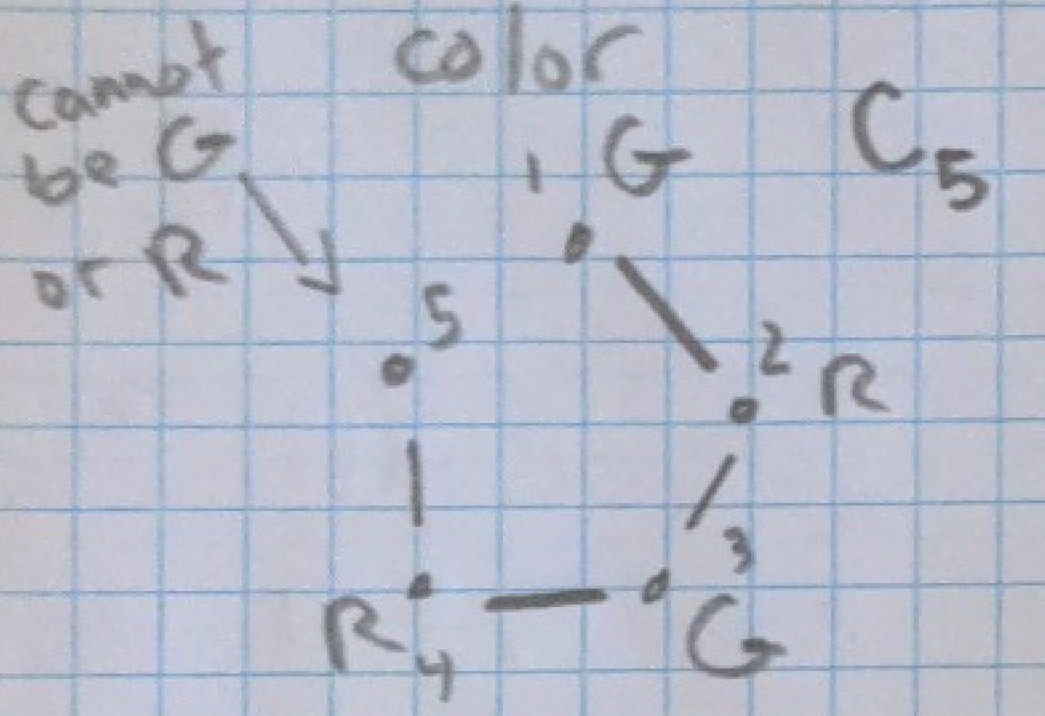
6) b) there will be equal vertices of each color color one and two. When there is an odd number of vertices there would be an unequal amount between the colors, this would require the final vertex to be a third color



V_5 must be neither G or R to be the final vertex in the cycle.

c) see a.

d) A tree is a connected graph with no cycles.

Claim: Every tree has chromatic number 2.

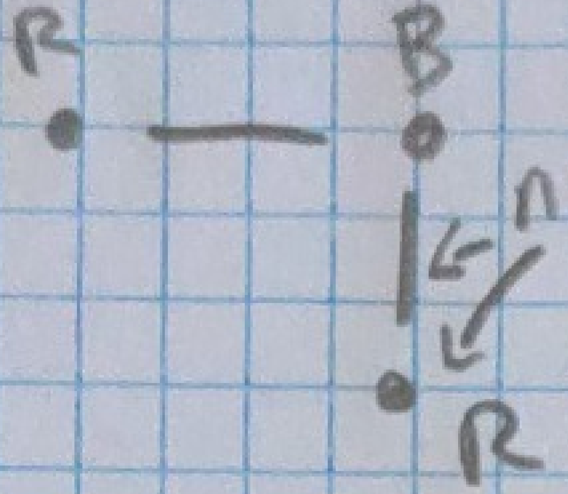
Proof: Base case: A tree with two vertices

In this scenario pick two colors, R + B. Pick a vertex and assign it a color, any vertex adjacent to this vertex will be the other. This tree has two vertices and has $X(G) = 2$

<u>Inductive Step</u>: Assume G is a tree with chromatic number of 2, G has K vertices.

Consider the K+1 vertex. When adding a vertex to a tree it must also be connected by 1 edge initially

6) d) Assign a color to this new leaf. It cannot be the same color as the adjacent vertex, since there are no more adjacent vertices the other color can be assigned to it. The chromatic # remains 2.



Therefore, by induction, every tree has chromatic # 2.

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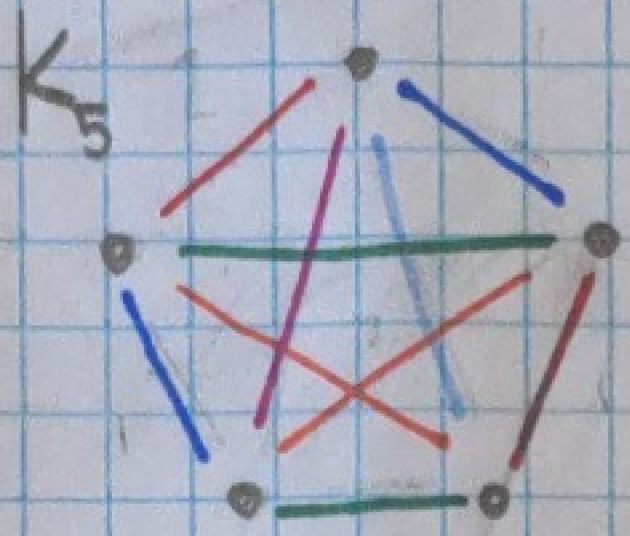
7. A Quidditch league has 5 teams. In a tournament every team will play every other team once. Each team can play one match per day. What is the fewest number of days over which the tournament can take place

K_5

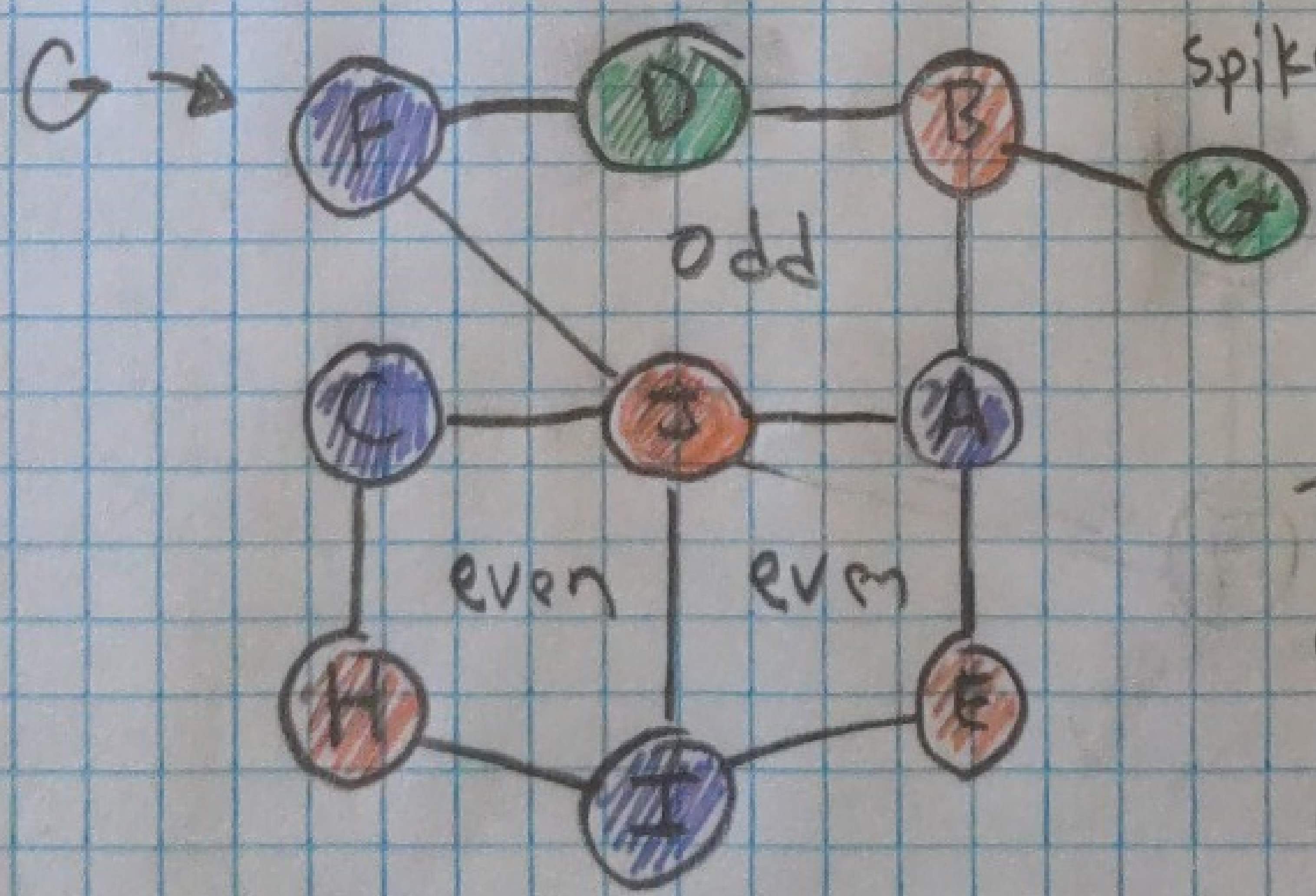
day 1
day 2
day 3
day 4
day 5
day 6



Color edges. Each edge is a day.

It will take 6 days to play the tournament

Color the vertices. Each color represents a car



Car 1

Car 2

Car 3

The minimum amount of cars required is 3. The odd cycle make it so $\chi(G)$ cannot be 2