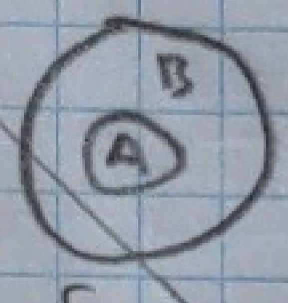
Chris Hunt DP5 MTH 231

SQ-5) Prove or disprove: if A is a subset of B, then $\wp(A)$ is a subset of $\wp(B)$

<u>Claim:</u> $A \subseteq B \rightarrow \wp(A) \subseteq \wp(B)$

<u>Proof:</u> ~~Assume $A \subseteq B$. That is $\forall x \in A$ is also $\in B$~~



~~The definition of a power set is the set that consists all of the possible subsets. To disprove the claim we need to find some set A that is a subset of B but $\wp(A)$ is not a subset of $\wp(B)$~~

~~Consider set A and B.~~

~~$A = \{X_1\}$ $B = \{X_1, X_2\}$~~

~~What are the subsets of B?~~

~~$\wp(B) = \{\{\}, \underline{\{X_1\}}, \{X_2\}, \{X_1, X_2\}\}$~~

~~$\{X_1\}$ is a subset in B, therefore $A \subseteq B$.~~

~~Now consider $\wp(A)$:~~

~~$\wp(A) = \{\{\}, \{X_1\}\}$~~

~~We can see that each element~~

$\neg(\wp(A) \subseteq \wp(B)) \rightarrow \neg A \subseteq B$

Consider the contrapositive $\neg(\wp(A) \subseteq \wp(B)) \rightarrow \neg(A \subseteq B)$

Assume $\neg(\wp(A) \subseteq \wp(B))$ is true. This would mean there is a subset of A which is not a subset of B. Since by the definition of a power set, every possible subset of A is in $\wp(A)$ and likewise for B that means there is some element X in A which is not in B. The contrapositive is true, therefore the original claim is also true.

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SQ-6) $D = \{ S \mid S \text{ not an element of } S \}$

A) Claim: $D \in D$

__Proof:__ If D were to be an element of D this would immediately break the definition of D that each element must be the set of all sets not containing themselves. Therefore $D \in D$ would be a contradiction

B) Claim: $D \notin D$

__Proof:__ Assume $D \in D$, that means D is a set that does not contain itself and thus be added to D. At which point it must be in D. This leads to a loop of contradictions.