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18. Suppose $P(x)$ is some predicate for which the statement $\forall x P(x)$ is true. Is it also the case that $\exists x P(x)$ is true? Is the converse always true? Assume the domain of discourse is non-empty.

$\forall x P(x) \rightarrow \exists x P(x)$ is always true because $P(x)$ is true for every x, therefore there ~~is~~ will always be some x such that $P(x)$ is true.

The converse, $\exists x P(x) \rightarrow \forall x P(x)$ is not always true because if we only know that $P(x)$ is true for some but not necessarily all, then the statement $\forall x P(x)$ might be false for some value of x.

20. Consider the statement, "For all natural numbers n, if n is prime, then n is solitary."

a. Converse: For all natural numbers n, if n is solitary, then n is prime.

Contrapositive: For all natural numbers n, if n is not solitary, then it is not prime.

b. Negation: For all natural numbers n, if n is not prime, then n is not solitary.

We would have to find a number that is not prime and is solitary.

c. "If 10 is prime, then 10 is solitary". This statement evaluates to true because 10 is not prime. The truth table for conditionals allows the "then statement" to be true or false when the "if statement" is false. Since we don't know what makes a number solitary then 10 could still be solitary

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20.

d. Original: Since 8 is not prime, the original statement is still true. The statement makes no claims about nonprimes.

Converse: Since 8 is not prime, but it is solitary, This makes the converse statement False

contrapositive: 8 is solitary and it is not prime, the original statement is still true. The statement makes no claims about solitary numbers.

e. If the original statement is true we can conclude that every prime number is contained within all numbers that are solitary.