Chris Hunt DP3 MTH231

7) b. For all integers a and b, if a is even and b is a multiple of 3, then ab is a multiple of 6.

<u>Let:</u> P: "a is even" Q: "b is a multiple of 3"

R: "$6 \mid ab$"

$\forall a,b \in \mathbb{Z}: P \wedge Q \to R$ converse: $\forall a,b \in \mathbb{Z}: R \to P \wedge Q$

<u>Proof:</u> ~~Suppose not. Then, $\exists a,b \in \mathbb{Z}$ $R \wedge \neg(P \wedge Q) \equiv R \wedge (\neg P \vee \neg Q)$~~

~~So $6 \mid ab$ and a is odd or b is not a multiple of 3.~~

~~Let $ab = 6n$, where n is some integer.~~

~~$ab = 6n = 2(3)(n)$~~

If a is odd it cannot be a multiple of 2

Proof by counter example

Suppose $6 \mid ab$. Then a must be even and b must be a multiple of 3.

Let $a = 6$ and $b = 2$.

$$a \cdot b = 6 \cdot 2 = 12 \text{ and } 6 \mid 12 = 2$$

Since the product of a and b is a multiple of 6 and b is not a multiple of 3, the converse is False.

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a) For all integers a, b, and c, if $a^2+b^2=c^2$, then a or b is even

$\forall a,b,c \in \mathbb{Z} (a^2+b^2=c^2 \rightarrow$ a or b is even$)$

Contradiction: $\exists a,b,c \in \mathbb{Z} (a^2+b^2=c^2 \wedge$ a and b are odd$)$

Suppose a and b are odd. $a=2k+1$ $b=2j+1$ where k, j are some integers.

Consider: $a^2+b^2=(2k+1)^2+(2j+1)^2=4k^2+4k+1+4j^2+4j+1$

$=4k^2+4j^2+4k+4j+2$

Now this value must be the square of some integer c to contradict this statement.

$c^2=4k^2+4j^2+4k+4j+2$

$c^2=2(2k^2+2j^2+2k+2j+1) \leftarrow$ even

Since c^2 is even, then c is even as well because the square of an odd number is always odd

$(2k+1)^2=4k^2+4k+1=2(2k^2+2k)+1 \leftarrow$ odd

$c=2p$

$(2p)^2=2(2k^2+2j^2+2k+2j+1)$

$4p^2=2(2k^2+2j^2+2k+2j+1)$

$2p^2=2k^2+2j^2+2k+2j+1$

$2(p^2)=2(k^2+j^2+k+j)+1$

$\uparrow$ even $\uparrow$ odd

This is a contradiction!

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12) Prove: $x = y$ if and only if $xy = \frac{(x+y)^2}{4}$

Let: $P(x,y): x = y$ $\quad Q(x,y): xy = \frac{(x+y)^2}{4}$

$\forall x,y\, (P(x,y) \to Q(x,y) \wedge Q(x,y) \to P(x,y))$

Suppose $P(x,y)$

Concider xy

$$xy = \frac{(x+y)^2}{4} = xx = \frac{(x+x)^2}{4} = \frac{4x^2}{4} = x^2 = xx$$

This is true for all values x and y

~~Now examine the second statement $Q(x,y) \to P(x,y)$~~

~~Suppose the contrapositive $\neg P(x,y) \to \neg Q(x,y)$~~

~~$x \neq y$: $xy \neq \frac{(x+y)^2}{4}$~~

~~$4xy = (x+y)(x+y)$~~

~~Hmm maybe not~~

~~How about examining the contradiction~~

Suppose $Q(x,y)$

$$xy = \frac{(x+y)^2}{4}$$

$$4xy = (x+y)^2$$

$$4xy = x^2 + y^2 + 2xy$$
$$-4xy$$

$$0 = x^2 + y^2 - 2xy$$

$$(0 = (x-y)^2)^{.5}$$

$$0 = x - y$$

$$x = y$$

Therefore the second statement is true