Chris Hunt DP7 MTH231

2.3) Arithmetic Sequence: If the terms of a sequence differ by a constant it is arithmetic

Recursive: $a_n = a_{n-1} + d \quad a_0 = a$

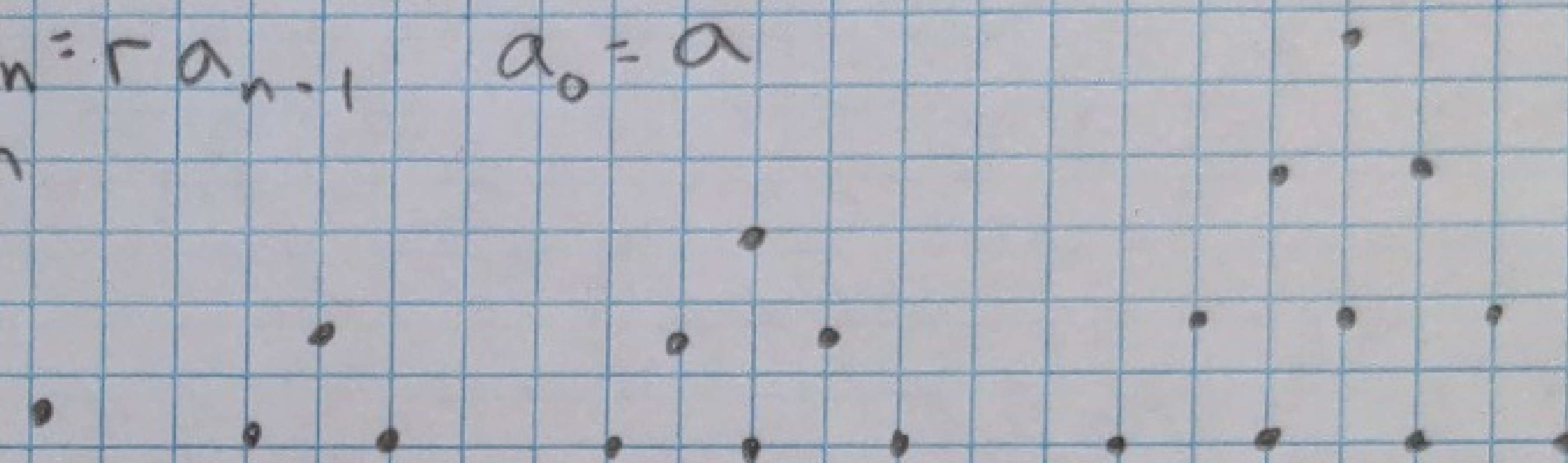
Closed: $a_n = a + dn$

Geometric Sequence: If the ratio between successive terms is constant r = common ratio

Recursive: $a_n = r a_{n-1} \quad a_0 = a$

Closed: $a \cdot r^n$

Triangle #'s:



$T_1 = 1 \quad T_2 = 3 \quad T_3 = 6 \quad T_4 = 10$

Not arithmetic

Not Geometric

The nth term of the sequence is the sum of the first n terms in the sequence of differences

T_n is said to be the sequence of partial sums

$T_1 = 1 = 1$

$T_2 = 3 = 1 + 2$

$T_3 = 6 = 1 + 2 + 3$

$T_4 = 10 = 1 + 2 + 3 + 4$

$\vdots$

$$T_n = \frac{n(n+1)}{2} \rightarrow \binom{n+1}{2}$$

$2 + 5 + 8 + 11 + 14 + \ldots + 470$

$+3$

$$\begin{aligned} S &= 2 + 5 \ldots \\ + \; S &= 470 + 467 \ldots \\ \hline 2S &= 472 + 472 + \end{aligned}$$

$2S = 157 \cdot 472 = 74104$

Find n

$a_n = 2 + 3n$

$470 = 2 + 3n$

$468 = 3n$

$156 = n \rightarrow 0 - 156 = 157$ items

Divide by 2 since it was doubled

$T_{156} = \rightarrow 37052$

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Summing Geometric Sequences:

$3 + 6 + 12 + 24 + \dots + 12288$

3 6 12

$6/3 = 2$ $12/6 = 2$

$r = 2 \rightarrow a_n = 3 \cdot 2^n$

To find the sum: multiply each term by 2 (the common ratio)

Then subtract S

$$
\begin{array}{rcl}
S & = & 3 + 6 + 12 + 24 + \dots + 12288 \\
-\ 2S & = & \quad 6 + 12 + 24 + \dots + 12288 + 24576 \\
\hline
-S & = & 3 + 0 + 0 + 0 + \dots + 0 - 24576
\end{array}
$$

$-S = -24573 \quad \cdot -1 \longrightarrow S = 24573$

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SQ-9: How many permutations of $\{1,2,3,4,5\}$ leave exactly one element fixed

order matters

$\underline{1} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 4! \cdot 5 = 5!$

can be done for each index

$\binom{5}{1}\binom{4}{1}\binom{3}{1}\binom{2}{1}\binom{1}{1}$

$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$ ... am I missing something?

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SQ-10: Ten ladies drop off hats. Hats given back randomly. How many ways can exactly six receive their own hat?

$\underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{1}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}$

$\binom{10}{6} = 210$ or $\binom{10}{4} = 210$

For this problem each lady can have two states (right hat, wrong hat) or (1,0). This then turns into a binomial problem

From 10 choose 6 to receive right hat

or

From 10 choose 4 to receive wrong hat