The Valley-reprise

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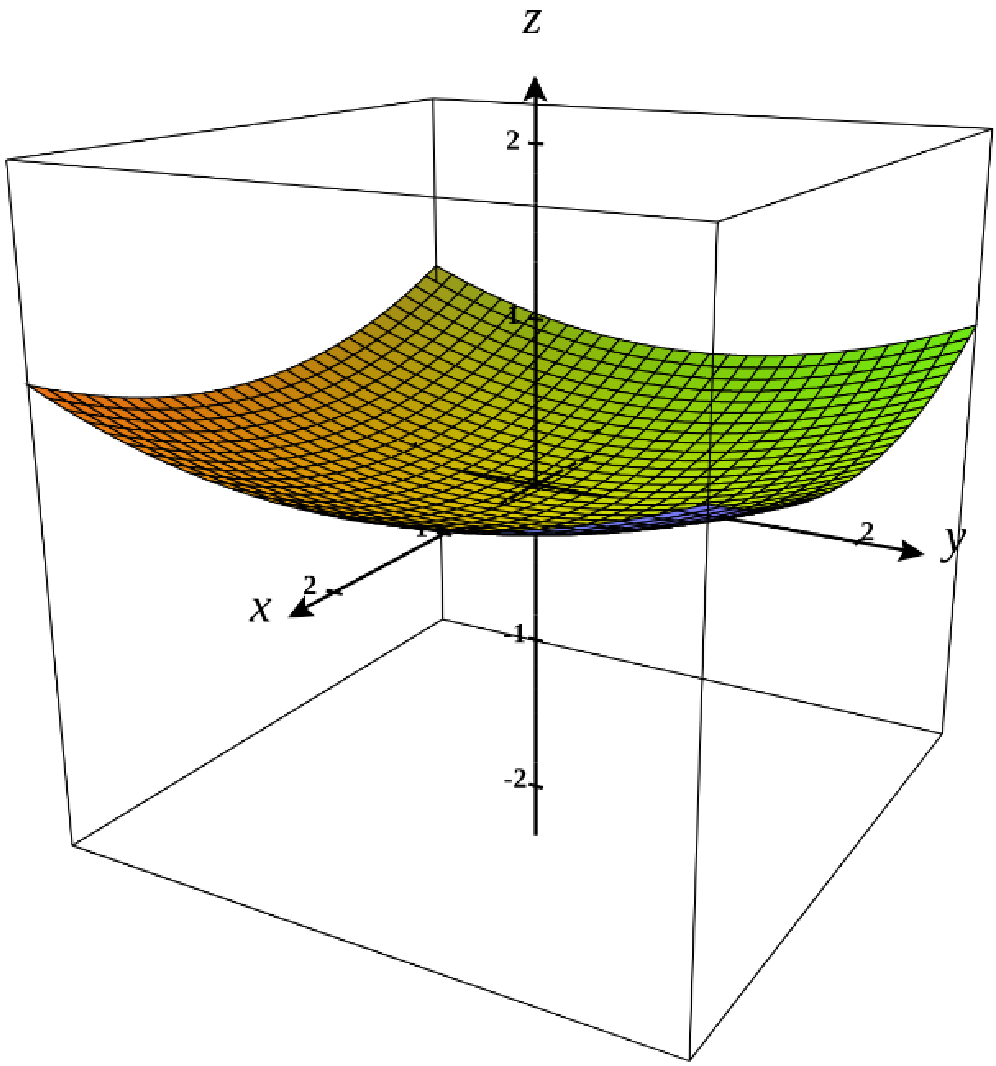
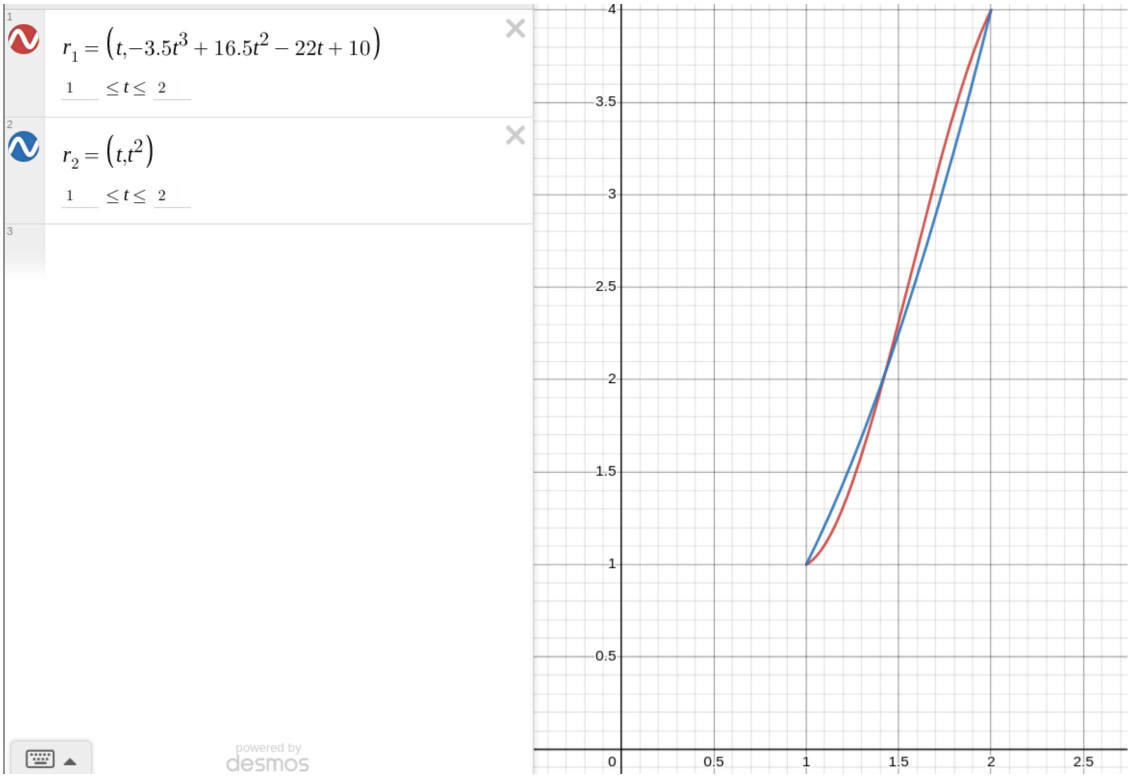
The valley can be described by the following function:

$$h(x, y) = \frac{x^2 + y^2}{10}$$



We will find the change of elevation between the point at (2,4), (1,1) along the following two paths:

$$\vec{r_1} = t\hat{i} + (-3.5t^3 + 16.5t^2 - 22t + 10)\hat{j} \qquad \vec{r_2} = t\hat{i} + t^2\hat{j} \qquad 2 \geq t \geq 1$$



To find the elevation change, ΔE, between the two paths a line integral will be used:

$$\Delta E = \int_c \vec{\nabla} h \cdot d\vec{r}$$

Path 1:

$$x = t \quad y = -3.5t^3 + 16.5t^2 - 22t + 10 \qquad 2 \geq t \geq 1$$

$$d\vec{r_1} = dt\hat{i} + (-10.5t^2 + 33t - 22)\, dt\hat{j} \qquad \vec{\nabla} h = \frac{2}{10}t\hat{i} + \frac{2}{10}(-3.5t^3 + 16.5t^2 - 22t + 10)\hat{j}$$

$$\Delta E_1 = \frac{2}{10}\int_c t + (-10.5t^2 + 33t - 22)(-3.5t^3 + 16.5t^2 - 22t + 10)\, dt$$

$$\Delta E_1 = \frac{2}{10}\int_2^1 -220 + 815t - 1194.t^2 + 852.5t^3 - 288.75t^4 + 36.75t^5\, dt \rightarrow \Delta E_1 = -1.8\ ft$$

Path 2:

$$\vec{\nabla} h = \frac{2}{10}x\hat{i} + \frac{2}{10}y\hat{j} \qquad x = t \quad y = t^2 \qquad 2 \geq t \geq 1$$

$$d\vec{r_2} = dt\hat{i} + 2t\, dt\hat{j} \qquad \vec{\nabla} h = \frac{2}{10}t\hat{i} + \frac{2}{10}t^2\hat{j}$$

$$\Delta E_2 = \frac{2}{10}\int_2^1 t + 2t^3 \; dt \rightarrow \Delta E_2 = -1.8 \; ft$$

The change in elevation between the two points is -1.8 ft. The two values are equal because the gradient we are integrating our path over is conservative. The line integral between two points within a conservative vector field will always be the same regardless of the path taken there. This is independence of path.