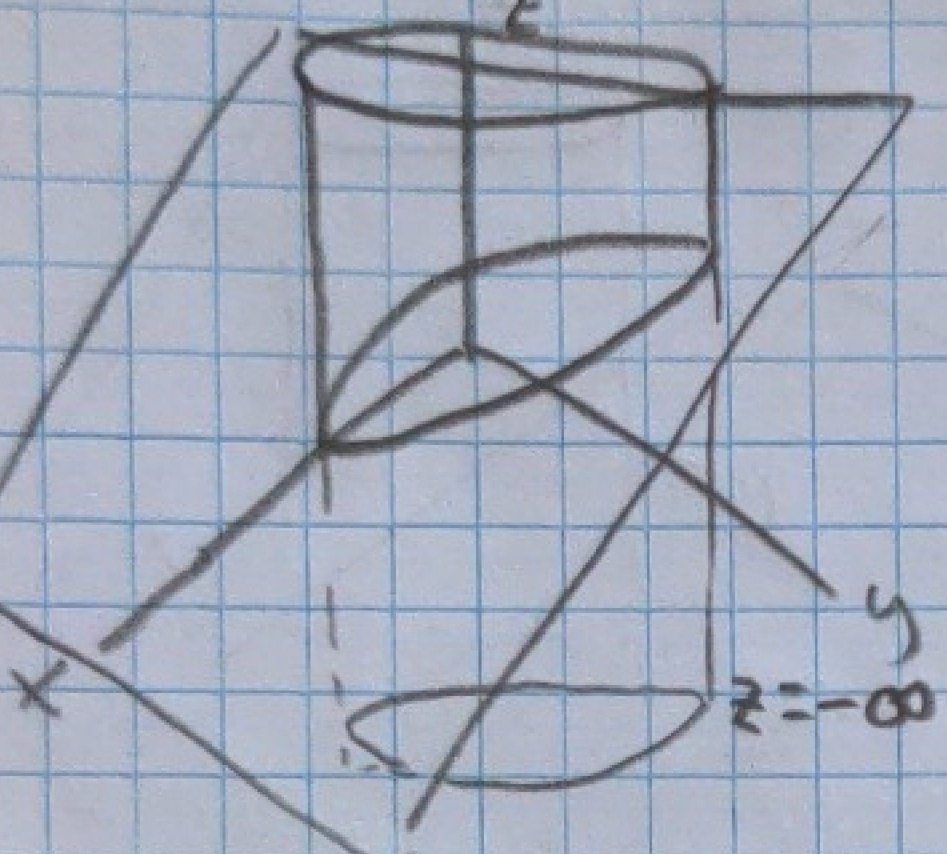
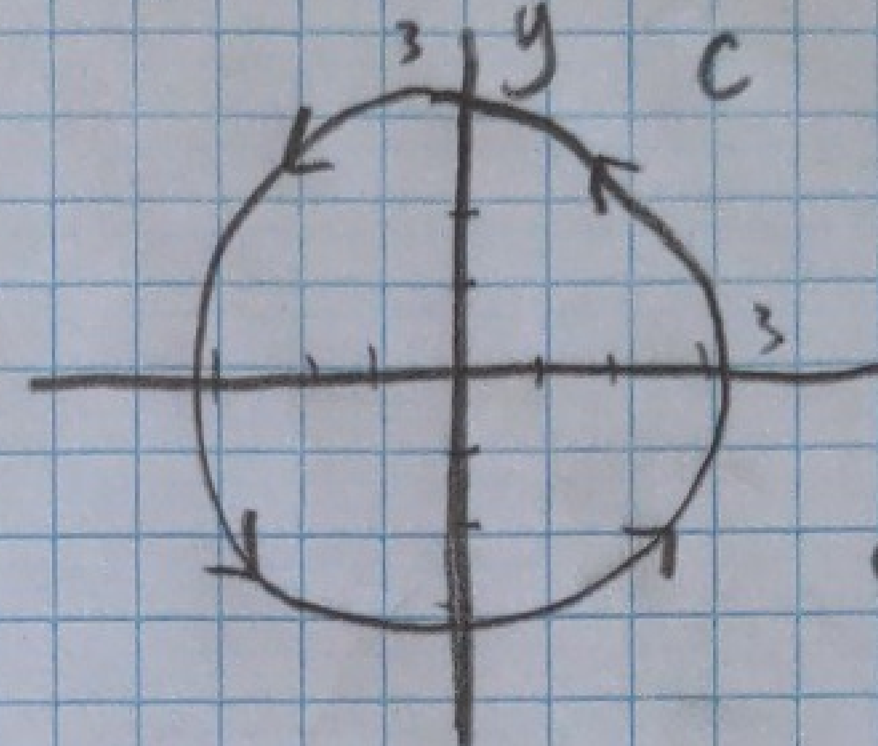
Chris Hunt HW10 MTH 255

1. b) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where:

$$\vec{F}(x,y,z) = xy\hat{\imath} + 2z\hat{\jmath} + 3y\hat{k}$$

and C is the curve of the intersection of the plane $x+z=5$ and the cylinder $x^2+y^2=9$, oriented counterclockwise as viewed from above





$x^2+y^2=9$

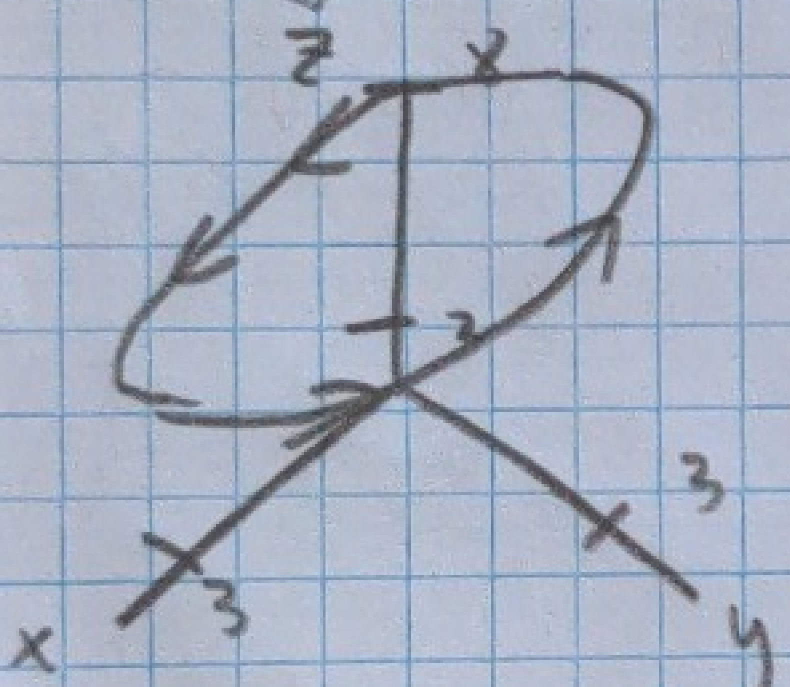
$r^2=9$

$r=3$

$0 \le \phi \le 2\pi$

$-3 \le x \le 3$ $\quad z = 5-x$

$-3 \le y \le 3$ $\quad 2 \le z \le 8$



Use cylindrical coordinates:

$x = r\cos\phi \quad y = r\sin\phi \quad z = 5 - r\cos\phi$

$\vec{r} = 3\cos\phi\,\hat{\imath} + 3\sin\phi\,\hat{\jmath} + (5-3\cos\phi)\hat{k} \quad dz = 3\sin\phi\, d\phi$

$d\vec{r} = (-3\sin\phi\,\hat{\imath} + 3\cos(\phi)\hat{\jmath} + 3\sin(\phi)\hat{k})\, d\phi$

$\vec{F}(\phi) = 3\cos(\phi)3\sin(\phi)\hat{\imath} + 2(5-3\cos(\phi))\hat{\jmath} + 9\sin(\phi)\hat{k}$

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^{2\pi} -27\cos(\phi)\sin^2(\phi) + 30\cos(\phi) - 18\cos^2(\phi) + 27\sin^2(\phi)\, d\phi$$

$$\int_C \vec{F} \cdot d\vec{r} = \boxed{9\pi}$$

Given a surface, S, with an open boundary, C, we can find the curl over the surface by taking the line integral around C.

$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{A} = \int_C \vec{F} \cdot d\vec{r}$$

In this case the surface has some arbitrary closed lower bound and a definite boundary, C, which is described by $\vec{r}$. Because of this we can use Stokes' Theorem to find the total curl over the surface. In this case the curl integrated over the surface would be 9π.