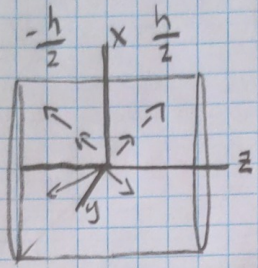
Chris Hunt — The Charge — MTH 255

Challenge:

$$\vec{E} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{\vec{r}}{|\vec{r}|^3}$$



$$-\frac{h}{2} \le z \le \frac{h}{2} \qquad 0 \le \phi \le 2\pi$$

$$r = a \qquad a > 0$$

$$\vec{r} = r\hat{r} + z\hat{k}$$

$$|\vec{r}| = (r^2+z^2)^{1/2}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left(\frac{r}{(r^2+z^2)^{3/2}}\hat{r} + \frac{z}{(r^2+z^2)^{3/2}}\hat{k}\right)$$

Cylinder Body:

$$d\vec{A} = r\,d\phi\,dz\,\hat{r} \rightarrow \boxed{d\vec{A} = a\,d\phi\,dz\,\hat{r}}$$

$$\vec{E}\cdot d\vec{A} = \left(\frac{q}{4\pi\varepsilon_0}\cdot\frac{a^2}{(a^2+z^2)^{3/2}}\right)d\phi\,dz$$

$$\text{Flux} = \int_S \left(\frac{q}{4\pi\varepsilon_0}\cdot\frac{a^2}{(a^2+z^2)^{3/2}}\right)d\phi\,dz$$

$$\frac{a^2q}{4\pi\varepsilon_0}\int_{-\frac{1}{2}}^{h/2}\int_0^{2\pi}\frac{1}{(a^2+z^2)^{3/2}}\,d\phi\,dz$$

int ϕ

$$\frac{a^2q}{2\varepsilon_0}\int_{-h/2}^{h/2}(a^2+z^2)^{-3/2}\,dz$$

$$\frac{a^2q}{2\varepsilon_0}\cdot\frac{2h}{a^2(h^2+4a^2)^{1/2}}$$

$$\boxed{\frac{q}{\varepsilon_0}\cdot\frac{h}{(h^2+4a^2)^{1/2}}}$$

Flux through Body

Chris Hunt The Charge MTH255

Challenge:

$$\vec{E} = \frac{q}{4\pi\epsilon_0}\left(\frac{r}{(r^2+z^2)^{3/2}}\hat{r} + \frac{z}{(r^2+z^2)^{3/2}}\hat{k}\right)$$

Cylinder Cap: $0 \le r \le a \quad 0 \le \phi \le 2\pi$

$r = r \quad \phi = \phi \quad z = \frac{h}{2}$ → Will just double this integral due to symmetry

$dr = dr \quad d\phi = d\phi$

$$\boxed{d\vec{A} = r\,d\phi\,dr\,\hat{k}} \qquad \vec{E}\cdot d\vec{A} = \frac{qh}{8\pi\epsilon_0}\cdot\frac{r}{\left(r^2+\frac{h^2}{4}\right)^{3/2}}\,d\phi\,dr$$

$$\text{Flux} = 2\int_S \left(\frac{qh}{8\pi\epsilon_0}\cdot\frac{r}{\left(r^2+\frac{h^2}{4}\right)^{3/2}}\right)d\phi\,dr$$

$$\downarrow$$

$$\frac{qh}{4\pi\epsilon_0}\int_0^a\int_0^{2\pi}\frac{r}{\left(r^2+\frac{h^2}{4}\right)^{3/2}}\,d\phi\,dr$$

$$\downarrow$$

$$\frac{qh}{2\epsilon_0}\int_0^a\frac{r}{\left(r^2+\frac{h^2}{4}\right)^{3/2}}\,dr$$

$$\downarrow$$

$$\frac{qh}{2\epsilon_0}\cdot 2\left(\frac{1}{h} - \left(4a^2+h^2\right)^{-1/2}\right)$$

$$\frac{q}{\epsilon_0} - \frac{qh}{\epsilon_0\left(h^2+4a^2\right)^{1/2}}$$

Challenge:

Now add the Flux from the body and caps

$$\text{Total Flux} = \frac{q}{\varepsilon_0} - \frac{qh}{\varepsilon_0(h^2+4a^2)^{1/2}} + \frac{qh}{\varepsilon_0(h^2+4a^2)^{1/2}}$$

$$\text{Total Flux} = \frac{q}{\varepsilon_0}$$ ← Gauss' Law!