Coulomb's Law

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Given the electrostatic field $\vec{E}$ at the point $\vec{r}$ due to a charge q at $P(0,0,0)$ is:

$$\vec{E}(\vec{r}) = \frac{q}{4\pi\epsilon_o} * \frac{\vec{r}}{|\vec{r}|^3}$$

a) Compute $div(\vec{E})$:

Using spherical components:

$$\vec{E} = \frac{q}{4\pi\epsilon_o} * \frac{1}{r^2}\,\hat{r}$$

$$div\vec{E} = \frac{1}{r^2}\frac{\delta}{\delta r}(r^2\vec{E}_{\hat{r}}$$

$$div(\vec{E}) = 0$$

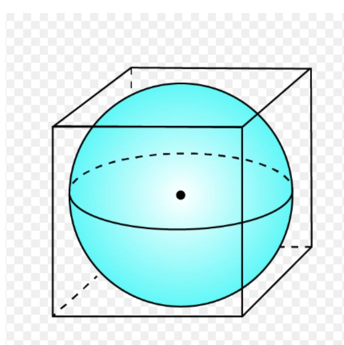
b) Let S_a be the sphere of radius a.

The electrostatic flux through the sphere of radius a is:

$$\Phi_{S_a} = \oint_S \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}\frac{Nm^2}{C}$$

By leveraging what we know about the flux through a sphere around a point charge from Guass's Law, the flux through any surface can be calculated using the divergence theorem if the surface area that the flux is being calculated through has a hollow sphere in the center which surrounds the point charge.

c) Consider an outward oriented arbitrary surface, S around a point charge q.



To find the flux through the cube, S we will place a sphere, S_a within it to form a new surface S_b. From this we can formulate the following equality:

$$\Phi_S - \Phi_{S_a} = div(\vec{E}) * dV_{S_b} = 0$$
$$\Phi_S - \frac{q}{\epsilon_0} = 0 \rightarrow \Phi_S = \frac{q}{\epsilon_0}$$

By using the divergence theorem, Gauss's Law is proven once again. For any arbitrary surface S enclosing a point charge the Electric Flux through that surface will be:

$$\Phi_S = \frac{q}{\epsilon_0} \frac{Nm^2}{C}$$