The Wire

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The Wire

A wire takes the shape of a semicircle.

$$x^2 + y^2 = 1 \qquad y \geq 0$$

Find the center of mass of the wire if the linear density of the wire is given as:

$$\rho(x, y) = k(1 - y)$$

To find the center of mass $(\bar{x}, \bar{y})$ we must first find the mass along the curve, then find the moment of mass around the x and y axes, M_x and M_y.

$$\bar{x} = \frac{M_x}{m} \qquad \bar{y} = \frac{M_y}{m}$$

To find the mass along the curve of wire we will parameterize the length of the curve and integrate the density function ρ along that curve.

Let

$$\vec{r}(t) = \hat{r} \qquad \phi(t) = t \qquad 0 \leq t \leq \pi$$
$$d\vec{r} = \hat{\phi} dt \qquad |d\vec{r}| = 1$$
$$\rho = k(1 - y) = k(1 - sin(t))$$

Solve for mass:

$$m = \int_0^\pi \rho |d\vec{r}| dt \rightarrow m = \int_0^\pi k(1 - sin(t)) dt \rightarrow m = k[t + cos(t)]|_0^\pi \rightarrow m = k\pi - 2k$$

Now find moments of mass $M_x = \int_c xk(1 - sin(t))dt$ and $M_y = \int_c yk(1 - sin(t))dt$ where $x = rcos(t)$ and $y = rsin(t)$

$$M_x = k \int_c cos(t)(1 - sin(t))dt \rightarrow \int_0^\pi cos(t) - cos(t)sin(t)dt$$
$$M_y = k \int_c sin(t)(1 - sin(t))dt \rightarrow \int_0^\pi sin(t) - sin^2(t)dt$$

Integrating these we get

$$M_x = 0 \qquad M_y = 0.4292$$

Now we can find center of mass

$$\bar{x} = 0 \qquad \bar{y} = 0.3760$$