Fluid Flow

Christopher Hunt

A fluid is flowing along a cylindrical pipe of radius a in the $\hat{i}$ direction. The velocity of the fluid at a radial distance r from the center of the pipe is $\vec{v} = u(1 - \frac{r^2}{a^2})\hat{i}\frac{cm}{sec}$.

(a)

When $r = a$ the velocity is $0\frac{cm}{sec}$. The value u is the max velocity of the fluid in the pipe which is reached when $r = 0$, at the center of the pipe.

(b)

To find the flux through a circular cross-section, begin by finding the surface differential:

$$x = 0 \quad 0 \leq r \leq a \quad 0 \leq \phi \leq 2\pi$$

$$d\vec{r} = dr\hat{r} + rd\phi\hat{\phi} + dx\hat{i}$$

$$d\vec{r_1} = dr\hat{r} \qquad d\vec{r_2} = rd\phi\hat{\phi}$$

$$d\vec{A} = d\vec{r_1} \times d\vec{r_2} = rdrd\phi\hat{i}$$

Now find the Flux:

$$Flux = \int_C \vec{G} \cdot d\vec{A} \rightarrow \int_0^{2\pi} \int_0^a (ur - \frac{ur^3}{a^2})drd\phi$$

Computing this double integral gives us:

$$Flux = \frac{ua^2\pi}{2} \frac{cm^3}{sec}$$