Flux Fooling

Christopher Hunt

Let D be the cylinder with diameter of 10 inches, height of 13 inches including the bottom but not the top. Presume that the z-axis is through the center of D parallel to the height.

a.1) If $\vec{F} = -3z\hat{k}$, use geometry to deduce whether the flux through D is positive, negative, or zero:

The vector field only has a $\hat{k}$ component, therefore no flux will opccur through the cylinder body. The bottom has a normal vector of $-\hat{k}$, therefore, it will be effected by the vector field $\vec{F}$.

If the bottom is $z = 0$ the flux will be zero.
If the bottom is $z > 0$ the flux will be positive.
If the bottom is $z < 0$ the flux will be negative.

b.1) Support this with explicit calculations:

Case 1: Body of cylinder

$$d\vec{A} = rd\phi dz\hat{r} \quad \vec{F} = -3z\hat{k} \quad r = 5\ in \quad h = 13\ in \quad 0 \le \phi \le 2\pi \quad b \le z \le h + b$$

$$\int\int_D \vec{F} \cdot d\vec{A} = 0$$

Case 2: Bottom, b is at $z = 0$

$$d\vec{A} = -rd\phi dr\hat{k} \quad \vec{F} = -3z\hat{k} \quad 0 \le r \le 5\ in \quad h = 13\ in \quad 0 \le \phi \le 2\pi \quad z = 0$$

$$\int\int_D \vec{F} \cdot d\vec{A} = 0$$

Case 3: Bottom, b, is at $z > 0$

$$d\vec{A} = -rd\phi dr\hat{k} \quad \vec{F} = -3z\hat{k} \quad 0 \le r \le 5\ in \quad h = 13\ in \quad 0 \le \phi \le 2\pi \quad z > 0$$

$$\int\int_D \vec{F} \cdot d\vec{A} \rightarrow \int_0^5 \int_0^{2\pi} 3brd\phi dr \rightarrow 75b\pi$$

Case 4: Bottom, b, is at $z < 0$

$$d\vec{A} = -rd\phi dr\hat{k} \quad \vec{F} = -3z\hat{k} \quad 0 \le r \le 5\ in \quad h = 13\ in \quad 0 \le \phi \le 2\pi \quad z < 0$$

$$\int\int_D \vec{F} \cdot d\vec{A} \rightarrow \int_0^5 \int_0^{2\pi} 3brd\phi dr \rightarrow -75b\pi$$

c.1) How would the flux change if the top were included:

To include the top all we need to do is take the flux through the bottom, invert the sign because the normal vector i opposite to the bottom and make $z = b + 13$.

Case 1: $b = 0$

$$\Phi = 0 - 75\pi(b + 13) = -975\pi$$

Case 1: $b > 0$

$$\Phi = 75\pi b - 75\pi(b + 13) = -975\pi$$

Case 1: $b < 0$

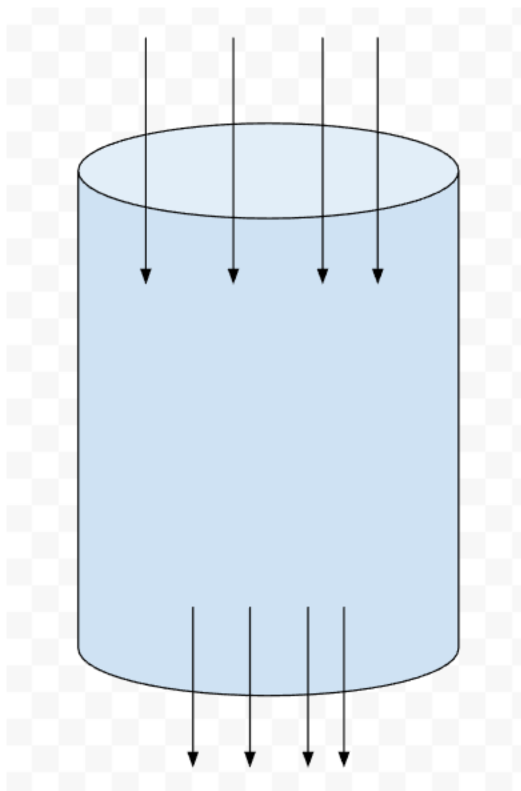
$$\Phi = -75\pi b - 75\pi(b + 13) = -975\pi$$

d.1) Use the Divergence Theorem to write down an equation relating the flux through D and the flux through the top

$$\Phi_c = div(\vec{F}) * V_c = Flux\ through\ the\ total\ cylinder$$

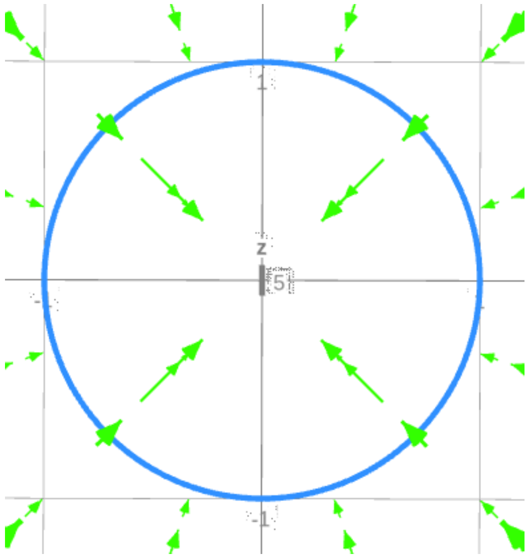
$$\Phi_c = -3 * \pi * r^2 * h = -975\pi\ when\ r = 5\ in\ and\ h = 13\ in$$

e.1) Create a visual equation expressing the same relationship:



a.2) If $\vec{G} = -4x\hat{i} - 4y\hat{j} = -4r\hat{r}$, use geometry to deduce whether the flux through D is positive, negative, or zero:

Since the vector field has no $\hat{k}$ component there is no flux through the bottom. The flux through the cylinder body will be negative. This can be demonstrated by the projection of the cylinder onto the xy-plane and the vector field.



b.2) Support this with explicit calculations:

Case 1: Bottom

$$d\vec{A} = -rd\phi dr\hat{k} \quad \vec{G} = -4r\hat{r} \quad 0 \le r \le 5\ in \quad h = 13\ in \quad 0 \le \phi \le 2\pi$$

$$\int\int_D \vec{F} \cdot d\vec{A} = 0$$

Case 2: Cylinder Body

$$d\vec{A} = rd\phi dz\hat{r} \quad \vec{G} = -4r\hat{r} \quad r = 5\ in \quad h = 13\ in \quad 0 \le \phi \le 2\pi \quad 0 \le z \le h$$

$$\int\int_c \vec{G} \cdot d\vec{A} = \int_0^{13}\int_0^{2\pi} -100 d\phi dz = -2600\pi$$

c.2) How would the flux change if the top were included:

Since there is no $\hat{k}$ component to the vector field the flux will be the same as above.

d.2) Use the Divergence Theorem to write down an equation relating the flux through D and the flux through the top

$$\Phi_c = div(\vec{F}) * V_c = Flux\ through\ the\ total\ cylinder$$

$$\Phi_c = -8 * \pi * r^2 * h = -2600\pi\ when\ r = 5\ in\ and\ h = 13\ in$$

e.2) Create a visual equation expressing the same relationship: