Estimating

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a) Suppose that $\vec{\nabla} \cdot \vec{F} = xyz^2$

i. Find $\vec{\nabla} \cdot \vec{F}$ at P(1,2,1):

$$div(\vec{F_P}) = 1 * 2 * 1^2 = 2$$

ii. Estimate the flux out of a box with side length 0.2 centered at P:

$$\Phi = div(\vec{F_P}) * V_{box} = 2 * .2^3 = 0.016$$

iii. Use the Divergence Theorem to calculate the exact flux out of the box:

$$.9 \leq x \leq 1.1 \quad 1.9 \leq y \leq 2.1 \quad .9 \leq z \leq 1.1$$

$$\Phi = \int_c div(\vec{F}) dV_{box} = \int_{.9}^{1.1} \int_{1.9}^{2.1} \int_{.9}^{1.1} xyz^2 \, dxdydz = 0.0160533$$

The estimation of the flux through the box had a percent error of 0.332%

b) Given a smooth vector field $\vec{G}$, at P(0,0,0), such that:

$$\vec{\nabla} \times \vec{G} = 2\hat{\imath} - 3\hat{\jmath} + 5\hat{k}$$

Recall that the circulation can be estimated using this equality: $\oint_C \vec{G} \cdot d\vec{r} = A_c(\vec{\nabla} \times \vec{G}) \cdot \vec{u}$, where $\vec{u}$ is the normal vector to the given circulation. Estimate the circulation $\oint_C \vec{G} \cdot d\vec{r}$ around a circle of radius r = 0.1, centered at P in each of the following planes.

i. xy-plane

$$A_c = .01\pi \quad \vec{u} = \hat{k} \quad (\vec{\nabla} \times \vec{G}) \cdot \vec{u} = 5$$

$$\oint_C \vec{G} \cdot d\vec{r} = A_c(\vec{\nabla} \times \vec{G}) \cdot \vec{u} = 0.05\pi$$

ii. yz-plane

$$A_c = .01\pi \quad \vec{u} = \hat{\imath} \quad (\vec{\nabla} \times \vec{G}) \cdot \vec{u} = 2$$

$$\oint_C \vec{G} \cdot d\vec{r} = A_c(\vec{\nabla} \times \vec{G}) \cdot \vec{u} = 0.02\pi$$

iii. xz-plane

$$A_c = .01\pi \quad \vec{u} = \hat{\jmath} \quad (\vec{\nabla} \times \vec{G}) \cdot \vec{u} = -3$$

$$\oint_C \vec{G} \cdot d\vec{r} = A_c(\vec{\nabla} \times \vec{G}) \cdot \vec{u} = -0.03\pi$$