The Ant

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The Ant

Part 1

An ant crawls along the radius from the center to the edge of a circular disk of radius 1 meter, moving at a constant rate of .01 m/sec. Meanwhile, the disk is turning counterclockwise about its center at 1 revolution/sec.

From this problem statement we acquire two equations and a range of time:

$$r(t) = .01t\ m \qquad \phi(t) = 2\pi t\ radians \qquad 0 \leq t \leq 100\ seconds$$

Where r(t) is describing the motion of the ant and $\phi(t)$ is describing the motion of the turntable as it spins counter clockwise.

We have previously found that the general equation for a parametric curve in polar is $\vec{r}(t) = r(t)\hat{r}$ and that $\hat{r} = cos(\phi(t))\hat{i} + sin(\phi(t))\hat{j}$ we can write our parametric curve in polar or rectangular as:

$$\vec{r}(t) = .01t\hat{r} = .01t\cos\phi(t)\hat{i} + .01t\sin\phi(t)\hat{j}$$

To find the velocity of the ant we can use the derivative of the polar form of $\vec{r}(t)$ which we have already found to be $\frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r(t)\frac{d\phi}{dt}\hat{\phi}$

Take the derivative of r(t) and $\phi(t)$: $\frac{dr}{dt} = .01\ \frac{m}{s} \qquad \frac{d\phi}{dt} = 2\pi\ rad$

$$\frac{d\vec{r}}{dt} = .01\hat{r} + .02\pi t\hat{\phi}\ \frac{m}{s}$$

This equation is the velocity of the ant as it moves, but what is it's speed? To find the speed we need to take the velocity and find it's magnitude using the Pythagorean Theorem.

$$|\frac{d\vec{r}}{dt}| = \sqrt{.0001 + .0004\pi^2 t^2}\ \frac{m}{s}$$

To find the ant's acceleration we take the general form for acceleration we defined in HW1 and plug in all of our expressions defined above:

$$\frac{d^2\vec{r}}{dt^2} = (\frac{d^2r}{dt^2} - r(t)(\frac{d\phi}{dt}^2))\hat{r} + (2\frac{dr}{dt}\frac{d\phi}{dt} + r(t)\frac{d^2\phi}{dt^2})\hat{\phi}$$

$$\frac{d^2r}{dt^2} = 0 \qquad \frac{d^2\phi}{dt^2} = 0$$

$$\frac{d^2\vec{r}}{dt^2} = (0 - .04\pi^2 t)\hat{r} + (.04\pi + 0)\hat{\phi}$$

$$\frac{d^2\vec{r}}{dt^2} = -.04\pi^2 t\hat{r} + .04\pi\hat{\phi}\ \frac{m}{s^2}$$

The magnitude of this acceleration is found the same way as above, by using the Pythagorean Theorem.

$$|\frac{d^2\vec{r}}{dt^2}| = \sqrt{.0016\pi^4 t^2 + .0004\pi^2}\ \frac{m}{s^2}$$

Part 2

Now consider the same problem but the turntable is also move upwards at .02 meters per second.

Our known values are:

$$r(t) = .01tm \qquad \phi(t) = 2\pi trad \qquad z(t) = .02tm \qquad 0 \le t \le 100$$

The differential, $d\vec{r}(t)$, in rectangular and cylindrical coordinates is known to be:

$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k} = dr\hat{r} + r(t)d\phi\hat{\phi} + dz\hat{z}$$

All we are missing is the dz $\to dz = .02\ m$

Plug everything into $d\vec{r}$ in the equation for cylindrical coordinates.

$$d\vec{r} = .01\hat{r} + .02\pi t\hat{\phi} + .02\hat{z}$$

This would then give us a speed of:

$$|\frac{d\vec{r}}{dt}| = \sqrt{.0005 + .0004\pi^2 t^2}\ \frac{m}{s}$$

Acceleration would be the second derivative of $r(t)$. Since the second derivative of the z(t) is 0 $\to \frac{d^2z}{dt^2} = 0$ the ant's acceleration would be the same as Part 1:

$$\frac{d^2\vec{r}}{dt^2} = -.04\pi^2 t\hat{r} + .04\pi\hat{\phi}\ \frac{m}{s^2}$$

$$|\frac{d^2\vec{r}}{dt^2}| = \sqrt{.0016\pi^4 t^2 + .0004\pi^2}\ \frac{m}{s^2}$$