Problem 1

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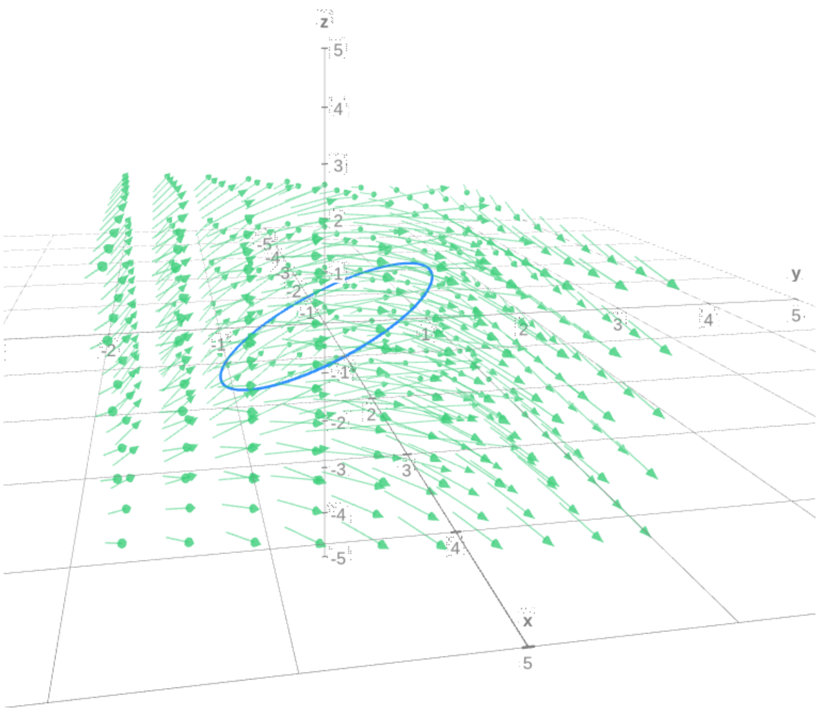
(a)

Find the circulation of $\vec{F}$ around the loop $\vec{r}(t)$.

$$\vec{F} = x\hat{i} + \hat{j} - y\hat{k} \qquad \vec{r}(t) = cos(t) + sin(t)\hat{j} + sin(t)\hat{k}$$



To find the circulation of a path through a vector field we will use the line integral, $\int_c \vec{F} \cdot d\vec{r}$. Begin by finding $d\vec{r}$ and the bounds of integration.

$$d\vec{r} = -sin(t)\hat{i} + cos(t)\hat{j} + cos(t)\hat{k}\ dt$$

$$0 \leq t \leq 2\pi \qquad x(t) = cos(t) \qquad y(t) = sin(t)$$

Solve the line integral:

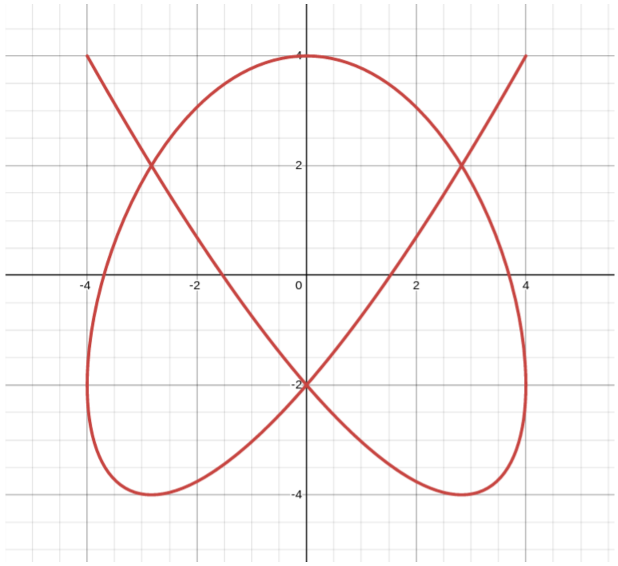
$$\oint_0^{2\pi} \vec{F} \cdot d\vec{r} \rightarrow \oint_0^{2\pi} -2sin(t)cos(t) + cos(t)\ dt$$

$$-2\left[\frac{sin^2(t)}{2}|_0^{2\pi}\right] + \left[sin(t)|_0^{2\pi}\right] = 0$$

The fact that the circulation is zero suggests that $\vec{F}$ is a conservative vector field but I'm not really sure about that because I could not find a potential function for the vector field...

(b)

The Pretzel



Find the total amount of chocolate on a pretzel whose shape is $x(t) = -4sin(3t)$ cm and $y(t) = 4cos(4t)$ cm. The density of the chocolate can be described by the function $\lambda(t) = 3(x^2 + y^2)\frac{g}{cm}$.

The mass of chocolate can be found by using $m = \int_c \lambda(t)|d\vec{r}|$

$$x(t) = -4sin(3)\ cm \qquad y(t) = 4cos(4t)\ cm \qquad 0 \leq t \leq \frac{\pi}{2}\ and\ \pi \leq t \leq \frac{3\pi}{2}$$

Find $\vec{r}(t)$, $|d\vec{r}|$ and $\lambda(t)$ in terms of $x(t)$ and $y(t)$:

$$\vec{r}(t) = -4sin(3t)\hat{i} + 4cos(4t)\hat{j}\ cm \rightarrow d\vec{r} = (-12cos(t)\hat{i} - 16sin(4t)\hat{j})\ dt$$

$$|d\vec{r}| = \sqrt{144cos^2(3t) + 256sin^2(4t)}\ dt$$

$$\lambda(t) = 3(x^2 + y^2)\ = \ 3(16sin^2(3t) + 16cos^2(4t))\frac{g}{cm}$$

Now set up the integral $m = \int_c \lambda(t)|d\vec{r}|$:

$$m = \int_0^{\frac{\pi}{2}} \Big(3(16sin^2(3t) + 16cos^2(4t))\Big) * \sqrt{144cos^2(3t) + 256sin^2(4t)}\ dt$$

$$+ \int_{\pi}^{\frac{3\pi}{2}} \Big(3(16sin^2(3t) + 16cos^2(4t))\Big) * \sqrt{144cos^2(3t) + 256sin^2(4t)}\ dt$$

$$m = 865.73159 + 865.73159 = 1731.46318\ g$$

This pretzel is heavily coated in 1731.46 g of chocolate.