The Farmer

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A farmer weighing 150 lbs carries a sack of grain weighing 20 lbs up a circular helical staircase around a silo of radius 25 feet. As the farmer climbs, the sack leaks grain at 1 lb per 10 feet of ascent. How much work is performed by the farmer in climbing through a vertical distance of 60 feet in exactly four revolutions?

Recall the formula for work:

$$w = \int_c \vec{\boldsymbol{F}} \cdot d\vec{r}$$

From this problem statement we can find $\vec{\boldsymbol{F}}$ and $d\vec{r}$

$$\vec{\boldsymbol{F}} = \vec{m}(x,y,z) = 150 - 20 - \frac{z}{10}\hat{z}\ lbs \qquad \vec{r} = 25\hat{r} + z\hat{z}\ ft \qquad \phi(z) = \frac{2\pi}{15}z\ ft \qquad 0 \leq z \leq 60\ ft$$

$$d\phi = \frac{2\pi}{15}dz \qquad d\vec{r} = (\frac{50\pi}{15}\hat{\phi} + \hat{z})\ dz$$

$$w = \int_0^{60} 130 - \frac{z}{10}dz \rightarrow w = 130z|_0^{60} - \frac{z^2}{20}|_0^{60} \rightarrow w = 7800 - 180$$

$$w = 7620\ lbf$$

At the top of the silo the farmer would have done 7620 pounds of force to overcome gravity.