

HIP 4

A wire bent to make the shape of an L .15 m on each side is charged to 50mC.

- a) Derive an analytical function that finds the potential everywhere.

The diagram shows an L-shaped wire in the xy-plane. The vertical segment is along the y-axis from y=0 to y=.15m, and the horizontal segment is along the x-axis from x=0 to x=.15m. A point P(x₀, y₀) is shown in the first quadrant. A small element of charge dq is shown on the vertical segment at position y. The distance from dq to P is r. The total charge is Q = 50 μC. The linear charge density is λ = Q / (2L). The potential at P is given by V = ∫ dV. The potential due to the vertical segment is V₁ = ∫ (1 / (4πε₀)) * (dq / r). The potential due to the horizontal segment is V₂ = ∫ (1 / (4πε₀)) * (dq / r). The total potential is V_{tot} = V₁ + V₂.

Diagram labels: $Q = 50 \mu C$, $\lambda = \frac{Q}{2L}$, $dq = \lambda dy$, $P(x_0, y_0)$, $r^2 = x_0^2 + (y - y_0)^2$, $V = \int dV$, $V_1 = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r}$, $V_2 = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r}$, $V_{tot} = V_1 + V_2$.

Equations:

$$V = \int dV$$

$$V_1 = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r}$$

$$V_1 = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda dy}{\sqrt{x_0^2 + (y - y_0)^2}}$$

$$V_2 = \int dV$$

$$V_2 = \int \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{r}$$

$$V_2 = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda dx}{\sqrt{y_0^2 + (x - x_0)^2}}$$

$$V_{tot} = V_1 + V_2$$

b) Create a VPython program that uses numerical integration to calculate the potential at positions one cm apart everywhere within a space 20cm by 20 cm large that contains the charged wire.

```
GlowScript 3.1 VPython
## Electric potential of a bent charged wire ##
## Constants ##
k=1/(4*pi*8.85*10**-12) #C2/N*m2
Q=50*10**-6 # C
L=15 #Cm
n=30
dq=Q/n #C
step=1
lx=0
ly=0

## Position of interest ##
iPos=vec(lx,ly,0)

## Numerical calculator for electric potential in wire along the x-axis ##
def dVxWire(A):
    dVx=0
    dx=0
    while dx<=L:
        dVx=dVx+((k*dq)/mag(A-vec(dx,0,0)))
        dx=dx+step

    return(dVx)

## Numerical calculator for electric potential in wire along the y-axis ##
def dVyWire(A):
    dVy=0
    dy=0
    while dy<=L:
        dVy=dVy+((k*dq)/mag(A-vec(0,dy,0)))
        dy=dy+step

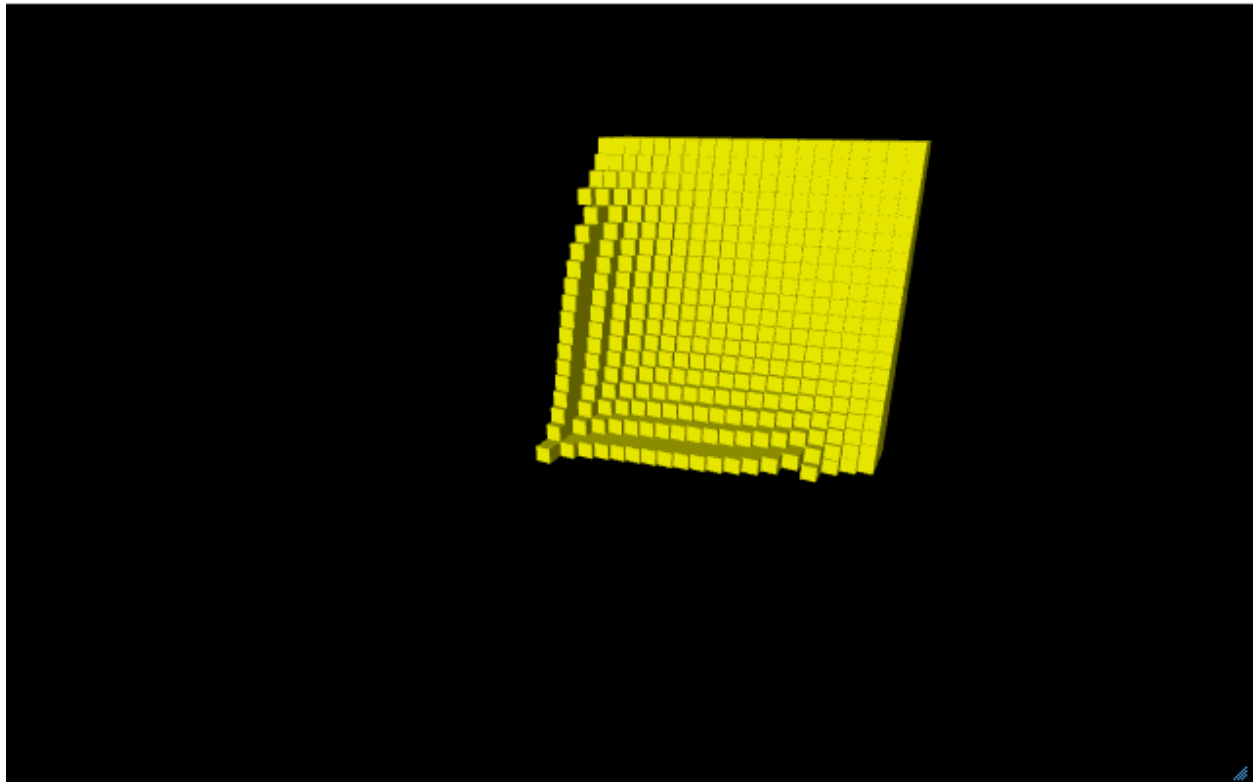
    return(dVy)

## Nested loop that moves the interested position along the positive x axis and ##
##positive y axis, finding the electric potential at that point##
for x in range(0,20,step):
    rate(100000000)
    for y in range(0,20,step):
        rate(100000000)
        iPos=vec(lx,ly,0)
        ePot=dVxWire(iPos)+dVyWire(iPos)
        box(pos=iPos+vec(0,0,ePot/2),size=vec(1,1,(ePot)),color=color.yellow)
        ly=ly+step
        print("Position: ",iPos," Electric Potential: ",ePot)
        ly=0
        lx=lx+step
```

```

1 GlowScript 3.1 VPython
2 ## Electric potential of a bent charged wire ##
3 ## Constants ##
4 k=1/(4*pi*8.85*10**-12) #C2/N*m2
5 Q=50*10**-6 # C
6 L=15 #Cm
7 n=30
8 dq=Q/n #C
9 step=1
10 Ix=0
11 Iy=0
12
13
14 ## Position of interest ##
15 iPos=vec(Ix,Iy,0)
16
17
18
19 ## Numerical calculator for electric potential in wire along the x-axis ##
20 def dVxWire(A):
21     dVx=0
22     dx=0
23     while dx<=L:
24         dVx=dVx+((k*dq)/mag(A-vec(dx,0,0)))
25         dx=dx+step
26
27     return(dVx)
28
29
30 ## Numerical calculator for electric potential in wire along the y-axis ##
31 def dVyWire(A):
32     dVy=0
33     dy=0
34     while dy<=L:
35         dVy=dVy+((k*dq)/mag(A-vec(0,dy,0)))
36         dy=dy+step
37
38     return(dVy)
39
40 ## Nested loop that moves the interested position along the positive x axis and ##
41 ##positive y axis, finding the electric potential at that point##
42 for x in range(0,20,step):
43     rate(100000000)
44     for y in range(0,20,step):
45         rate(100000000)
46         iPos=vec(Ix,Iy,0)
47         ePot=dVxWire(iPos)+dVyWire(iPos)
48         box(pos=iPos+vec(0,0,ePot/2),size=vec(1,1,(ePot)),color=color.yellow)
49         Iy=Iy+step
50         print("Position: ",iPos,"    Electric Potential: ",ePot)
51     Iy=0
52     Ix=Ix+step
53
54
55

```



Position: < 4, 10, 0 >	Electric Potential: 6.34968e+4 J/C
Position: < 4, 11, 0 >	Electric Potential: 6.08174e+4 J/C
Position: < 4, 12, 0 >	Electric Potential: 5.79543e+4 J/C
Position: < 4, 13, 0 >	Electric Potential: 5.48393e+4 J/C
Position: < 4, 14, 0 >	Electric Potential: 5.14452e+4 J/C
Position: < 4, 15, 0 >	Electric Potential: 4.78262e+4 J/C
Position: < 4, 16, 0 >	Electric Potential: 4.41417e+4 J/C
Position: < 4, 17, 0 >	Electric Potential: 4.06061e+4 J/C
Position: < 4, 18, 0 >	Electric Potential: 3.73836e+4 J/C
Position: < 4, 19, 0 >	Electric Potential: 3.45386e+4 J/C
Position: < 5, 0, 0 >	Electric Potential: Infinity J/C
Position: < 5, 1, 0 >	Electric Potential: 1.13009e+5 J/C
Position: < 5, 2, 0 >	Electric Potential: 9.44013e+4 J/C
Position: < 5, 3, 0 >	Electric Potential: 8.44369e+4 J/C
Position: < 5, 4, 0 >	Electric Potential: 7.78853e+4 J/C
Position: < 5, 5, 0 >	Electric Potential: 7.30934e+4 J/C
Position: < 5, 6, 0 >	Electric Potential: 6.93123e+4 J/C
Position: < 5, 7, 0 >	Electric Potential: 6.61371e+4 J/C
Position: < 5, 8, 0 >	Electric Potential: 6.33213e+4 J/C
Position: < 5, 9, 0 >	Electric Potential: 6.07008e+4 J/C
Position: < 5, 10, 0 >	Electric Potential: 5.8158e+4 J/C
Position: < 5, 11, 0 >	Electric Potential: 5.56043e+4 J/C
Position: < 5, 12, 0 >	Electric Potential: 5.29732e+4 J/C
Position: < 5, 13, 0 >	Electric Potential: 5.02232e+4 J/C
Position: < 5, 14, 0 >	Electric Potential: 4.73466e+4 J/C
Position: < 5, 15, 0 >	Electric Potential: 4.43814e+4 J/C

c) Use your results in part a and part b to conduct a reasonableness test.

To test the reasonableness I will exam the point at $P = (.05, .05)$ m

The calculation done numerically gives a result of $V = 7.30934 \times 10^4$ J/C

The calculation done analytically gives a result of $V = 6.97 \times 10^6$ J/C

$$\begin{aligned} P &= (.05, .05) \\ V_{\text{tot}} &= \frac{\lambda}{4\pi\epsilon_0} \left(\int_0^L \frac{1}{\sqrt{.05^2 + (y-.05)^2}} dy + \int_0^L \frac{1}{\sqrt{.05^2 + (x-.05)^2}} dx \right) \\ V_{\text{tot}} &= \frac{\lambda}{4\pi\epsilon_0} \cdot \left(2 \cdot \left(\sinh^{-1}(10(2x-.1)) \right) \Big|_0^{.15} \right) \\ \boxed{V_{\text{tot}} &= 6.97 \times 10^6 \frac{\text{J}}{\text{C}} \text{ at point } P} \end{aligned}$$

CATEGORY	PLARY (1.5)	MPLEISHED (1)	LOPING (0.5)	AGENT (0)
Statement and tion	arning tool for our class is written	blem is clearly presented for reader in n words.	blem is directly copied or is hard	p into some calculation
	etch could be dropped into a novel as it stands.	a clear sketch, larger than a credit the problem set up with important and data noted	some sketch of the problem	etch?
s Tools	ate physics tools are correlated ercise in textbook quality and	ate physics tools are correlated to the . Appropriate tools include: pictures, bservational laws utilized, etc...	ysics tools are correlated to the	e a few equations written.
m Solution tation	is very clearly presented with g asides or annotations	is complete and clearly presented o significant intuitive demands on the	solution I have to read between	es version of solution with only hts present
	ution can serve as solution	g is larger than a credit card, tion is fluid, notation used is clear.	gure the path of your solution with	read it.
		correctly given	ions & quantities are presented s	hits at the results
n			close	ot reasonable
ant Figures		Sig Figs	ffort to use correct significant	he number from the calculator
hableness	s more than one type of ableness check.	he clear rationale for appropriateness of ion in the setting	that the answer is reasonable but asn't given any evidence	assion
Graded			e	ut your self-assessment is from mine by at least two steps.