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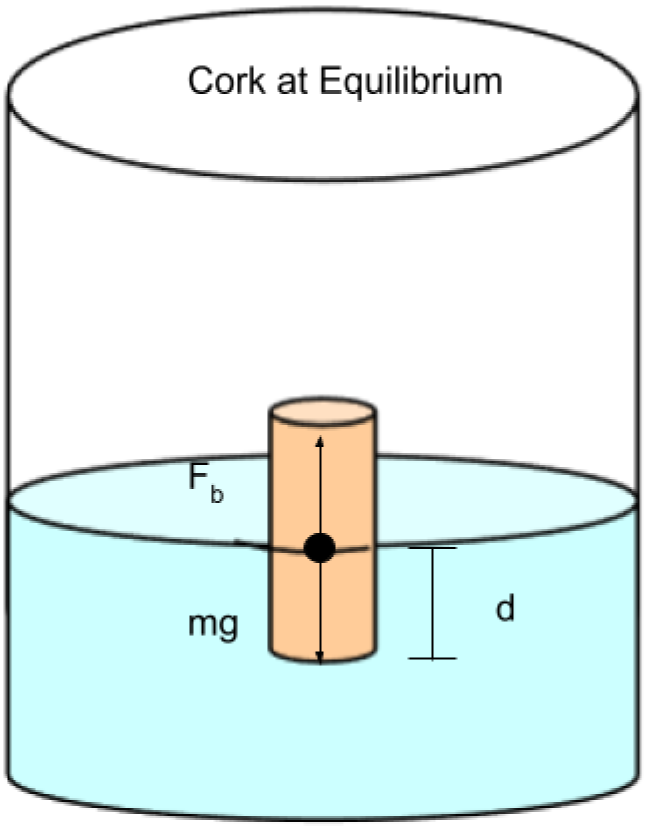
PH 212

HIP 5

a. A cylindrical cork of mass m, radius r, and height h is floating in a tub of water as shown. The density of water is water. You push down on the cork a distance s at it starts bobbing up and down. What is the period of oscillation?State your answer in terms of givens: m, r, h, s, ρ_{water} and g.



$F_{net} = 0 = F_b - mg$

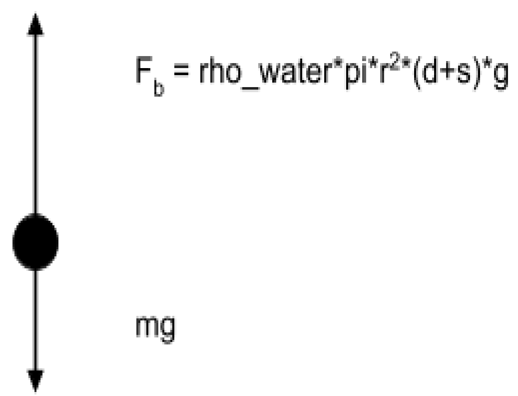
$rho_{water}*pi*r^2*d*g = mg$

First we draw a free body diagram of the cork at equilibrium.

We find that mg = $\rho_{water} * \pi * r^2 * d * g$

Then we draw a free body diagram of the cork the instant it was released after being submerged a distance s.

FBD Cork after submersion



F_{net} = ma = ρ_{water} * π*r²*d*g + ρ_{water} * π*r²*s*g - mg → mg = ρ_{water} * π*r²*d*g, therefore they cancel each other out → $F_{net} = ma = \rho_{water} * \pi * r^2 * g * s$

Solve for a → a = (ρ_{water} * π*r²)/m*s → since we know that the motion that will occur will be harmonic motion the distance s will be a negative value (away from equilibrium) → $a = d^2s/dt^2 = -(\rho_{water} * \pi * r^2 * g)/m * s$

This is very close to the equation $d^2s/dt^2 = -\omega^{2} * s$ → (ρ_{water} * π*r²)/m will equal our ω

We know that $\omega^2 = 4\pi^2/T^2 = (\rho_{water} * \pi * r^2 * g)/m$ → solve for T to find the period → $T = ((4*\pi*m)/(\rho_{water} * r^2 * g))^{.5}$

Unit check: $(kg * m^3/kg * 1/m^2 * s^2/m)^{1/2} = s$

The unit checks out. This solution passes the unit check reasonableness check.

b. You push the cork down a distance of 5.0cm and let go. At the end of the third complete oscillations, you find that the cork's maximum descent is now only 3cm. Create and print out a graph (using Excel, VPython, Matlab, by hand, or whatever you like) for 10 cycles for this damped situation where m=25g, r=1.2cm, and h=6.2cm.

We have the equation these four equations for our system:

$x(t) = A^*e^{-t/\tau}\cos(\omega t+\phi_o)$

$\omega= (\rho_{water} * \pi^*r^2{}^*g)/m$

$T = ((4^*\pi^*m)/(\rho_{water} * r^2{}^*g))^{.5}$

A = s

In order to find the constant for the exponential decay we use the information provided.

$-.03 = -.05\ e^{-t/\tau}\cos(\omega t+\phi_o)$ → we are looking for a minimum so we can set $\cos(\omega t+\phi_o) = 1$ → $-.03 = -.05\ e^{-t/\tau}$ → $.03/.05 = e^{-t/\tau}$ → perform the natural log to both sides → $\ln(.6) = -t/\tau$ → We know that the time will be at the 3rd oscillation, so t = 3T, divide both sides by 3T → $\ln(.6)/3T = -1/\tau$ → this is our exponential decay constant

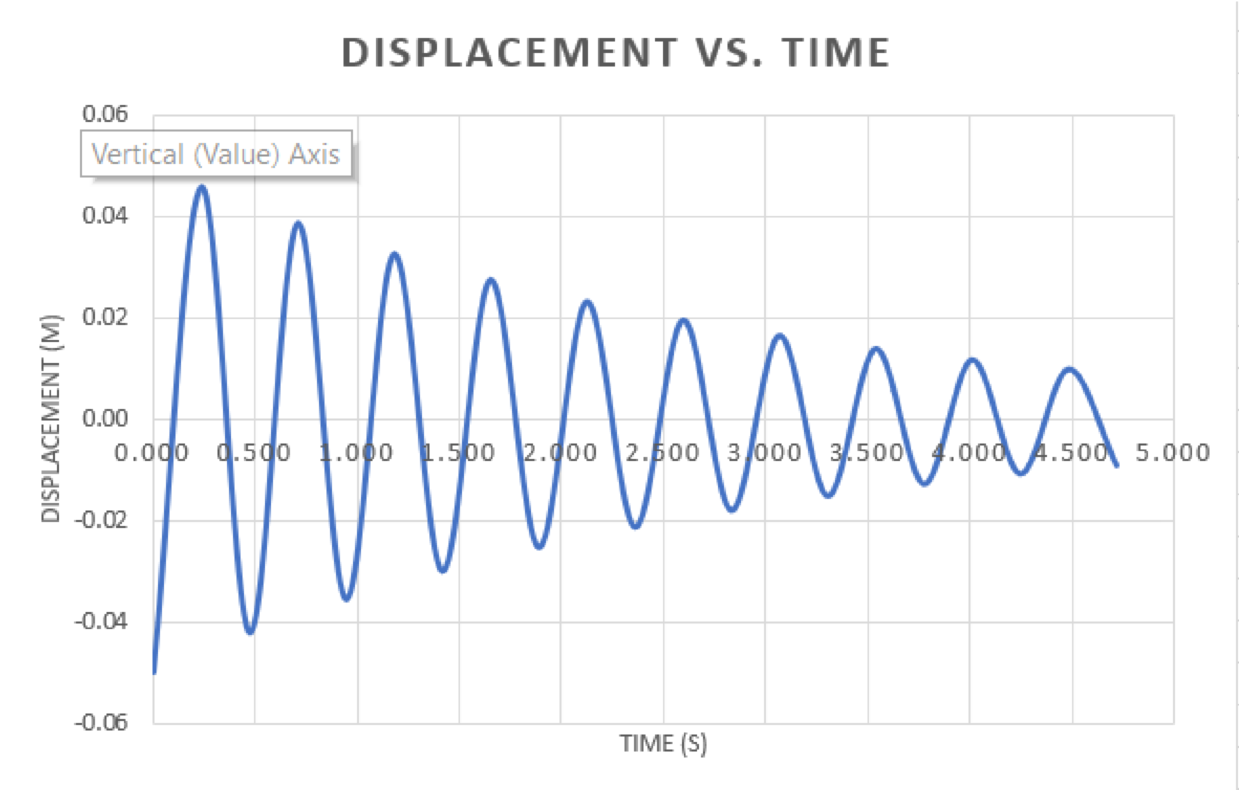
We can now do our calculations in excel to determine the descent at the tenth oscillation. We define our constants and then calculate the values at a time of ½ the period in order to find the maximum and the minimum of our harmonic motion.

The tenth oscillation occurs at 4.72 seconds with an amplitude of .0091 m below equilibrium.

The cork bobs up and down for approximately 20 seconds before coming to rest, from conventional experience this is a reasonable approximation for the time a cork would bob up and down from just one encounter of displacement.

E2 =B6*EXP(((LN(0.6))/(3*B7))*D2)*COS(B8*D2)

	A	B	C	D	E
1	Gravity (m/s^2)	9.81		Time	Displacement
2	Density of Fluid (kg/m^3)	1000		0.000	-0.0500
3	Radius of cork (m)	0.012		0.236	0.0459
4	Height of cork (m)	0.062		0.472	-0.0422
5	Mass of cork (kg)	0.025		0.707	0.0387
6	Depth Submerged (m)	-0.05		0.943	-0.0356
7	Period (s)	0.472		1.179	0.0327
8	Omega (rad/s)	13.32		1.415	-0.0300
9				1.651	0.0276
10				1.886	-0.0253
11				2.122	0.0232
12				2.358	-0.0213
13				2.594	0.0196
14				2.830	-0.0180
15				3.065	0.0165
16				3.301	-0.0152
17				3.537	0.0139
18				3.773	-0.0128
19				4.008	0.0118
20				4.244	-0.0108
21				4.480	0.0099
22				4.716	-0.0091


DISPLACEMENT VS. TIME
Vertical (Value) Axis
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
DISPLACEMENT (M)
0.000
0.500
1.000
1.500
2.000
2.500
3.000
3.500
4.000
4.500
5.000
TIME (S)

Lecture Time: Name:

CATEGORY	EXEMPLARY (1.5)	ACCOMPLISHED (1)	DEVELOPING (0.5)	EMERGENT (0)
Problem Statement and Introduction	A new learning tool for our class is written	The problem is clearly presented for reader in your own words.	The problem is directly copied or is hard to follow.	You jump into some calculation
Picture	Your sketch could be dropped into a graphic novel as it stands.	There is a clear sketch, larger than a credit card, of the problem set up with important features and data noted	There is some sketch of the problem setup	What sketch?
Physics Tools	Appropriate physics tools are correlated to the exercise in textbook quality and size	Appropriate physics tools are correlated to the exercise. Appropriate tools include: pictures, FBDs, conservational laws utilized, etc...	Some physics tools are correlated to the exercise.	There are a few equations written.
Problem Solution Presentation	Solution is very clearly presented with intriguing asides or annotations	Solution is complete and clearly presented making no significant intuitive demands on the reader.	In your solution I have to read between the lines	Cliff notes version of solution with only high points present
Form	Your solution can serve as solution manual.	Drawing is larger than a credit card, organization is fluid, notation used is clear.	I could figure the path of your solution with effort.	You can read it.
Units		All units correctly given	Calculations & quantities are presented with units	Some units at the results
Solution		Correct	You are close	None/Not reasonable
Significant Figures		Correct Sig Figs	Makes effort to use correct significant figures	Copies the number from the calculator
Reasonableness	Provides more than one type of Reasonableness check.	Gives one clear rationale for appropriateness of the solution in the setting	Asserts that the answer is reasonable but really hasn't given any evidence	No discussion
All Self Graded		Done	Not Done	Done, but your self-assessment is different from mine by at least two steps.