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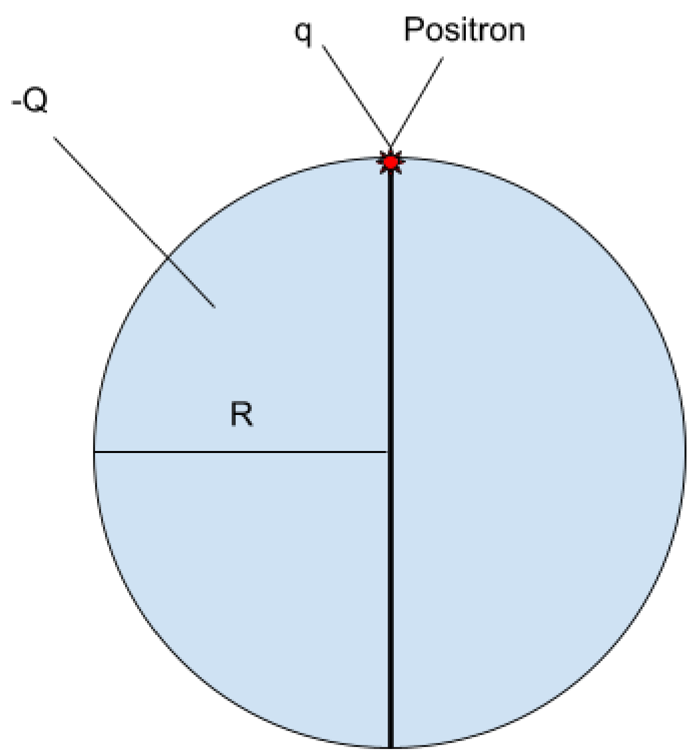
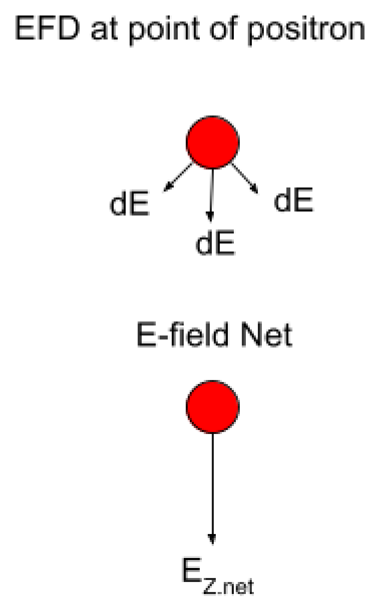
PH 213

# HIP 3

You have a homogeneous sphere of radius R and total charge –Q. You drill a thin hole directly through the diameter of the sphere. When you let go of a positron at one end of a hole, the positron oscillates back and forth through the diameter of the sphere. Find the period of oscillation of the positron.




For this problem we are assuming that the charge of the sphere and the charge of the positron are both symmetric. Also that R≫width of the drilled hole and does not impact the symmetry or the charge of the sphere.

We'll begin by finding the electric field of the charged sphere. In order to find the E-field that the charged sphere creates from its surface to its core, we will use Gauss's Law.


Volumetric Charge Density $= \rho_{tot} = \dfrac{Q_{tot}}{V_{tot}}$

$$\Phi_{enc} = \int \vec{E}\, d\vec{A} \rightarrow E \cdot A \rightarrow \boxed{E(4\pi r^2) = \Phi_{enc}}$$

$$\rho_{tot} = \frac{Q_{tot}}{V_{tot}} = \frac{q_{enc}}{V_{enc}} \rightarrow \frac{Q_{tot}}{\frac{4}{3}\pi R^3} = \frac{q_{enc}}{\frac{4}{3}\pi r^3} \rightarrow \text{solve for } q_{enc}$$

$$\boxed{q_{enc} = Q_{tot} \cdot \frac{r^3}{R^3}}$$

$$E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0} \rightarrow E(4\pi r^2) = \frac{Q_{tot}\, r^3}{\varepsilon_0 R^3}$$

$$\boxed{E = \frac{Q_{tot}\, r}{4\pi \varepsilon R^3}}$$

This equation seems reasonable because as the distance, r, goes towards 0, the overall charge drops linearly.

As r approaches R, we see the equation turn into $\rightarrow$ $E=Q/(4*\pi*\varepsilon_o*R^2)$. This is the equation for the E-field of a point charge, precisely what we would expect.

Now that we have an equation that accurately describes the E-field from the surface of the charge to it's core we can use this to find the $F_{electric}$ that the positron is experiencing at the moment it is placed and released at a distance R along this drilled path through the charge. From this we can deduce the period.

$$\vec{F}_{znet} = \vec{E}q \rightarrow m\vec{a} = \frac{Qq}{4\pi\varepsilon_0 R^3}\cdot r(t)$$

$$\vec{a} = \frac{Qq}{4\pi\varepsilon_0 R^3 m}\cdot r(t)$$ → looks like simple harmonic motion.

$$\sqrt{\omega^2} = \sqrt{\frac{Qq}{4\pi\varepsilon_0 R^3 m}} \rightarrow \omega = \frac{2\pi}{T} = \sqrt{\frac{Qq}{4\pi\varepsilon_0 R^3 m}}$$

$$\boxed{T = \frac{2\pi}{\sqrt{\frac{Qq}{4\pi\varepsilon_0 R^3 m}}}}$$ Period for a positron

The period of the positron is estimated to be $T=(2*\pi)/((Q*q)/(4*\pi*\varepsilon_o*R^3*m))^{1/2}$

Reasonableness can be checked through a unit check: $1/(C^2*kg*m^3/C^2*m^3*kg*s^2)^{½} = s$

| GORY | PLARY (1.5) | MPLISHED (1) | LOPING (0.5) | GENT (0) |
|---|---|---|---|---|
| Statement and<br>tion | arning tool for our class is written | blem is clearly presented for reader in<br>words. | blem is directly copied or is hard<br>. | p into some calculation |
| | tch could be dropped into a<br>novel as it stands. | a clear sketch, larger than a credit<br>the problem set up with important<br>and data noted | some sketch of the problem | etch? |
| Tools | ate physics tools are correlated<br>ercise in textbook quality and | ate physics tools are correlated to the<br>. Appropriate tools include: pictures,<br>nservational laws utilized, etc… | ysics tools are correlated to the<br>. | e a few equations written. |
| n Solution<br>tation | is very clearly presented with<br>asides or annotations | is complete and clearly presented<br>o significant intuitive demands on the | olution I have to read between | s version of solution with only<br>ts present |
| | ution can serve as solution | is larger than a credit card,<br>tion is fluid, notation used is clear. | gure the path of your solution with | read it. |
| | | correctly given | ons & quantities are presented<br>s | its at the results |
| n | | | close | t reasonable |
| ant Figures | | Sig Figs | ffort to use correct significant | he number from the calculator |
| ableness | more than one type of<br>bleness check. | e clear rationale for appropriateness of<br>ion in the setting | that the answer is reasonable but<br>sn't given any evidence | ssion |
| Graded | | | e | t your self-assessment is<br>from mine by at least two steps. |