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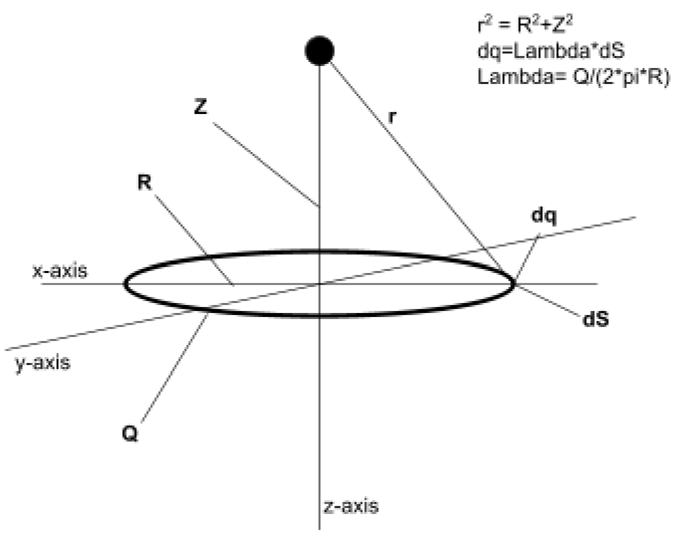
PH 213

HIP 2

You have a hoop of charge of radius R and total charge -Q. You place a positron at the center of the hoop and give it a slight nudge in the direction of the central axis that is normal to the plain of the hoop. Due to the negative charge on the hoop, the positron oscillates back and forth.

Step 1: Clearly show how to use integration to find the z-axis E-field of a ring charge.



Electric Field Diagram

Each dE is of the same magnitude and pointing at the same angle, theta, above the xy-plane

Theta = arcsin $Z/(Z^2+R^2)^{1/2}$

Due to the symmetry of the scenario, E_{net} is pointing parallel the z-axis, therefore in deriving our equation we only need to account for the dE in the z-axis.

$$\vec{E}_{net} = \int d\vec{E}_z = \int d\vec{E}\,\sin\theta$$

$$\vec{E}_{net} = \frac{z}{(z^2+R^2)^{1/2}} \int d\vec{E}$$

$$\vec{E}_{net} = \frac{z}{(z^2+R^2)^{1/2}} \int \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq}{r^2}$$

$$\vec{E}_{net} = \frac{z}{(z^2+R^2)^{1/2}} \cdot \frac{1}{4\pi\varepsilon_0} \cdot \frac{1}{z^2+R^2} \int dq$$

$$\vec{E}_{net} = \frac{z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \cdot \int_0^{2\pi R} \frac{Q}{2\pi R}\, dS$$

$$\vec{E}_{net} = \frac{z}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}} \cdot \frac{Q \cdot 2\pi R}{2\pi R}$$

$$\boxed{\vec{E}_{net} = \frac{Qz}{4\pi\varepsilon_0 (z^2+R^2)^{3/2}}}$$

Step 2: Place a positron a small distance above the plane of the ring and calculate the period of oscillation.

To find the period of oscillation of the positron we must first friend it's acceleration. This can be done with the equation → **a**=(qE)/m

a = (qE)/m → **a** = $((q*Q*Z)/(4*\pi * \varepsilon_o{}^* m*(Z^2+R^2)^{3/2}))$ → if we consider R≫Z we can rewrite the formula as such → **a** = $((q*Q*Z)/(4*\pi * \varepsilon_o{}^* m*R^3))$ → or it can be written as such → $d^2Z/dt^2 = ((q*Q)/(4*\pi * \varepsilon_o{}^* m*R^3)*Z(t)$ → we expect to see simple harmonic motion therefore we can equate this to our equation for SHM → $-\omega^2 * Z(t) = ((q*Q)/(4*\pi * \varepsilon_o{}^* m*R^3)*Z(t)$ → from this we can solve for ω and then find the period.

$\omega^2 = (4\pi^2)/T^2 = ((q*Q)/(4*\pi * \varepsilon_o{}^* m*R^3)$ → we will equate $1/(4*\pi * \varepsilon_o)$ to k → $(4\pi^2)/T^2 = ((q*Q*k)/(m*R^3)$ → $T^2 = (4*\pi^2*m*R^3)/(q*Q*k)$ → take the square root of both sides to find the period→ $T = ((4*\pi^2*m*R^3)/(q*Q*k))^{½}$

Unit Check: $(kg*m^3*C^2*s^2/C^2*kg*m^3)^{½}$ → $(s^2)^{½}$ → s

Through a unit check this equation seems reasonable.

Step 3: Use VPython to find the force on a positron a distance d=0.13mm above a center of a ring of R=5.2cm and charge Q=-3.7×10^{-9}C. Use this result as a reasonableness test for this HIP. Include a printout of your program with what you turn in.

```
GlowScript 3.1 VPython
#Variables
q1=-3.7*10**-9 #Charge of ring in C
q2=1.6*10**-19 #Charge of positron in C
m=9.11*10**-31 #Mass of positron in kg
R=.052 #Radius of ring in meters
d=.00013 #Initial Displacement of positron on axis of ring
k=8.99*10**9 #Nm^2/C^2
n=100000 #number of segments
theta=0.0 #radians
dtheta=(2*pi)/n #radians
dq=q1/n #C

#Initial vectors
E_sum=vec(0,0,0) #N/C
de=vec(0,0,0) #N/C
p1=vec(0,0,d) #m


E=((k*q1*d)/((R**2+d**2)**(3/2))) #Derived equation for E-field of a charged ring
T=((4*(pi**2)*m*(R**3))/(abs(q1)*q2*k))**.5 #Period of oscillation of positron


#Begin while loop using the Riemann Sum method to approximate the Electric Field
while theta<=(2*pi):
    rate(115200)
    s=vec(R*cos(theta),R*sin(theta),0) #Position of the charge dq
    de=((k*dq)/((mag(p1-s))**2))*norm(p1-s) #Electric field equation
    E_sum=E_sum+de #Adding the dE to the total sum of the electric field
    theta=theta+dtheta #advancing to the next position along the ring

print("Electric Field vector at point of interest: ",E_sum, "N/C")
print("Magnitude of Electric Field vector: ",mag(E_sum),"N/C")
print("Electric Field using derived equation: ",E,"N/C")
print("Period of oscillation of positron: ",T,"Seconds")
```

Results:

```
Electric Field vector at point of interest:  < 0.123013, -1.32184e-8, -30.7535 > N/C
Magnitude of Electric Field vector:  30.7538 N/C
Electric Field using derived equation:  -30.7532 N/C
Period of oscillation of positron:  3.0825e-8 Seconds
```

Comparing the results of the Riemann Sum method to calculate the electric field compared to the derived equation from step 1, we can conclude that the derived equation is indeed reasonable.

GORY	PLARY (1.5)	MPLISHED (1)	LOPING (0.5)	GENT (0)
Statement and tion	arning tool for our class is written	blem is clearly presented for reader in words.	blem is directly copied or is hard .	p into some calculation
	tch could be dropped into a novel as it stands.	a clear sketch, larger than a credit the problem set up with important and data noted	some sketch of the problem	etch?
Tools	ate physics tools are correlated ercise in textbook quality and	ate physics tools are correlated to the . Appropriate tools include: pictures, nservational laws utilized, etc…	ysics tools are correlated to the .	e a few equations written.
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Graded			e	t your self-assessment is from mine by at least two steps.