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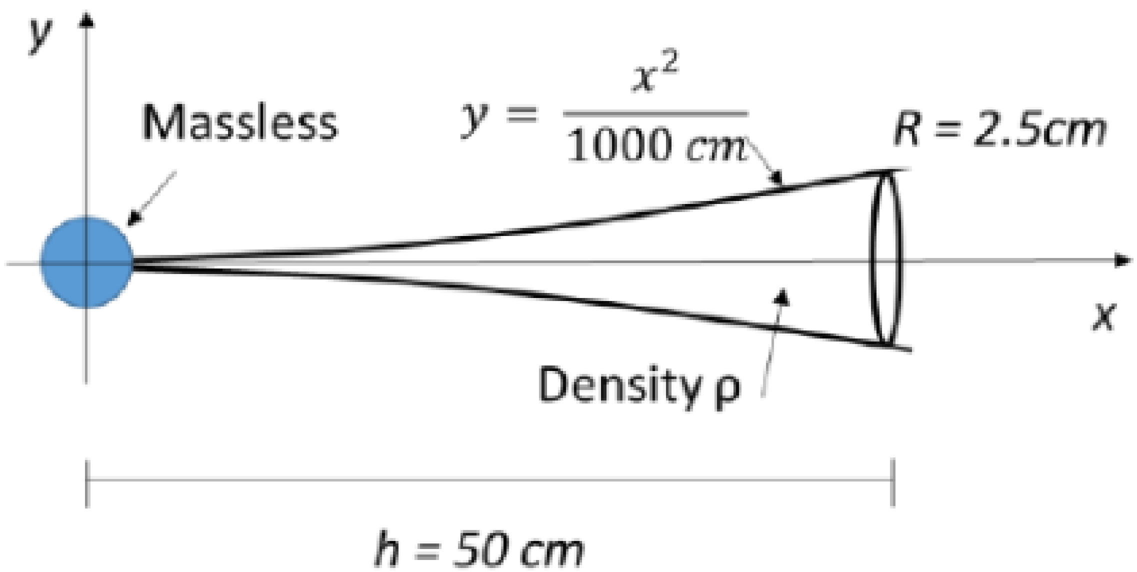
PH 212

HIP 3

Consider a juggling cone whose sides can be represented by the function $f(x)=bx^2$ where b = 1000 cm^{-1}. The object is 50 cm in length with a base diameter of 5 cm. The perfectly spherical top is made of an ultralight foam. The density of the cone is ρ=1g/cm.



a) Determine the mass of the juggling cone.

To find the mass of the cone we must use what we know about density, ρ=m/V.

If we can find volume, we can then find the mass of the cone.

We know that the cones sides are defined by the function above. We must first, then, use that as our value for the radius of a circle and then integrate that area of a circle from 0 to 50 cm to find the volume

$r = x^2/1000$

$V(x) = \int_0^{50} \pi(x^2/1000)^2\, dx \rightarrow V(x) = \int_0^{50} (\pi/(1*10^6))*x^4\, dx \rightarrow$

$V(x) = (\pi/(1*10^6)) \int_0^{50} x^4\, dx \rightarrow V(x) = (\pi/(1*10^6)) * x^5/5 \,\Big|_0^{50} \rightarrow V(50) = (50^5\pi)/(5*10^6)$

$\rightarrow$ $V = 196.35\ cm^3$

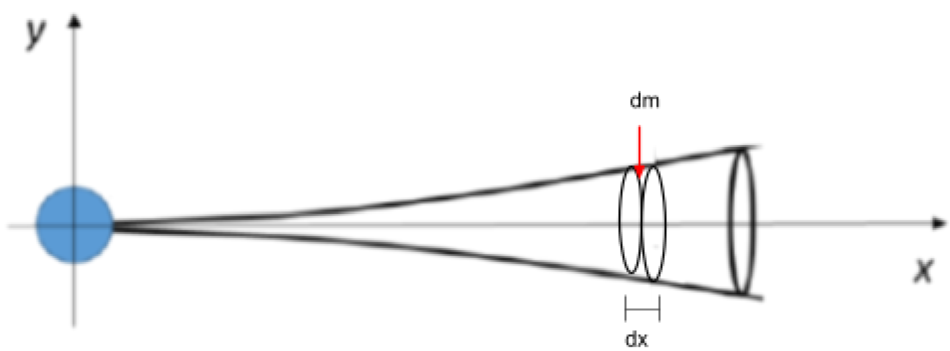
Now that we know the volume, we can solve for mass using our density equation.

$m = \rho * V \rightarrow V = \pi*h^5/(5*10^6) \rightarrow 196\ cm^3$; $\rho = 1\ g/cm^3 \rightarrow 1\ g/cm^3 * 196.35\ cm^3 \rightarrow 196.35\ g \rightarrow 196\ g$

The mass of the cone is 196 g.

This is reasonable because the density being 1 gram per cm^3 would make the mass the same as the volume. The volume seems reasonable considering it is 50 cms long and at the largest diameter it is 5 cm.

b) What is the center of mass of the cone?



To find center of mass we use the equation $x_{cm} = 1/m_{total} \int x\, dm$

In order to integrate this function we need to replace dm with something

This juggling cone is made of many thinly sliced discs, dx, whose radii can be found by the equation $r = x^2/1000$

Each disc will have a volume defined as $dV = \pi r^2 * dx$; to get the dm of this dV all we must do is multiply dV by our density → $dm = \rho * \pi r^2 * dx$

Now we take out equation for the center of mass and substitute what we just got for dm

$x_{cm} = 1/m_{total} \int_0^h x*\rho*\pi r^2*dx$ → we know that $r = x^2/1000$; substitute r →

$x_{cm} = 1/m_{total} \int_0^h x*\rho*\pi(x^2/1000)^2*dx$ → square what is in parentheses and move our constants outside of the integral → $x_{cm} = 1/m_{total} *\rho*\pi*(1/(1*10^6)*\int_0^h x^5*dx$

Plug in our equation for the mass, $\rho*\pi*h^5/(5*10^6)$ → $x_{cm} = 1/(\rho*\pi*h^5/(5*10^6)) *\rho*\pi *(1/(1*10^6)*\int_0^h x^5*dx$ → the rho's, pi's, and 10^6's cancel and we are left with → $x_{cm} = 5/h^5 \int_0^h x^5*dx$ → solve the integral → $x_{cm} = 5/h^5 *h^6/6$ → $x_{cm} = ⅚*h$

When we plug 50 cm in for h we get that the center of mass of the juggling cone is at 41.67 cm.

This seems reasonable because most of the mass is located at the end away from the origin making ⅚ the length of the object to be accurate for the center of mass.

c) What is the moment of inertia about the x-axis?
To find the moment of inertia we must recall the formula for the moment ofinertia of a disk $I_{disk} = ½MR^2$. Since the radius changes over the interval x= [0,50] cm we must use an integral to solve for this change in r that occurs. We get this equation --> $I_x = \int 1/2 * r^2 dm$

We know an equation for r and an equation for dm. Substitute them in.

$I_x = \int 1/2 * (x^2/1000)^2 * \rho * \pi * (x^2/1000)^2 \, dx \rightarrow$

$I_x = (\rho * \pi)/(2 * 10^{12}) * \int x^8 \, dx \rightarrow$ integrate $\rightarrow$

$I_x = (\rho * \pi)/(2 * 10^{12}) * x^9/9 \,|_0^h \rightarrow I_x = ((\rho * \pi)h^9)/(18 * 10^{12}) \rightarrow$ plug in 50 cm for h $\rightarrow I_x = 340.88 \, g * cm^2 \rightarrow I_x = 341 \, g * cm^2$

The moment of inertia for this juggling cone about the x-axis is 341 $g*cm^2$.

Since the formula for the moment of inertia of a disk about an axis of rotation is the one half the mass times the radius squared we can get an idea of what the moment of inertia for this object would be if it were a similar object. So when you solve for the moment of inertia for a cylinder with a length of 50 cm and a radius of 2.5 cm about twice as large. This makes sense since we have a cone-like object whose mass is contained more towards the axis of rotation which would give it a smaller moment of inertia.

Lecture Time: Name:

GORY	PLARY (1.5)	MPLISHED (1)	LOPING (0.5)	GENT (0)
Statement and tion	arning tool for our class is written	blem is clearly presented for reader in n words.	blem is directly copied or is hard .	p into some calculation
	tch could be dropped into a novel as it stands.	a clear sketch, larger than a credit the problem set up with important and data noted	some sketch of the problem	etch?
s Tools	iate physics tools are correlated ercise in textbook quality and	iate physics tools are correlated to the . Appropriate tools include: pictures, onservational laws utilized, etc…	hysics tools are correlated to the .	e a few equations written.
n Solution tation	is very clearly presented with g asides or annotations	is complete and clearly presented no significant intuitive demands on the	solution I have to read between .	es version of solution with only nts present
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Graded			e	ut your self-assessment is from mine by at least two steps.