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PH212

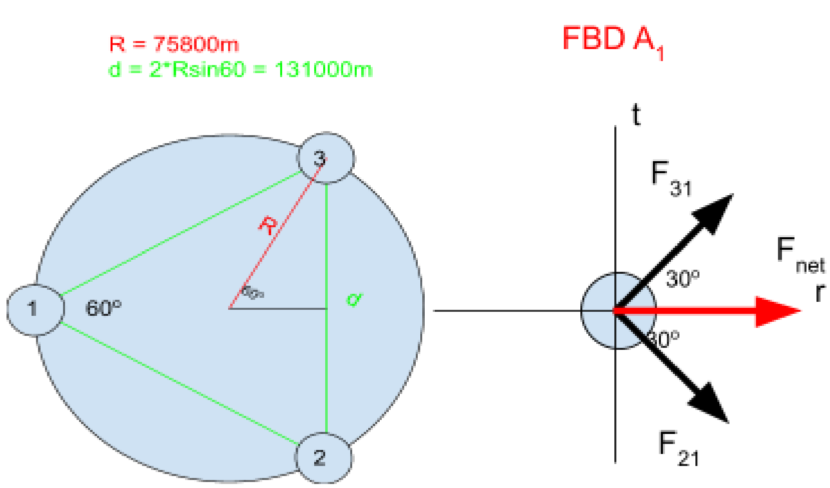
Ph 212 -- HIP 1 -- Universal Gravitation

Three asteroids of identical mass ($M = 1.03\times10^{12}$ kg) are orbiting their common center of mass in a perfect circle of radius R=75800 m. $G = 6.67\text{x}10^{-11}$ Nm^2/kg^2

a. What is the period of orbit of one of these asteroids?

We begin with Newton's Law of Universal Gravitation - $\mathbf{F_{12}}=(Gm_1m_2)/r^2$, the equation of a centripetal force - $\mathbf{F_{cent}}=(mv^2)/r$, and the equation that equates linear velocity to angular velocity - $v = (2\pi r)/\ \tau$ - in order to derive the equation for τ.

First draw a diagram of the system and a free body diagram of one of the asteroids



<u>Known values:</u>

$m_1 = 1.03\text{x}10^{12}$ R = 75800 m d= 131000 m $G = 6.67\text{x}10^{-11}$ Nm^2/kg^2

$\mathbf{F_{21}}$= <$(Gm_1m_2)/d^2\cos30$, $(Gm_1m_2)/d^2\sin30$> $\mathbf{F_{31}}$ = <$(Gm_1m_2)/d^2\cos30$, $-(Gm_1m_2)/d^2\sin30$>

$\mathbf{F_{21}} + \mathbf{F_{31}} = \mathbf{F_{net}}$ = <$2(Gm_1m_2)/d^2\cos30$, 0> → $|\mathbf{F_{net}}| = 2(Gm_1m_2)/d^2\cos30$

We know that the asteroids are undergoing an orbital motion, therefore the Force acting on them will be a centripetal one. We begin by equating this force to the equation of a centripetal force.

$|\mathbf{F}_{\mathbf{net}}| = 2(Gm_1m_2)/d^2\cos30 = \mathbf{F}_{\mathbf{cent}}=(m_1v^2)/R \rightarrow (m_1v^2)/R = 2(Gm_1m_2)/d^2\cos30 \rightarrow m_1$ cancels out $\rightarrow$ solve for $v^2 \rightarrow v^2 = ((2Gm)\cos30^*R)/d^2 \rightarrow v^2 = ((2\pi R)/T)^2 \rightarrow (4\pi^2R^2)/T^2 = ((2Gm)\cos30^*R)/d^2 \rightarrow$ solve for $T \rightarrow T= ((4\pi^2d^2R)/(2Gm\cos30))^{1/2}$

Solve for T

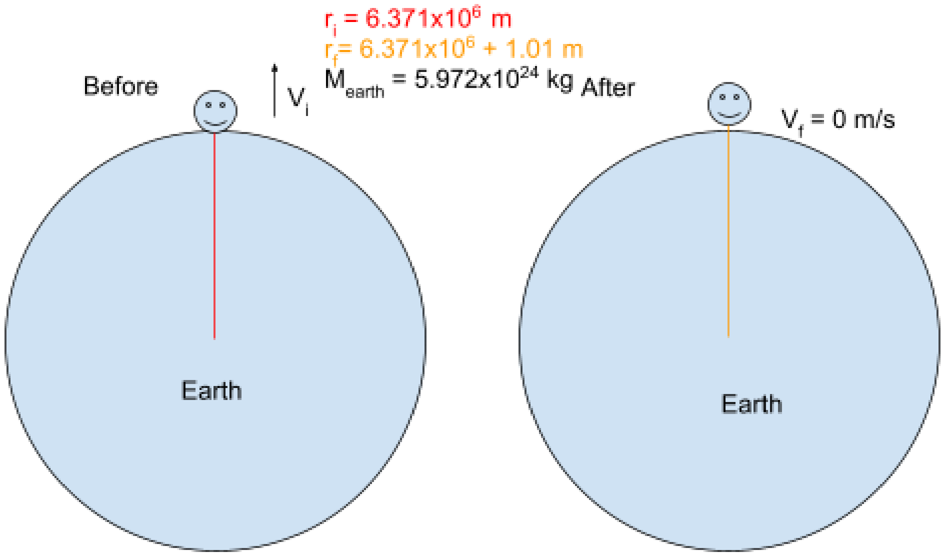
The period for one revolution is $T = 2.08 \times 10^7$ s

Considering the relationships of the masses and the radius of the orbital path, this period seems somewhat reasonable. But considering the fact that G is of such a small magnitude that it takes either very large masses or small values of r to create a powerful force.

b. You are standing on one of the asteroids pointing outward from the center of orbit with a goal of jumping off the asteroid and escaping the entire 3-asteroid system. The radius of the asteroid upon which you are standing is 710 meters.

If on Earth you can jump 1.01 meters, what final speed will you have when you escape the Asteroid cluster?

Since we know how high we can jump on Earth, we are able to use the Law of Conservation of Energy and Newton's Law of Universal Gravitation to determine our gravitational potential energy at the peak of the jump which will allow us then to find the initial energy of the system which will be 100% kinetic energy. This will give us our initial velocity.



$U_{gi}+KE_i = U_{gf}+KE_f$ $\quad$ $U_g = -(Gm_1m_2)/r$ $\quad$ $KE = 1/2m_1v^2$ $\quad$ $m_1 = m_1$

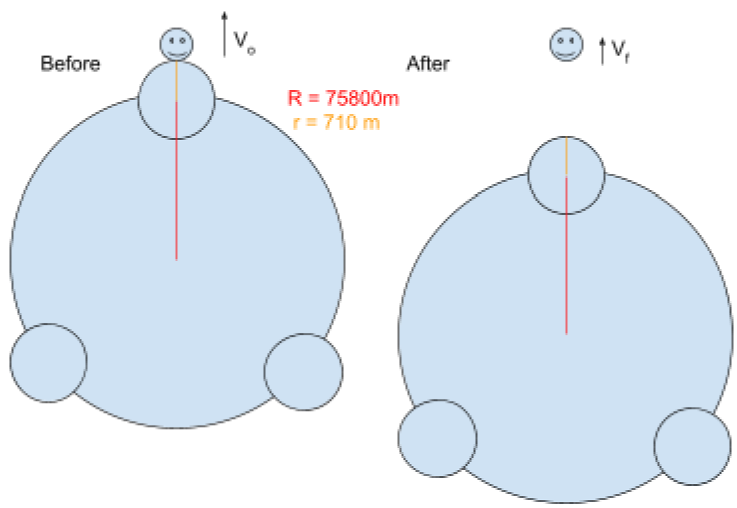
$1/2m_1v_i^2 + (-(Gm_1m_2)/r_i) = 1/2m_1v_f^2 - (Gm_1m_2)/r_f \rightarrow$ m_1 cancel out $\rightarrow$ $KE_f = 0 \rightarrow$ subtract U_{gi} to both sides$\rightarrow$ $1/2v_i^2 = (-(Gm_2)/r_f) + ((Gm_2)/r_i)$

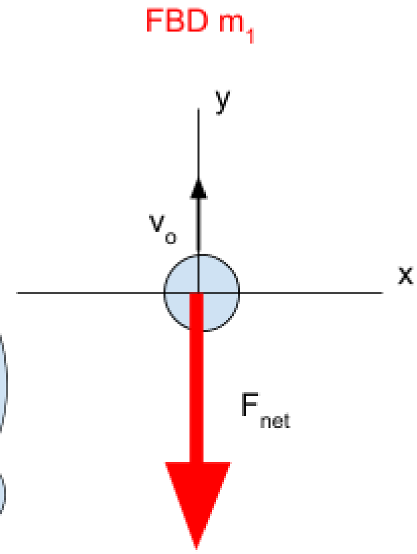
Solve for v $\rightarrow$ $v = ((-2Gm_2)/(r_f) + (2Gm_2)/(r_i))^{½}$ $\rightarrow$ plug in G, mass of the Earth, the radius of the Earth and the radius of the Earth plus the jump height. $\rightarrow$ $v_i = 4.45236604154 \rightarrow v_i = 4.45$ m/s

Now that we know the initial velocity of our jump we can use these same laws to find our final velocity after escaping the asteroid cluster. Since we are standing on the surface of the asteroid we must take into account not only the radius of the asteroid but also the radius from the common center of mass to find the appropriate r_i. And the mass will be the total mass of all asteroids

Radius of common mass = 75800 m Radius of asteroid = 710 m $\rightarrow$ r_i=76510 m $\quad r_f = \lim_{rf \rightarrow \text{infinity}}$

$m_1 = m_1$ $\qquad$ $m_2 = 3.09\text{x}10^{12}$ kg





$1/2m_1v_i^2 + (-(Gm_1m_2)/r_i) = 1/2m_1v_f^2 + (-(Gm_1m_2)/r_f) \rightarrow$ m_1 cancels out $\rightarrow$ subtract U_{gi} to both sides$\rightarrow$ $\lim_{rf \rightarrow \text{infinity}}$ of $(-(Gm_2)/r_f) = 0 \rightarrow 1/2v_i^2 = 1/2v_f^2 - (-(Gm_2)/r_i) \rightarrow$ two negatives make a positive $\rightarrow 1/2v_i^2 = 1/2v_f^2 + (Gm_2)/r_i) \rightarrow$ solve for $v_f \rightarrow v_f = (v_i^2 - (2(Gm_2)/r_i))^{½}$

Solve for $v_f \rightarrow$ $v_f = 4.44939 \rightarrow v_f = 4.45$ m/s

The final velocity will be 4.45 m/s.

When rounding to 3 sig figs the speed is the same as the initial velocity but in fact you lose a very very small amount of velocity, approximately .00298 m/s. This is reasonable because the mass of the asteroids are and it's radius are not large enough or small enough to create a large enough force to keep someone from jumping away from the system.

Lecture Time: MTWF 11:00 Name: Christopher Hunt

CATEGORY	EXEMPLARY (1.5)	ACCOMPLISHED (1)	DEVELOPING (0.5)	EMERGENT (0)
Problem Statement and Introduction	A new learning tool for our class is written	The problem is clearly presented for reader in your own words.	The problem is directly copied or is hard to follow.	You jump into some calculation
Picture	Your sketch could be dropped into a graphic novel as it stands.	There is a clear sketch, larger than a credit card, of the problem set up with important features and data noted	There is some sketch of the problem setup	What sketch?
Physics Tools	Appropriate physics tools are correlated to the exercise in textbook quality and size	Appropriate physics tools are correlated to the exercise. Appropriate tools include: pictures, FBDs, conservational laws utilized, etc…	Some physics tools are correlated to the exercise.	There are a few equations written.
Problem Solution Presentation	Solution is very clearly presented with intriguing asides or annotations	Solution is complete and clearly presented making no significant intuitive demands on the reader.	In your solution I have to read between the lines	Cliff notes version of solution with only high points present
Form	Your solution can serve as solution manual.	Drawing is larger than a credit card, organization is fluid, notation used is clear.	I could figure the path of your solution with effort.	You can read it.
Units		All units correctly given	Calculations & quantities are presented with units	Some units at the results
Solution		Correct	You are close	None/Not reasonable
Significant Figures		Correct Sig Figs	Makes effort to use correct significant figures	Copies the number from the calculator
Reasonableness	Provides more than one type of Reasonableness check.	Gives one clear rationale for appropriateness of the solution in the setting	Asserts that the answer is reasonable but really hasn't given any evidence	No discussion
All Self Graded		Done	Not Done	Done, but your self-assessment is different from mine by at least two steps.