Part 1: Multiple Choice Questions   Solutions

Choose the best answer for each question.

An integer n is *incredible* provided that $2|(n^2 + 2n + 1)$. Which of the following statements is true?

○ A. **There exists an odd integer that is incredible.**

○ B. **All even integers are incredible.**

○ C. **All incredible integers are nonnegative.**

○ D. **If n is incredible, then n is divisible by 2.**

Answer Point Value: 2 points

Answer Key: A

A proposition and a proof are presented below.

Proposition. For all $x, y \in \mathbb{Z}$, if x is even and y is odd, then xy is even.

Proof. Let $x, y \in \mathbb{Z}$. Suppose that x is even and y is odd. We will show that xy is even by direct proof. Since x is even and y is odd, there exists an integer k such that $x = 2k$ and $y = 2k + 1$ by definition. We observe that the product $xy = (2k)(2k + 1) = 4k^2 + 2k = 2(2k^2 + k)$. Thus, there exists an integer ℓ, namely $\ell = 2k^2 + k$, such that $xy = 2\ell$. Therefore, xy is even. $\square$

If there is a logical error in the proposition or the proof, what is it?

A. **The proof presented is correct.**

B. **The proposition is not true.**

C. **The proof presented is incorrect because it assumes that the conclusion is true.**

D. **The proof is incorrect because it uses the same variable for two different values.**

Answer Point Value: 2 points

Answer Key: D

Which of the following values of c is a counterexample to the statement, "if c is composite, then $3c + 1$ is prime"?

A. 6

B. 2

C. 8

D. 5

Answer Point Value: 2 points

Answer Key: C

Let x, y be Boolean variables. Which of the following Boolean expressions is a tautology?

A. $\neg(x \wedge y) \leftrightarrow (\neg x \vee \neg y)$

B. $(x \rightarrow y) \wedge (y \rightarrow x)$

C. $(x \vee y) \wedge (\neg x) \wedge (\neg y)$

D. $(\neg x \wedge \neg y) \vee (x \wedge y)$

Let x, y be Boolean variables. Which of these Boolean expressions is logically equivalent to $x \leftrightarrow y$?

A. $(x \wedge y) \vee (\neg x \wedge \neg y)$

B. $(x \wedge y) \wedge (\neg x \wedge \neg y)$

C. $(x \wedge y) \leftrightarrow (\neg x \wedge \neg y)$

D. $(x \wedge y) \rightarrow (\neg x \wedge \neg y)$

We define the set $A = \{X \subseteq \mathbb{Z} : |X| = 3\}$. Which of the following statements is true?

A. $\{3, 3, 3\} \in A$

B. $\forall X \in A, \exists Y \in A, X \cap Y = \emptyset$

C. $(3, 3, 3) \in A$

D. $\exists X \in A, \displaystyle\prod_{x \in X} x = 3$

How many 3-digit positive integers are there that have no repeated digits or all digits are odd?

○ A. $9 \cdot 9 \cdot 8 + 5^3 - 5 \cdot 4 \cdot 3$

○ B. $9 \cdot 9 \cdot 8 + 5 \cdot 4 \cdot 3 - 5^3$

○ C. $9 \cdot 9 \cdot 8 + 5^3$

○ D. $10 \cdot 9 \cdot 8 + 5^3 - 5 \cdot 4 \cdot 3$

Answer Point Value: 2 points

Answer Key: A

What is the negation of the statement "Every good boy deserves fudge"?

○ A. **No good boy deserves fudge.**

○ B. **Every good boy does not deserve fudge.**

○ C. **One good boy does not deserve fudge.**

○ D. **Any good girl deserves ice cream.**

Answer Point Value: 2 points

Answer Key: C

The notation $\exists!$ is used to indicate that there exists exactly one object to satisfy the condition. For example, $\exists! x \in \mathbb{N}, x^2 = 1$ means there is a natural number x whose square is 1 *and there is only one such* x.

Which of the following statements is true?

A. $\exists! y \in \mathbb{N}, \forall x \in \mathbb{N}, x < y$

B. $\exists! x \in \mathbb{N}, \forall y \in \mathbb{N}, xy = y$

C. $\forall x \in \mathbb{N}, \exists! y \in \mathbb{N}, xy = 1$

D. $\forall x \in \mathbb{N}, \exists! y \in \mathbb{N}, x + y = 0$

Answer Point Value: 2 points

Answer Key: B

Given $A = \{1, 2\}$ and $B = \{2, 3\}$, what is the cardinality of $\mathcal{P}(A \triangle B)$? Recall that $\mathcal{P}(X)$ is the *power set* of the set X.

A. 0

B. 8

C. 4

D. 2

Answer Point Value: 2 points

Answer Key: C

Part 2: Long Answer/Proofs

Handwrite your answers in complete English sentences on a separate sheet of blank paper. Write your name and section number on the first page of your handwritten solutions and use mathematical notation and terminology discussed in this course. Create one PDF of your write-up to both questions and upload it. Be sure to check that your upload is clear, legible and complete. Do not write your answers in LaTeX/Overleaf or on a tablet.