Spring 2021 MATH-UA 120 Discrete Math
Exam 2 Sec 14 - 23
Administered online with a 1 hour time limit

Part 1: Multiple Choice Questions

Solutions

Choose the best answer for each question.

Let $A = \{1, 2, 3\}$. The relation $R = \{(1, 1), (1, 3)\}$ on A is not symmetric. What is the least number of additional ordered pairs we need to include in R to ensure it is symmetric?

- A. 3
- B. 1
- C. 4
- D. 2

Answer Point Value: 2 points

Answer Key: B

Consider the equivalence relation R corresponding to the partition $\mathcal{P} = \{\{1, 2\}, \{3\}, \{4\}\}$ of $A = \{1, 2, 3, 4\}$. What is R?

- A. $\{(1, 1), (2, 2), (3, 3), (3, 4), (4, 3), (4, 4)\}$
- B. $\{(1, 1), (1, 2), (1, 3), (3, 4), (4, 3), (4, 4)\}$
- C. $\{(1, 1), (1, 2), (2, 1), (2, 2), (3, 3), (4, 4)\}$
- D. $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$

Answer Point Value: 2 points

Answer Key: C

How many different letter arrrangements can be made from the word CINNAMON?

- A. $8!$
- B. $8!/3!$

C. 8^3

D. $\binom{8}{3}$

Answer Point Value: 2 points

Answer Key: B

Let n be a positive integer. Which of the following is NOT equal to $\binom{n}{2}$?

A. **The number of ways to select two puppies from a litter of n puppies.**

B. **The sum integers $1 + 2 + \cdots + n - 1$.**

C. **The cardinality of the set $\{S \subseteq \{1, 2, \ldots, n\} \colon |S| = 2\}$**

D. **The cardinality of the set $\{(a_1, a_2, \ldots, a_n) \colon \forall i \in \{1, 2, \ldots, n\}, a_i \in \{0, 1\}\}$.**

Answer Point Value: 2 points

Answer Key: D

In proving the statement, "The sum of two negative integers is negative" by contradiction, what are the first sentences?

A. **Let x and y be nonnegative integers. Suppose, for the sake of contradiction, that $x + y$ is nonnegative.**

B. **Let x and y be negative integers. Suppose, for the sake of contradiction, that $x + y$ is nonnegative.**

C. **Let x and y be nonnegative integers. Suppose, for the sake of contradiction, that $x + y$ is positive.**

D. **Let x and y be negative integers. Suppose, for the sake of contradiction, that $x + y$ is positive.**

Answer Point Value: 2 points

Answer Key: B

Suppose we wish to prove the following statement by contrapositive: "$\forall x \in \mathbb{Z}$, if $x^2 - 6x + 5$ is even, then x is odd." Which of these would be the best first line of a proof?

- A. **Suppose that $\forall x \in \mathbb{Z}$, if $x^2 - 6x + 5$ is even, then x is even.**
- B. **Let $x \in \mathbb{Z}$ be such that $x^2 - 6x + 5$ is even.**
- C. **Let $x \in \mathbb{Z}$ be even. We will show that $x^2 - 6x + 5$ is odd.**
- D. **Suppose $\exists x \in \mathbb{Z}$ such that $x^2 - 6x + 5$ is even, and x is even.**

Answer Point Value: 2 points

Answer Key: C

Which of the following statements in quantifier notation is equivalent to the Well-Ordering Principle?

- A. $\forall S, S \subseteq \mathbb{N} \wedge S \neq \emptyset \rightarrow \exists x \in S, x \text{ is the least element.}$
- B. $\exists S, S \subseteq \mathbb{N} \wedge S \neq \emptyset \rightarrow \forall x \in S, x \text{ is the least element.}$
- C. $\forall S, S \subseteq \mathbb{N} \wedge S \neq \emptyset \rightarrow \forall x \in S, x \text{ is the least element.}$
- D. $\exists S, S \subseteq \mathbb{N} \wedge S \neq \emptyset \rightarrow \exists x \in S, x \text{ is the least element.}$

Answer Point Value: 2 points

Answer Key: A

In which line is the there a logical error in the proof by induction?

Proposition. All horses are the same color.

Proof. Let n be the number of horses such that $n \geq 1$. We proceed by induction on n.

Base case. If $n = 1$, then all horses have the same color.
Induction hypothesis. For $n = k$, we assume that a group of k horses has the same color.
Inductive step. We want to show that a group of $n = k + 1$ horses has the same color. Consider a group of $k + 1$ horses. The first k of the $k + 1$ horses are the same color by the induction hypothesis. Likewise, the last k of the $k + 1$ horses are the same color. Therefore the first horse in the group and the last horse in the group are the same color. Hence, the first horse, middle horses and last horse in the group have the same color.

Thus, all horses are the same color by induction. $\square$

- A. **2**
- B. **3**
- C. **1**
- D. **The proof by induction presented is correct.**

Answer Point Value: 2 points

Answer Key: B

What is the solution to the recurrence relation?

$$a_n = 2a_{n-1} - a_{n-2}; a_0 = 1, a_1 = 2$$

- A. **none of the these**
- B. $a_n = n + 1$
- C. $a_n = (-1)^n + n$
- D. $a_n = 1$

Answer Point Value: 2 points

Answer Key: B

What is the degree of the polynomial with the polynomial sequence $\{-5, 3, 13, 25, 39, 55, 73, 93, 115, \ldots\}$?

- A. **4**
- B. **5**
- C. **2**
- D. **3**

Answer Point Value: 2 points

Answer Key: C