DOING PHYSICS WITH MATLAB

COMPUTATIONAL OPTICS

RAYLEIGH-SOMMERFELD DIFFRACTION
CROSS SHAPED APERTURES

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op_rs_rxy_04_man.m

Calculation of the energy density in a plane perpendicular to the optical axis for a double slit

simpson2d.m
Function to calculate the value of a two-dimensional integral using the Simpson's [2D] method.

fn_distancePQ.m
Function to calculate distance between two points

Review the following website for more detail of the Rayleigh-Sommerfeld diffraction integral.

Scalar Diffraction Theory

Diffraction from rectangular apertures

Surface [2D] integration

RAYLEIGH DIFFRACTION INTEGRAL OF THE FIRST KIND

The **Rayleigh-Sommerfeld region** includes the entire space to the right of the aperture. It is assumed that the Rayleigh-Sommerfeld diffraction integral of the first kind is valid throughout this space, right down to the aperture. There are no limitations on the maximum size of either the aperture or observation region, relative to the observation distance, because **no approximations have been made**.

The Rayleigh-Sommerfeld diffraction integral of the first kind (RS1) can be expressed as

$$E_P = \frac{1}{2\pi}\iint_{S_A} E_Q \frac{e^{j\,k\,r_{PQ}}}{{r_{PQ}}^3} z_p\,(j\,k\,r_{PQ} - 1)\;\mathrm{d}S \qquad (4)$$

where E_P is the electric field at the observation point P, E_Q is the electric field within the aperture and r_{PQ} is the distance from an aperture point Q to the point P. The double integral is over the area of the aperture S_A.

The [2D] integration is performed over a rectangular ($a_x \times a_y$) with integration limits ($-a_x/2$ to $+a_x/2$) and ($-a_y/2$ to $+a_y/2$). The aperture space is made up of a grid on n_Q x n_Q points.

1. The maximum energy density u_{Qmax} [W.m^{-2}] in the aperture space is specified

```
uQmax = 1e-3;
```

2. The electric field E_Q is calculated at each grid point

```
EQmax = sqrt(2*uQmax/(cL*nR*eps0));
EQ = EQmax .* ones(nQ,nQ);
```

3. By setting a subset of the E_Q values to zero, the shape of the aperture can be established.

The code for the mscript **op_rs_rxy_04a.m** needs to be modified for different shaped apertures by changing: values for the input parameters, the setting of the values E_Q to 0, the output parameters, the Figure Windows, etc.

CROSS SHAPED APERTURE

Apertures with a uniform illumination and a cross shape are modelled using the mscript **op_rs_rxy_04_man. m** . The irradiance (energy density) in observation planes (XY plane) which are parallel to the aperture space are calculated in the near and far field.

There is a transition from Fraunhofer diffraction (far field) to Fresnel diffraction (near field) as the distance between the aperture and observation planes decreases. The distance dividing the two regimes is known as the **Rayleigh distance** d_{RL}

$$d_{RL} = \frac{a^2}{\lambda} \qquad \text{where } a \text{ is the maximum of } a_x \text{ and } a_y$$

Fraunhofer diffraction (far field) $\quad z_P > d_{RL}$

Fresnel diffraction (near field) $\quad z_P < d_{RL}$

Figure (1) shows the dimensions of a cross shaped aperture in an opaque screen.

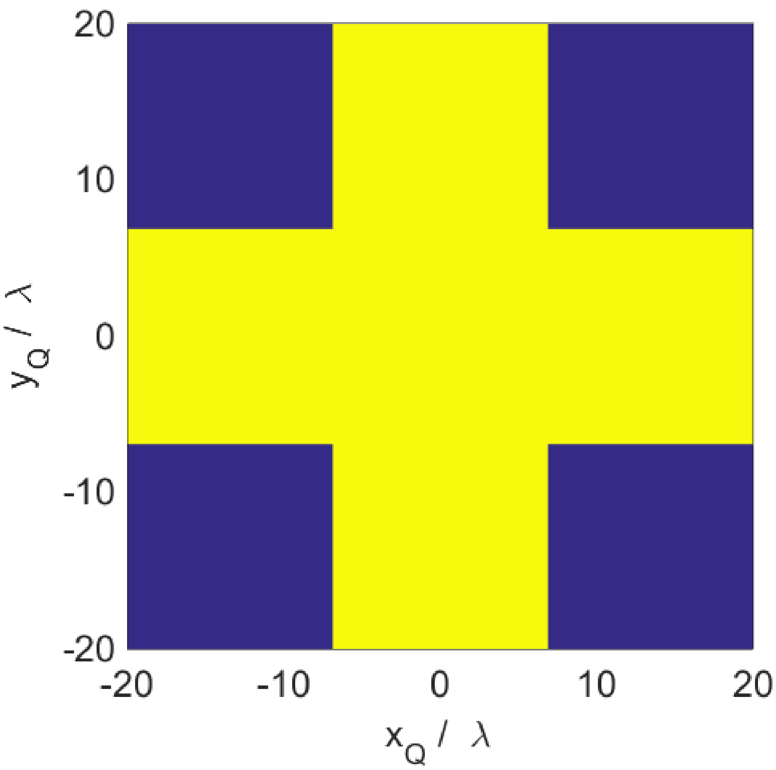

Fig. 1. Cross shaped aperture of width and height equal to 40λ where λ = 650 nm in an opaque screen. Dark blue region $E_Q = 0$ and yellow region E_Q = constant > 0.

Far field calculations $z_P = 6000\,\lambda > d_{RL}$ $\qquad d_{RL} = \dfrac{a^2}{\lambda} = 1600\,\lambda$ $\qquad a = 40\,\lambda$

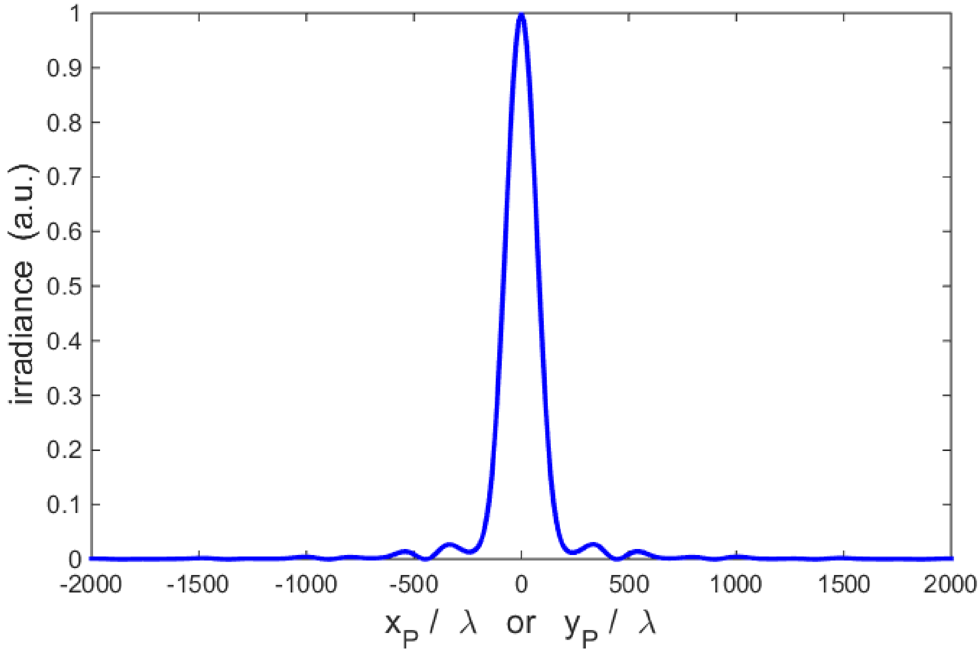

Fig. 2. Variation in the irradiance along the X axis or Y axis in the far field.

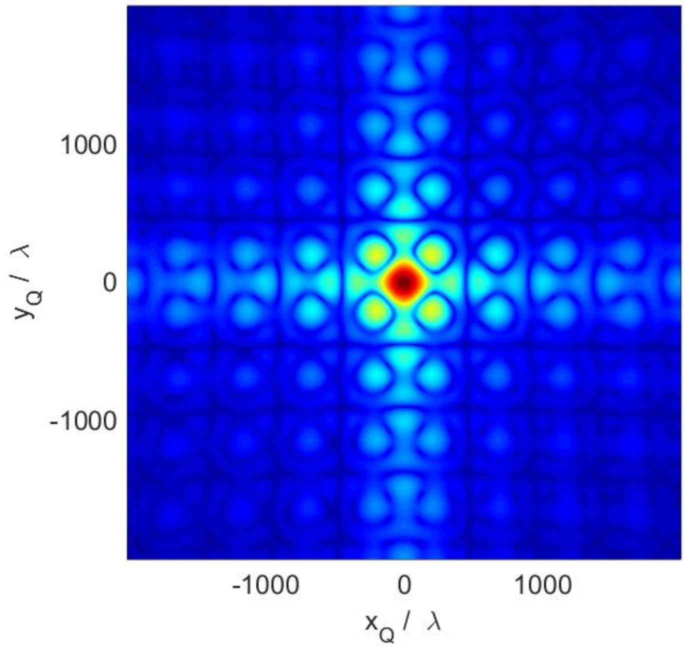

Fig 3. Scaled irradiance plot in the XY observation plane in the far field.

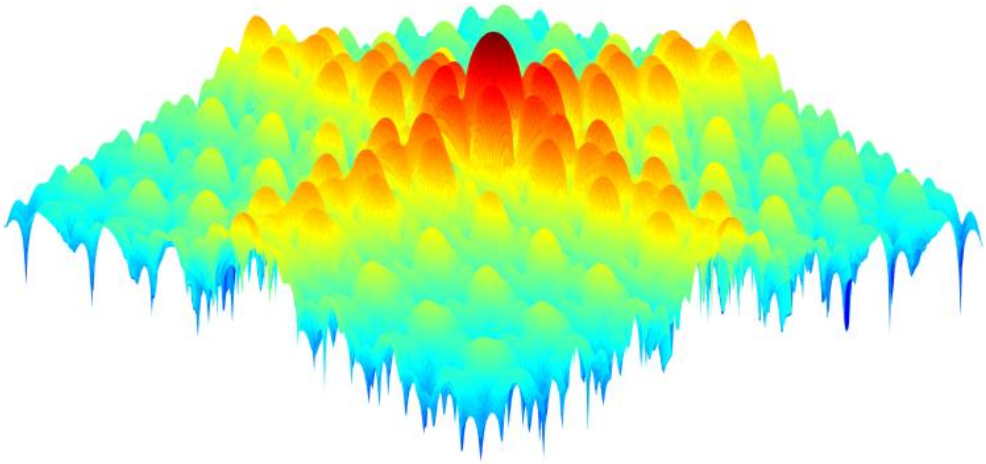

Fig 4. Scaled irradiance surf-plot in the XY observation plane in the far field.

Near field calculations $z_P = 600\lambda > d_{RL}$ $d_{RL} = \dfrac{a^2}{\lambda} = 1600\lambda$ $a = 40\lambda$

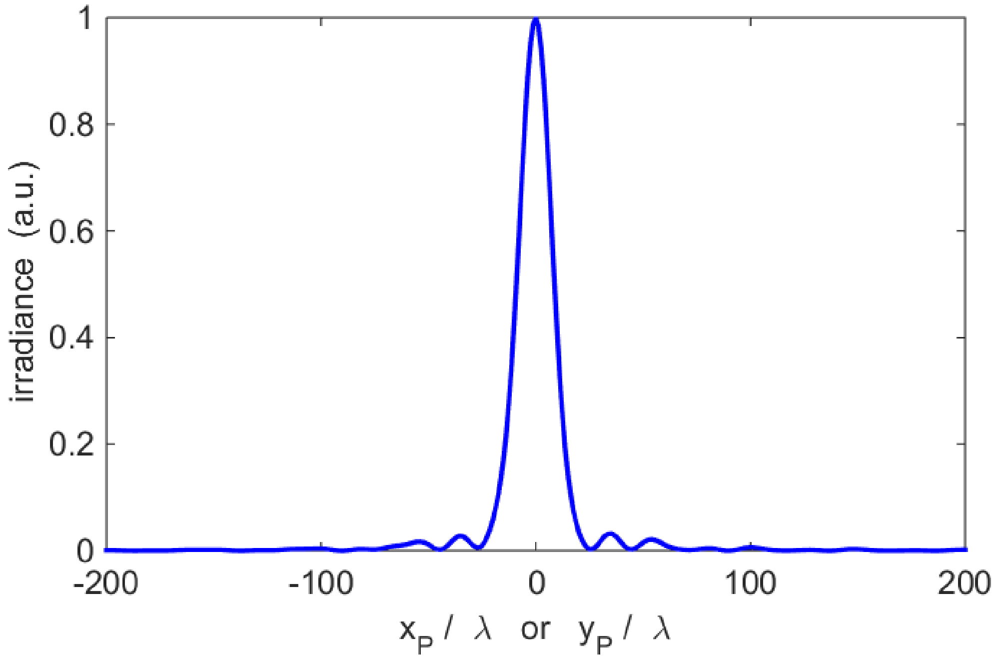

Fig. 5. Variation in the irradiance along the X axis or Y axis in the nearr field.

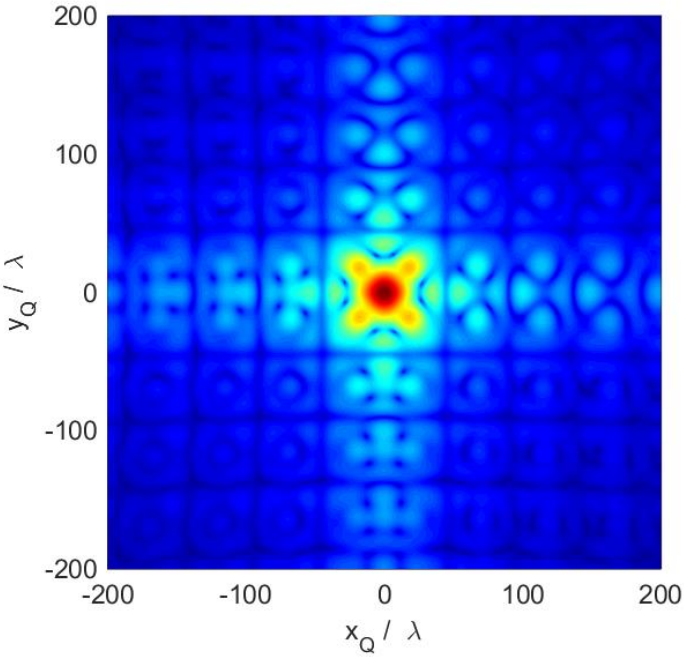

Fig 6. Scaled irradiance plot in the XY observation plane in the far field.

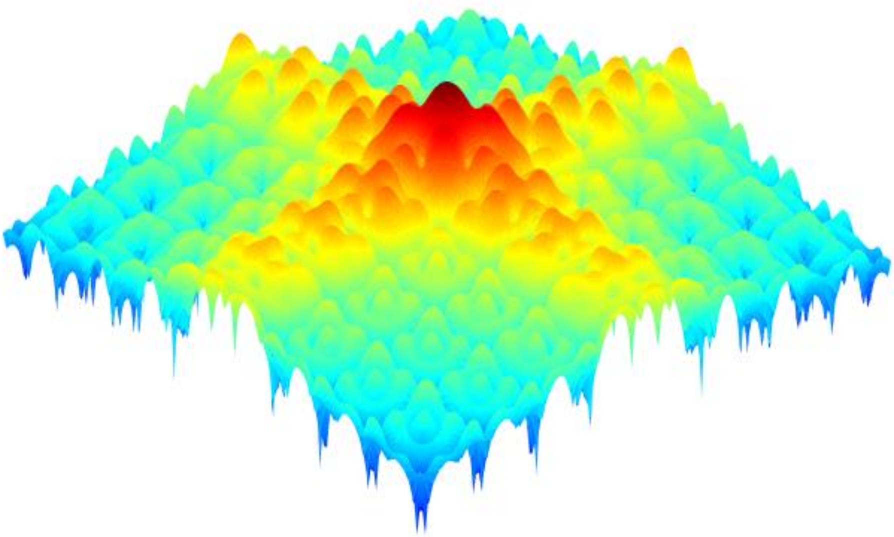

Fig 7. Scaled irradiance surf-plot in the XY observation plane in the far field.