DOING PHYSICS WITH MATLAB

COMPUTATIONAL OPTICS

RAYLEIGH-SOMMERFELD DIFFRACTION INTEGRAL OF THE FIRST KIND ANNULAR APERTURES

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op_rs_annular.m

Calculation of the irradiance in a plane perpendicular to the optical axis for uniformly illuminated circular - annular apertures. It uses Method 3 – one-dimensional form of Simpson's rule for the integration of the diffraction integral. Function calls to:

simpson1d.m (integration)

fn_distancePQ.m (calculates the distance between points P and Q)

turningPoints.m (max, min and zero values of a function)

Background documents

- Scalar Diffraction theory: Diffraction Integrals

- Numerical Integration Methods for the Rayleigh-Sommerfeld Diffraction Integral of the First Kind

- Circular apertures

RAYLEIGH-SOMMERFELD DIFFRACTION INTEGRAL OF THE FIRST KIND

UNIFORMLY ILLUMINATED ANNULAR APERTURES

The Rayleigh-Sommerfeld diffraction integral of the first kind states that the electric field at an observation point P can be expressed as

$$(1) \qquad E(\mathrm{P}) = \frac{1}{2\pi} \iint_{S_{\mathrm{A}}} E(\vec{r}) \frac{e^{jkr}}{r^3} z_p \, (jkr - 1) \, \mathrm{d}S$$

It is assumed that the Rayleigh-Sommerfeld diffraction integral of the first kind is valid throughout the space in front of the aperture, right down to the aperture itself. There are no limitations on the maximum size of either the aperture or observation region, relative to the observation distance, because **no approximations have been made**.

The **irradiance** or more generally the term **intensity** has S.I. units of W.m^{-2}. Another way of thinking about the irradiance is to use the term **energy density** as an alternative. The use of the letter *I* can be misleading, therefore, we will often use the symbol *u* to represent the irradiance or energy density.

The irradiance or energy density *u* of a monochromatic light wave in matter is given in terms of its electric field *E* by

$$(2) \qquad u = \frac{c\, n\, \varepsilon_0}{2} |E|^2$$

where *n* is the refractive index of the medium, *c* is the speed of light in vacuum and ε_0 is the permittivity of free space. This formula assumes that the magnetic susceptibility is negligible, i.e. $\mu_r \approx 1$ where μ_r is the magnetic permeability of the light transmitting media. This assumption is typically valid in transparent media in the optical frequency range.

The integration can be done accurately using any of the numerical procedures based upon Simpson's rule to compute the energy density in the whole space in front of the aperture.

Numerical Integration Methods for the Rayleigh-Sommerfeld Diffraction Integral of the First Kind

The geometry for the diffraction pattern from circular type apertures is shown in figure (1).

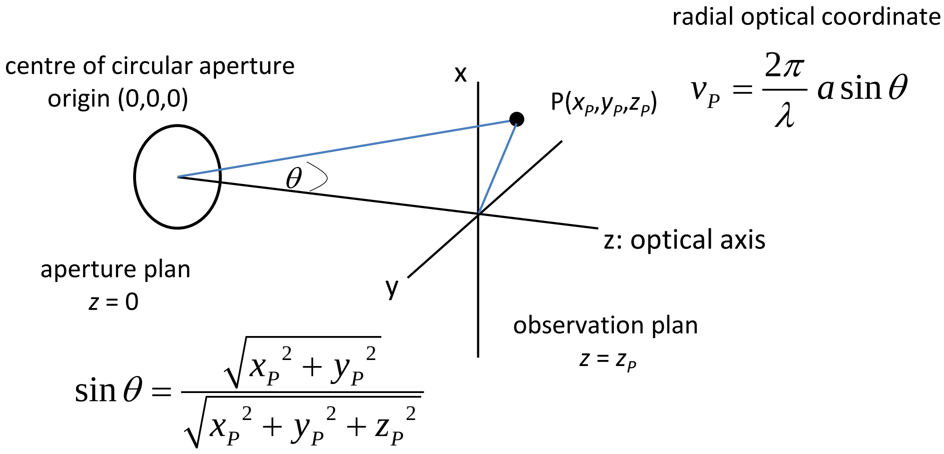


Fig. 1. Circular aperture geometry.

The radial optical coordinate v_P is a scaled perpendicular distance from the optical axis.

(3) $$v_P = \frac{2\pi}{\lambda} a \sin\theta \qquad \sin\theta = \frac{\sqrt{x_P{}^2 + y_P{}^2}}{\sqrt{x_P{}^2 + y_P{}^2 + z_P{}^2}}$$

Numerical integration of the Rayleigh-Sommerfeld diffraction integral of the first kind given by equation (1) for annular apertures can be done using a one-dimensional form of Simpson's rule (Method 3). The aperture space is partitioned into a series of rings and values of the electric field E_Q are set either to zero or E_{Qmax} for each ring as shown in figure (2)

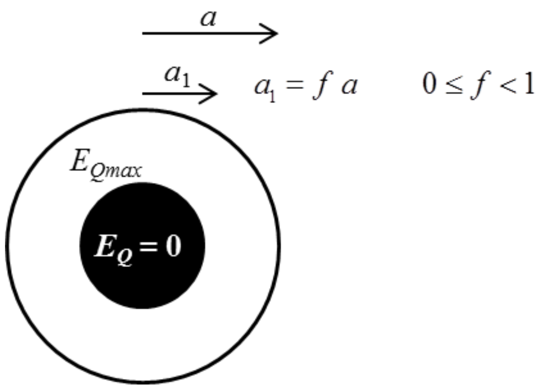


Fig. 2. An annular aperture. The radius of the aperture is a and the radius of the opaque disk is a_1 where $a_1 = f\ a \qquad 0 \le f < 1$.

Consider the diffraction from an aperture with the following default parameters:

Wavelength $\lambda = 6.328\times10^{-7}$ m
Aperture space grid points $n_Q = 360800$
Observation space grid points $n_P = 509$

Aperture Space
radius of aperture $a = 1.000\times10^{-4}$ m
Energy density $u_{Qmax} = 1.000\times10^{-3}$ W.m^{-2}
Energy from aperture $U_Q(\text{theory}) = 2.356\times10^{-11}$ J.s^{-1}

Observation Space
Max radius $r_P = 2.000\times10^{-2}$ m
Distance aperture to observation plane $z_P = 1.000$ m
Rayleigh distance $d_{RL} = 6.321\times10^{-2}$ m
Energy: aperture to screen $U_P = 2.214\times10^{-11}$ J.s^{-1}

Tables 1 and 2 give a summary of the optical coordinates v_P for the dark rings, the percentage of the energy that is radiated from the aperture that is enclosed by the first dark ring on the observation screen, and the relative strengths of the peaks in the diffraction pattern. The figures show the diffraction pattern for the annular apertures modelled in Tables 1 and 2.

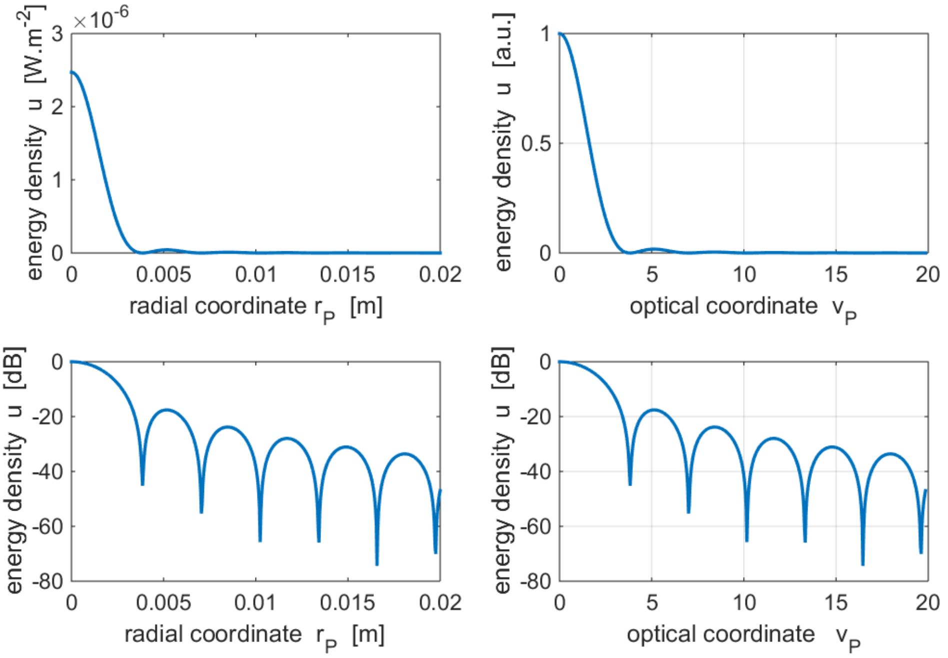
Table 1. Optical coordinate v_P for the dark rings and percentage of the energy enclosed within the first dark ring

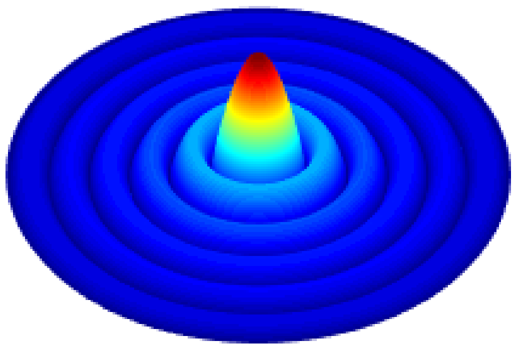
f	**0**	**0.20**	**0.40**	**0.60**	**0.80**	**0.98**
1st	3.83	3.68	3.32	2.97	2.66	2.42
2nd	7.00	7.34	7.50	6.80	6.10	5.59
3rd	13.33	9.69	10.36	10.63	9.58	8.72
4th	16.46	13.72	12.67	14.19	13.06	11.92
% energy	84	77	59	37	17	1.6

Table 2. Optical coordinate v_P for the peaks and their relative strengths

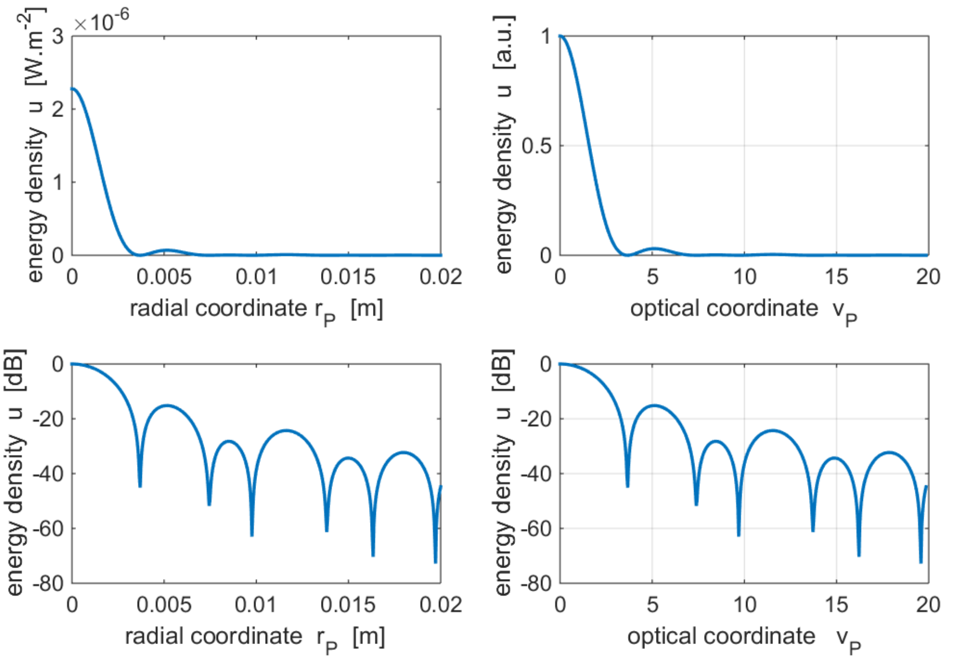
f	**0**	**0.20**	**0.40**	**0.60**	**0.80**	**0.98**
1st	5.12 0.0175	5.12 0.0303	4.97 0.0706	4.65 0.1202	4.22 0.1527	3.87 0.1621
2nd	8.40 0.0042	8.44 0.0015	8.68 0.0033	8.44 0.0305	7.740 0.0734	7.08 0.0899
3rd	11.61 0.0016	11.53 0.00376	11.49 0.0007	12.08 0.0044	11.22 0.0401	10.28 0.0621
4th	14.78 0.0008	14.89 0.0004	14.62 0.0028	15.05 0.0001	14.70 0.0109	13.45 0.0474

f **= 0 full circular aperture**

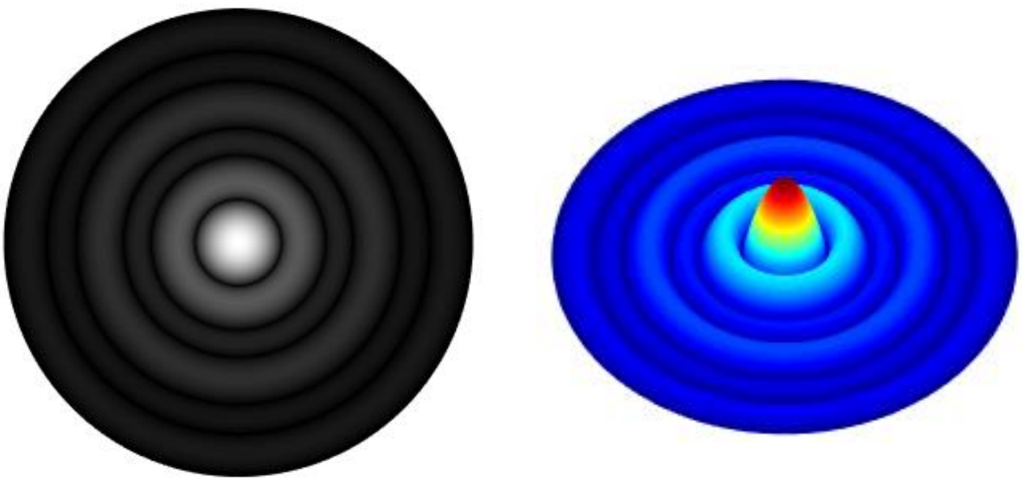




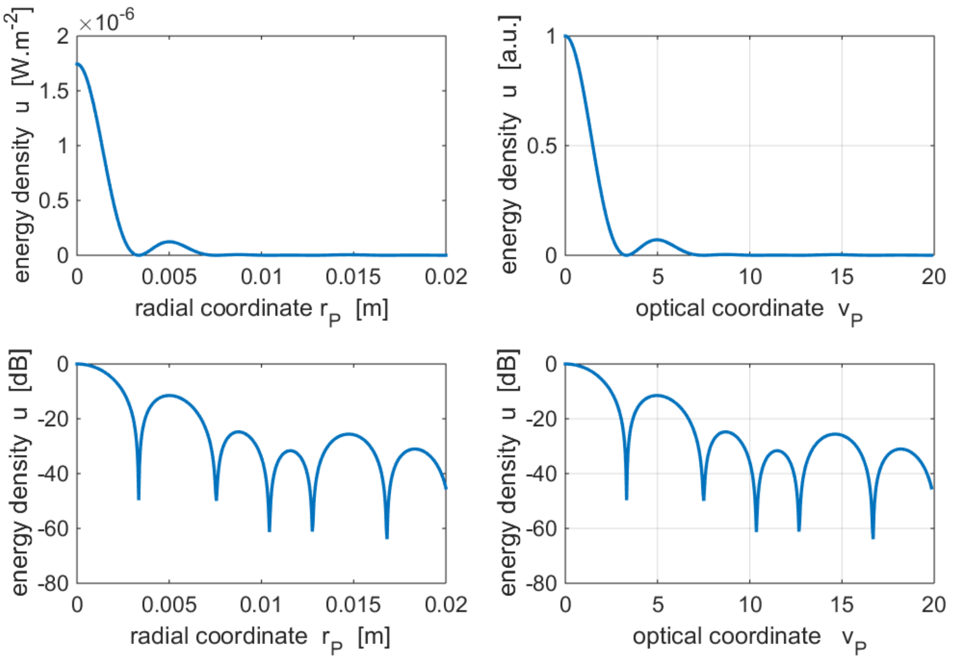
f = 0.20

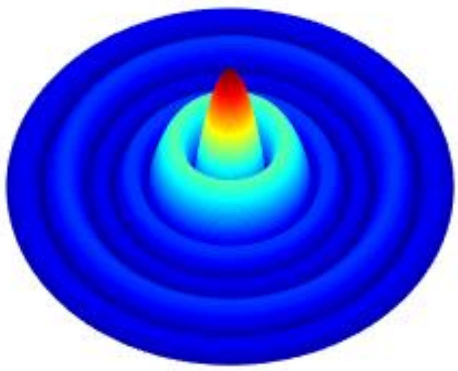


×10^-6
energy density u [W.m^-2]
radial coordinate r_P [m]
energy density u [a.u.]
optical coordinate v_P
energy density u [dB]
radial coordinate r_P [m]
energy density u [dB]
optical coordinate v_P

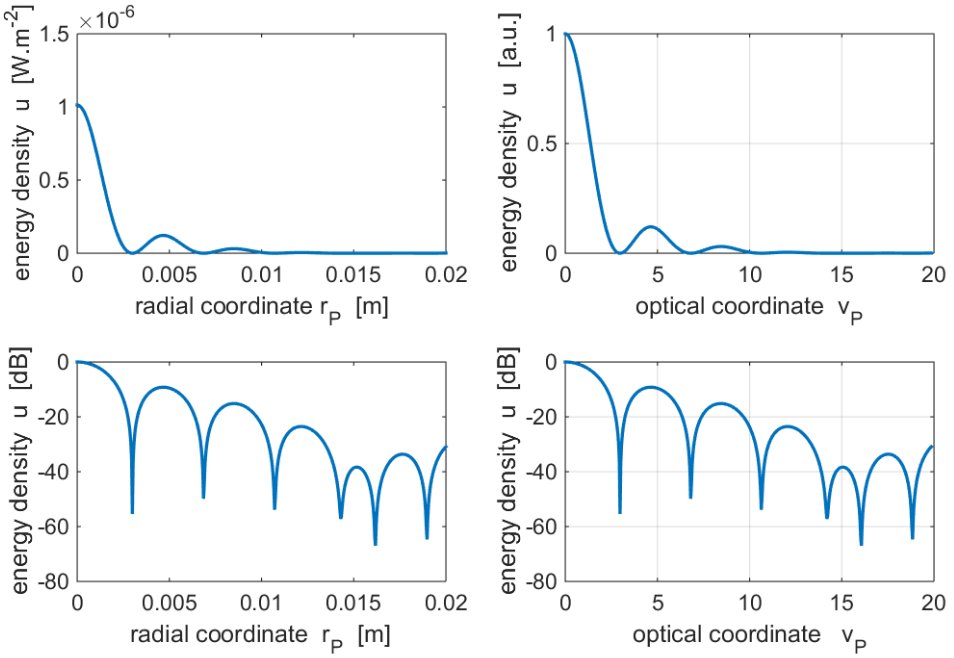


***f* = 0.40**

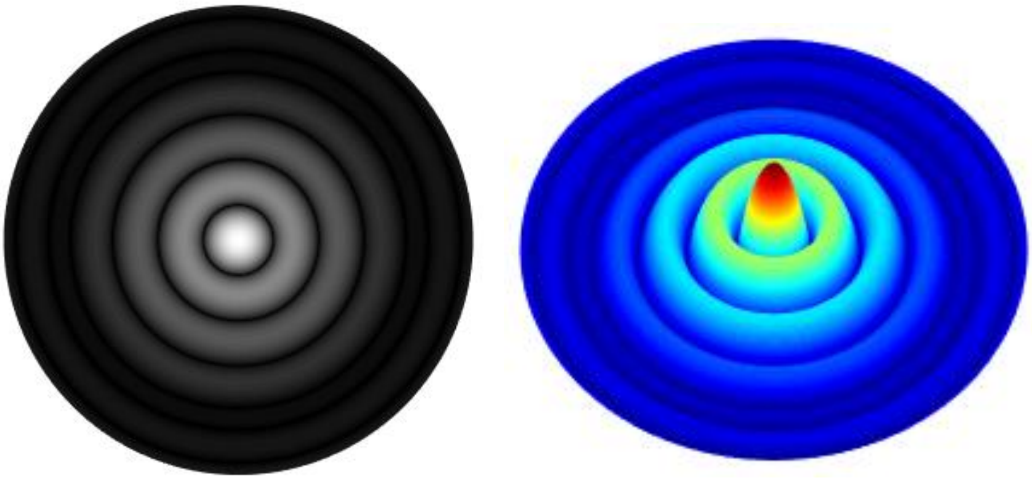

energy density u [W.m^-2]
×10^-6
radial coordinate r_P [m]
energy density u [a.u.]
optical coordinate v_P
energy density u [dB]
radial coordinate r_P [m]
energy density u [dB]
optical coordinate v_P



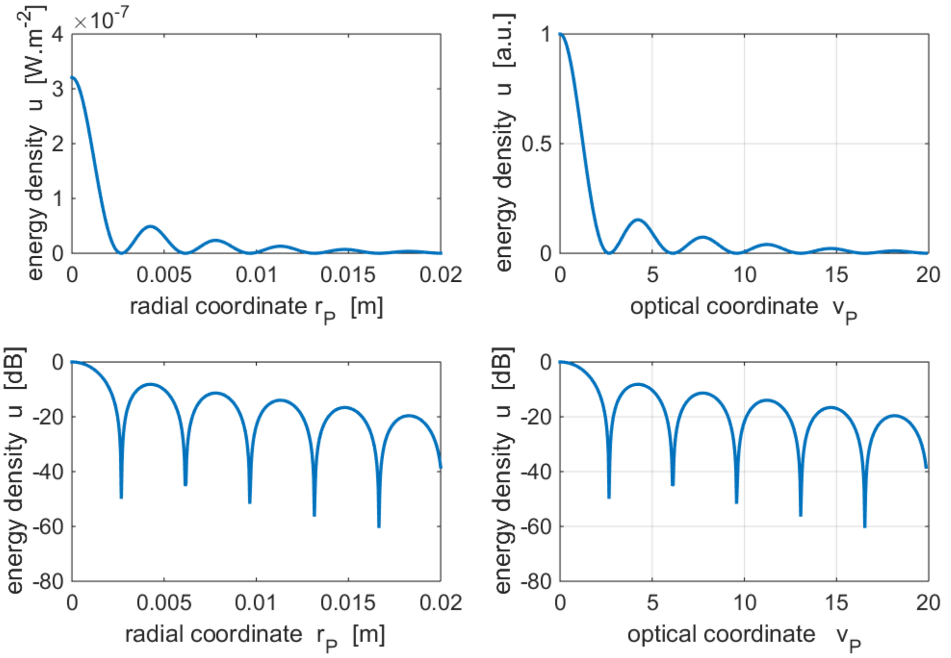
f = 0.60

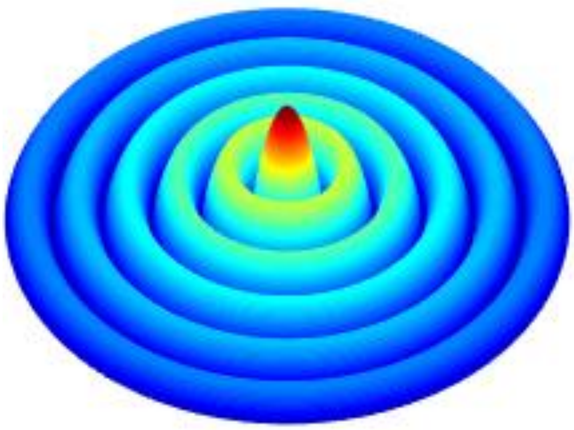


energy density u [W.m^{-2}]
$\times 10^{-6}$
radial coordinate r_P [m]
energy density u [a.u.]
optical coordinate v_P
energy density u [dB]
radial coordinate r_P [m]
energy density u [dB]
optical coordinate v_P

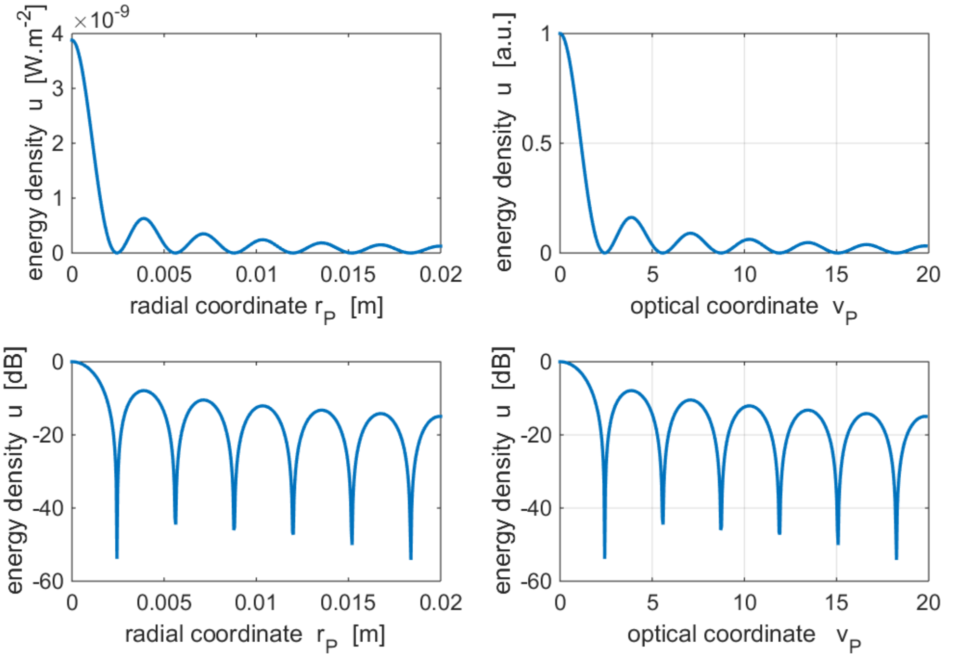


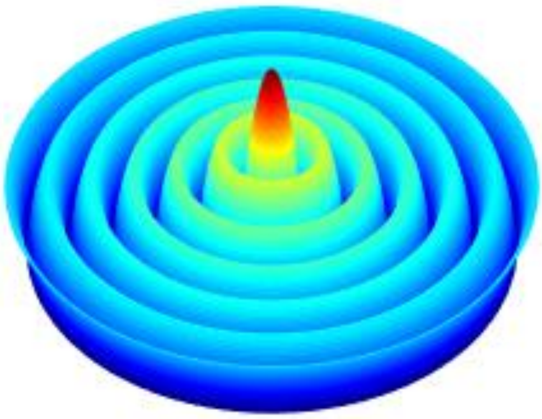
***f* = 0.80**


×10-7
energy density u [W.m-2]
radial coordinate rP [m]
energy density u [a.u.]
optical coordinate vP
energy density u [dB]
radial coordinate rP [m]
energy density u [dB]
optical coordinate vP



f **= 0.98**





As the radius of the opaque disk increases from 0 to 1:

- The size of the Airy Disk decreases.
- Reduction in the percentage of the energy within the Airy Disk decreases.
- The relative strengths of the peaks do not necessarily decrease.
- Uneven spacing between minima and maxima.

Double annular aperture