DOING PHYSICS WITH MATLAB

VISUALIZATION OF QUANTUM MECHANICAL PHENOMENA

COMPLEX VALUED FUNCTIONS

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DOWNLOAD DIRECTORY FOR MATLAB SCRIPTS

Download and inspect the scripts and make sure you can follow the structure of the programs.

visualization

wm_spectrum.m

Color plot of the visible spectrum for the wavelength range from 380 nm to 780 nm.

Calls the function **ColorCode.m**

Colorcode.m

Function to give the RGB values for a given wavelength color.

Is it assumed the supplied lambda is within the range 380-780 nm.

Smaller or higher values are set notionally to the extreme values.

The script **wm_spectrum.m** can be used to produce a plot of the visible spectrum for the wavelength range from 380 nm to 780 nm. The script uses the **area** plot function to give the spectrum. The color for each wavelength is calculated from the function **ColorCode.m**.

We start our study of visualizing quantum mechanical phenomena by considering a complex periodic function representing a plane wave in [1D]. A stationary plane wave can be presented by the wavefunction $\psi_n(x)$ with wave number k_n

(1) $$\psi_n(x) = \frac{1}{\sqrt{2L}} \exp\left(i k_n x\right) \quad k_n = \frac{n\pi}{L} \quad n = 0, \pm 1, \pm 2, \ldots$$

Each wavefunction $\psi_n(x)$ has a spatial period of $2L$, therefore, you only need to consider the interval [-L L].

$$\psi_n(x + 2L) = \psi_n(x)$$

The term $\left(1/\sqrt{2L}\right)$ is a normalizing factor

(2) $$\int_{-L}^{L} \psi_n^{\,*}(x) \psi_n(x)\, dx = 1$$

The complex valued plane function can be visualized in several ways using the Matlab script **vqm0001.m**. A color is assigned to the phase in the interval $[-\pi \; +\pi]$ as shown in figure 1.

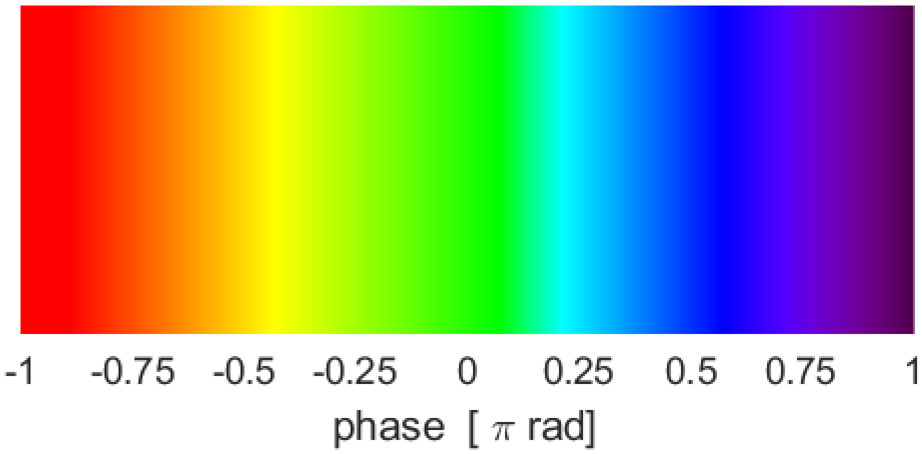

Fig. 1. The phase of a complex function is assigned a color code.

The real and imaginary parts of the wavefuction and its absolute value for n = 4 and L = 50 are shown in figure 2 as separate plots.

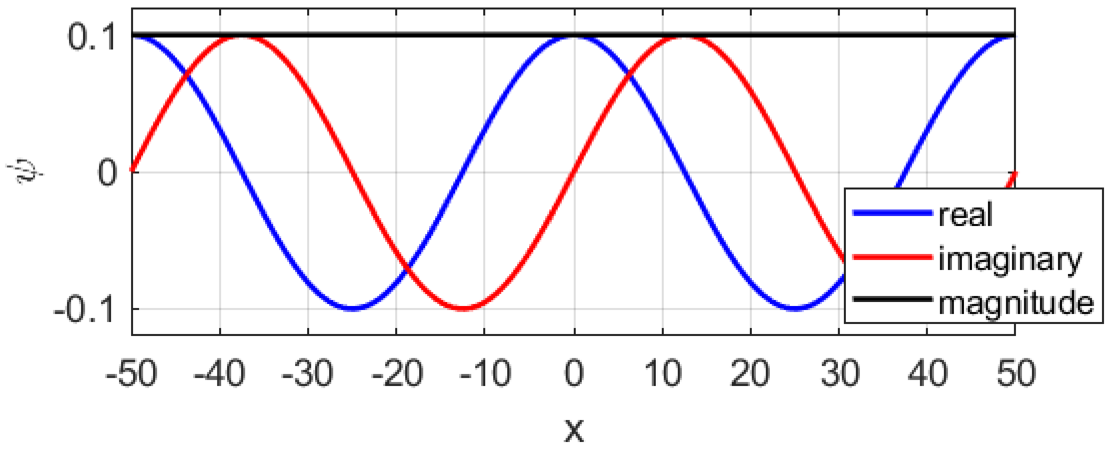

Fig. 2. The real and imaginary parts of the complex function $\psi_4(x)$ and its absolute value.

The wavefunction $\psi_4(x)$ can be displayed as space curve in a [3D] as shown in figure 3.

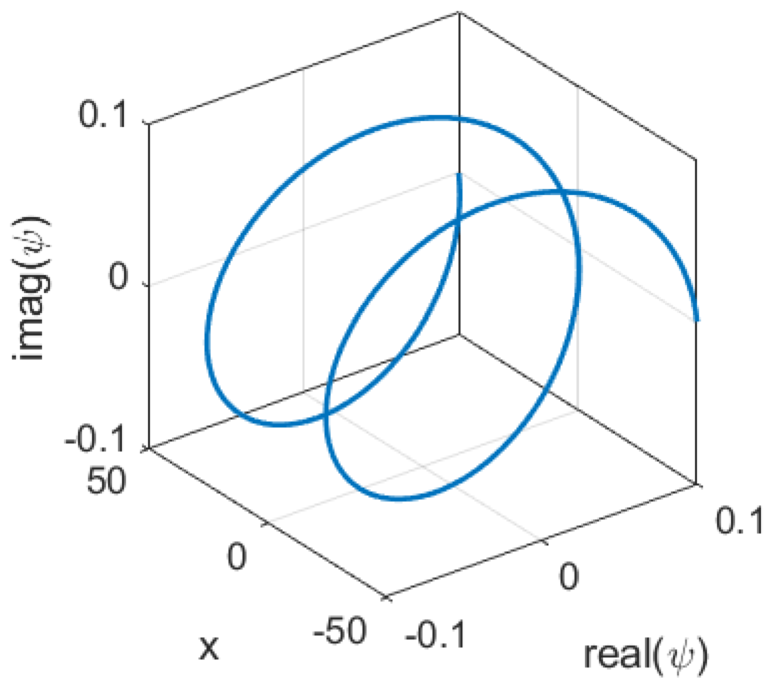

Fig.3. Space curve of the wavefunction $\psi_4(x)$ in the interval $\left[-50 \ \ +50\right]$.

The phase of the complex function can be computed using the Matlab function **angle**. A plot of the variation in the phase of the wavefunction $\psi_4(x)$ in the interval $[-50 \;\; +50]$ is shown in figure 4. The lower plot gives the value of the absolute of the wavefunction $\psi_4(x)$ and the phase is given by the color as shown in figure 1.

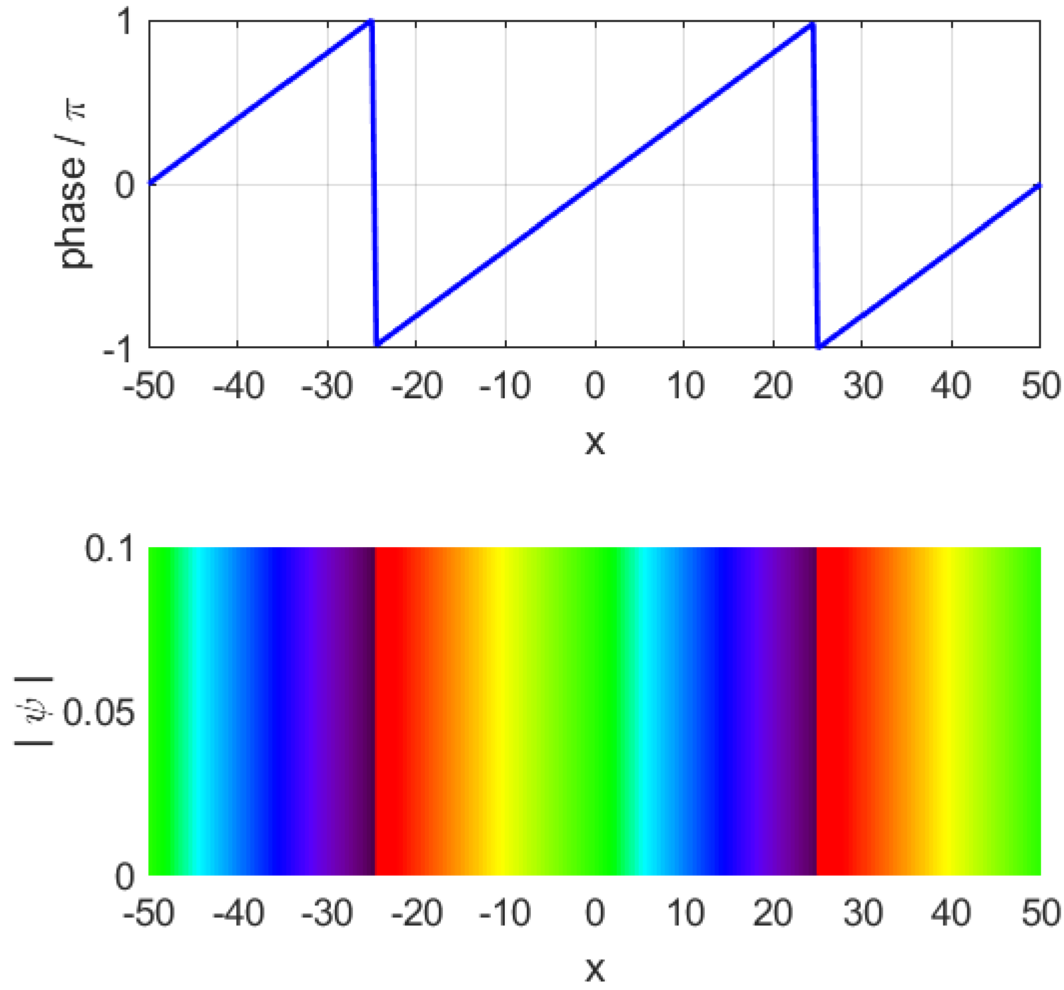

Fig. 4. The phase $\psi_4(x)$ of the wavefunction as a function of position.

Examination of figures 2 and 4 shown that the spatial period (wavelength λ) is equal to 50.

(3) $$k_n = \frac{2\pi}{\lambda_n}$$

$$k_4 = \frac{4\pi}{2L} \quad \rightarrow \quad \lambda_4 = L$$

You can check that the wavefunction is normalized by integrating the function given by equation 2 using the script **simpson1d.m**

```
CN = simpson1d(conj(psi).*psi,-L,L);
                    % check normalization
```

The value CN computed is displaced in the Command Window:

```
CN = 1.000
```