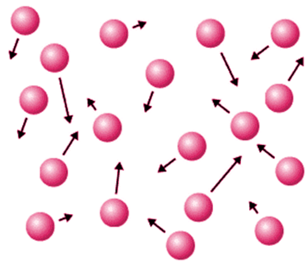
Maxwell Distribution of Speeds



In this simulation of the Maxwell distributioin of speeds for an idea gas, you can vary both the temperature and the molecular mass of the gas and oberseve the changes in the probability density function.   script tp_LE_Mawell.mlx

For an ideal gas, the meaning of the **temperature** measured in kelvin is that it gives the **average translational kinetic energy** of the gas molecules

$$\frac{3}{2}\,k\,T = \frac{1}{2}\,m\,v^2$$

Temperature is a direct measure of the molecular translational kinetic energy of a gas. When two ideal gases with molecules of different mass are at the same temperature, then they will have the same average molecular translational kinetic energy.

However, we would not expect all the molecules in a gas to be travelling at the same speed. The way in which the speed of the molecules vary was first proposed by **James Clerk Maxwell** in 1860. At this time, no experimental evidence was possible because of the nonexistence of proper vacuum equipment. It was not until 1926 that Otto Stern was able to partially confirm Maxwell's predictions.

Maxwell predicted that the speeds of all the gas molecules would be distributed according to his Maxwell distribution of speeds function. The prediction gives a probability density function which is called the **Maxwell-Boltzmann Distriution**. It gives the fraction of molecules with speed v in the range $v \pm dv$.

$$f(v)\,dv = 4\pi\, v^2 \left(\frac{M}{2\pi\, R\,T}\right)^{3/2} \exp\left(-\frac{M\,v^2}{2\,R\,T}\right) dv$$

$$f(v)\,dv = 4\pi\, v^2 \left(\frac{m}{2\pi\, k\,T}\right)^{3/2} \exp\left(-\frac{m\,v^2}{2\,k\,T}\right) dv$$

where

v = speed of a gas molecule

$f(v)\,dv$ = fraction of molecules with speeds in the range $v \pm\, dv$

T = temperature [K]

k = Boltzmann constant

R = Universal gas constant

M = molecular mass of the gas

m = mass of a gas molecule

We can express the Maxwell-Boltzmann Distribution in terms of a **probability density function** p_D where $p_D\,dv$ gives the probability of the gas molecules of having a speed in the range $v \pm\, dv$.

$$p_D = N\,\left(\frac{M}{T}\right)^{3/2} \exp\left(-\frac{M\,v^2}{2\,R\,T}\right)$$

where N is the normalizing constant such that

$$\int_0^{\infty} p_D dv = 1$$

And the probability to find a gas molecule with speed in the range $v \pm\, dv$ is

$$\text{Probability}(v \pm dv) = \int_{v-dv}^{v+dv} p_D dv$$

The average speed of the molecules can be expressed in a number of different ways;

Most probable speed $$v_p = \sqrt{\frac{2RT}{M}}$$

Average speed $$\langle v \rangle = \sqrt{\frac{8RT}{\pi M}}$$

Root-mean-square speed $$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Average translational kinetic energy of a molecule $$\langle E \rangle = \frac{3}{2}kT$$

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close all
clc
clear
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Input variables: temperature T [K] and molecular mass M [g]

```
T = 300;

M =   8;
```

```
% velocity limits:  probability(v1 < v < v2)
v1 = 1000;
v2 = 2000;
```

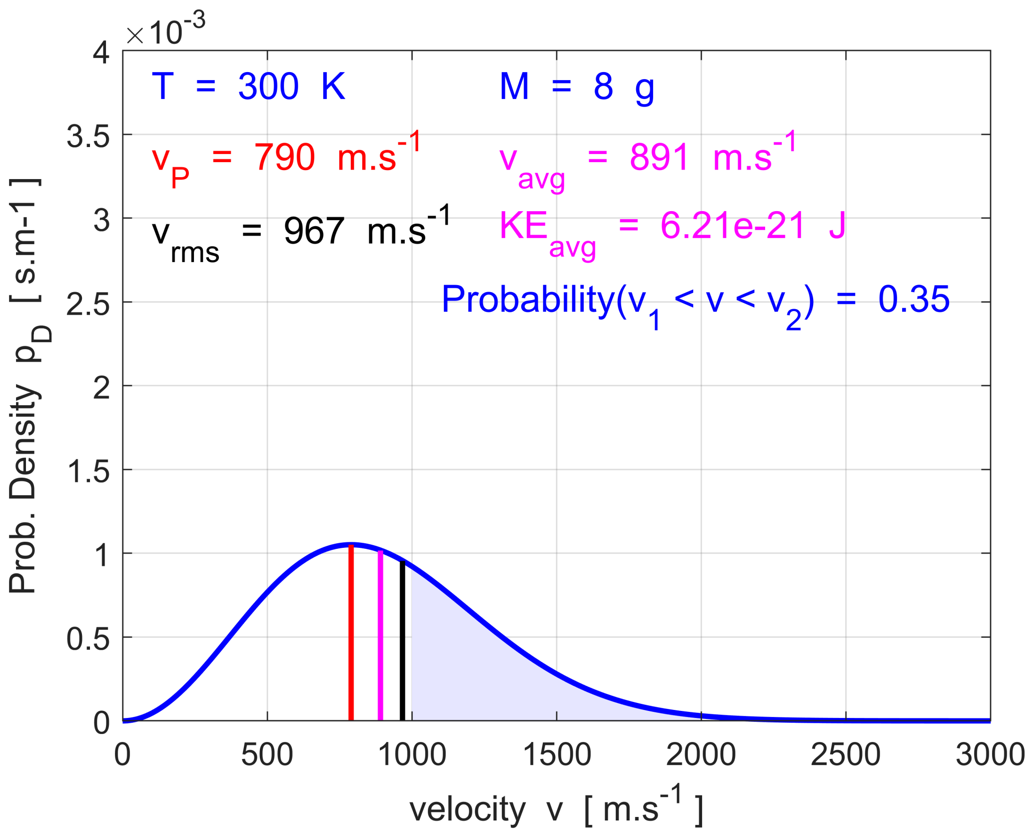
Setup and Calculations

```
R = 8.3145;     % Universal Gas constant
k = 1.38e-23;   % Boltzmann constant

vMin = 0;       % velocity range [m/s]
vMax = 3000;
nV = 1001;
v = linspace(vMin,vMax,nV);
indexv1 = find(v>v1,1);
indexv2 = find(v>v2,1);
indexv = indexv1:indexv2;
if mod(length(indexv),2) == 0; indexv = indexv1:indexv2+1; end

M = M*1e-3;     % molecular mass [kg]

% Maxwell-Boltzmann probability density funtion
fn = (M/T)^(1.5) .*v.^2 .* exp(-(M.*v.^2) ./ (2*R*T));

N = simpson1d(fn,vMin,vMax);   % normalization function

pD = fn./N;                    % probability density

% Velocities and average translational kinetic energy
v_P = sqrt(2*R*T/M);           index1 = find(v>=v_P,1);
v_avg = sqrt(8*R*T/(pi*M));    index2 = find(v>=v_avg,1);
v_rms = sqrt(3*R*T/M);         index3 = find(v>=v_rms,1);
KE_avg = 1.5*k*T;
prob12 = simpson1d(pD(indexv),v1,v2);
```

Graphics and output variables

```
figure(1)
Ylim = 4e-3;
xP = v(indexv1:indexv2); yP = pD(indexv1:indexv2);
Harea = area(xP, yP);
set(Harea,'FaceColor',[0.9 0.9 1],'EdgeColor',[0.9 0.9 1])
hold on
xP = v; yP = pD;
plot(xP,yP,'b','linewidth',2)
xlabel('velocity  v  [ m.s^{-1} ]')
ylabel('Prob. Density  p_D  [ s.m{-1} ]')

xP = [v_P v_P]; yP = [0 pD(index1)];
```

```
plot(xP,yP,'r','linewidth',2)
xP = [v_avg v_avg]; yP = [0 pD(index2)];
plot(xP,yP,'m','linewidth',2)
xP = [v_rms v_rms]; yP = [0 pD(index3)];
plot(xP,yP,'k','linewidth',2)
grid on
set(gca,'fontsize',12)
ylim([0 Ylim])
%legend('p_D','v_P','v_{avg}','v_{rms}')

tm1 = 'T  =  '; tm2 = num2str(T,'%4.0f '); tm3 = '  K';
tm = [tm1 tm2 tm3];
Htext = text(100,0.95*Ylim,tm);
set(Htext,'fontsize',14,'color','b')

tm1 = 'M  =  '; tm2 = num2str(M*1000,'%4.0f '); tm3 = '  g';
tm = [tm1 tm2 tm3];
Htext = text(1300,0.95*Ylim,tm);
set(Htext,'fontsize',14,'color','b')

tm1 = 'v_P  =  '; tm2 = num2str(v_P,'%4.0f '); tm3 = '  m.s^{-1}';
tm = [tm1 tm2 tm3];
Htext = text(100,0.84 *Ylim,tm);
set(Htext,'fontsize',14,'color','r')

tm1 = 'v_{avg}  =  '; tm2 = num2str(v_avg,'%4.0f '); tm3 = '  m.s^{-1}';
tm = [tm1 tm2 tm3];
Htext = text(1300,0.84 *Ylim,tm);
set(Htext,'fontsize',14,'color','m')

tm1 = 'v_{rms}  =  '; tm2 = num2str(v_rms,'%4.0f '); tm3 = '  m.s^{-1}';
tm = [tm1 tm2 tm3];
Htext = text(100,0.73 *Ylim,tm);
set(Htext,'fontsize',14,'color','k')

tm1 = 'KE_{avg}  =  '; tm2 = num2str(KE_avg,'%2.2e '); tm3 = '  J';
tm = [tm1 tm2 tm3];
Htext = text(1300,0.73 *Ylim,tm);
set(Htext,'fontsize',14,'color','m')

tm1 = 'Probability(v_1 < v < v_2)  =  '; tm2 = num2str(prob12,'%2.2f '); tm3 = '  ';
tm = [tm1 tm2 tm3];
Htext = text(1100,0.62 *Ylim,tm);
set(Htext,'fontsize',14,'color','b')
```



DOING PHYSICS WITH MATLAB

MATLAB DOWNLOAD DIRECTORY

Need to download the function **simpson1d.m** as well as the script **tp_LEMaxwell.mlx**

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