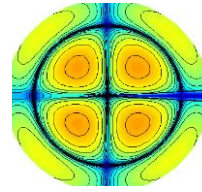


DOING PHYSICS WITH MATLAB

THERMAL PHYSICS

BLACKBODY RADIATION



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MATLAB SCRIPTS (download files)

The Matlab scripts are used to create colour spectrums of the radiation emitted from hot objects, to model the Sun as a blackbody and to model the radiation emitted from a tungsten filament.

- | | |
|---------------------|---|
| sun.m | m-script to model the radiation emitted from the Sun |
| filament.m | m-script to model the radiation emitted by a hot tungsten filament |
| black_temp.m | m-script to compare the radiation emitted from a hot object at four temperatures |
| simpson1d.m | function to evaluate the area under a curve using Simpson's 1/3 rule |
| Colourcode.m | function to return the appropriate colour for a wavelength in the visible range from 380 nm to 780 nm |

THERMAL RADIATION AND BLACKBODIES

PARTICLE NATURE OF ELECTROMAGNETIC RADIATION

The wave nature of electromagnetic radiation is demonstrated by interference phenomena. However, electromagnetic radiation also has a particle nature. For example, to account for the observations of the radiation emitted from hot objects, it is necessary to use a particle model, where the radiation is considered to be a stream of particles called *photons*. The energy of a photon, E is

$$E = hf \tag{1}$$

The electromagnetic energy emitted from an object's surface is called **thermal radiation** and is due a decrease in the internal energy of the object. This radiation consists of a continuous spectrum of frequencies extending over a wide range. Objects at room temperature emit mainly infrared and it is not until the temperature reaches about 800 K and above that objects glows visibly.

A **blackbody** is an object that completely absorbs all electromagnetic radiation falling on its surface at any temperature. It can be thought of as a perfect absorber and emitter of radiation. The power emitted from a blackbody, P is given by the **Stefan-Boltzmann law** and it depends only on the surface area of the emitter, A and its surface temperature, T

$$P = A \sigma T^4 \quad (2)$$

A more general form of equation (2) is

$$P = \varepsilon A \sigma T^4 \quad (3)$$

where ε is the **emissivity** of the object. For a blackbody, $\varepsilon = 1$. When $\varepsilon < 1$ the object is called a **graybody** and the object is not a perfect emitter and absorber.

The amount of radiation emitted by a blackbody is given by **Planck's radiation law** and is expressed in terms of the **spectral intensity (radiant emittance)** R_λ or R_f

$$R_\lambda = \frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp\left(\frac{h c}{k_B T \lambda}\right) - 1} \quad [\text{W.m}^{-2}.\text{m}^{-1}] \quad (4)$$

or

$$R_f = \frac{2\pi h f^3}{c^2} \frac{1}{\exp\left(\frac{h f}{k_B T}\right) - 1} \quad [\text{W.m}^{-2}.\text{s}^{-1}] \quad (5)$$

The power radiated per unit surface of a blackbody, P_A within a wavelength interval or bandwidth, (λ_1, λ_2) or frequency interval or bandwidth (f_1, f_2) are given by equations (6) and (7)

$$P_A = \int_{\lambda_1}^{\lambda_2} R_\lambda d\lambda = \int_{\lambda_1}^{\lambda_2} \left(\frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp\left(\frac{h c}{k_b T}\right) - 1} \right) d\lambda \quad [\text{W.m}^{-2}] \quad (6)$$

and

$$P_A = \int_{f_1}^{f_2} R_f df = \int_{f_1}^{f_2} \left(\frac{2\pi h f^3}{c^2} \frac{1}{\exp\left(\frac{hf}{k_b T}\right) - 1} \right) df \quad [\text{W.m}^{-2}] \quad (7)$$

The equations (6) and (7) give the Stefan-Boltzmann law, equation (2) when the bandwidths extend from 0 to ∞ .

Wien's displacement law states that the wavelength, λ_{peak} corresponding to the peak of the spectral intensity given by equation (4) is inversely proportional to the temperature of the blackbody and the frequency, f_{peak} for the spectral intensity given by equation (5) is proportional to the temperature

$$\lambda_{peak} = \frac{b_\lambda}{T} \quad f_{peak} = b_f T \quad (8)$$

The peaks in equations (4) and (5) occur in different parts of the electromagnetic spectrum and so

$$f_{peak} \neq \frac{c}{\lambda_{peak}} \quad (9)$$

The Wien's displacement law explains why long-wave radiation dominates more and more in the spectrum of the radiation emitted by an object as its temperature is lowered.

When classical theories were used to derive an expression for the spectral densities R_λ and R_f , the power emitted by a blackbody diverged to infinity as the wavelength became shorter and shorter. This is known as the **ultraviolet catastrophe**. In 1901 Max Planck proposed a new radical idea that was completely alien to classical notions, electromagnetic energy is **quantized**. Planck was able to derive the equations (6) and (7) for blackbody emission and these equations are in complete agreement with experimental measurements. The assumption that the energy of a system can vary in a continuous manner, i.e., it can take any arbitrary close consecutive values fails. Energy can only exist in integer multiples of the lowest amount or quantum, hf . ***This step marked the very beginning of modern quantum theory.***

Simulation The Sun and the Earth as Blackbodies

Inspect and *run* the m-script **sun.m** so that you are familiar with what the program and the code does. The m-script calls the functions **simpson1d.m** and **Colorcode.m**.

The Sun can be considered as a blackbody, and the total power output of the Sun can be estimated by using the Stefan-Boltzmann law, equation (2), and by finding the area under the curves for R_λ and R_f using equations (6) and (7). From observations on the Sun, the peak in the electromagnetic radiation emitted has a wavelength, $\lambda_{peak} = 502.25$ nm (yellow). The temperature of the Sun's surface (photosphere) can be estimated from the Wien displacement law, equation (8).

The distance from the Sun to the Earth, R_{SE} can be used to estimate of the surface temperature of the Earth, T_E if there was no atmosphere. The intensity of the Sun's radiation reaching the top of the atmosphere, I_0 is known as the *solar constant*

$$I_0 = \frac{P_s}{4\pi R_{SE}^2} \quad (10)$$

The power absorbed by the Earth, P_{Eabs} is

$$P_{Eabs} = (1 - \alpha) \pi R_E^2 I_0 \quad (11)$$

where α is the albedo (the reflectivity of the Earth's surface). Assuming the Earth behaves as a blackbody then the power of the radiation emitted from the Earth, P_{Erad} is

$$P_{Erad} = 4\pi R_E^2 \sigma T_E^4 \quad (12)$$

It is known that the Earth's surface temperature has remained relatively constant over many centuries, so that the power absorbed and the power emitted are equal, so the Earth's equilibrium temperature is

$$T_E = \left(\frac{(1 - \alpha) I_0}{4\sigma} \right)^{0.25} \quad (13)$$

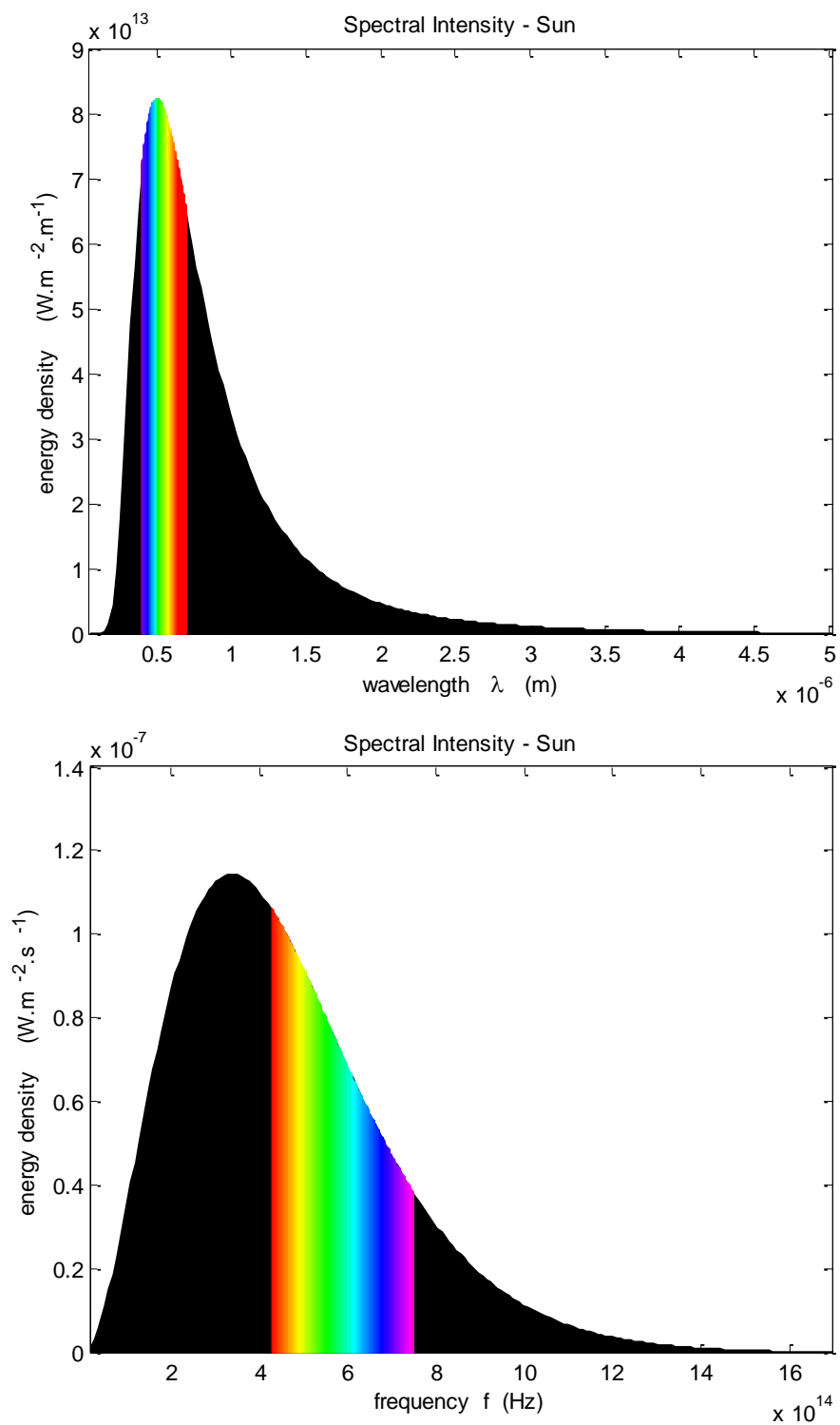
Table 1 is a summary of the physical quantities, units and values of constants used in the description of the radiation from a hot object.

Variable	Interpretation	Value	Unit
E	energy of photon		J
h	Planck's constant	6.62608×10^{-34}	J.s
c	speed of electromagnetic radiation	2.99792458×10^8	m.s^{-1}
f	frequency of electromagnetic radiation		Hz
λ	wavelength of electromagnetic radiation		
T	surface temperature of object		K
A	surface area of object		m^2
σ	Stefan-boltzmann constant	5.6696×10^{-8}	$\text{W.m}^{-2}.\text{K}^{-4}$
P	power emitted from hot object		W
ε	emissivity of object's surface		
R_λ	radiant emittance (spectral intensity): power radiated per unit area per unit wavelength interval		$(\text{W.m}^{-2}).\text{m}^{-1}$
R_f	radiant emittance (spectral intensity): power radiated per unit area per unit frequency interval		$(\text{W.m}^{-2}).\text{s}^{-1}$
k_B	Boltzmann constant	1.38066×10^{-23}	J.K^{-1}
b_λ	Wien constant: wavelength	2.898×10^{-3}	m.K
b_f	Wien constant: frequency	$2.83 \text{ k}_B T / h$	$\text{K}^{-1}.\text{s}^{-1}$
λ_{peak}	wavelength of peak in solar spectrum	5.0225×10^{-7}	m
R_S	radius of the Sun	6.96×10^8	m
R_E	radius of the Earth	6.96×10^6	m
R_{SE}	Sun-Earth radius	6.96×10^{11}	m
I_0	Solar constant	1.36×10^3	W.m^{-2}
α	Albedo of Earth's surface	0.30	

Table 1. Summary of the physical quantities used in describing the radiation from hot objects.

Sample results using sun.m

Plots of the spectral intensity curves



Matlab screen output for sun.m

Sun: temperature of photosphere, $T_S = 5770 \text{ K}$

Peak in Solar Spectrum

Theory: Wavelength at peak in spectral intensity, $wL = 5.02e-007 \text{ m}$

Graph: Wavelength at peak in spectral intensity, $wL = 4.98e-007 \text{ m}$

Corresponding frequency, $f = 6.02e+014 \text{ Hz}$

Theory: Frequency at peak in spectral intensity, $f = 3.39e+014 \text{ Hz}$

Graph: Frequency at peak in spectral intensity, $f = 3.36e+014 \text{ Hz}$

Corresponding wavelength, $wL = 8.93e-007 \text{ m}$

Total Solar Power Output

$P_{\text{Stefan_Boltzmann}} = 3.79e+026 \text{ W}$

$P(wL)_{\text{total}} = 3.77e+026 \text{ W}$

$P(f)_{\text{total}} = 3.79e+026 \text{ W}$

IR visible UV

$P_{\text{IR}} = 1.92e+026 \text{ W}$

Percentage IR radiation = 51.0

$P_{\text{visible}} = 1.39e+026 \text{ W}$

Percentage visible radiation = 36.8

$P_{\text{UV}} = 4.61e+025 \text{ W}$

Percentage UV radiation = 12.2

Sun - Earth

Theory: Solar constant $I_O = 1.360e+003 \text{ W/m}^2$

Computed: Solar constant $I_E = 1.342e+003 \text{ W/m}^2$

Surface temperature of the Earth, $T_E = 254 \text{ K}$

Surface temperature of the Earth, $T_E = -19 \text{ deg C}$

Questions

- 1 How do the peaks in the plots R_λ and R_f compare with the predictions of the Wien displacement law and $\lambda_{\text{peak}} = 502.25 \text{ nm}$ (yellow).
- 2 Compare the total solar power emitted by the Sun calculated from the Stefan-Boltzmann law and by the numerical integration to find the area under the spectral intensity (R_λ and R_f) curves.
- 3 Compare the percentage the radiation in the ultraviolet, visible and infrared parts of the solar spectrum.
- 4 How does the computed value of the intensity of the radiation reaching the Earth's surface, I_E compare with the solar constant, I_0 ?
- 5 From our simple model, the surface temperature of the Earth was estimated to be -19°C . Is this sensible? What is the surface temperature on the moon? The average the temperature of the Earth is much higher than this, about $+15^\circ\text{C}$. Explain the difference.
- 6 What changes occur in the calculations if the Sun was hotter (peak in the blue part of the spectrum) or cooler (peak in the red) part of the spectrum?
- 7 What would be wavelength λ_{peak} and the temperature of the Sun's surface if the Earth's equilibrium temperature was -15°C instead -19°C ? (In the m-script, increase the value of λ_{peak} until you reach the required equilibrium temperature of the Earth.)

M-script highlights

- 1 Suitable values for the wavelength and frequency integration limits for equations (6) and (7) are determined so that the spectral intensities at the limits are small compared to the peak values.

- 2 The Matlab function `area` is used to plot the spectral intensity curves, for example, in plotting the R_λ curve:

```
h_area1 = area(wL,R_wL);  
set(h_area1,'FaceColor',[0 0 0]);  
set(h_area1,'EdgeColor','none');
```

- 3 The color for the shading of the curve matches that of the wavelength in the visible part of the spectrum. A call is made to the function `ColorCode.m` to assign a color for a given wavelength band. For the shading of the R_λ curve:

```
thisColorMap = hsv(128);  
for cn = 1 : num_wL-1  
    thisColor = ColorCode(wL_vis(cn));  
    h_area = area(wL_vis(cn:cn+1),R_wL_vis(cn:cn+1));  
    set(h_area,'FaceColor',thisColor);  
    set(h_area,'EdgeColor',thisColor);  
end
```

- 4 Simpson's 1/3 rule is used for the numerical integration (`simpson1d.m`) to find the area under the spectral intensity curves. For the R_λ curve, the total power radiated by the Sun:

```
P_total = A_sun * simpson1d(R_wL,wL1,wL2);
```

- 5 The peaks in spectral intensities are calculated using Matlab logical functions:

```
wL_peak_graph = wL(R_wL == max(R_wL));  
f_peak_graph = f(R_f == max(R_f));
```

Simulation

How efficient is a hot tungsten filament?

Inspect and *run* the m-script **filament.m** so that you are familiar with what the program and the code does. The m-script calls the functions **simpson1d.m** and **Colorcode.m**.

Blackbodies do not exist in nature. However, simple models are often used that assume an object such as the Sun or an incandescent lamp behave as a blackbody.

Some car headlights use a hot tungsten filament to emit electromagnetic radiation. We can estimate the percentage of this radiation in the visible part of the electromagnetic spectrum for a hot tungsten filament that has a surface temperature of 2400 K and an electrical power of 55 W (the thermal power radiated is also 55 W). The first step is to calculate the thermal power radiated by a hot object using equation (14)

$$P_\lambda = A R_\lambda = \frac{N}{\lambda^5 \left(\exp\left(\frac{hc}{k_B T \lambda}\right) - 1 \right)} \quad [\text{W.m}^{-1}] \quad (14)$$

where N is a normalizing constant and includes a factor for the surface area. Its value is initially set to $N = 1$. The second step is to numerically integrate P_λ given by equation (14) with the limits λ_1 and λ_2 so that $P_{\lambda 1} \rightarrow 0$ and $P_{\lambda 2} \rightarrow 0$.

$$P = N \int_{\lambda_1}^{\lambda_2} \frac{1}{\lambda^5 \left(\exp\left(\frac{hc}{k_B T \lambda}\right) - 1 \right)} d\lambda \quad [\text{W}] \quad (15)$$

Determine new values of N until the thermal power radiated is, $P = 55$ W, i.e., the area under the curve for P_λ is 55 W. The area under the P_λ curve is shaded yellow to show the visible part of the spectrum. Lastly, the function P_λ is numerical integrated for the limits corresponding to only the visible part of the electromagnetic spectrum

$$\lambda_1 = 700 \text{ nm (red)} \quad \text{and} \quad \lambda_2 = 400 \text{ nm (blue)}$$

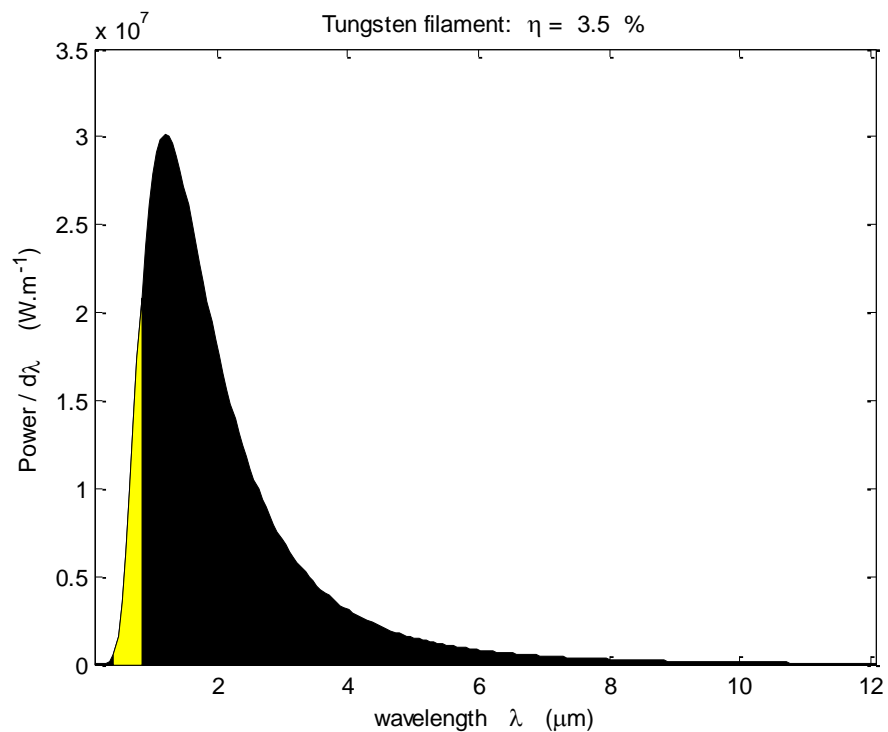
This gives only the total power radiated in the visible part of the electromagnetic spectrum, P_{visible} .

The filament efficiency, η given as percentage of visible radiation emitted by the hot tungsten filament to the power consumed by the filament

$$\eta = \left(\frac{P_{\text{visible}}}{P} \right) \left(\frac{100}{1} \right). \quad (16)$$

Sample Results for filament.m

Plot of the spectral intensity curve



Matlab screen output

wavelength at peak = 1.21e-006 m
wavelength at peak = 1.21 μm

P_total = 55.0 W

P_visible = 1.9 W

efficiency (percentage) = 3.5

Check normalization

P_check = 55.0 W

Questions

- 1 Are you surprised by the efficiency of the tungsten filament used in a light globe?
- 2 What part of the electromagnetic spectrum does the peak in the spectral intensity curve occur?
- 3 Most of the energy emitted from the light globe is not emitted in the visible part of the electromagnetic spectrum. What happens to most of the electrical energy supplied to the light globe?
- 4 What temperature would the filament have to be at so that the peak is in the visible part of the spectrum? Is this possible?
- 5 What is the minimum temperature of the filament so that the globe just starts to glow?
- 6 How do the results change if the power emitted by the hot tungsten filament was 75 W?

Simulation

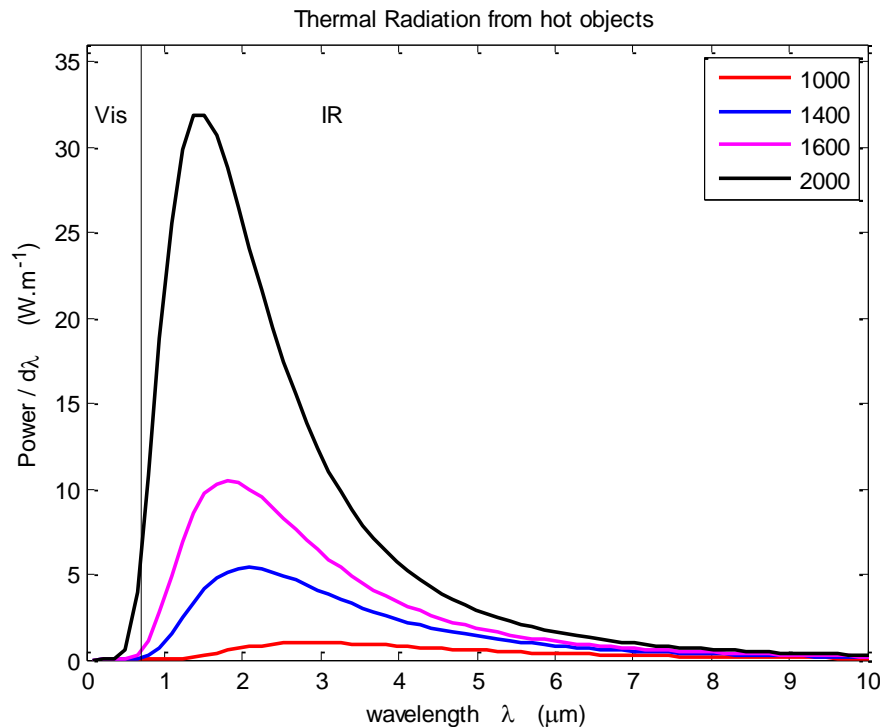
Thermal radiation emitted from hot objects?

Inspect and *run* the m-script **black_temp.m** so that you are familiar with what the program and the code does. The m-script calls the function **simpson1d.m**.

The thermal radiation emitted by a blackbody at four different temperatures is modeled. The spectral intensity curves for each temperature are plotted. The results of the modeling confirm the prediction of the Wien displacement law, equation (8) and the Stefan-Boltzmann Law, equation (2). Notice that at temperatures as high as 2000 K, only a small amount of radiation is emitted in the visible part of the electromagnetic spectrum.

Sample results using black_temp.m

Plot of the spectral intensity curves



Matlab screen output

Temperatures

1000 K 1400 K 1600 K 2000 K

Relative Peak wavelengths

1.0 0.7 0.6 0.5

Relative total power radiated (area under curves)

1.0 3.9 6.6 16.1