DOING PHYSICS WITH MATLAB

COMPUTATIONAL OPTICS

BESSEL FUNCTION OF THE FIRST KIND
Fraunhofer diffraction – circular aperture

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op_bessel1.m

- Bessel function of the first kind – calls Matlab function besselj

```
J1 = besselj(1,v);
```

- Fraunhofer diffraction pattern for a uniformly illuminated circular aperture – linear and log scale plots

```
IRR = (J1 ./ v).^2;
```

Calls the function **turningPoint.m** to find the zeros, minima and maxima of a function

```
[indexMin indexMax] = turningPoints(xData, yData);
```

BESSEL FUNCTION OF THE FIRST KIND J_1

The Bessel function of the first kind J_1 oscillates somewhat like the sine function as shown in figure 1. One difference is that the oscillations attenuate as its argument increases.

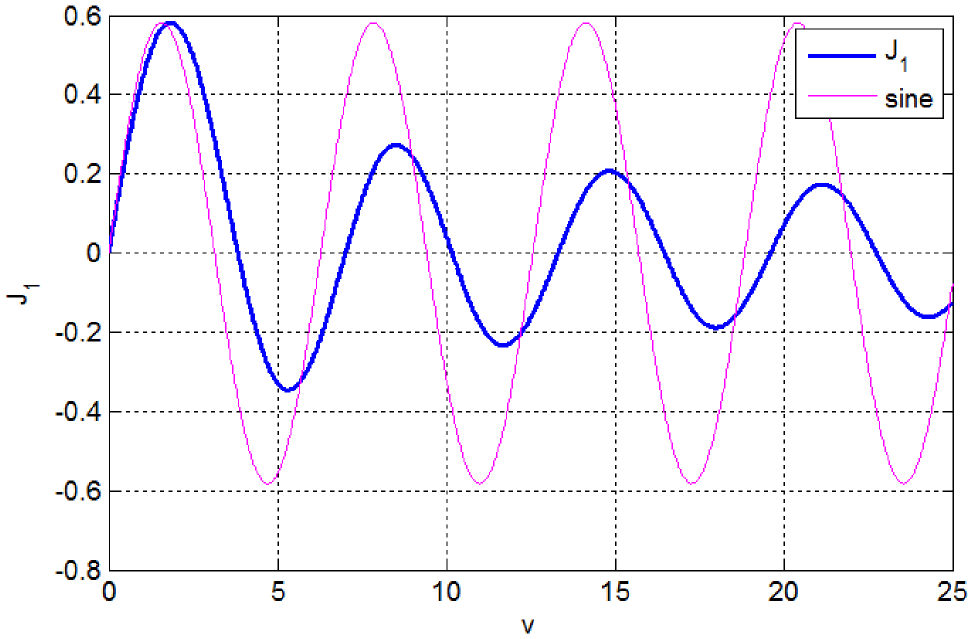


Fig.1. The Bessel function $J_1(v)$ and the sine function $\sin(v)$.

The mscript **turningPoint. m** is called to estimate the argument v for the minimum, maximum and the zero crossings of the Bessel function. The values are displayed in the Command Window:

J1 BESSEL FUNCTION OF THE FIRST KIND
MINs: Radial coordinate / J1 value

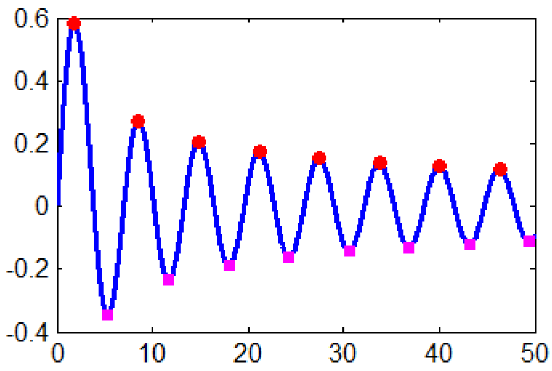
5.332	-0.346
11.706	-0.233
18.015	-0.188
24.311	-0.162
30.602	-0.144
36.890	-0.131
43.177	-0.121
49.462	-0.113

MAXs: Radial coordinate / J1 value

1.841	0.582
8.536	0.273
14.864	0.207
21.164	0.173
27.457	0.152
33.746	0.137
40.033	0.126
46.319	0.117

ZEROs: Radial coordinate   J1 = 0

3.832
7.016
10.174
13.324
16.471
19.616
22.760
25.904
29.047
32.190
35.332
38.475
41.617
44.759
47.901



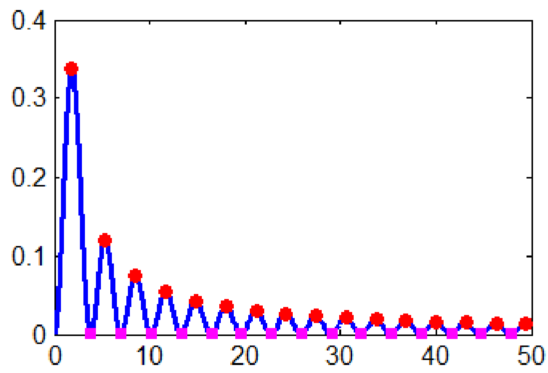


Fig. 2. The max, min and zero crossing for J_1. The plot on the left is for J_1 and the plot on the right is for J_1^2. **turningPoints.m**

FRAUNHOFER DIFFRACTION:
UNIFORMLY ILLUMINATED CIRCULAR APERTURE

Diffraction in its simplest description is any deviation from geometrical optics (light travels in straight lines) that result from an obstruction of a wavefront of light. A hole in an opaque screen represents an obstruction. On an observation screen placed after the hole, the pattern of light may show a set of bright and dark fringes around a central bright spot.

Fraunhofer diffraction occurs when both the incident and diffracted waves are effectively plane. This occurs when the distance from the source to the aperture is large so that the aperture is assumed to be uniformly illuminated and the distance from the aperture plane to the observation plane is also large. This means that the curvatures of the incident wave and diffracted waves can be neglected.

The **Fraunhofer diffraction pattern for a uniformly illuminated circular aperture** is described by the **Bessel function of the first kind J_1**. The geometry for the diffraction pattern from a circular aperture is shown in figure (3).

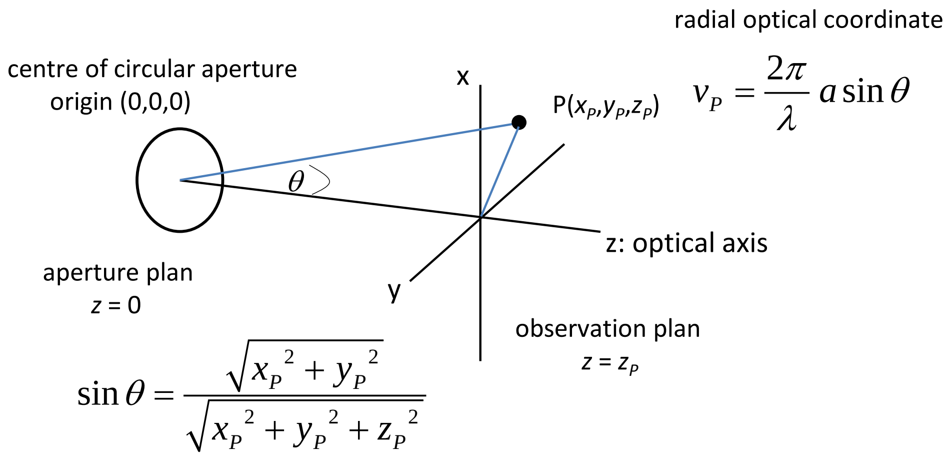


Fig. 3. Circular aperture geometry.

Irradiance *I* is the power of electromagnetic radiation per unit area (radiative flux) incident on a surface and its S.I. unit is watts per square meter [W.m^{-2}]. A more general term for irradiance that you can use is the term **intensity**.

The irradiance I_0 of a monochromatic light plane-wave in matter is given in terms of its electric field E by

(1) $$I = I_0 |E|^2$$

where E is the complex amplitude of the wave's electric field and I_0 is a normalizing constant.

The irradiance I in a plane parallel to the plane of the aperture is given by

(2) $$I = I_o \left(\frac{J_1(v_P)}{v_P} \right)^2$$ **Fraunhofer diffraction**

where v_P is a radial optical coordinate

(3) $$v_P = \frac{2\pi}{\lambda} a \sin\theta \qquad \sin\theta = \frac{\sqrt{x_P^{\,2} + y_P^{\,2}}}{\sqrt{x_P^{\,2} + y_P^{\,2} + z_P^{\,2}}}$$

The radial coordinate v_P is a scaled perpendicular distance from the optical axis. Figure (4) shows the irradiance as a function of the radial coordinate v_P. In the upper plot the irradiance is normalized to 1. The lower figure shows the irradiance as a decibel scale $I_{dB} = 10\log_{10}(I)$. The diffraction pattern is characterized by a strong central maximum and very weak peaks of decreasing magnitude.

The function **turningPoint.m** is called within the mscript to find the radial coordinates for the zeros and maxima in the intensity distribution., the results are displayed in the Command Window:

IRR Fraunhofer Diffraction MAX ZEROS
MAXs: Radial coordinate / IRR value

5.136	0.01750
8.417	0.00416
11.620	0.00160
14.796	0.00078
17.960	0.00044
21.117	0.00027
24.270	0.00018
27.421	0.00012
30.569	0.00009
33.717	0.00007
36.863	0.00005
40.008	0.00004
43.153	0.00003
46.298	0.00003
49.442	0.00002

ZEROs: Radial coordinate IRR = 0

3.832
7.016
10.174
13.324
16.471
19.616
22.760
25.904
29.047
32.190
35.332
38.475
41.617
44.759
47.901

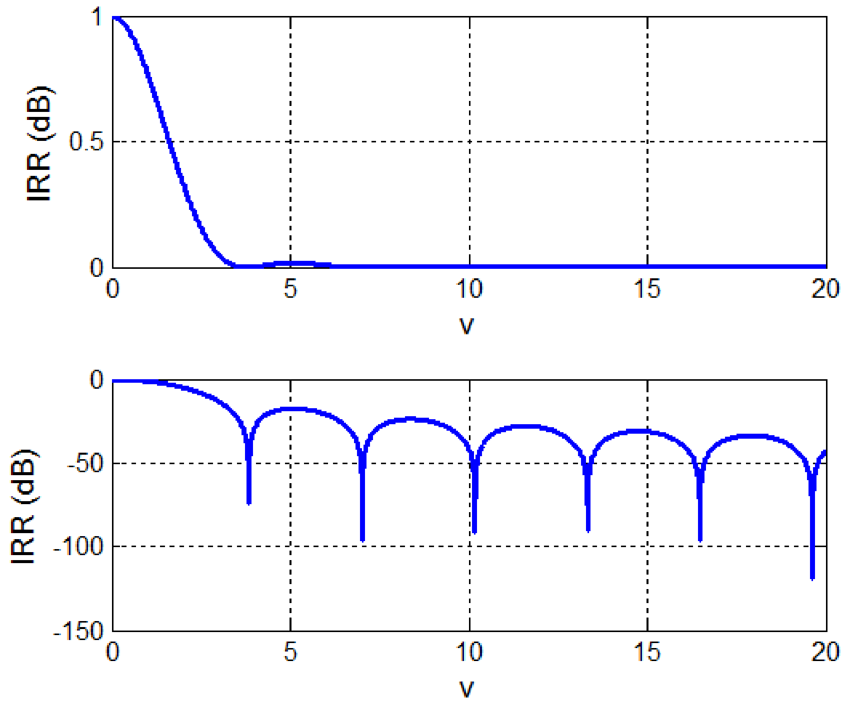


Fig. 4. Fraunhofer irradiance pattern for a circular aperture. The lower plot has a log scale for the irradiance $I_{dB} = 10\log_{10}(I)$.

The Fraunhofer diffraction pattern for the circular aperture is circularly symmetric and consists of a bright central circle surrounded by series of bright rings of rapidly decreasing strength between a series of dark rings. The bright and dark rings are not evenly spaced. The bright central region is known as the **Airy disk**. It extends to the first dark ring at v_P = 3.831 (the first zero of the Bessel function).

The spread of the Airy disk is determined by the radial coordinate v = 3.831. The radial coordinate for the first dark ring is

$$\text{First dark ring} \quad v_P = \frac{2\pi}{\lambda} a \sin\theta = 3.831 \quad \Rightarrow \quad \sin\theta = 0.61\frac{\lambda}{a}$$

1. The larger the wavelength λ the greater the width of the diffraction pattern on a detection screen $\sin\theta \propto \lambda$

2. The larger the radius a of the circular aperture, the narrower the diffraction pattern on a detection screen $\sin\theta \propto \dfrac{1}{a}$