CS 5003: Parameterized Algorithms

Lecture 18

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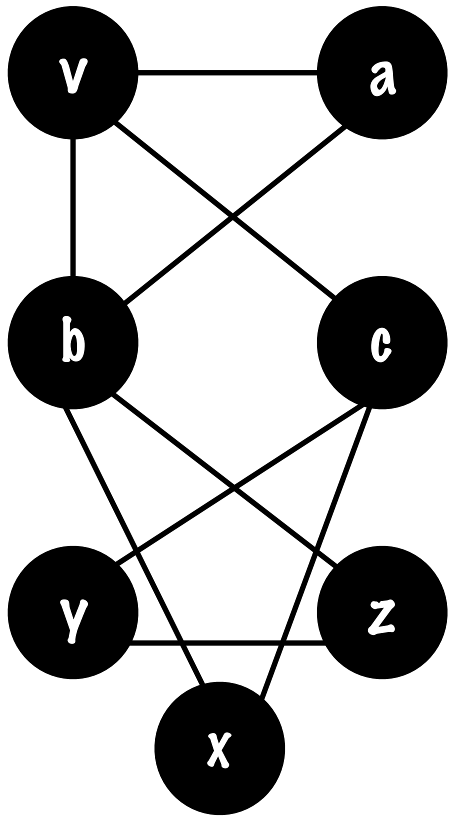
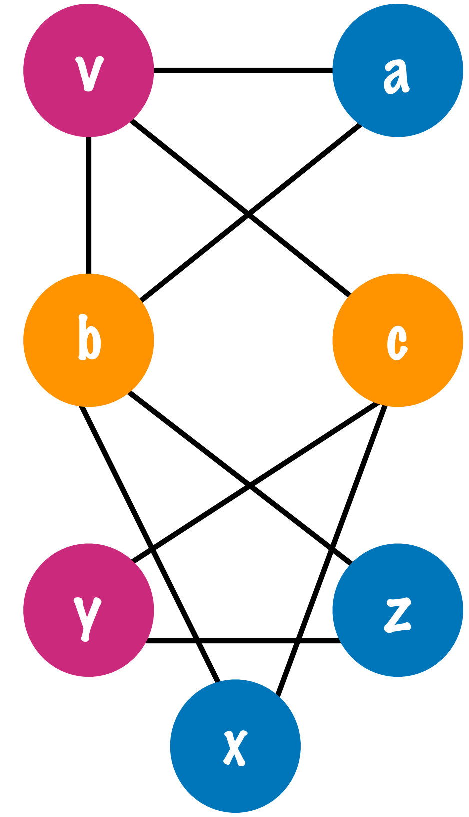
Reference Books: Parameterized Algorithms by Cygan et al. and Kernelization by Fomin et al.

Proper Vertex Coloring





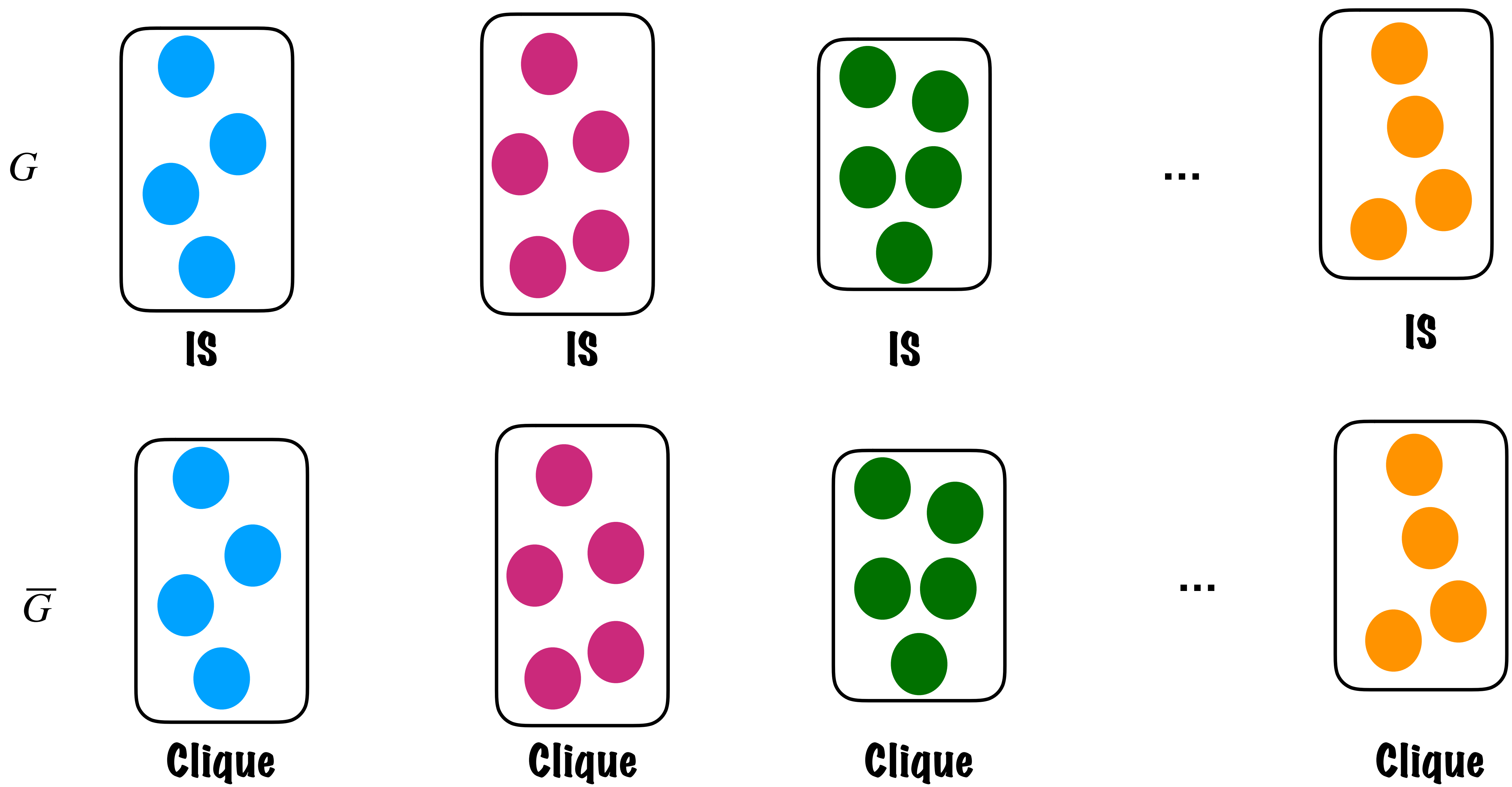
Coloring
<u>Instance:</u> A graph G on n vertices and integer k
<u>Question:</u> Does G have a proper colouring using k colors?
<u>Parameter:</u> k

* 2-coloring = Bipartite Checking
* 3-coloring is NP-hard
    * Not FPT w.r.t no. of colours as parameter

Proper Vertex Coloring

Dual of Coloring

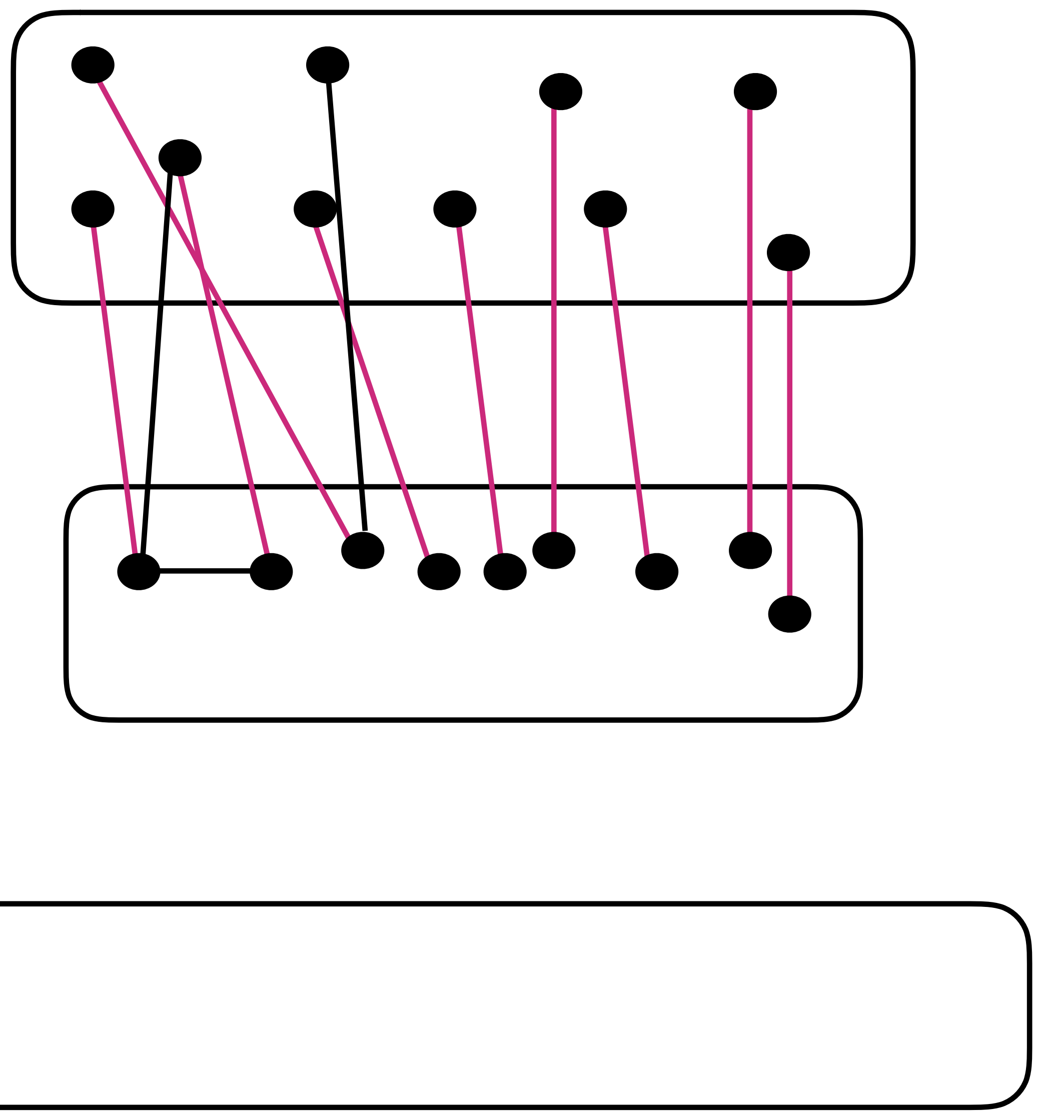
<u>Instance:</u> A graph G on n vertices and integer k
<u>Question:</u> Does G have a proper colouring using n-k colors?
<u>Parameter:</u> k



Crown Decomposition of $\overline{G}$

Crown C

Independent set



Head H

$N(C) \subseteq H$

Rest R

Matching saturating H

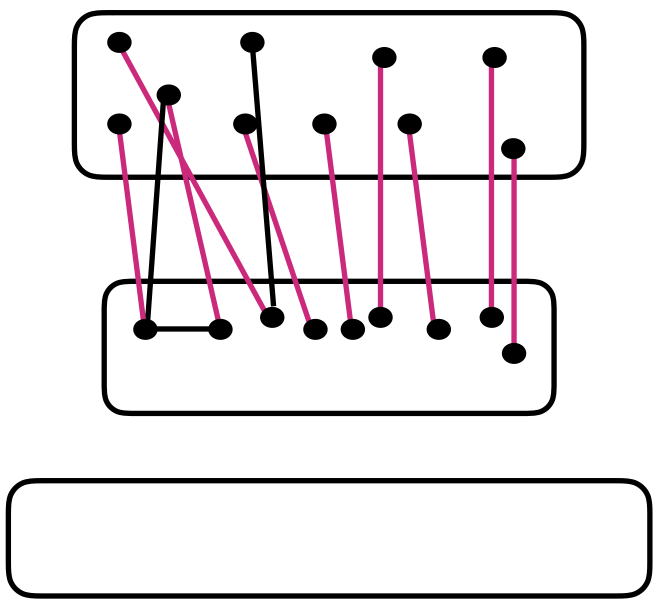
(G, k) is an yes-instance iff (G[R], k-|H|) is an yes-instance

G is (n-k)-colorable iff G[R] is (r-k+h)-colorable

Crown Decomposition of $\overline{G}$

Crown C

Clique in G



Head H

Every vertex in H has a non-neighbour in C

Rest R

Every vertex in C is adjacent to every vertex in R

- Suppose (G, k) is an yes-instance
- G is (n-k)-colorable
  - Every vertex in C has a distinct color
  - None of these c colors can be used for R
  - No. of colors used for R is n-k-c=n-(h+c)-(k-h)=r-(k-h)
- (G[R], k-h) is an yes-instance

Crown Decomposition of $\overline{G}$

Crown C

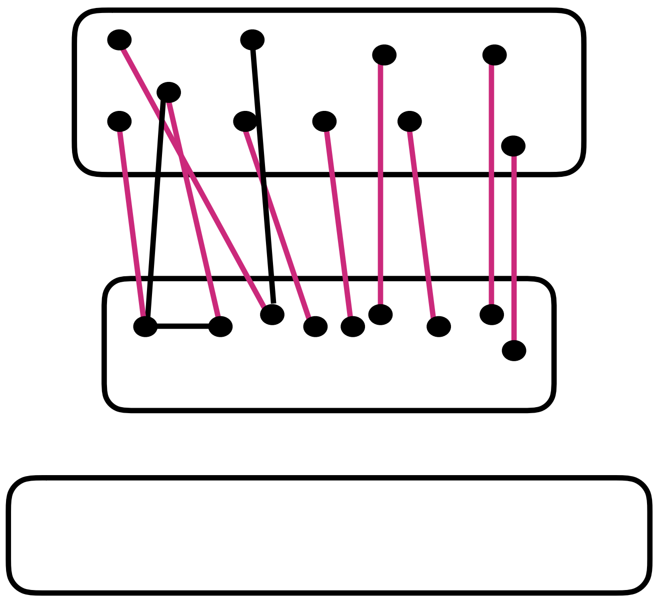
Clique in G

Head H

Every vertex in H has a non-neighbour in C

Rest R

Every vertex in C is adjacent to every vertex in R



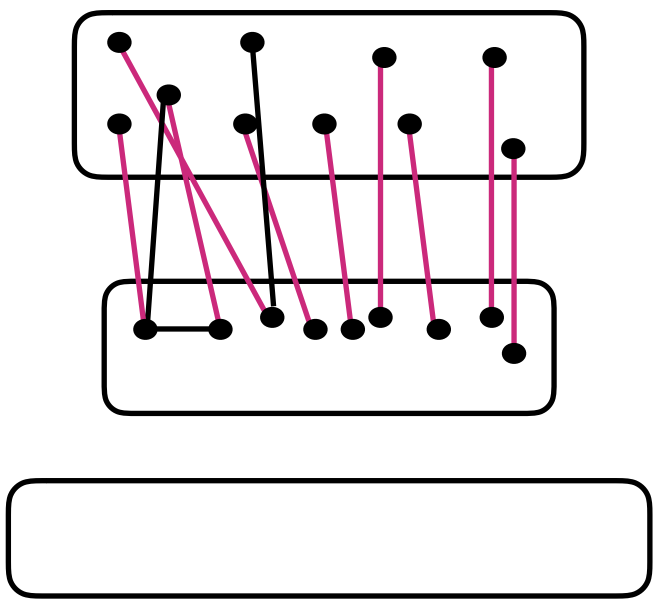
- Suppose (G[R], k-h) is an yes-instance
- G[R] is (r-k+h)-colorable
  - Use c new colors for C
  - Reuse these colors for H
  - No. of colors used for G is r-k+h+c = n-k
- (G, k) is an yes-instance

A Linear Kernel for Dual of Coloring

Crown C

Head H

Rest R



Clique in G

Every vertex in H has a non-neighbour in C

Every vertex in C is adjacent to every vertex in R

- Instance: (G, k)
- If there is a vertex v that is adjacent to all vertices, delete v As a result, \overline{G} will not ha
  - G is (n-k)-colorable iff (G-v) is (n-k-1)-colourable
- $\overline{G}$ has no isolated vertices. Apply Crown Lemma if no. of vertices > 3(k-1)
- If $\overline{G}$ has a matching of size k, then G is (n-k)-colorable Consider endpoints of matching in Set A such t
  - Endpoints of matching edges can be given same color in G
- Else, (C,H,R) is a crown in $\overline{G}$
  - Return (G[R],r-k+h)

Connected Vertex Cover

U is the set of vertices of degree atleast k + 1. Clearly they must be part of the solution. Let I denote the vertices in V(G) \ U that h

I

U

R

$I = \{v : N(v) \subseteq H\}$

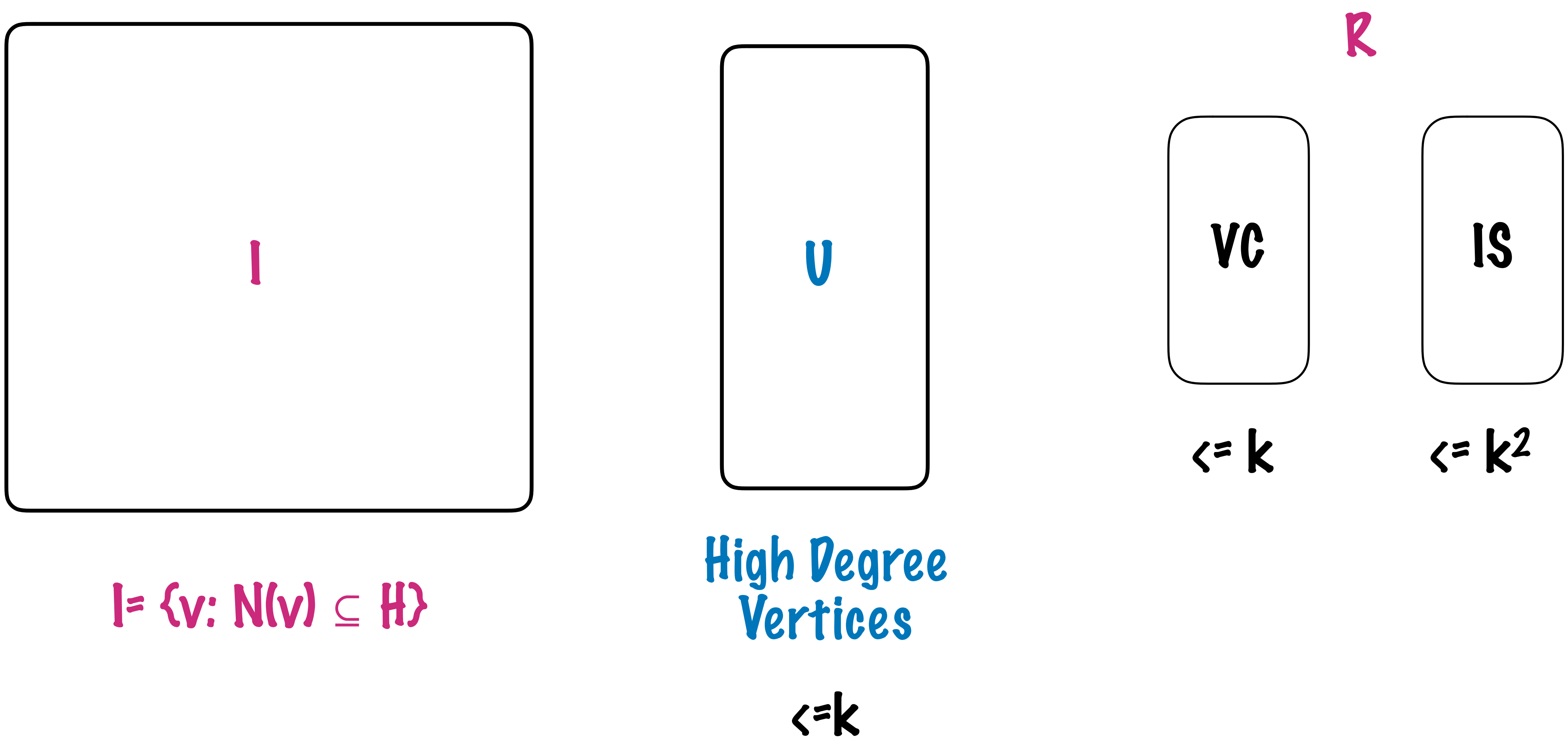
High Degree Vertices

$<= k$

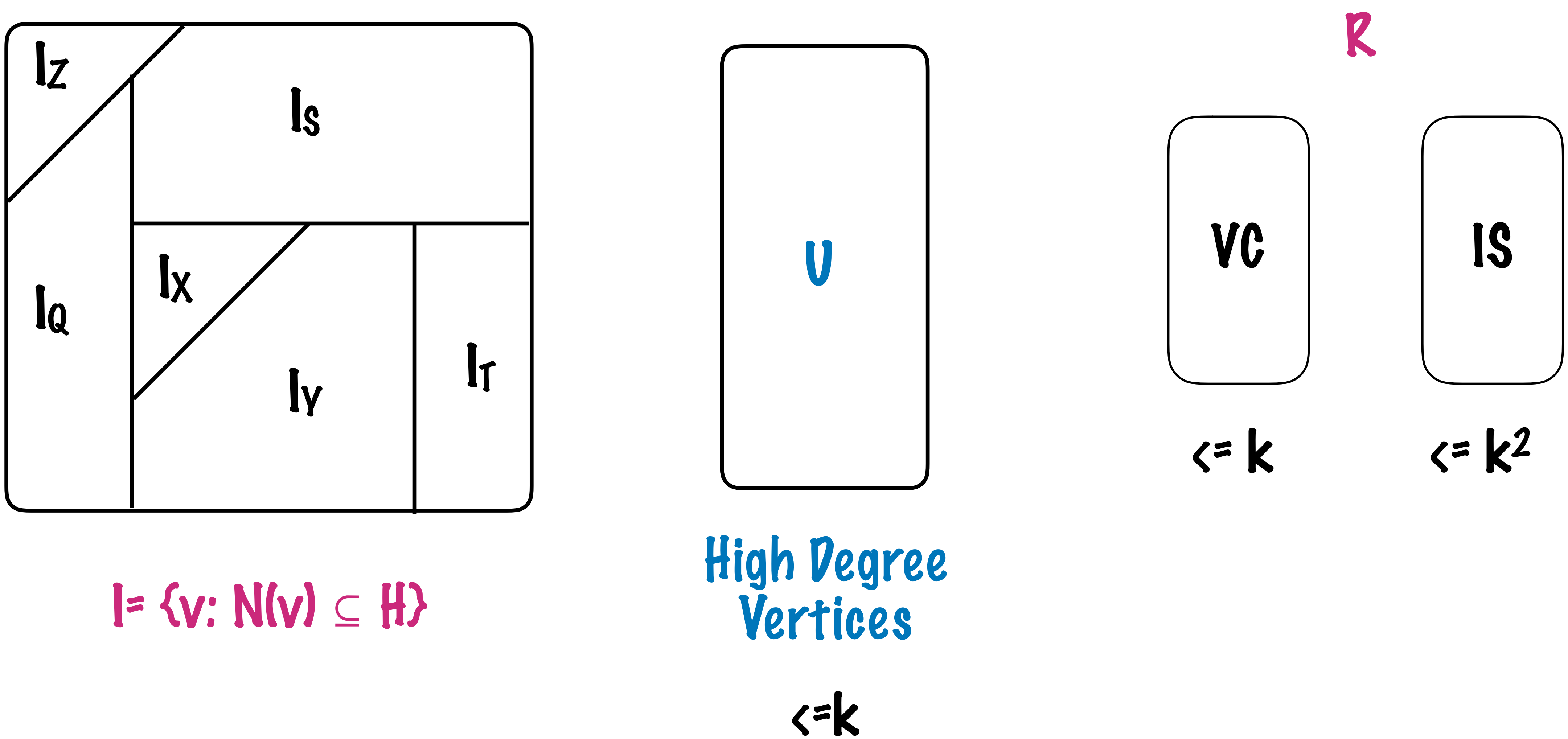
$\forall x \text{ in } R,\ N(x) \cap R \neq \emptyset$

$<= k + k^2$

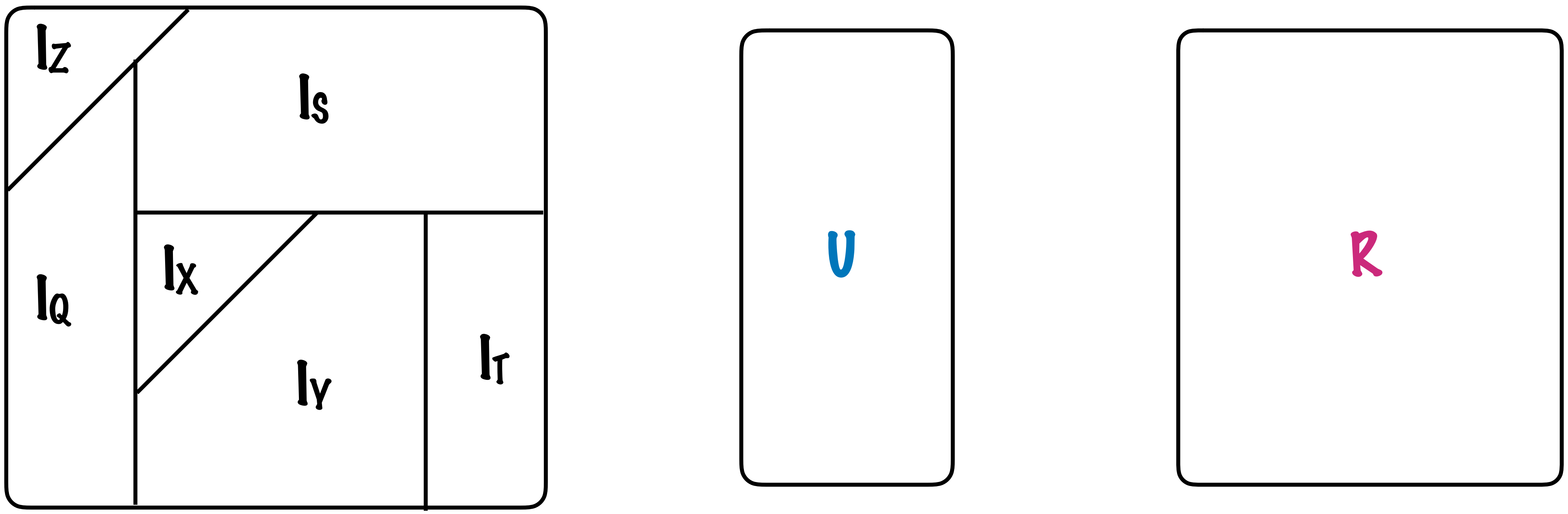
Connected Vertex Cover

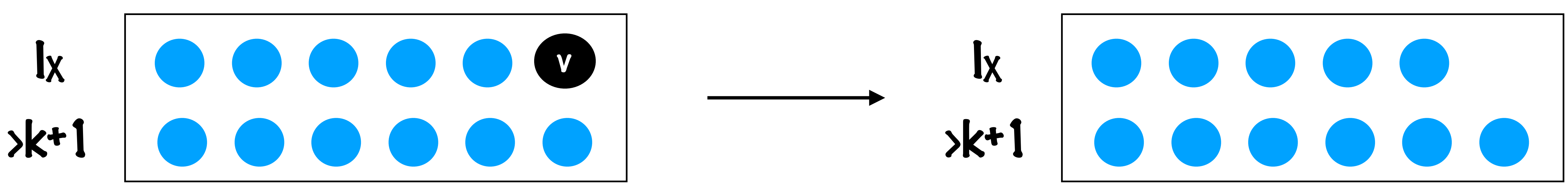


Connected Vertex Cover



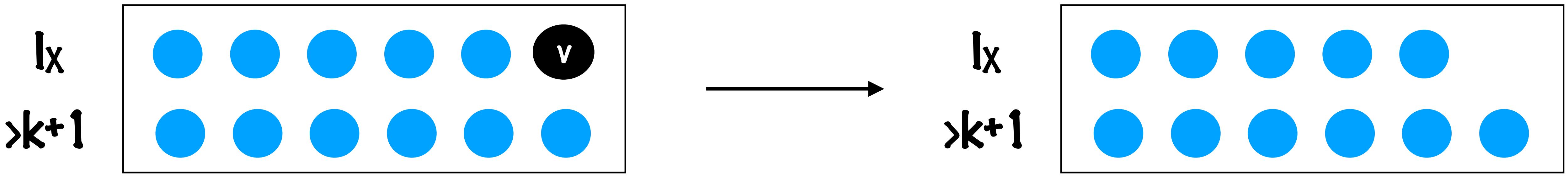
Connected Vertex Cover





(G, k) is an yes-instance iff (G-v, k) is an yes-instance

Connected Vertex Cover

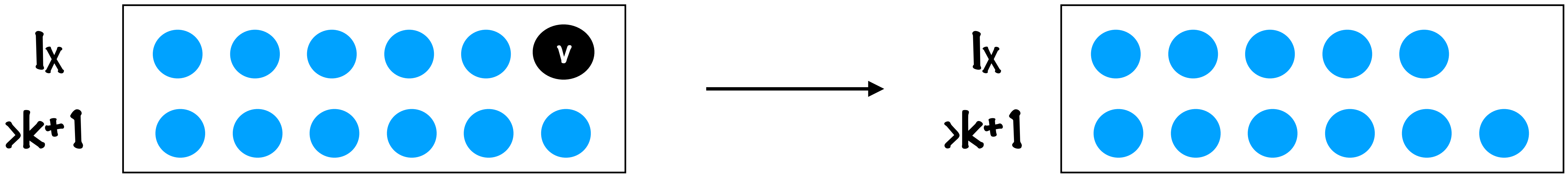


Suppose (G, k) is an yes-instance

- (G, k) is an yes-instance: S is a k size connected vertex cover
- $\exists$ a in I_X that is not in S => $X \subseteq S$
- If v is in S, then delete v from S and add a to S
  - S is a connected vertex cover of G-v
- If v is not in S, then S is a connected vertex cover of G-v

(G-v, k) is an yes-instance

Connected Vertex Cover

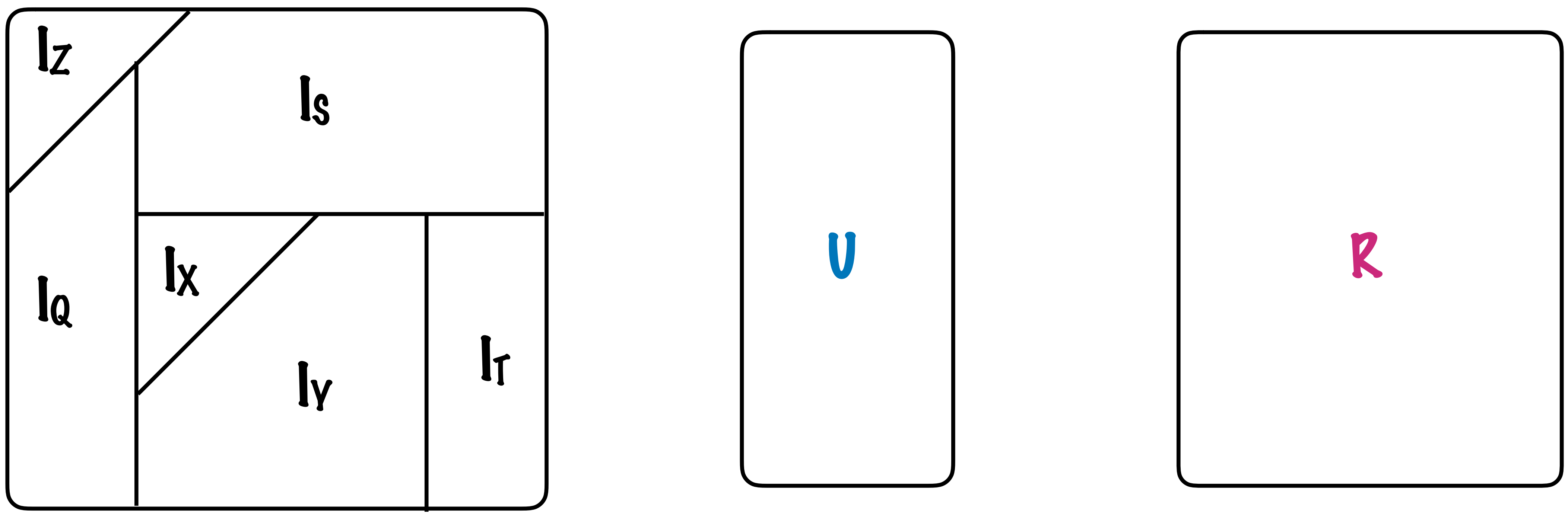


Suppose (G-v, k) is an yes-instance

* (G-v, k) is an yes-instance: S is a k size connected vertex cover
* ∃ a in I_X that is not in S => $X \subseteq S$
* S is a connected vertex cover of G

(G, k) is an yes-instance

Connected Vertex Cover



$\forall X \subseteq U, |I_X| <= k+1$

$|U| <= k$

$|R| <= k+k^2$

$|I| <= 2^{|U|} (k+1) <= 2^k (k+1)$

$O(k.2^k + k^2)$ kernel