CS 5003: Parameterized Algorithms

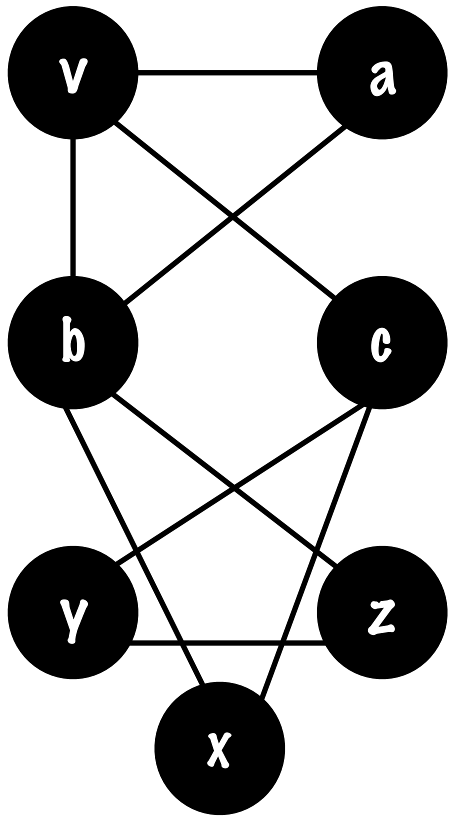
Lecture 18

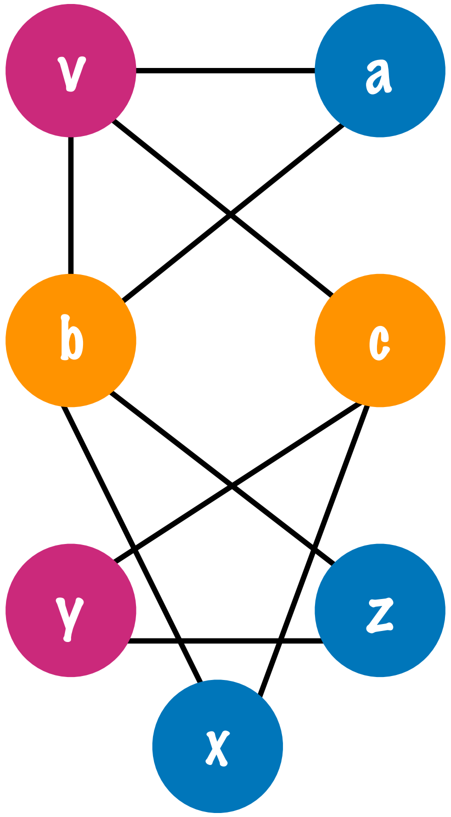
Krithika Ramaswamy

IIT Palakkad

Reference Books: Parameterized Algorithms by Cygan et al. and Kernelization by Fomin et al.

Proper Vertex Coloring





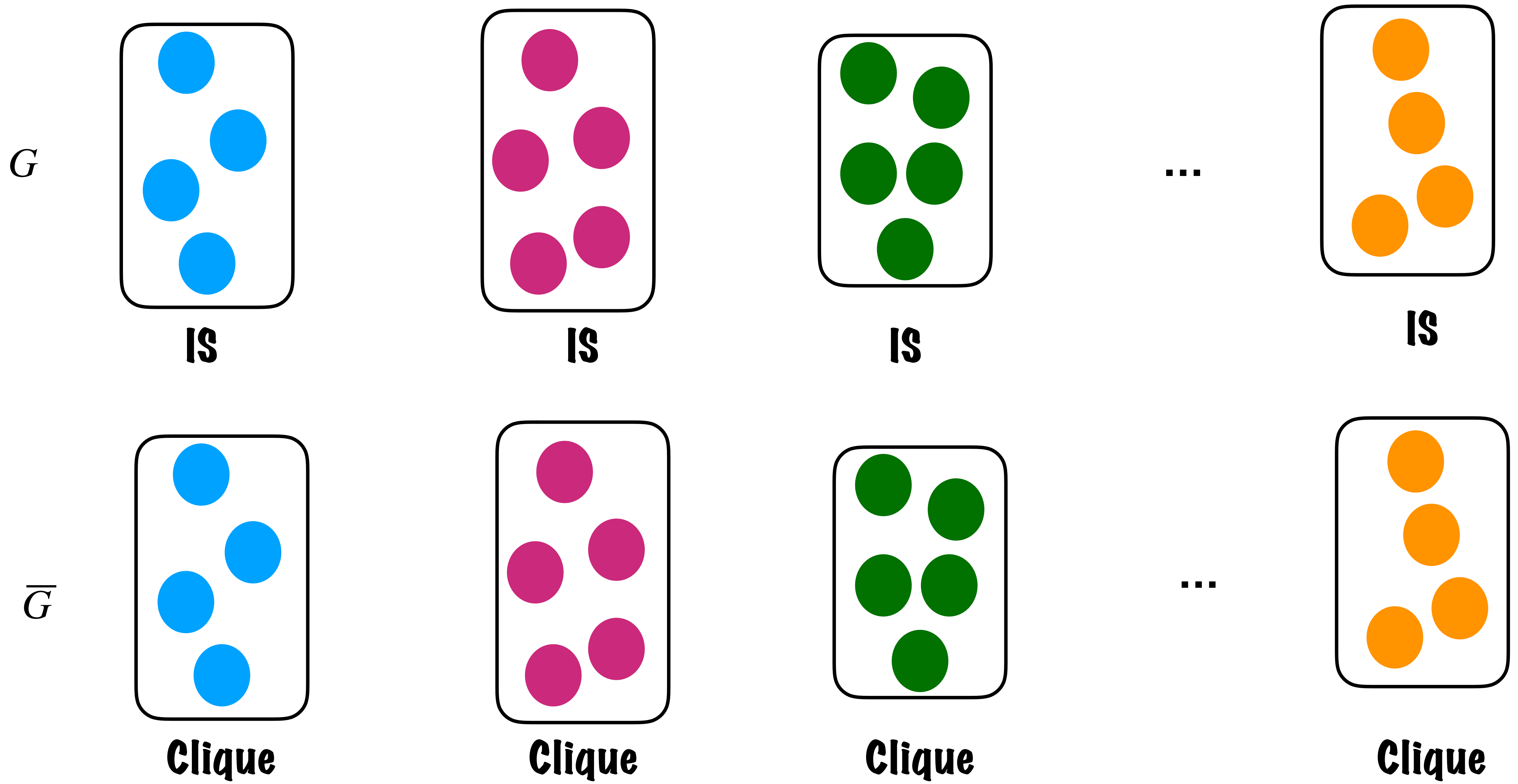
Coloring
Instance: A graph G on n vertices and integer k
Question: Does G have a proper colouring using k colors?
Parameter: k

* 2-coloring = Bipartite Checking
* 3-coloring is NP-hard
    * Not FPT w.r.t no. of colours as parameter

Proper Vertex Coloring

Dual of Coloring

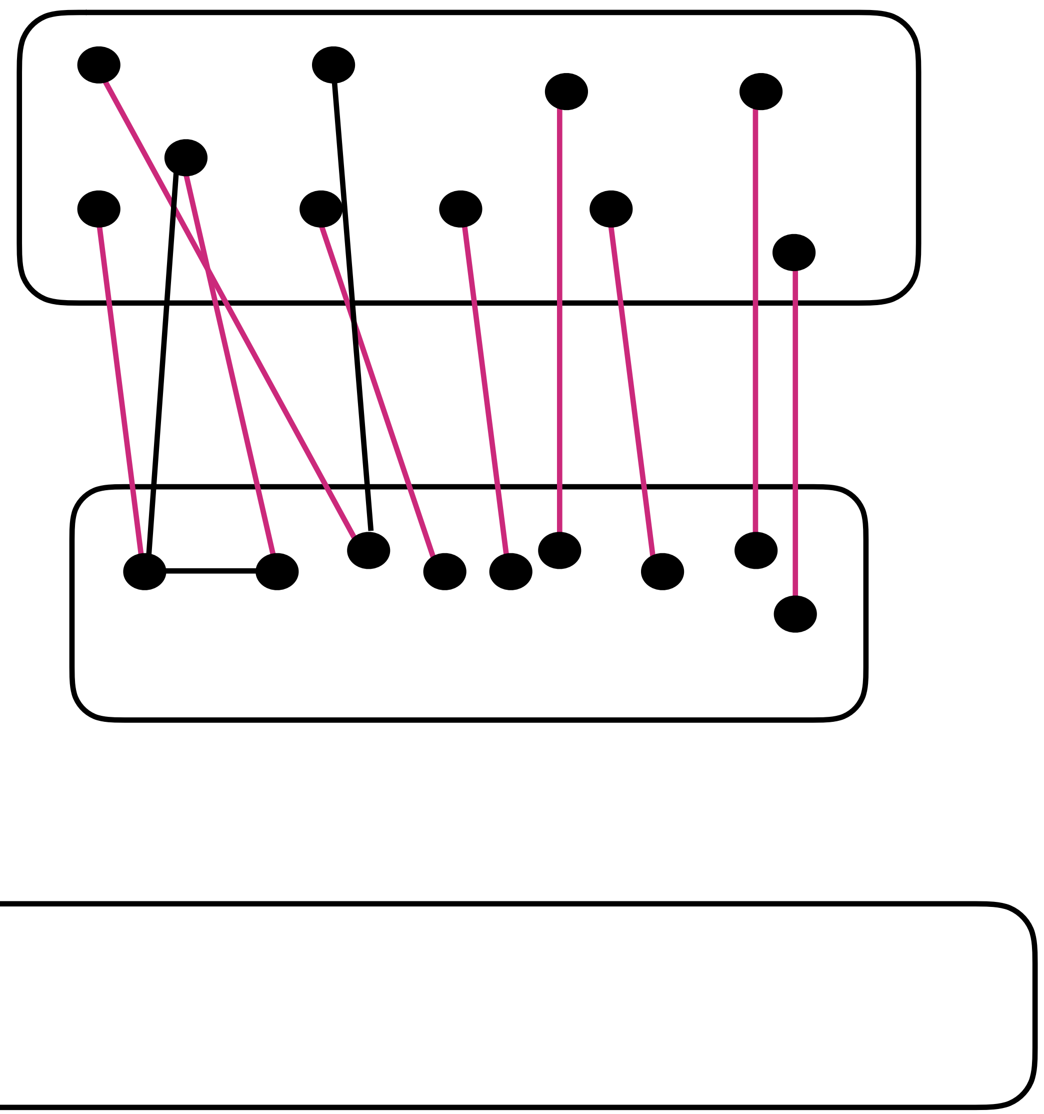
<u>Instance:</u> A graph G on n vertices and integer k
<u>Question:</u> Does G have a proper colouring using n-k colors?
<u>Parameter:</u> k



Crown Decomposition of $\overline{G}$

Crown C

Independent set



Head H

$N(C) \subseteq H$

Rest R

Matching saturating H

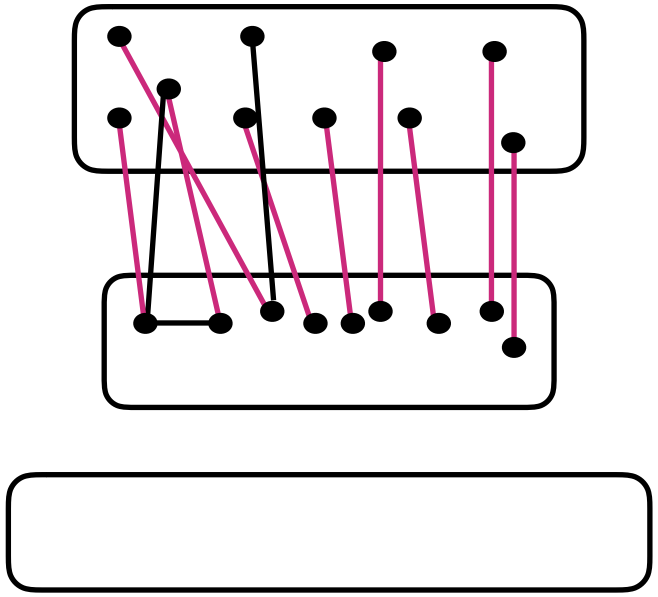
(G, k) is an yes-instance iff (G[R], k-|H|) is an yes-instance

G is (n-k)-colorable iff G[R] is (r-k+h)-colorable

Crown Decomposition of $\overline{G}$

Crown C

Clique in G



Head H

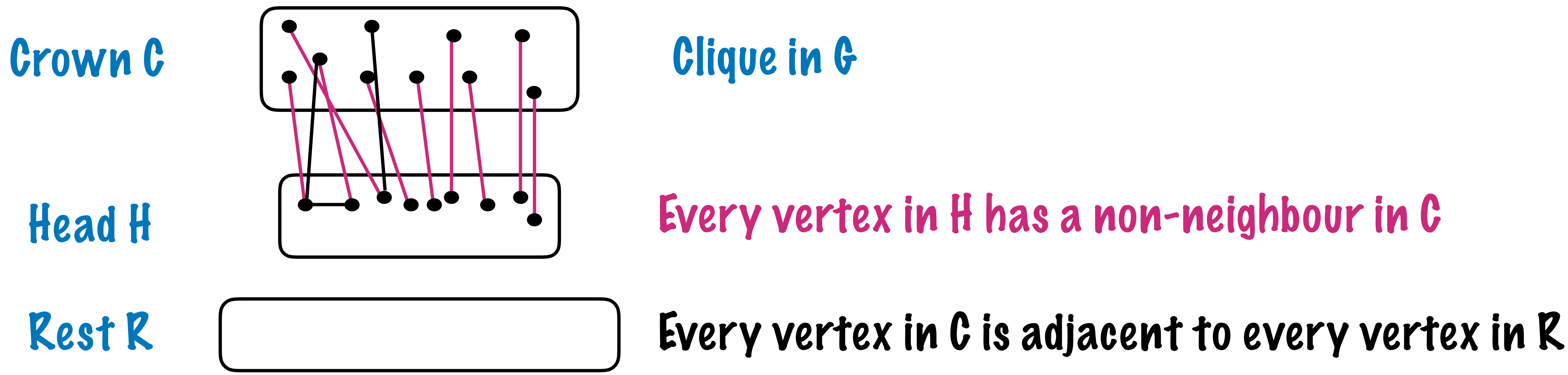
Every vertex in H has a non-neighbour in C

Rest R

Every vertex in C is adjacent to every vertex in R

- Suppose (G, k) is an yes-instance
- G is (n-k)-colorable
  - Every vertex in C has a distinct color
  - None of these c colors can be used for R
  - No. of colors used for R is n-k-c=n-(h+c)-(k-h)=r-(k-h)
- (G[R], k-h) is an yes-instance

Crown Decomposition of $\overline{G}$

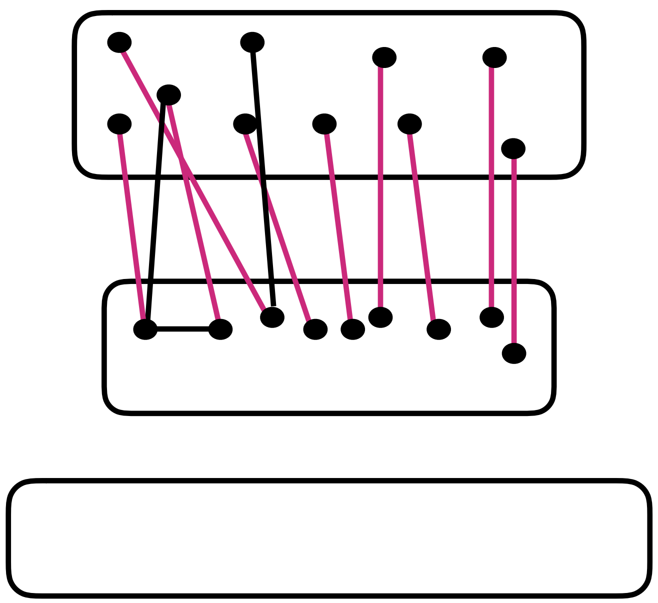


- Suppose (G[R], k-h) is an yes-instance
- G[R] is (r-k+h)-colorable
  - Use c new colors for C
  - Reuse these colors for H
  - No. of colors used for G is r-k+h+c = n-k
- (G, k) is an yes-instance

A Linear Kernel for Dual of Coloring

Crown C

Clique in G



Head H

Every vertex in H has a non-neighbour in C

Rest R

Every vertex in C is adjacent to every vertex in R

reverse dirn is trivial.forward dirn: neighbours of v should use atmost n - k - 1 colors a

* Instance: (G, k)
* If there is a vertex v that is adjacent to all vertices, delete v
  * G is (n-k)-colorable iff (G-v) is (n-k-1)-colourable

As a result, \overline{G} will not ha

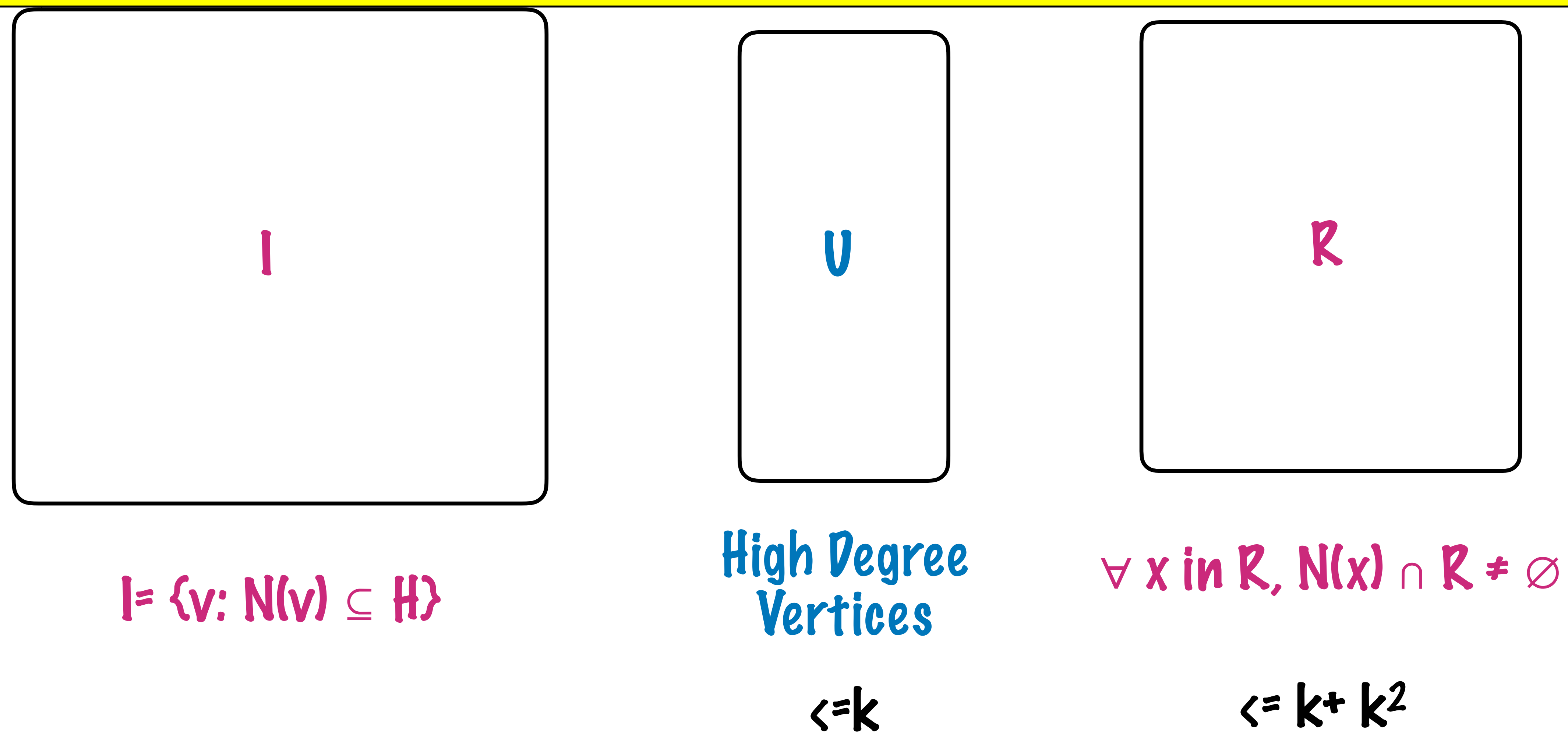
* $\overline{G}$ has no isolated vertices. Apply Crown Lemma if no. of vertices > 3(k-1)
* If $\overline{G}$ has a matching of size k, then G is (n-k)-colorable

Consider endpoints of matching in Set A, colou

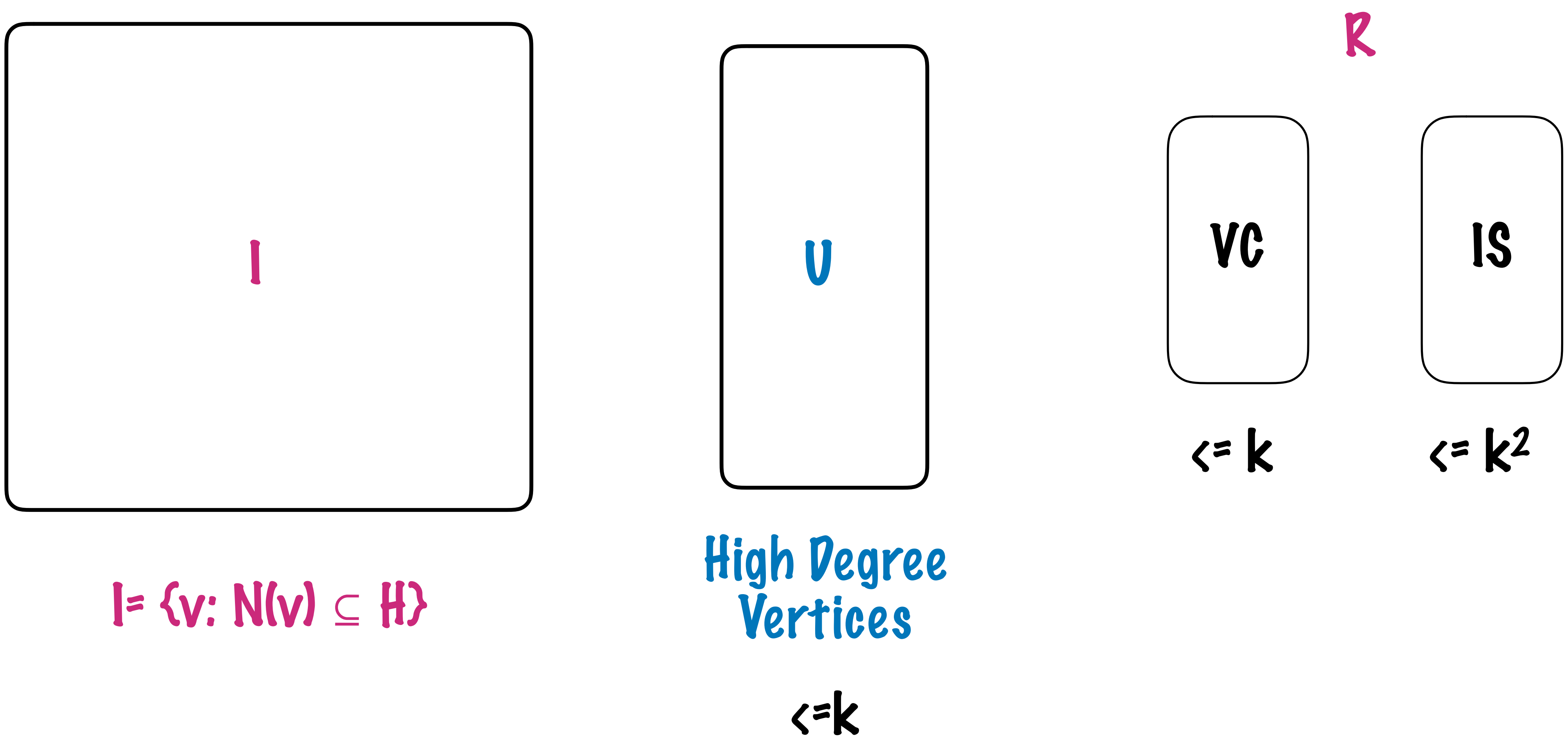
  * Endpoints of matching edges can be given same color in G
* Else, (C,H,R) is a crown in $\overline{G}$
  * Return (G[R],r-k+h)

Connected Vertex Cover

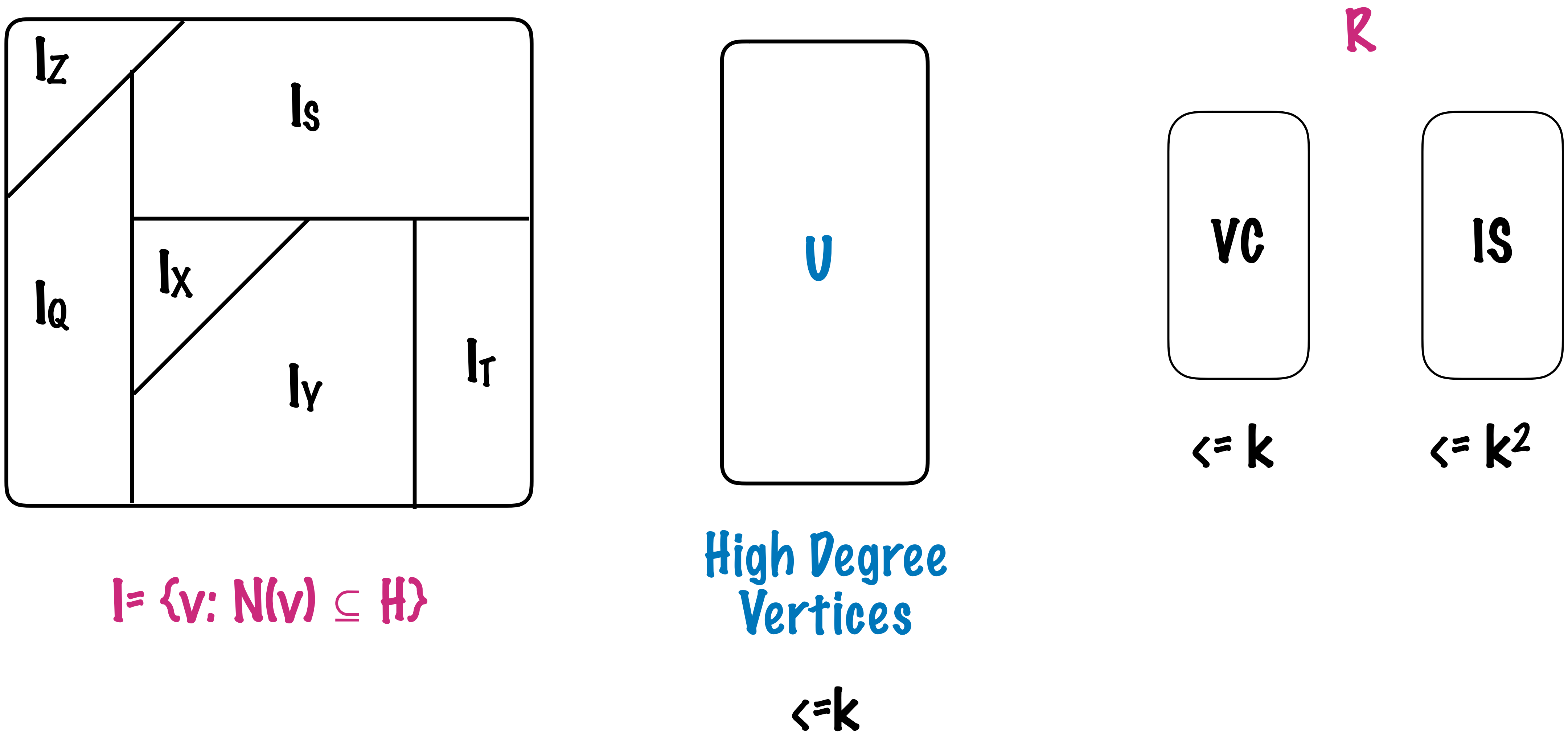
U is the set of vertices of degree atleast k + 1. Clearly they must be part of the solution. Let I denote the vertices in V(G) \ U that h



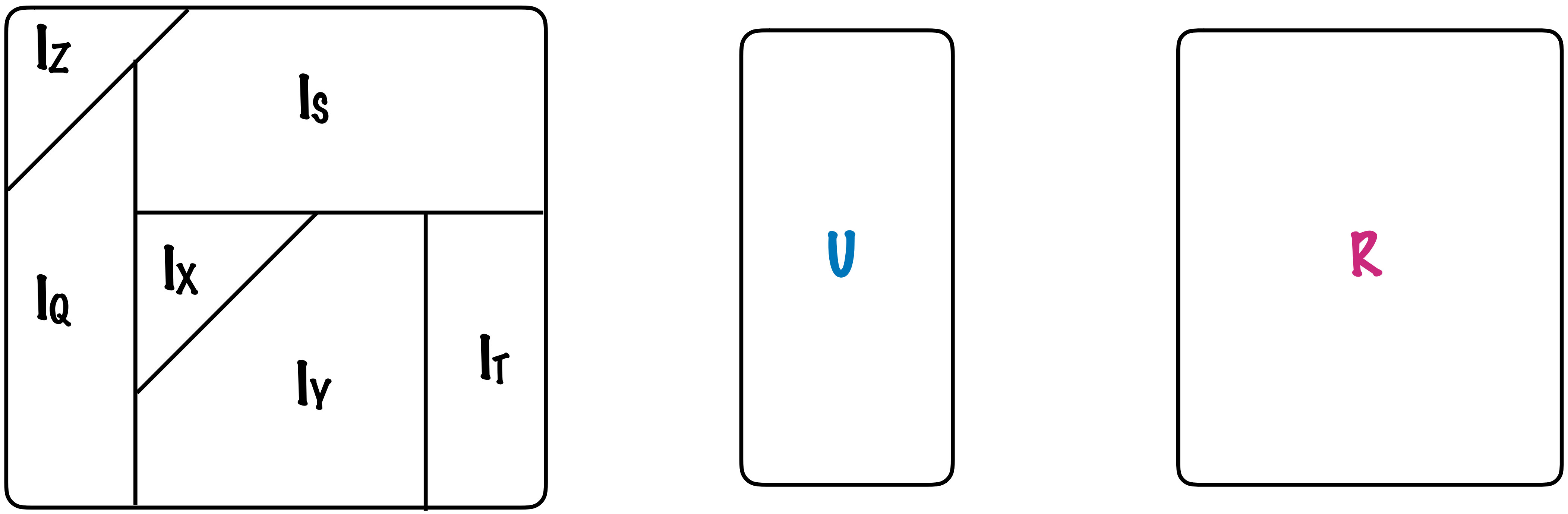
Connected Vertex Cover

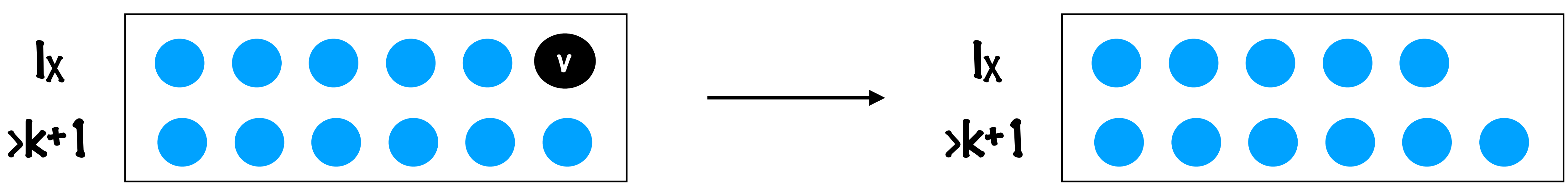


Connected Vertex Cover



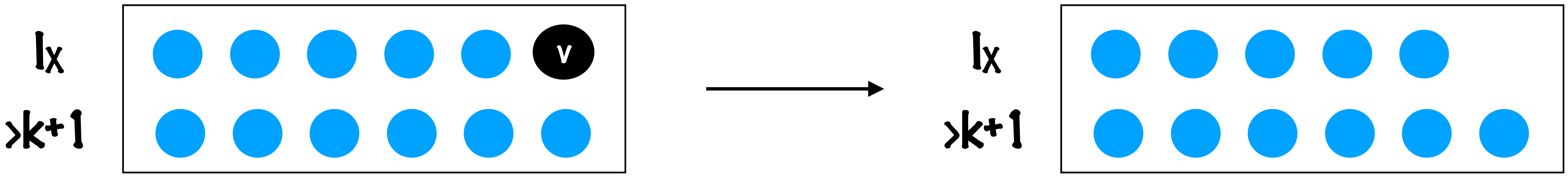
Connected Vertex Cover





(G, k) is an yes-instance iff (G-v, k) is an yes-instance

Connected Vertex Cover

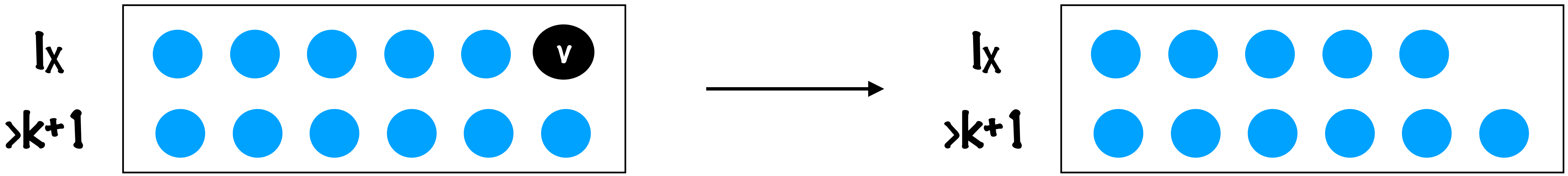


Suppose (G, k) is an yes-instance

* (G, k) is an yes-instance: S is a k size connected vertex cover
* $\exists$ a in I_X that is not in S => $X \subseteq S$
* If v is in S, then delete v from S and add a to S
  * S is a connected vertex cover of G-v
* If v is not in S, then S is a connected vertex cover of G-v

(G-v, k) is an yes-instance

Connected Vertex Cover

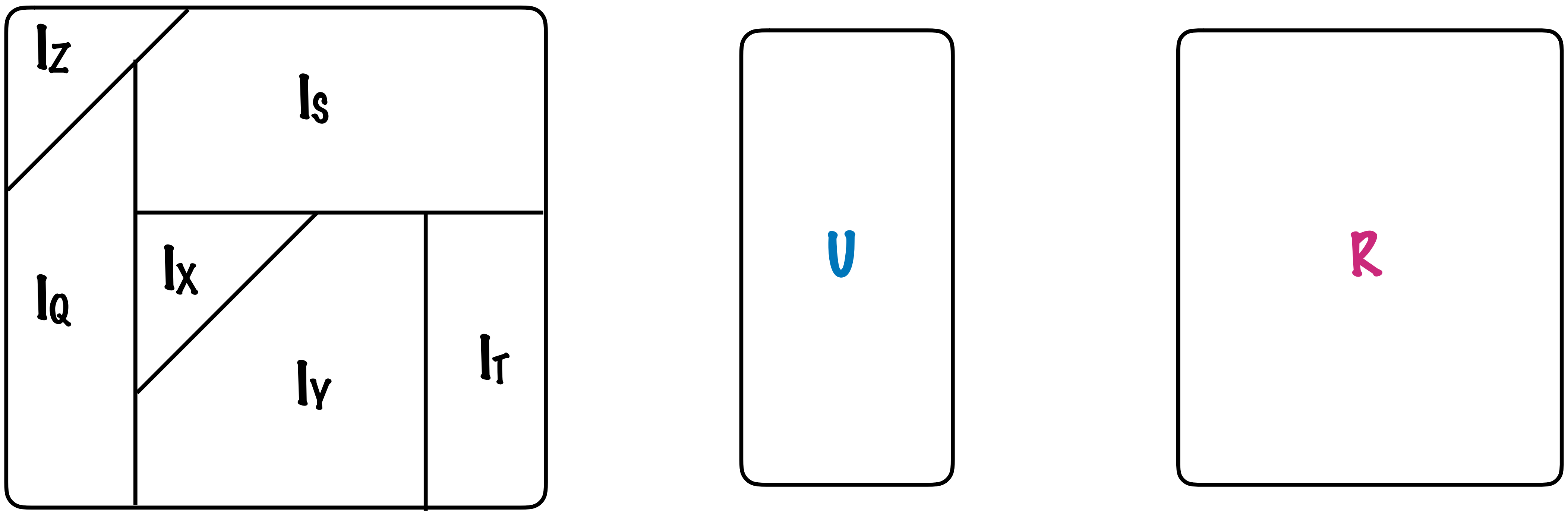


Suppose (G-v, k) is an yes-instance

- (G-v, k) is an yes-instance: S is a k size connected vertex cover
- $\exists$ a in I_X that is not in S => $X \subseteq S$
- S is a connected vertex cover of G

(G, k) is an yes-instance

Connected Vertex Cover



$\forall\, X \subseteq U, |\, I_X \,| <= k+1$

$|U| <= k$

$|R| <= k+k^2$

$|\, I \,| <= 2^{|U|}\,(k+1) <= 2^k\,(k+1)$

$O(k.2^k + k^2)$ kernel