(CS 5008) Reinforcement Learning : Assignment 5

Q1) Consider the problem of approximating the Y value in terms of X values, i.e., $Y_i \approx X_i\theta$. For a given set of data points (actual value not important) such as the one shown in Figure 1 write down the expression for the squared loss $L(\theta)$.

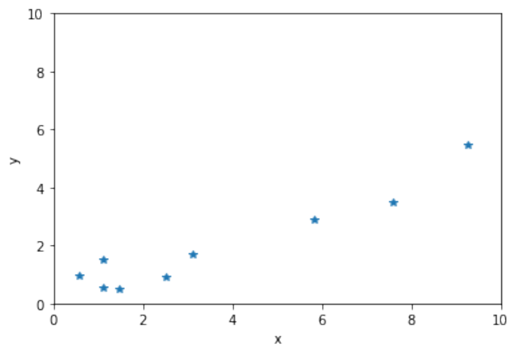

Figure 1: Fit a line through origin.

Q2) Now consider the approximation where $Y_i \approx \theta(0) + \theta(1)X_i$ (say to fit a line for the data set in Figure 2). Suppose we are interested in minimising the squared loss i) what is the expression of $L(\theta)$? (note that $\theta = (\theta(0), \theta(1)) \in \mathbb{R}^2$ has two co-ordinates), ii) what is the matrix formulation of the same problem, in particular what is the X matrix?

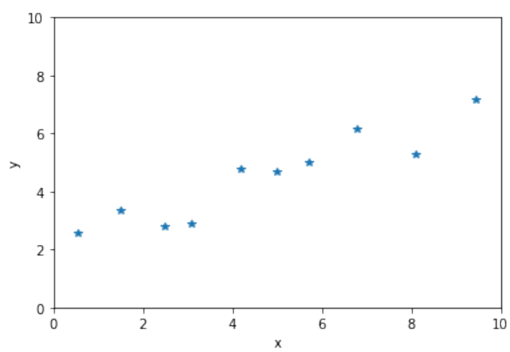

Figure 2: Fit a line that does not pass through origin.

Q3) Give an example of $(X_i, Y_i)_{i=1}^{1}0 \in \mathbb{R}^2 \times \mathbb{R}$, such that the approximation error $Y_i \approx X_i^\top \theta$.

Q4) Consider a 2×2 grid with 4 states, and let the feature of positions be $X(1) = (1,1), X(2) = (1,2), X(3) = (2,1), X(4) = (2,2)$. Write down the loss function to project $V(1) = 2$, $V(2) = 3$, $V(3) = 3$, $V(4) = 4$, i..e, appoximate $V \approx X^\top \theta$. Find $\theta_* = \arg\min_{\theta \in \mathbb{R}^2} \|V - X^\top \theta\|_2^2$.

Q5) What are the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, and an eigenvector?

Q6) Consider a 3 state Markov Chain where the probability of going from any state to any state is $\frac{1}{3}$. Write down the probability transition matrix P. What is the stationary distribution $d^\top = d^\top P$. What are the eigenvalues of P?

Q7) For any real matrix $A \in \mathbb{R}^{d\times d}$ and vector $x^\top$, show that $x^\top Ax = x^\top A^\top x$.

Q8) Let $X = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, Y = \begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$. Verify that $XY = X_1Y_1 + X_2Y_2$, where X_1, X_2 are columns of X and Y_1 and Y_2 are rows of Y.

Q9) Consider a 2×2 grid with 4 states, and let the feature of positions be $X_1 = (1, 1), X_2 = (1, 2), X_3 = (2, 1), X_4 = (2, 2)$. The probability of going from any state to any other state is equal. Let $s = s_t$ be the current state and $s' = s_{t+1}$ be the next state, then calculate $\mathbb{E}[X(s)X_(^\top s')]$ for all $s = 1, 2, 3, 4$.

Q10) Consider learning the mean of random variable $Uniform[0, 1]$. We are given samples $Y_t \overset{iid}{\sim} Uniform[0, 1]$, and consider the following update rule:

$$V_{t+1} = V_t + \alpha_t(Y_t - V_t) \tag{1}$$

Estimate the expected squared error in θ_t after $t = 100$ for a constant step size $\alpha = 0.1$.