CS 5003: Parameterized Algorithms

Lecture 16

Krithika Ramaswamy

IIT Palakkad

Reference Books: Parameterized Algorithms by Cygan et al. and Kernelization by Fomin et al.

Vertex Cover Above LP

Vertex Cover Above LP

Instance: A graph G on n vertices m edges and integer k
Question: Does G have a vertex cover of size at most k?
Parameter: k-lp(G)

$lp(G) >= |M|$

Vertex Cover Above Matching

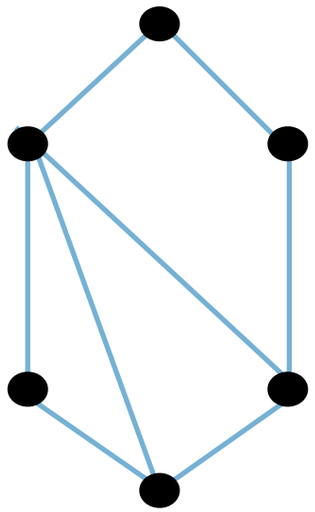
Instance: A graph G on n vertices m edges, integer k, a matching M
Question: Does G have a vertex cover of size at most k?
Parameter: k-|M|

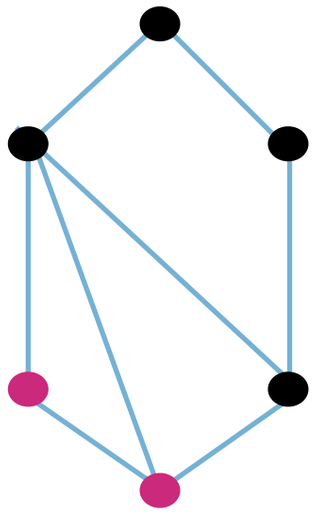
We have lp(G) >= M (think about it) and thus this makes sense.

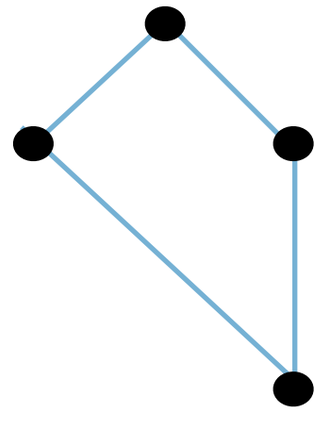
$4^{(k-lp(G))} n^{O(1)}$ time algorithm is a $4^{(k-|M|)} n^{O(1)}$ time algorithm

Odd Cycle Transversal

OCT - set of vertices that has at least one vertex of every odd length cycle







bipartite

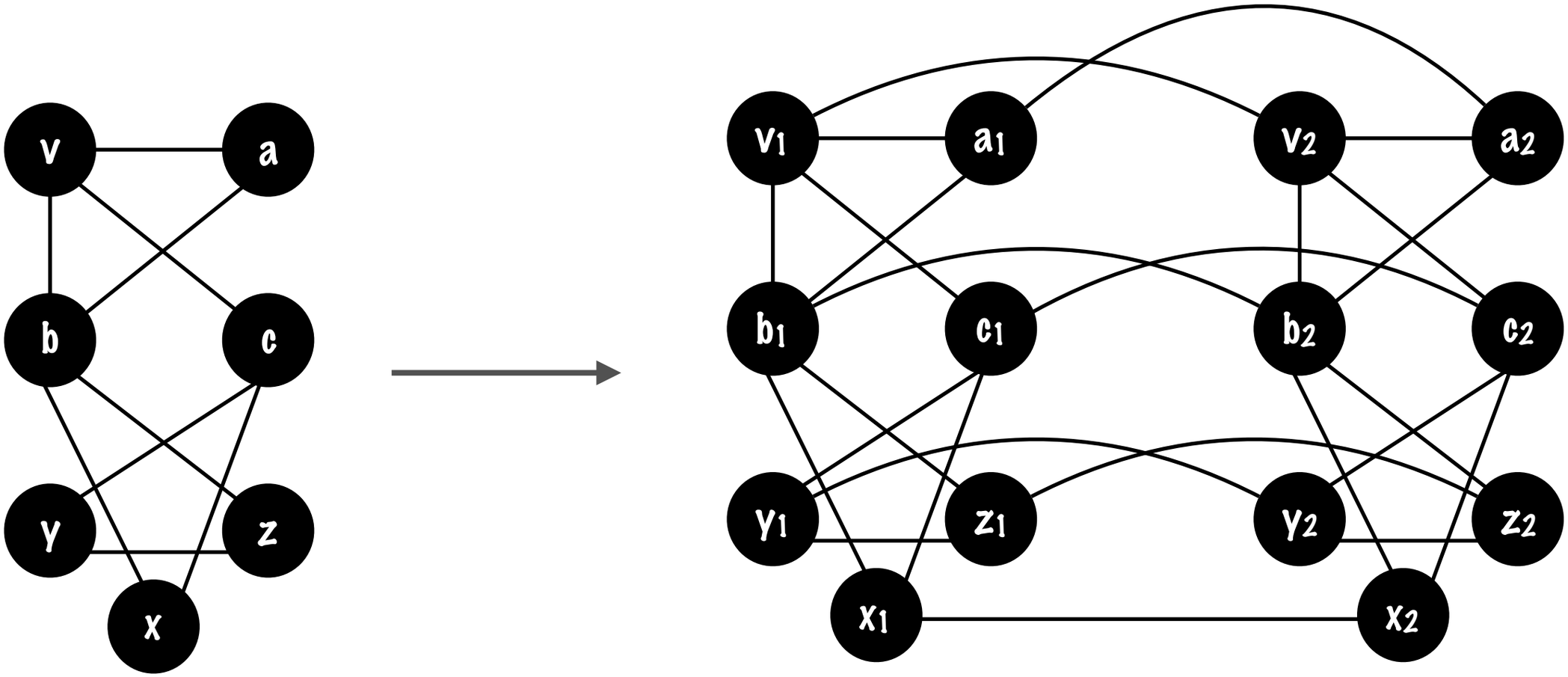
A graph is bipartite iff it has no odd cycle

forward dirn can be proved by contradiction, for reverse dirn, let v be any vertex in G, define X = {x | d_G(v, x) is ev

Odd Cycle Transversal

Instance: A graph G on n vertices m edges and integer k
Question: Does G have an oct of size at most k?
Parameter: k

OCT Reduces to VC Above LP



(G, k) OCT

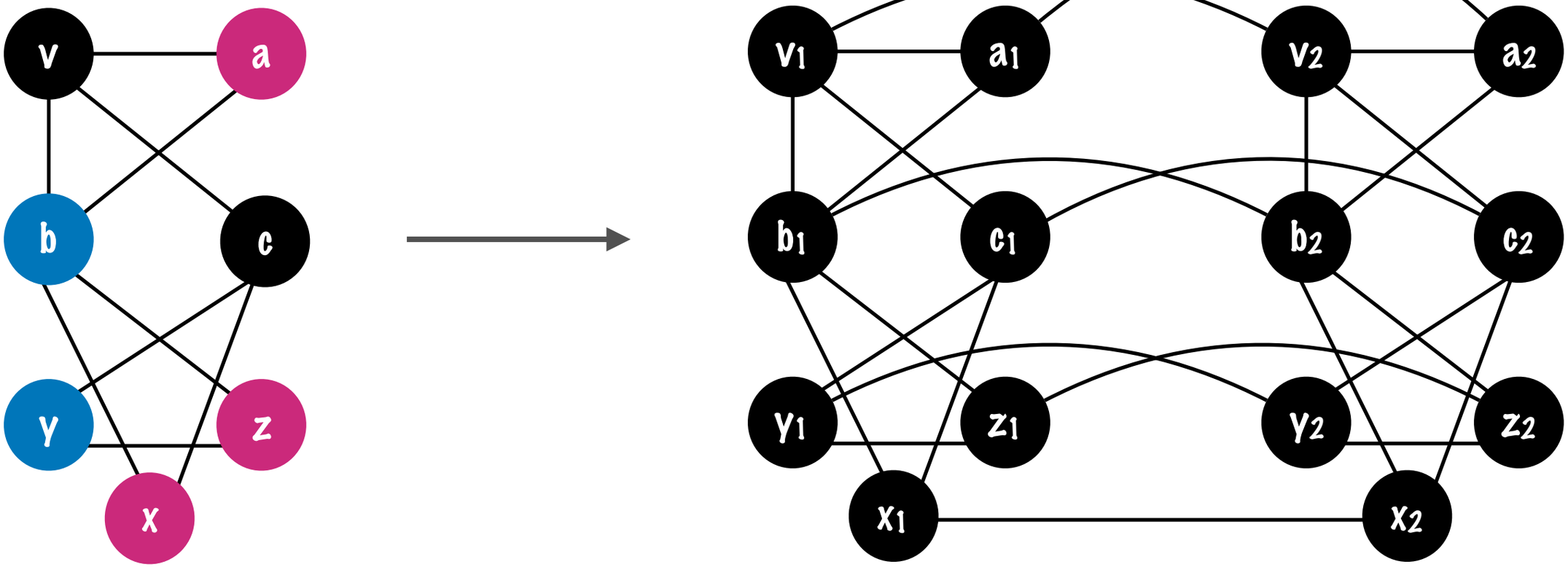
(H, |V(G)|+k) Vertex Cover

G has an OCT of size k iff H has a VC of size |V(G)|+k

OCT Reduces to VC Above LP

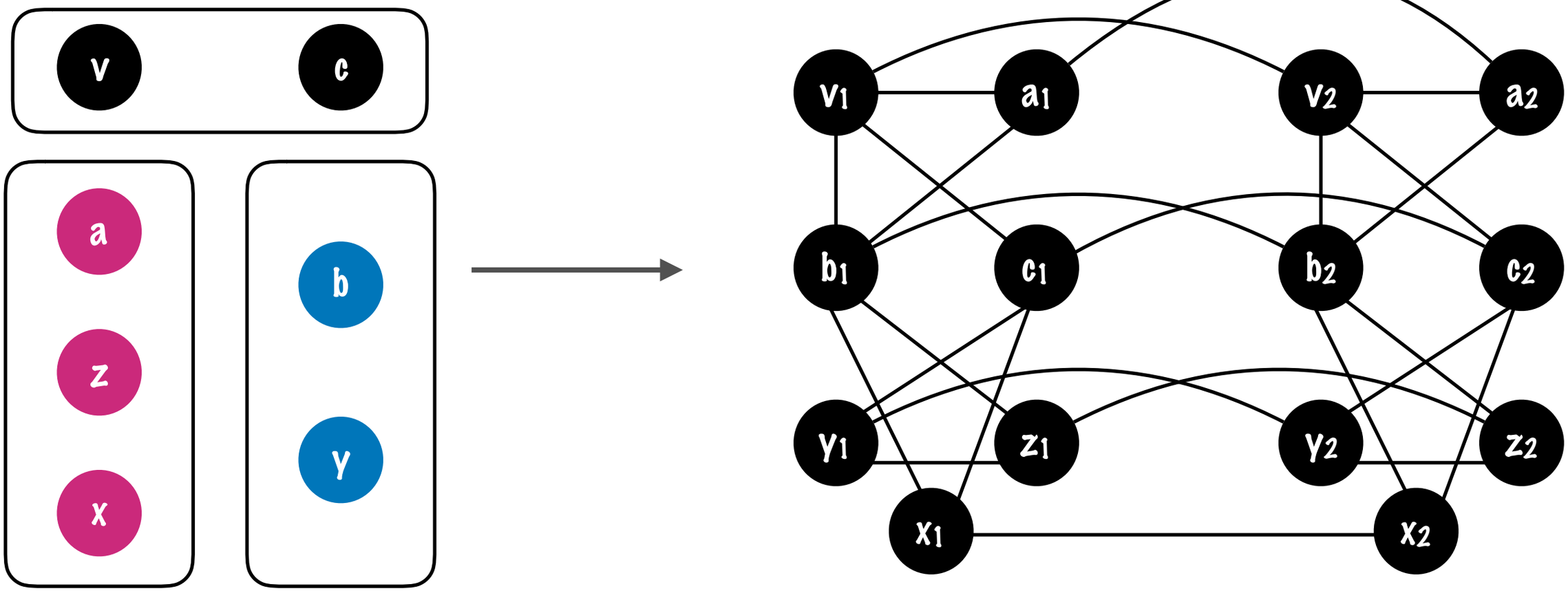
black denotes OCT and other 2 colors are for b

Suppose G has an OCT of size k



OCT Reduces to VC Above LP

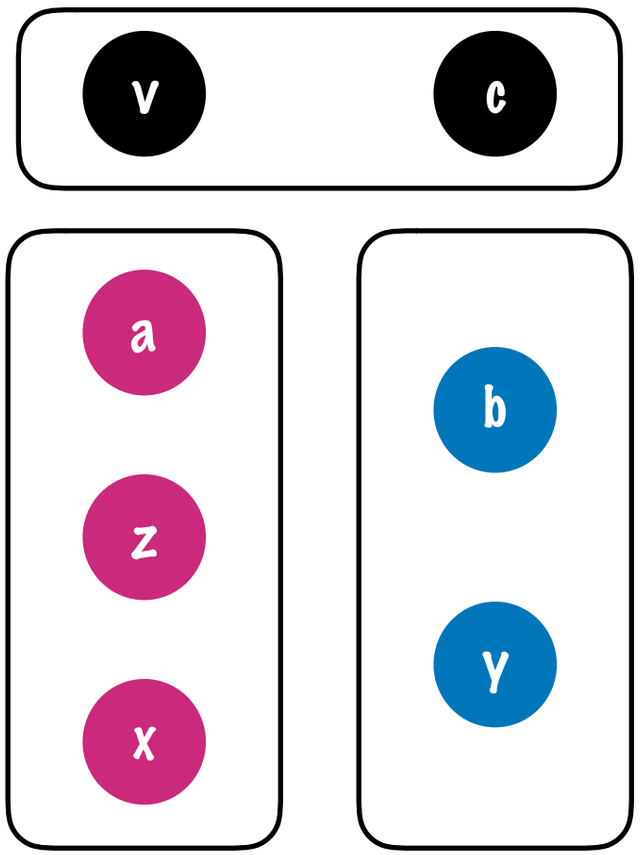
Suppose G has an OCT of size k

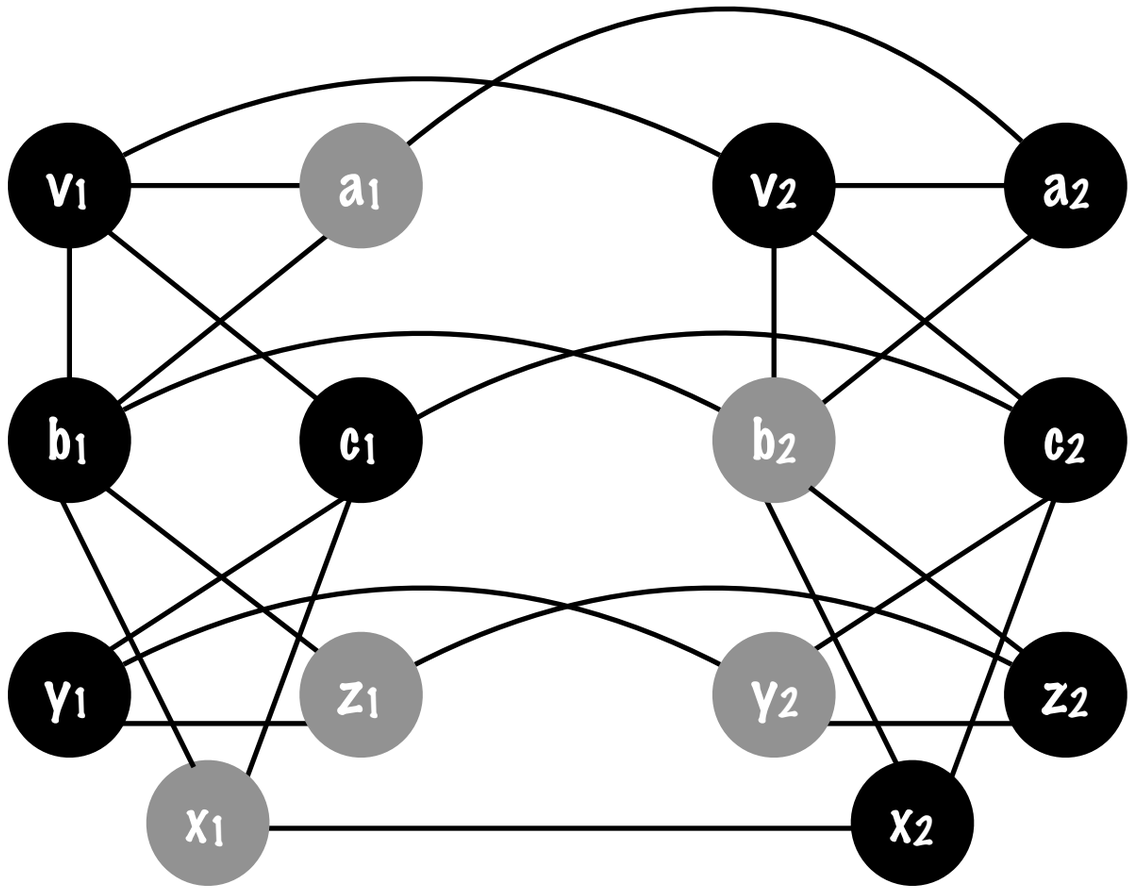


G has a bipartite graph of size |V(G)|-k

OCT Reduces to VC Above LP

Suppose G has an OCT of size k

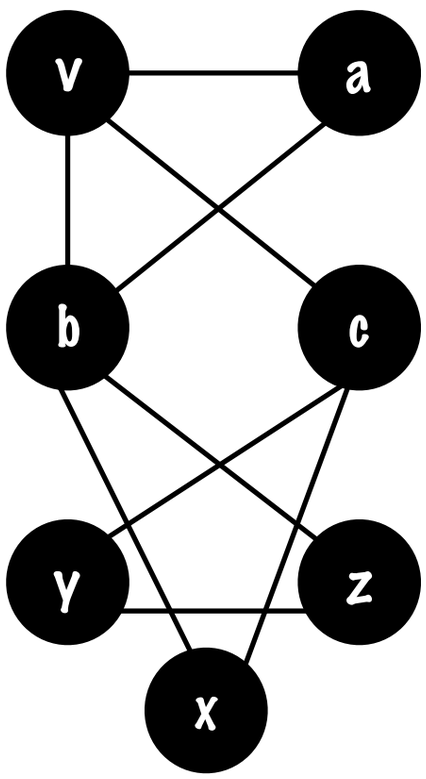




H has an IS of size |V(G)|-k

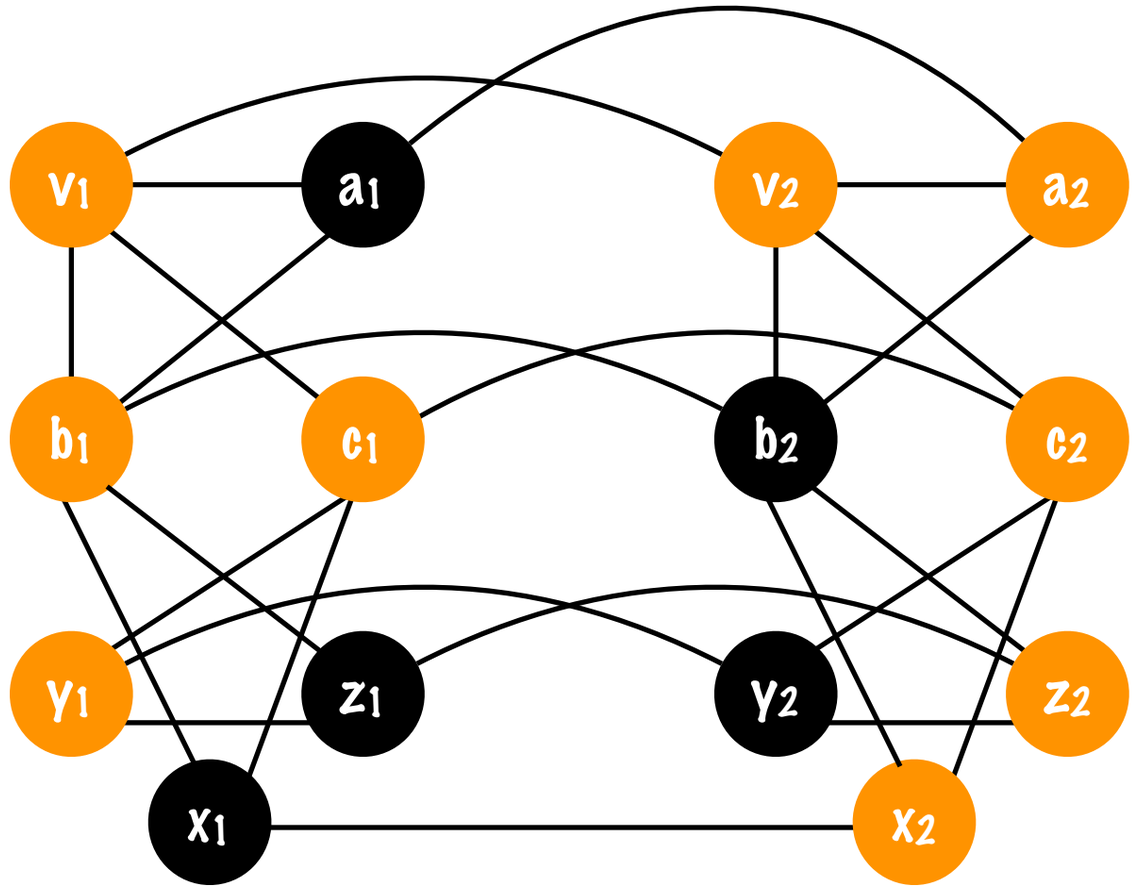
H has a VC of size |V(G)|+k

OCT Reduces to VC Above LP



Suppose H has a VC of size |V(G)|+k

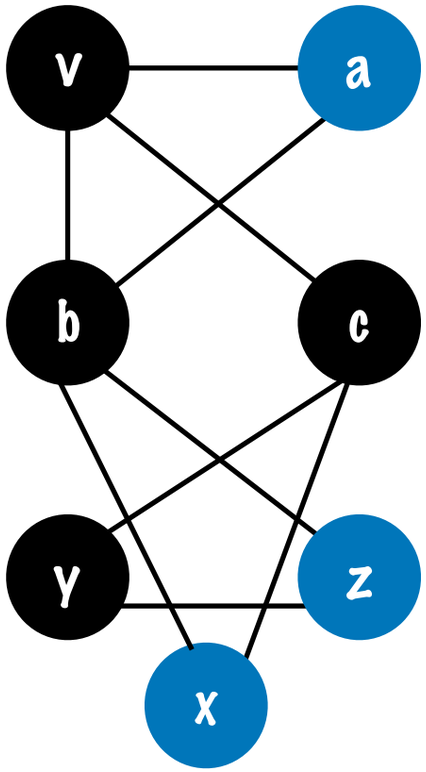
orange denotes VC.

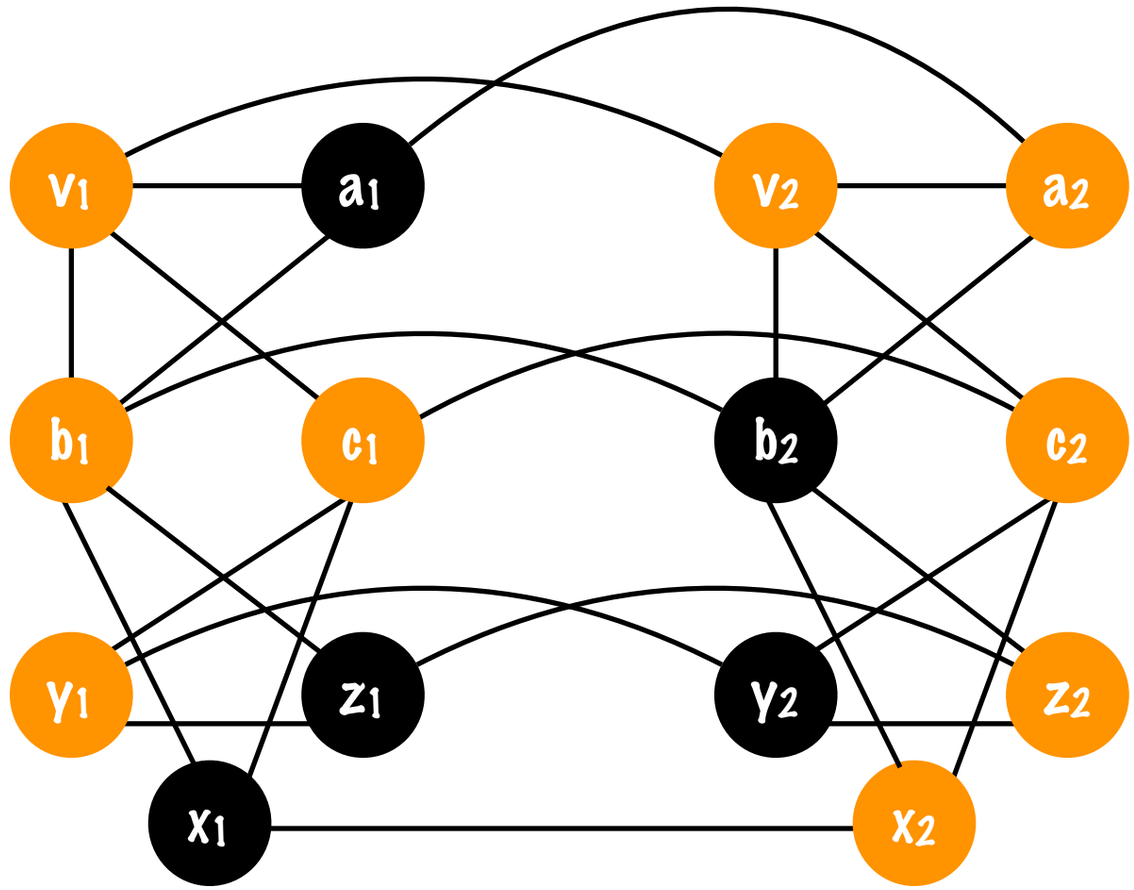


H has an IS of size |V(G)|-k

OCT Reduces to VC Above LP

Suppose H has a VC of size |V(G)|+k

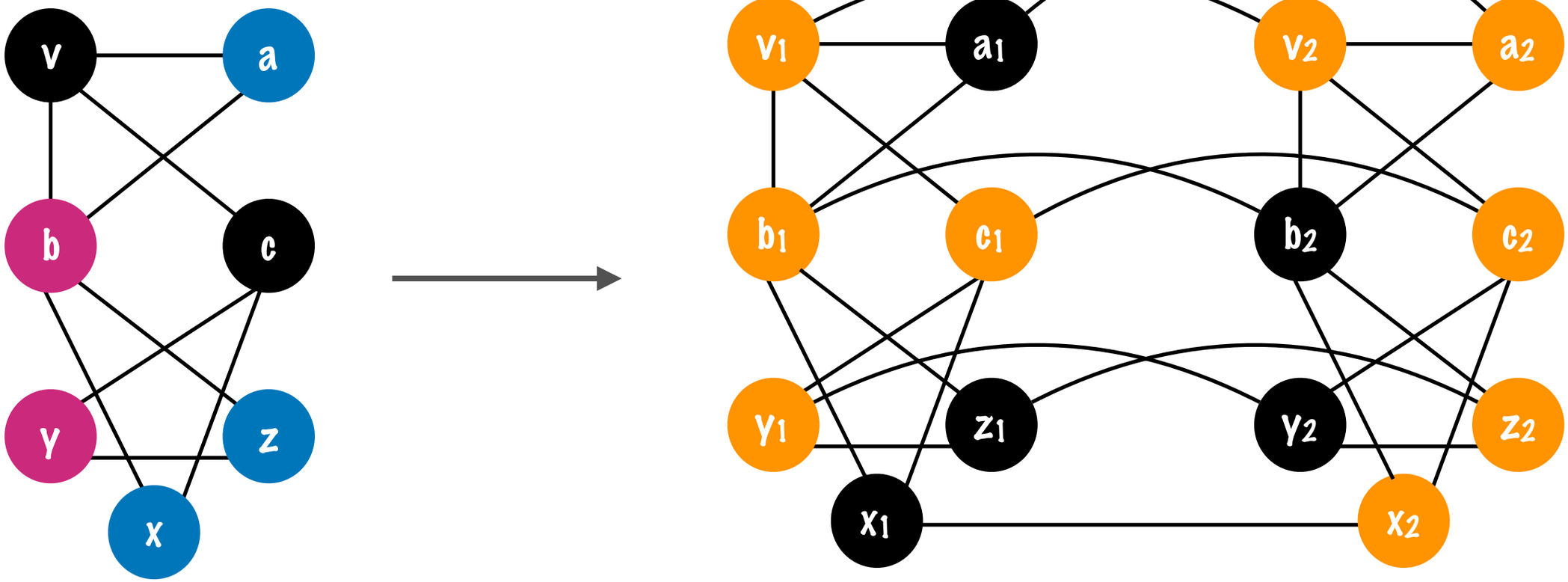




H has an IS of size |V(G)|-k

OCT Reduces to VC Above LP

Suppose H has a VC of size |V(G)|+k



G has a bipartite graph of size |V(G)|-k

G has an OCT of size k

OCT Reduces to VC Above LP

G has an OCT of size k iff H has a VC of size |V(G)|+k

* To determine if G on n vertices has an OCT of size k,
  * Construct H from G
  * Matching M of size n in H
  * Determine if H has a VC of size n+k
    * Use the Vertex Cover Above LP algorithm on H that has matching M
      * ($4^{n+k-|opt(H)}$ $n^{O(1)}$ time which is $4^{n+k-|M|=k}$ $n^{O(1)}$ time)