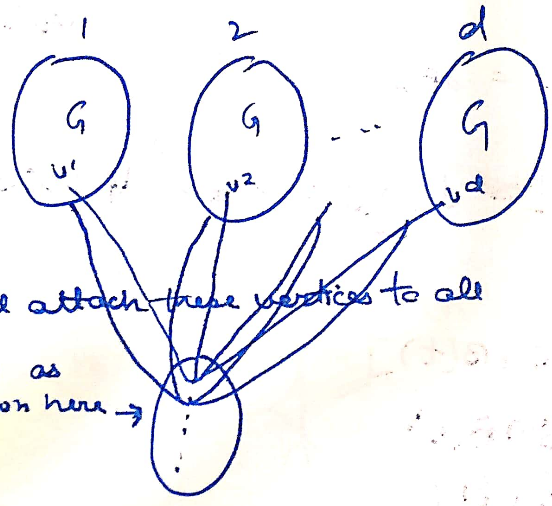
A graph is regular

Clique $\le$ clique in regular graph.

Let d = max deg.

Now take d copies of G:

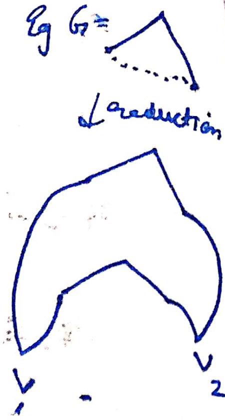
time taken $d \times \text{poly}(n,m)$



look at vertex v in G,

Add $d-\deg_G(v)$ vertices, and attach these vertices to all the d copies as shown here →

Eg G =



(G,k) yes $\iff$ (H,k) is a yes inst.

→ easy

← Assume $k \ge 3$, no vertex in newly added vertices can be part of clique as copies of v ~~are not~~ dont have edge in b/w. Similarly vertex in G_i & vertex in G_j, $i \neq j$ can't be part of clique as they don't have edge b/w them.

And $(H,k) \le (H^c, K)$

Clique in reg. graphs ↳ Independent set in reg. graphs.

Partial vertex: Input (G, k, x) — undirected, simple — Ques: $\exists$? $S, |S| \le k$ that covers $\ge x$ edges

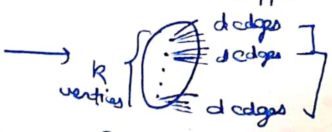
Param: k — we can make it $= k$.

$\ge k$ — v.c. of size $\ge n-k$

Red #3: $\longrightarrow (G, k, dk)$

Suppose G has an I.S. of size k.

Dir^n: $\longrightarrow$



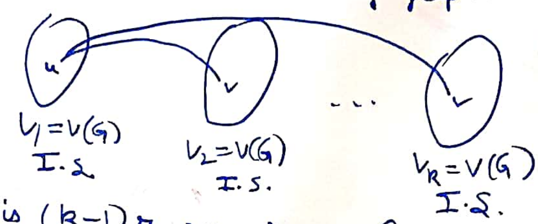
Clique in Reg graph $\le_{FPT}$ M.C. clique in reg. graph.

say x-regular

Dir^n: $\longleftarrow$ Suppose G has a partial v.c that covers $\ge dk$ edges $= dk$ edges as $> dk$ is not possible in this case.

$\Rightarrow$ it is an I.S.

Given (G, k), do



$\Rightarrow$ ~~No of edges~~ it is $(k-1)x$ regular graph.

Claim: G has k clique iff H has multicolored k clique.

$(\longrightarrow)$ Suppose G has a k-clique $(v_1, v_2, \ldots, v_k)$

$\Rightarrow$ ~~take~~ Consider $(v_1^1, v_2^2, \ldots, v_k^k)$ which is our multicolored clique.

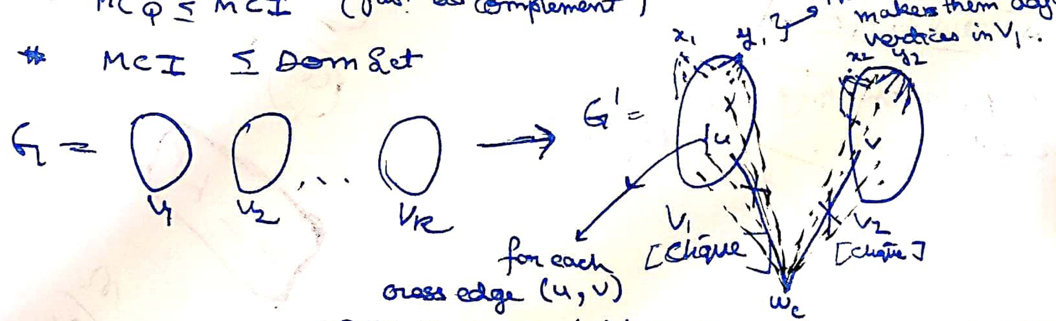
$(\longleftarrow)$ Suppose H has a multicolored k clique

$\Rightarrow$ that clique has exactly one vertex from each of the partition, & those edges are there in original graph as well (by defn)

$\Rightarrow$ We have k clique in original graph.

* MCϕ $\le$ MCI (just do complement) — ϕ denotes clique

* MCI $\le$ Dom Set



where u, v are in different partition, add w_e as shown, which is adjacent to everything in V_1 except u & adjacent to everything in V_2 except v.

$(\rightarrow)$ Suppose (G, k) is a Yes instance of MCI. these same vertices form a Dom. set in this new graph. [If w_e is not dominated $\Rightarrow$ both its end points are in dom. set $\Rightarrow$ These vertices are adjacent $\Rightarrow\Leftarrow$.

$(\leftarrow)$ Suppose (G', k) is a Yes instance of Dom. set. this dom. set doesn't contain any w_e as o/w $(x_1, y_1), (x_2, y_2)$ will not be dominated unless we add one vertex from V_1, V_2 & in this case Dom. set size will become greater than k.

- Now suppose there is an edge (c, d) in original graph. then both c, d cannot be in dom. set as then who will dominate w_f.
- Alone x_i cannot be in sol^n as then who will dominate y_i & both cannot be in sol^n as size will inc..
- Both I.S. & Dominating Set are not likely to be FPT But their difficulty is not same.

Similarly there cannot be two vertices picked from any $V_i \cup \{x_i, y_i\}$ as o/w dom. set size will become $> k$. Similarly if x_i or y_i is in dom. set, take any other vertex in V_i.