(CS 5008) Reinforcement Learning : Assignment 1

Probability Basics

1 Single Random Variable

Q 1) Consider a box containing chits with numbers 1 to 6 written on them. Random variable is X is the result of sampling a chit at random from the box. Now i) What is the probability mass function $p_X = Prob(X = x), x = 1, \ldots, 6$? ii) Say we would like to simulate an *unbiased* coin toss, how will you use the above random experiment. iii) Say someone wants to generate random samples such that $Prob(X = x)$ is equal for all $x = 1, \ldots, 6$, however there are $1, \ldots, 7$ chits instead of just 6 chits. How will you modify the procedure?

Q 2) Consider the village with 50% rain and 50% wind. Let $W = 0$ and $W = 1$ stand for no-wind and wind respectively, similarly $R = 0$ and $R = 1$ stand for no-rain and rain.
i) What is p_R? and what is p_W?

Q 3) Consider the village with 20% rain and 30% wind. Let $W = 0$ and $W = 1$ stand for no-wind and wind respectively, similarly $R = 0$ and $R = 1$ stand for no-rain and rain.
i) What is p_R? and what is p_W?

2 Joint Random Variables

Q 4) In the rain or no-rain, wind or no-wind, let p_{RW} stand for joint probability. Assign values for p_{RW} such that
a) 50% rain and 50% wind, R and W are independent.
b) 80% rain and 20% wind, R and W are independent.
c) 50% rain and 50% wind, R and W are mutually exclusive. What are $P(R|W)$, $P(R|NW)$, $P(NR|NW)$ and $P(NR|W)$?
d) 80% rain and 20% wind, R and W are mutually exclusive. What are $P(R|W)$, $P(R|NW)$, $P(NR|NW)$ and $P(NR|W)$?
e) 50% rain and 50% wind, $P(R|W) = 1$, what are $P(R|NW)$, $P(NR|NW)$ and $P(NR|W)$?
f) 80% rain and 20% wind, $P(R|W) = 1$, what are $P(R|NW)$, $P(NR|NW)$ and $P(NR|W)$?
g) 50% rain and 50% wind, $P(R|W) = 0.5$ and $P(NR|NW) = 0.2$, what are $P(NR|NW)$ and $P(NR|W)$?
h) 20% rain and 80% wind, $P(R|W) = 0.9$ and $P(NR|NW) = 0.1$, what are $P(NR|NW)$ and $P(NR|W)$?

3 Expectation

Q 5) Consider a box containing chits with numbers 1 to 6 written on them. Random variable is X is the result of sampling a chit at random from the box. Now a) What is $\mathbb{E}[X]$, and $\mathbb{E}[X^2]$? b) What is variance? and why is it always positive?