CS 5003: Parameterized Algorithms

Lectures 8-9

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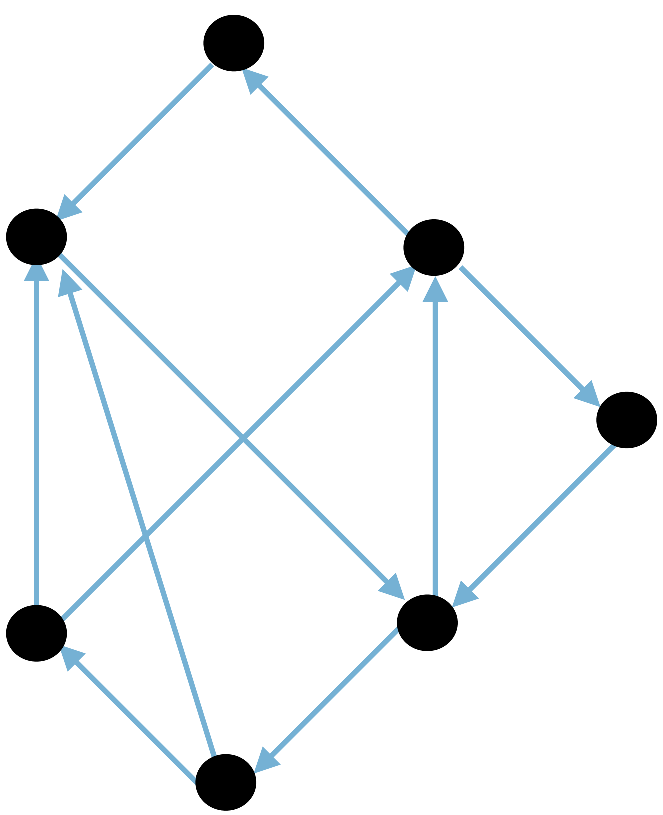
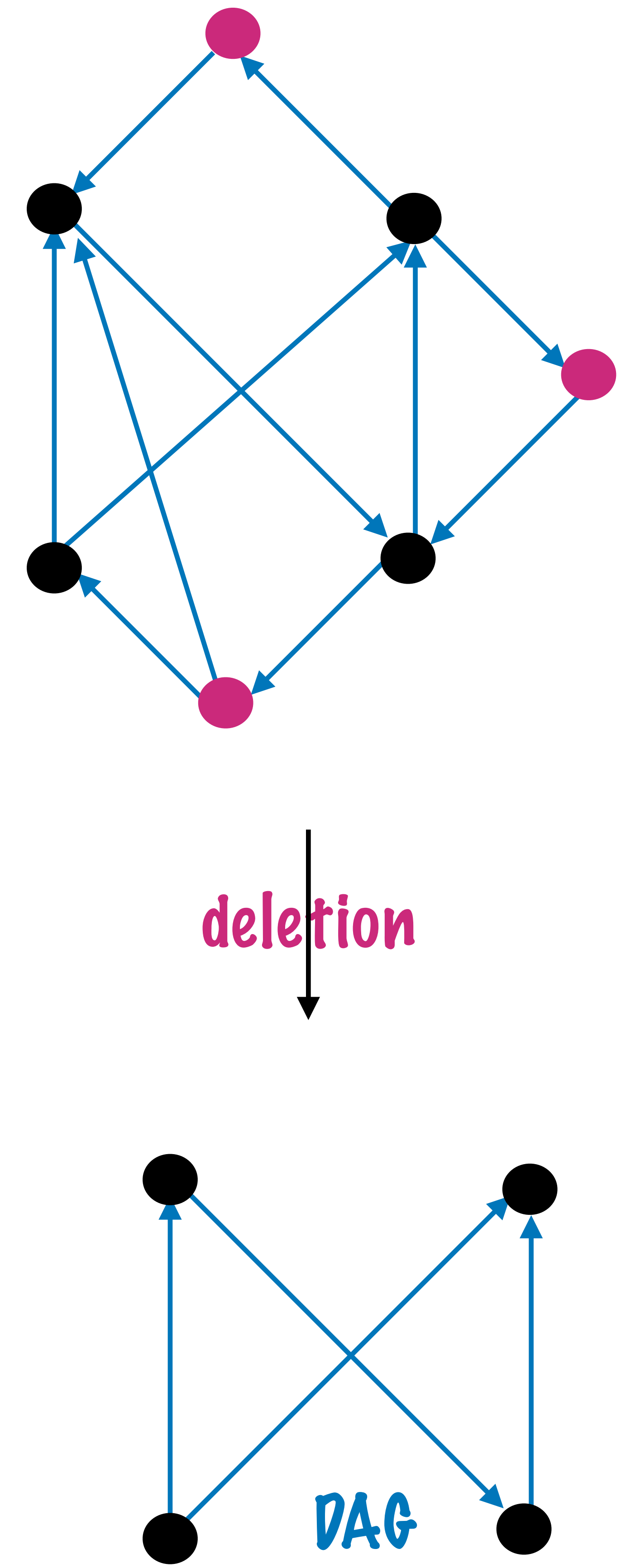
Reference Books: Parameterized Algorithms by Cygan et al. and Kernelization by Fomin et al.

Feedback Vertex Set

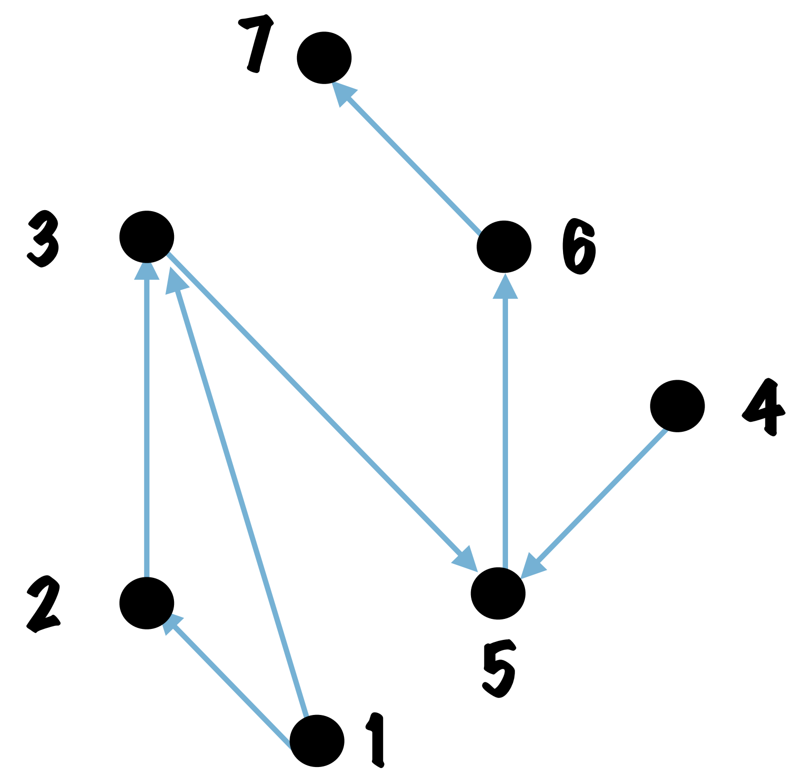
FVS - set of vertices that has at least one vertex of every directed cycle



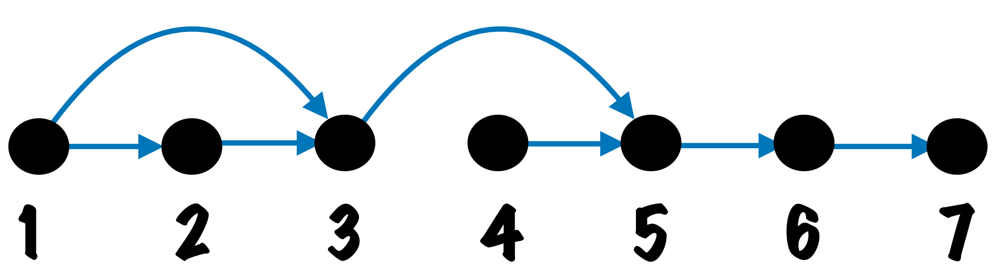
Directed graph (digraph)



Feedback Vertex Set

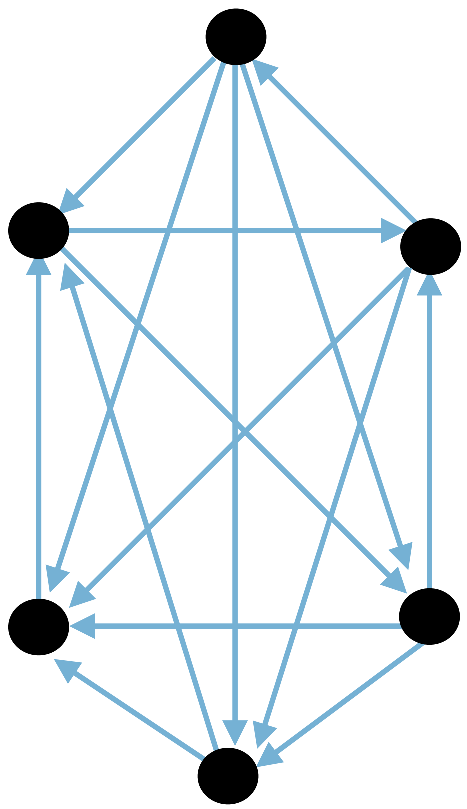


Lemma: A digraph is a DAG iff it has a topological ordering



Topological ordering

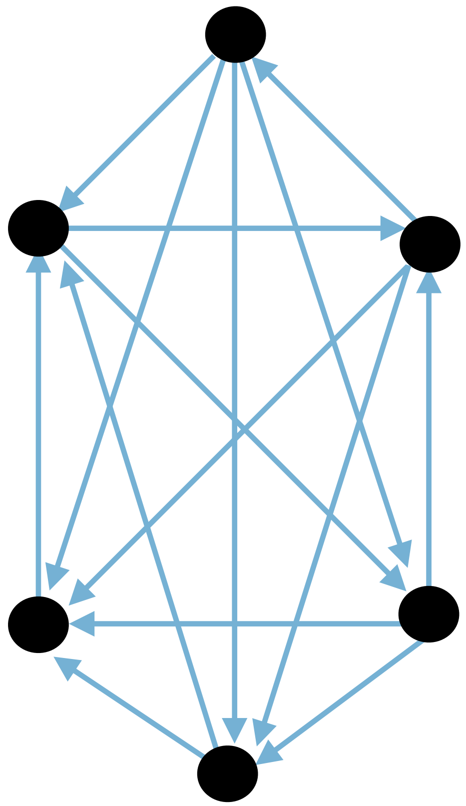
Feedback Vertex Set in Tournaments



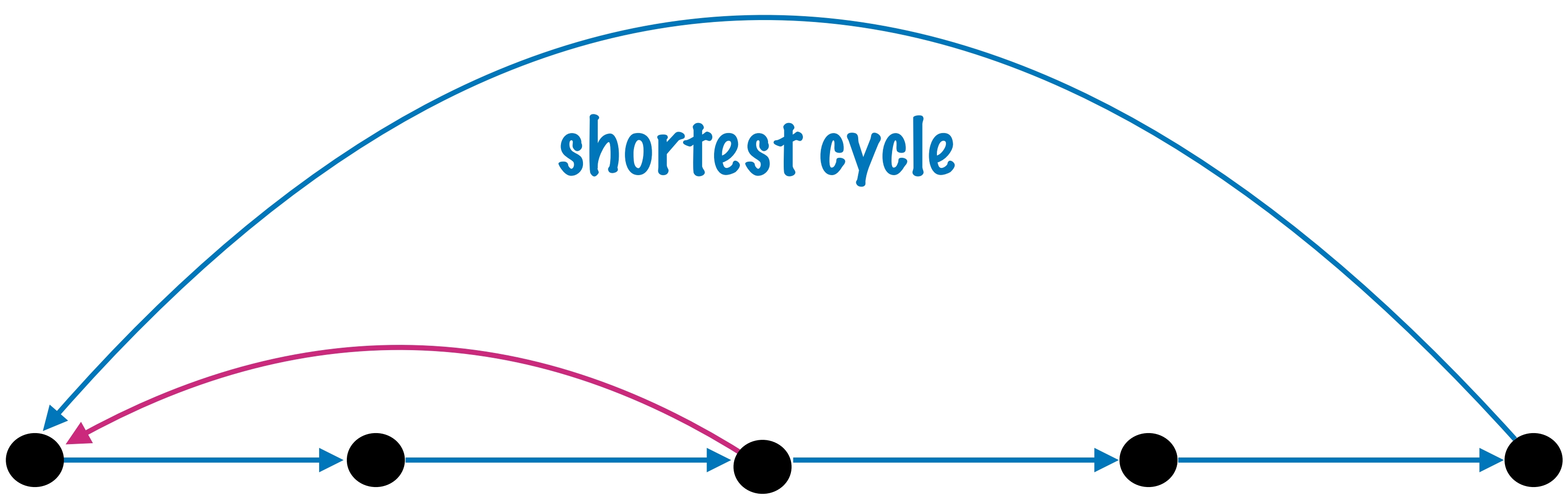
Feedback Vertex Set in Tournaments
Instance: A tournament T and an integer k
Question: Does there exist a feedback vertex set of T of size at most k?

Lemma: Acyclic tournaments have unique topological ordering

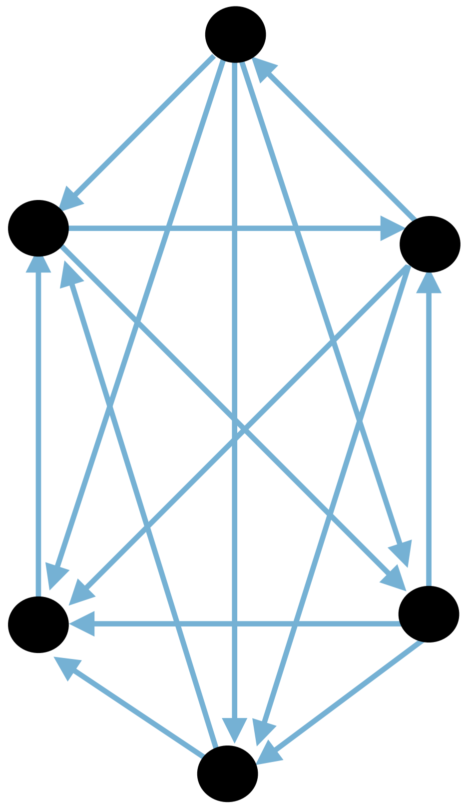
Feedback Vertex Set in Tournaments



Lemma: A tournament is acyclic iff it has no triangle



Feedback Vertex Set in Tournaments



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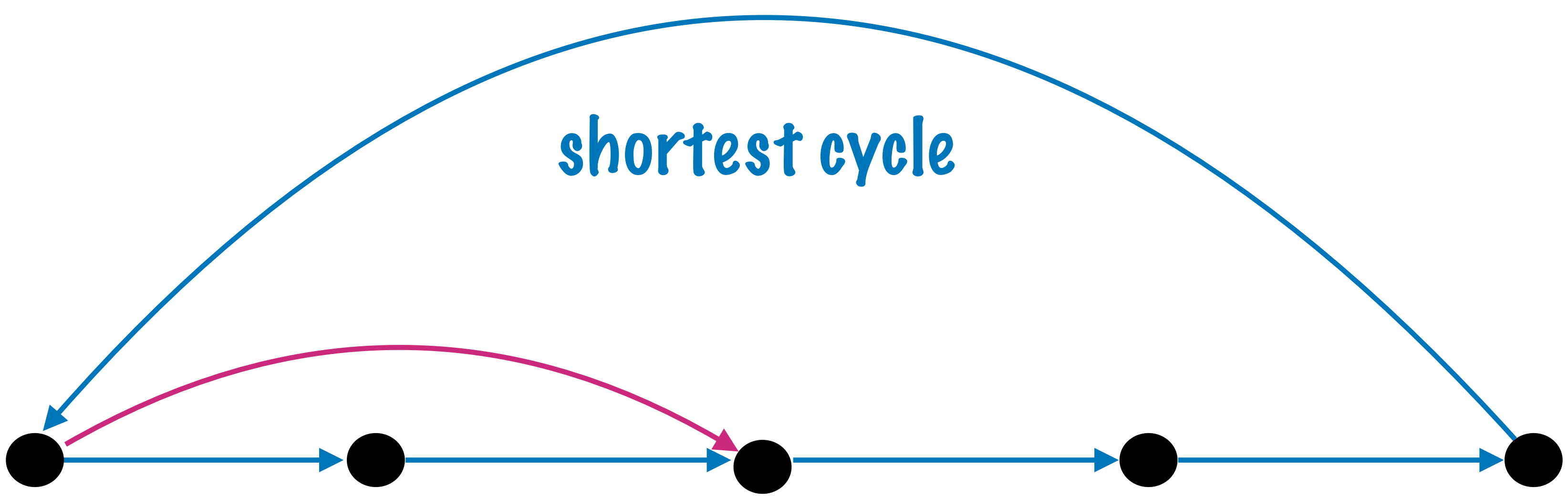


Feedback Vertex Set in Tournaments

Lemma: A tournament is acyclic iff it has no triangle

Branching Algorithm?

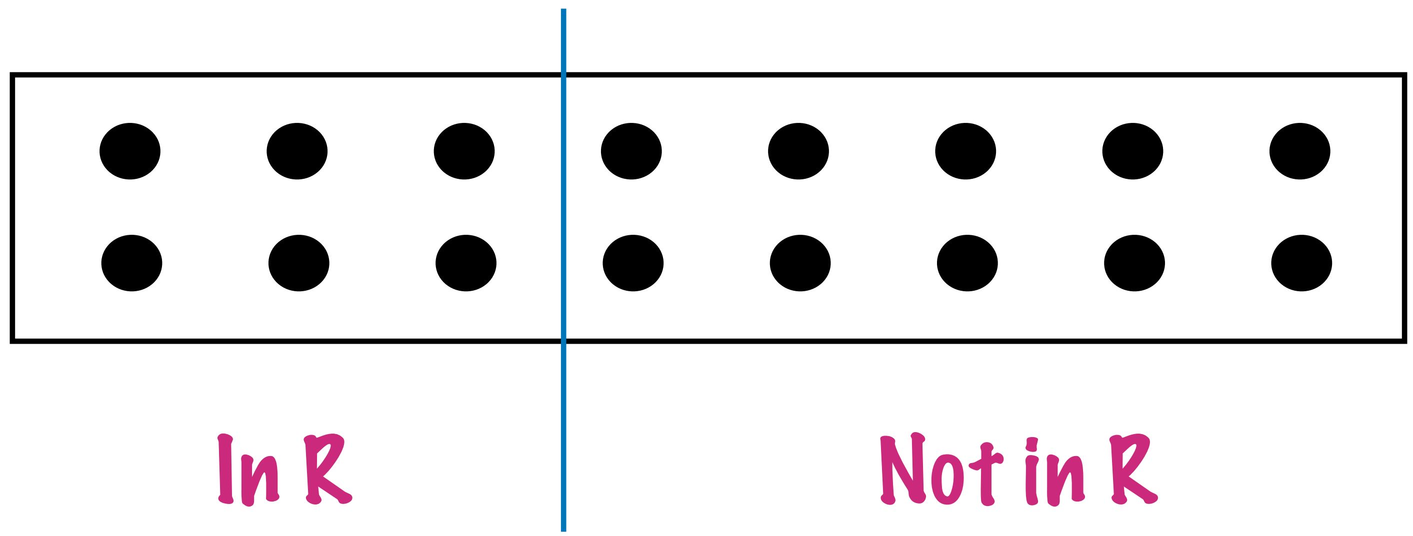
What are the recursive subproblems?

$O^*(3^k)$ algorithm

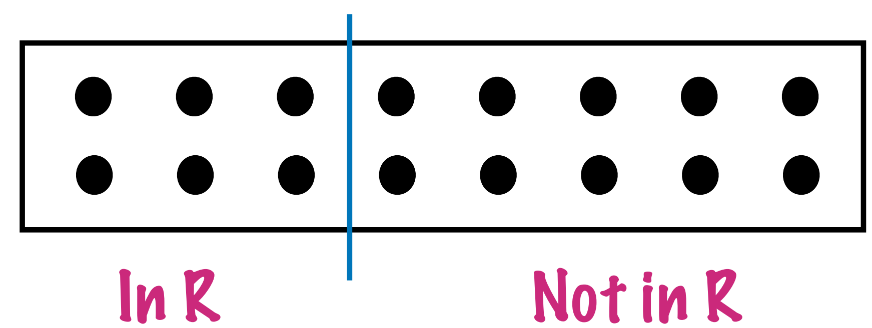
Theorem: FVST is FPT with respect to the solution size as parameter

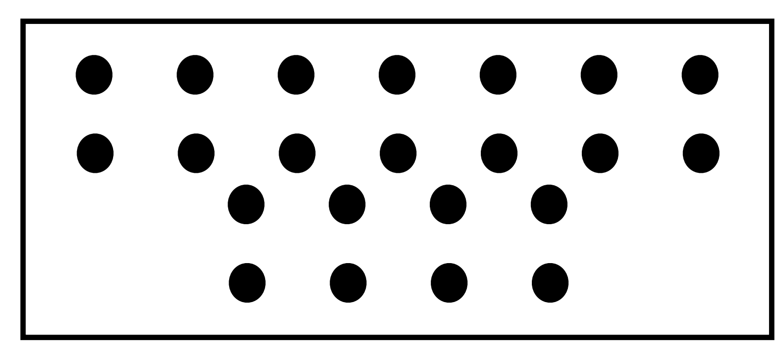
Feedback Vertex Set in Tournaments

Suppose we have a (k+1)-size solution S



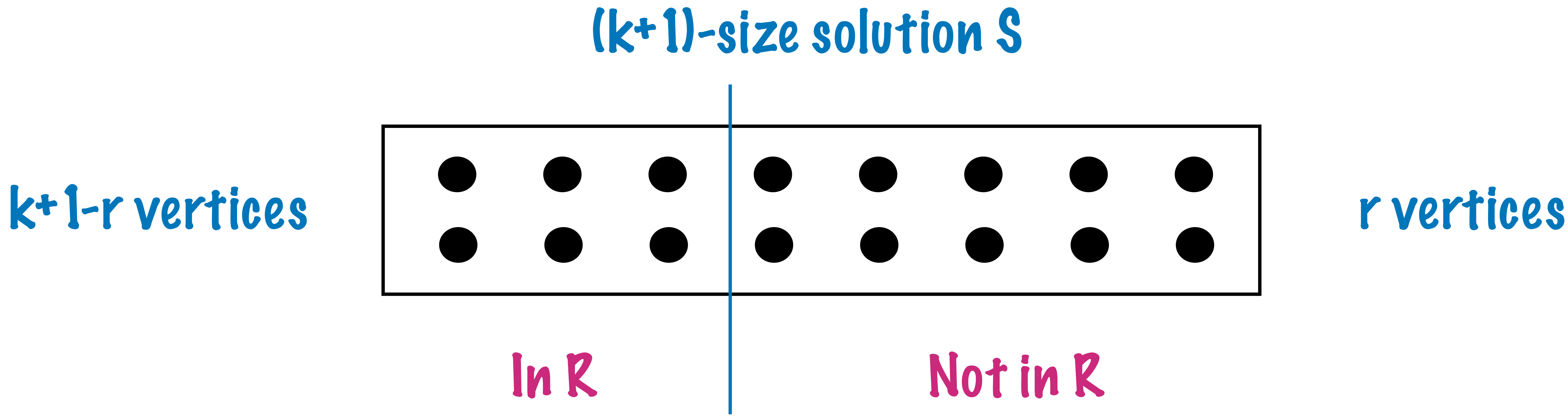
* We want <= k size solution R
* Suppose we know $S \cap R$
  * If we don't know $S \cap R$, guess!
    * 2^{k+1} choices

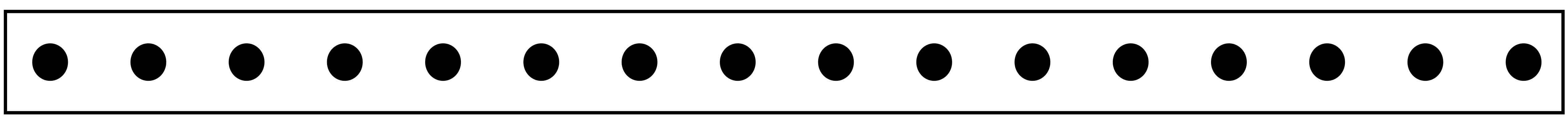




DAG (unique topo ordering)

Feedback Vertex Set in Tournaments



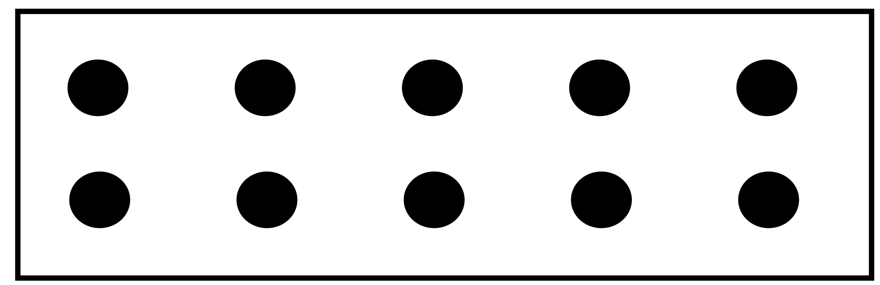


DAG with unique topological ordering

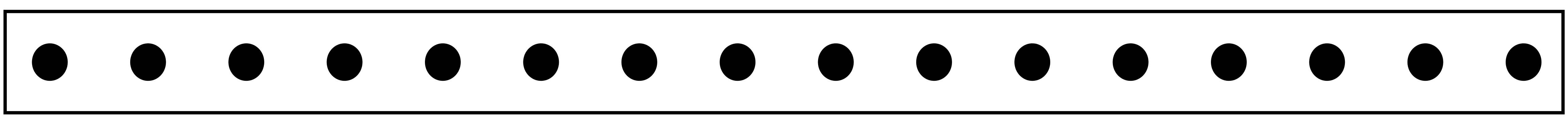
To find a set of <= r-1 vertices here

Feedback Vertex Set in Tournaments

r-size solution



DAG (unique topo ordering)

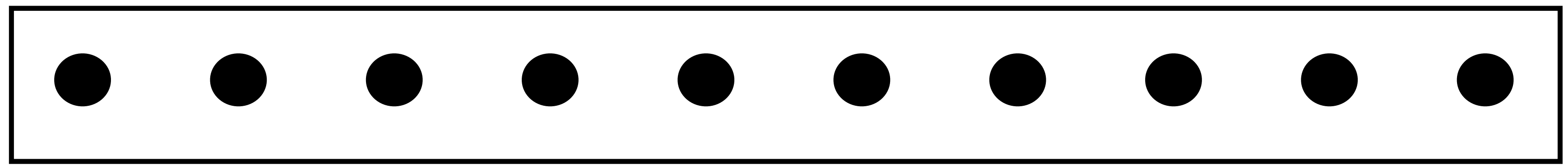


DAG (unique topo ordering)

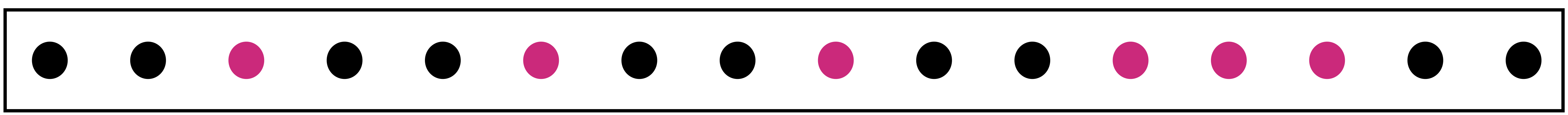
To find a set of <= r-1 vertices here

Feedback Vertex Set in Tournaments

r-size solution



DAG (unique topo ordering)



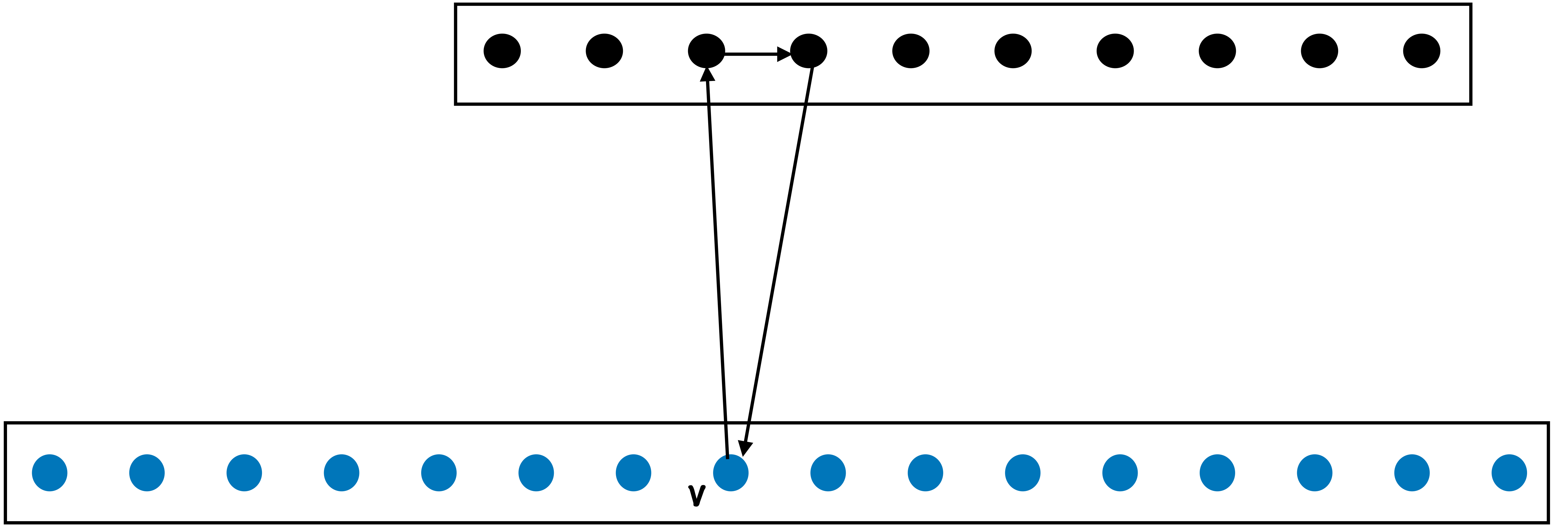
DAG (unique topo ordering)

Find a disjoint (r-1) size solution

Disjoint Compression

Feedback Vertex Set in Tournaments

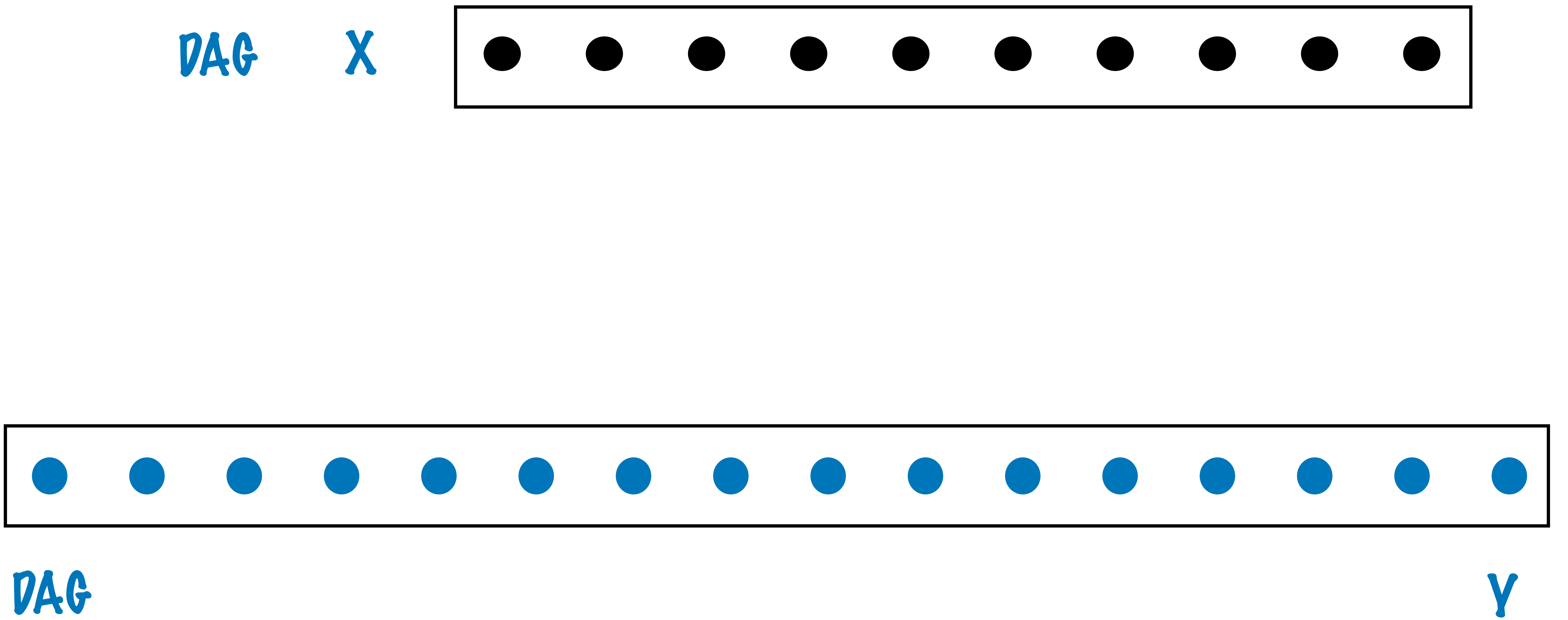
Disjoint Compression



Add v into solution and reduce parameter by 1

Feedback Vertex Set in Tournaments

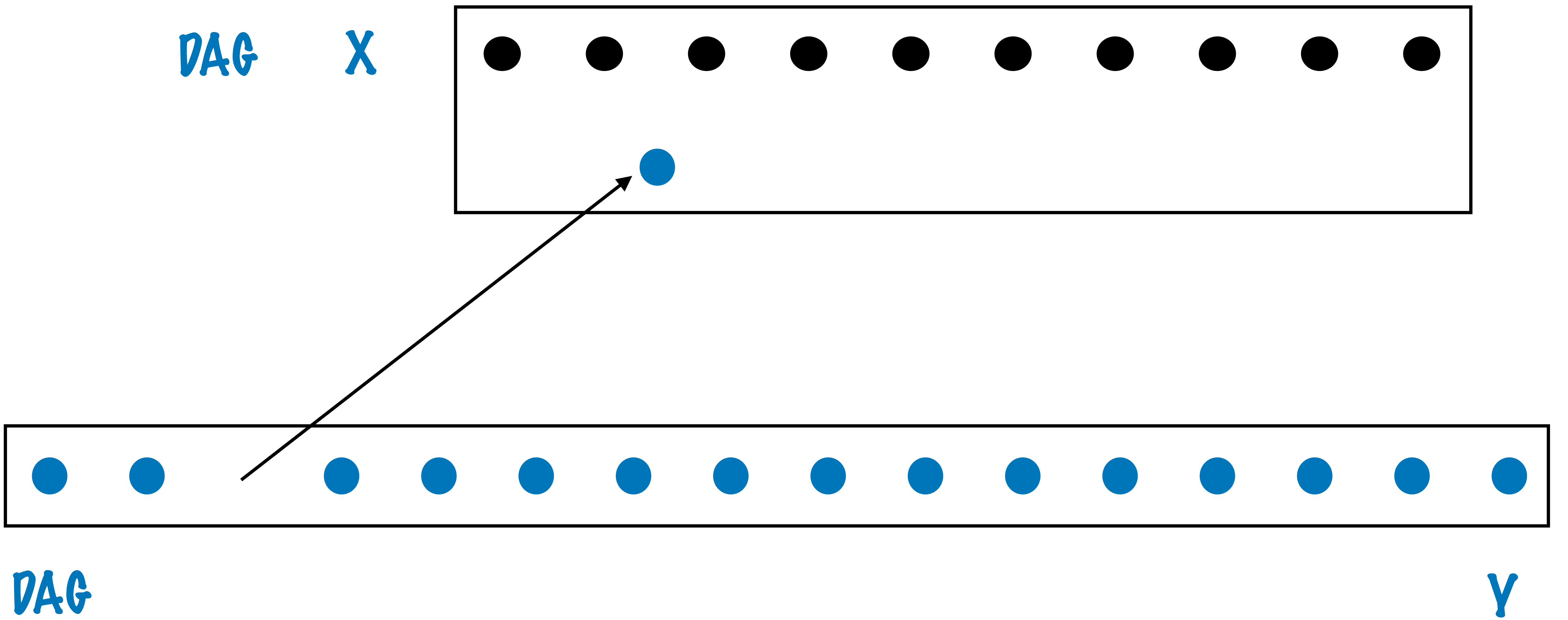
Disjoint Compression



Any triangle has two vertices from Y

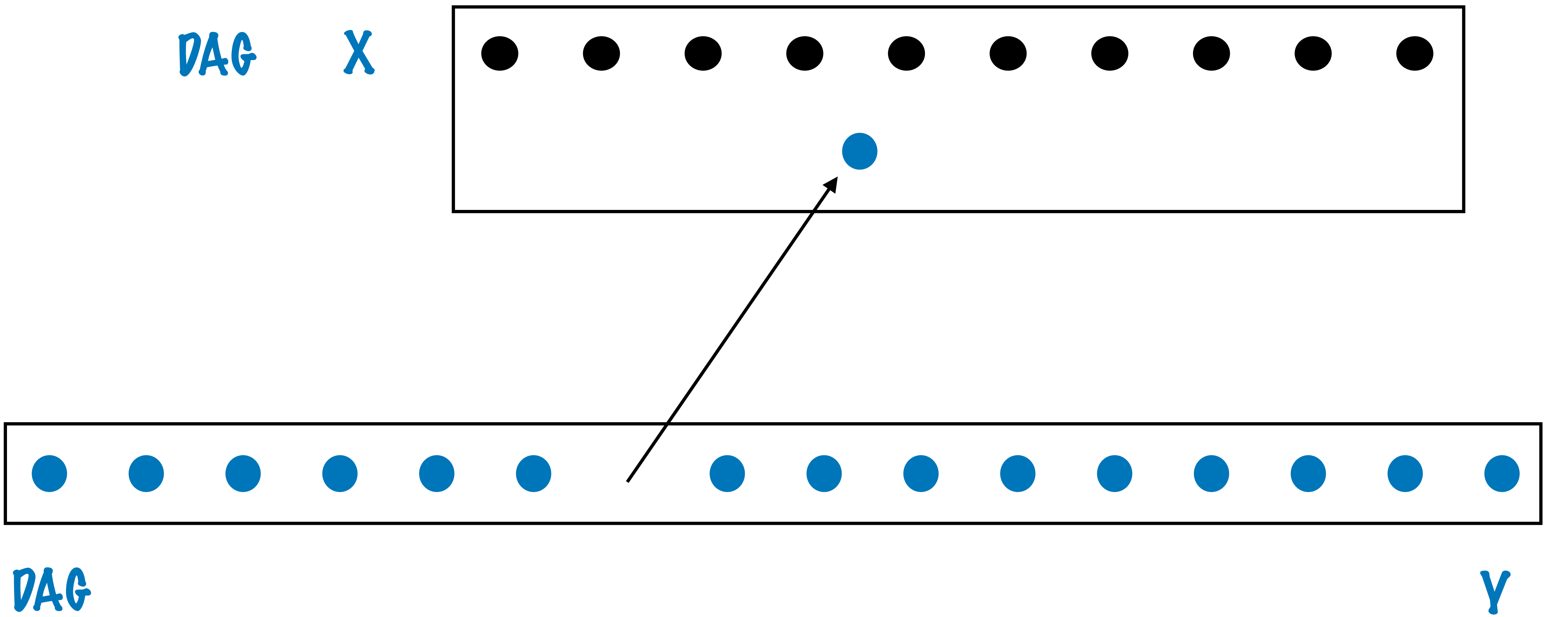
Feedback Vertex Set in Tournaments

Disjoint Compression



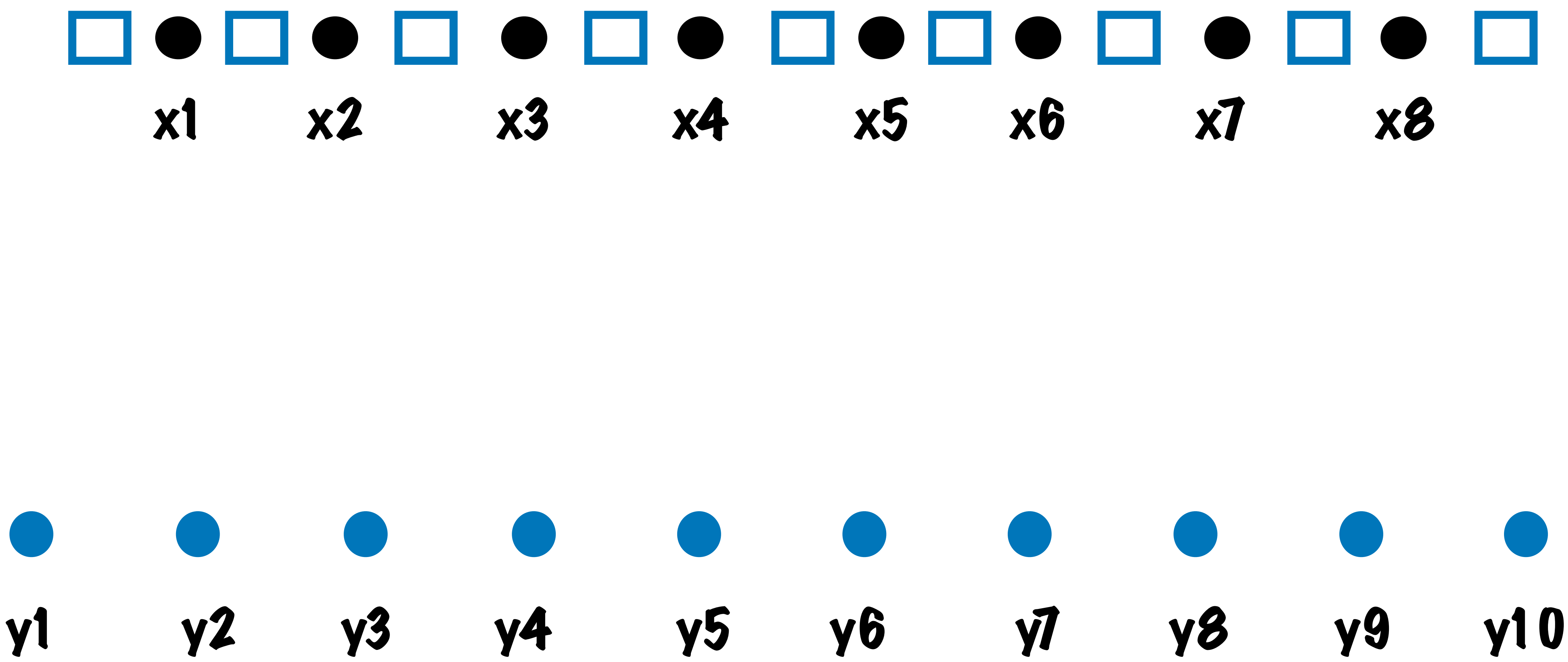
Feedback Vertex Set in Tournaments

Disjoint Compression



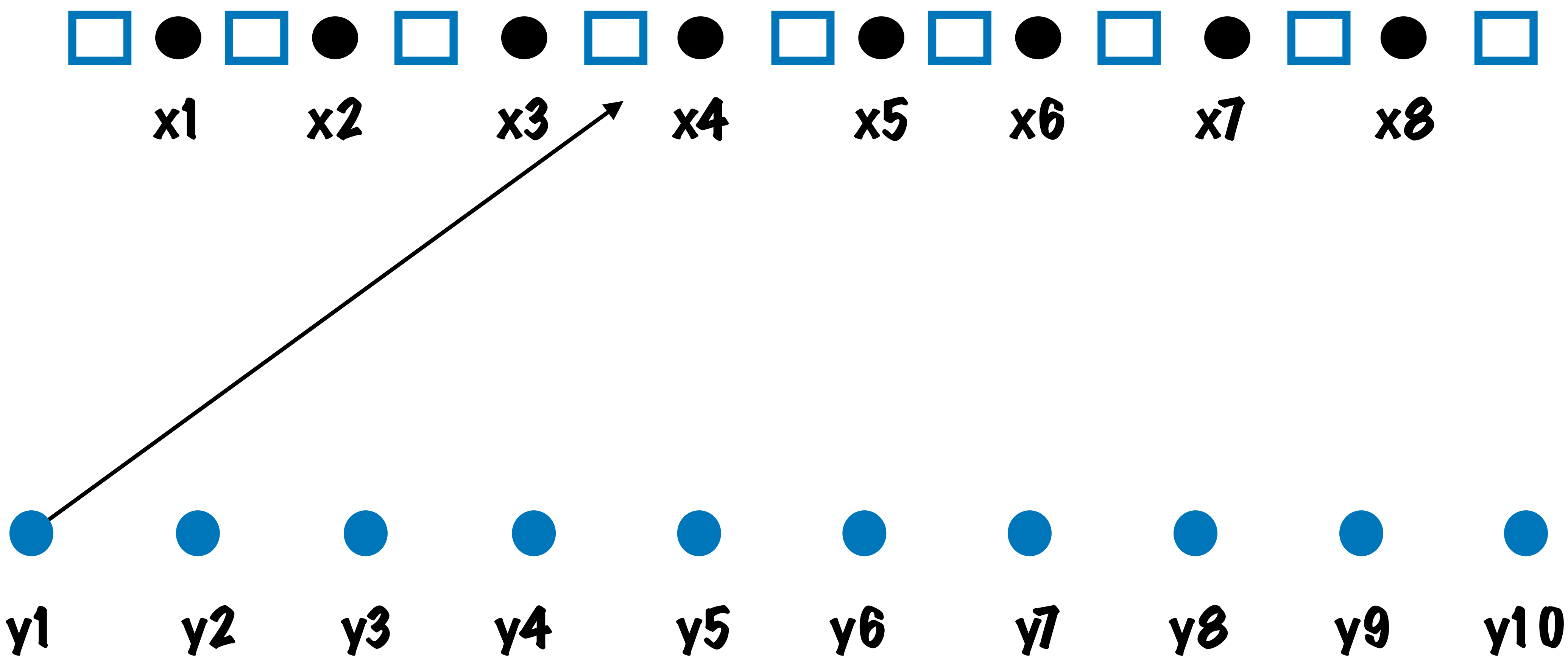
Feedback Vertex Set in Tournaments

Disjoint Compression



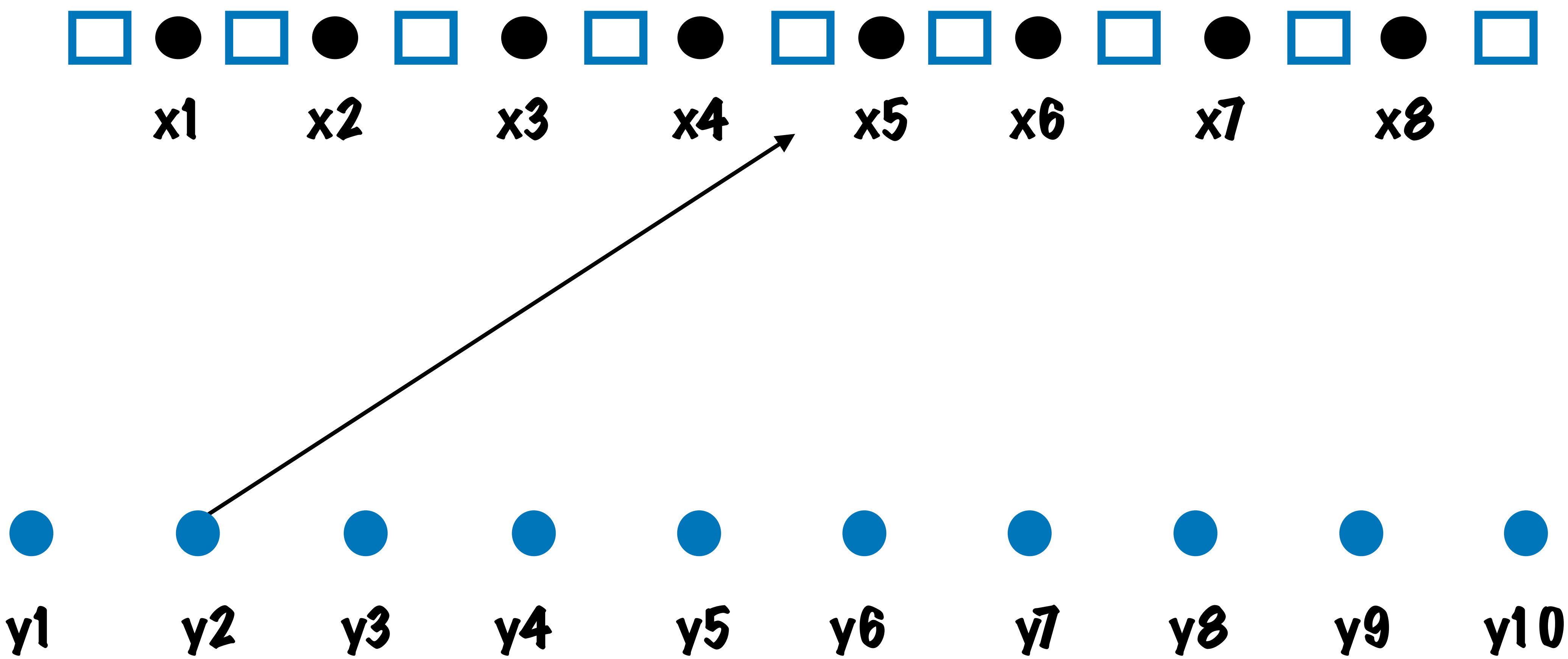
Feedback Vertex Set in Tournaments

Disjoint Compression



Feedback Vertex Set in Tournaments

Disjoint Compression



Feedback Vertex Set in Tournaments

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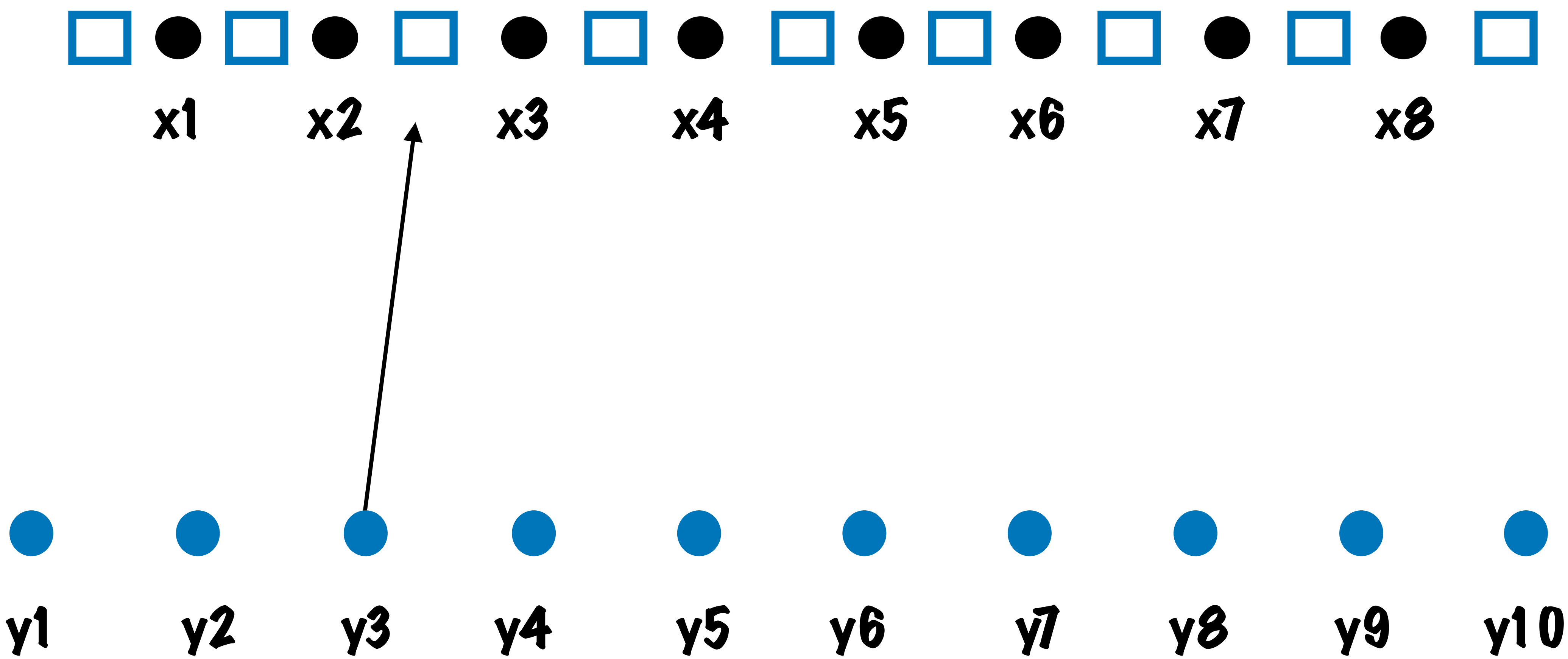

Feedback Vertex Set in Tournaments

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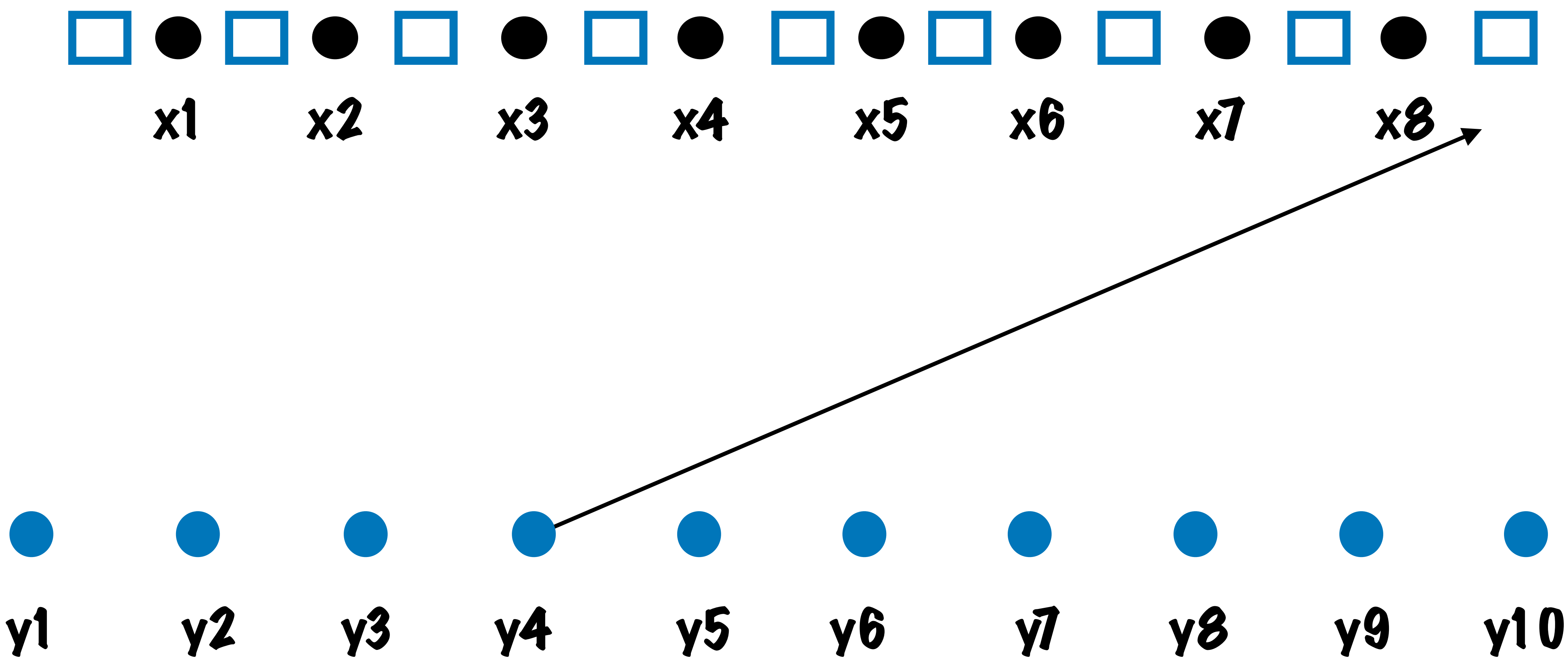

Feedback Vertex Set in Tournaments

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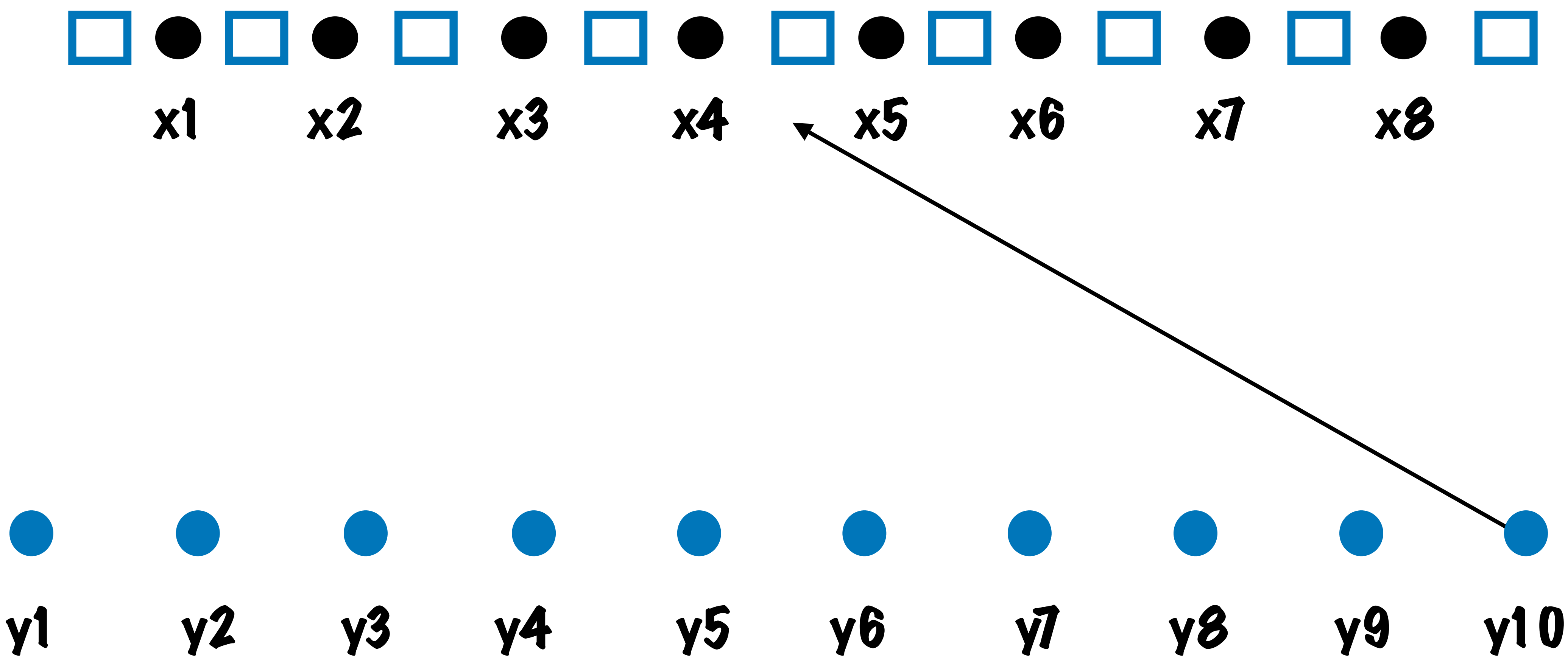

Feedback Vertex Set in Tournaments

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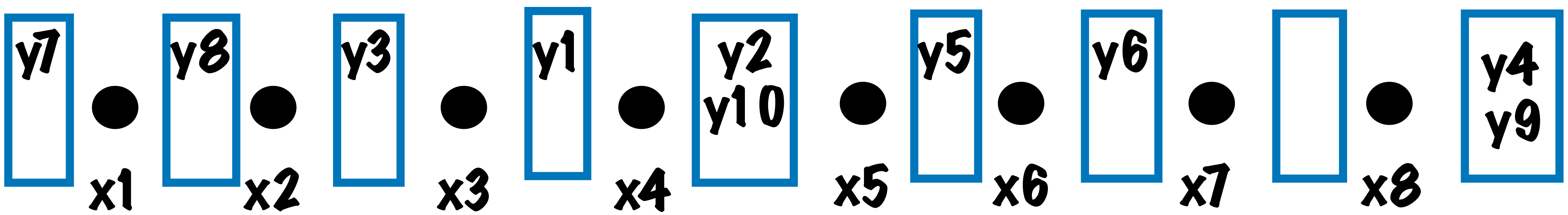

Feedback Vertex Set in Tournaments

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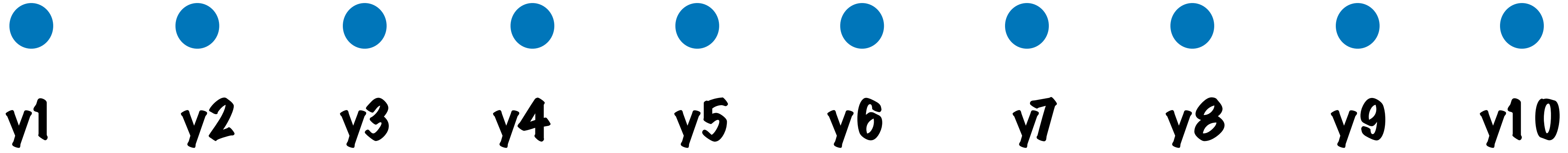

Feedback Vertex Set in Tournaments

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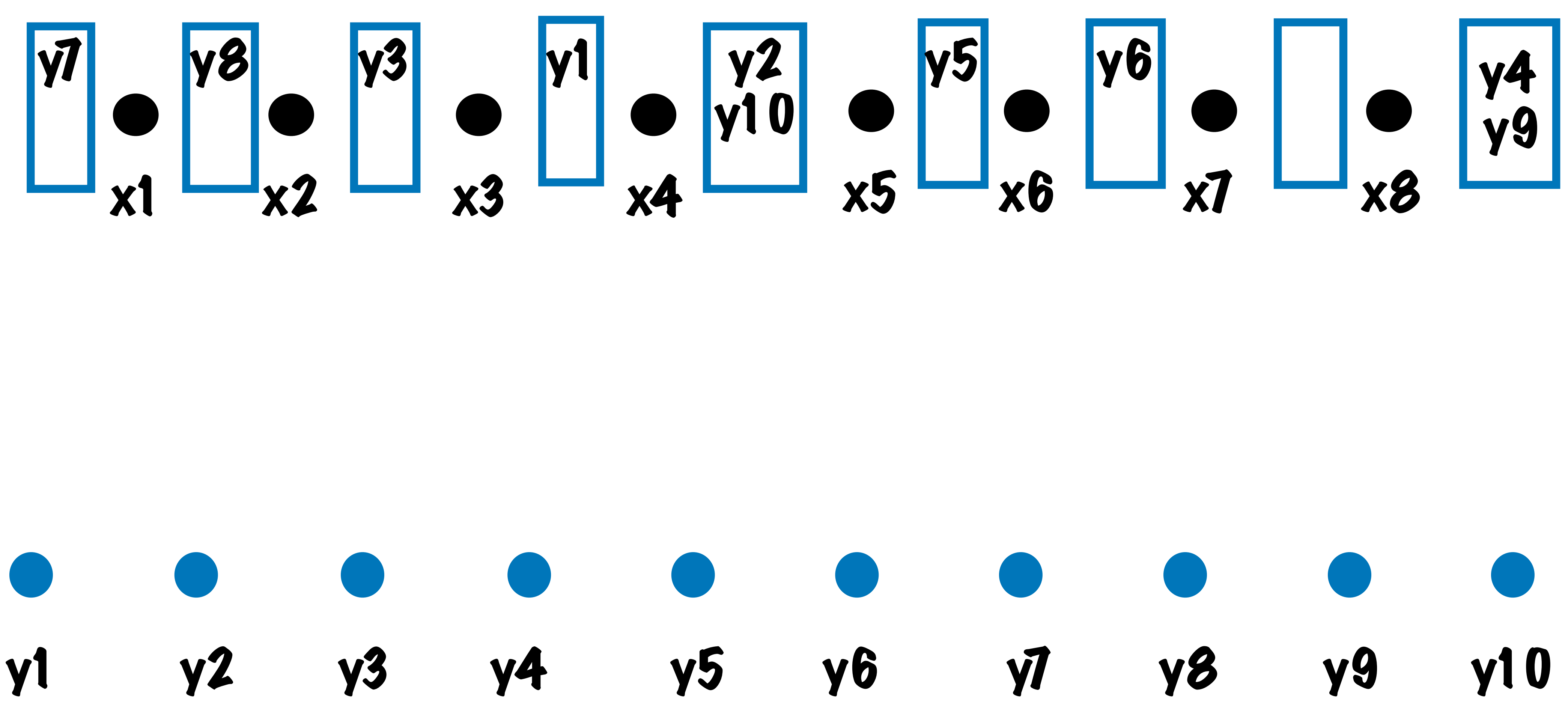
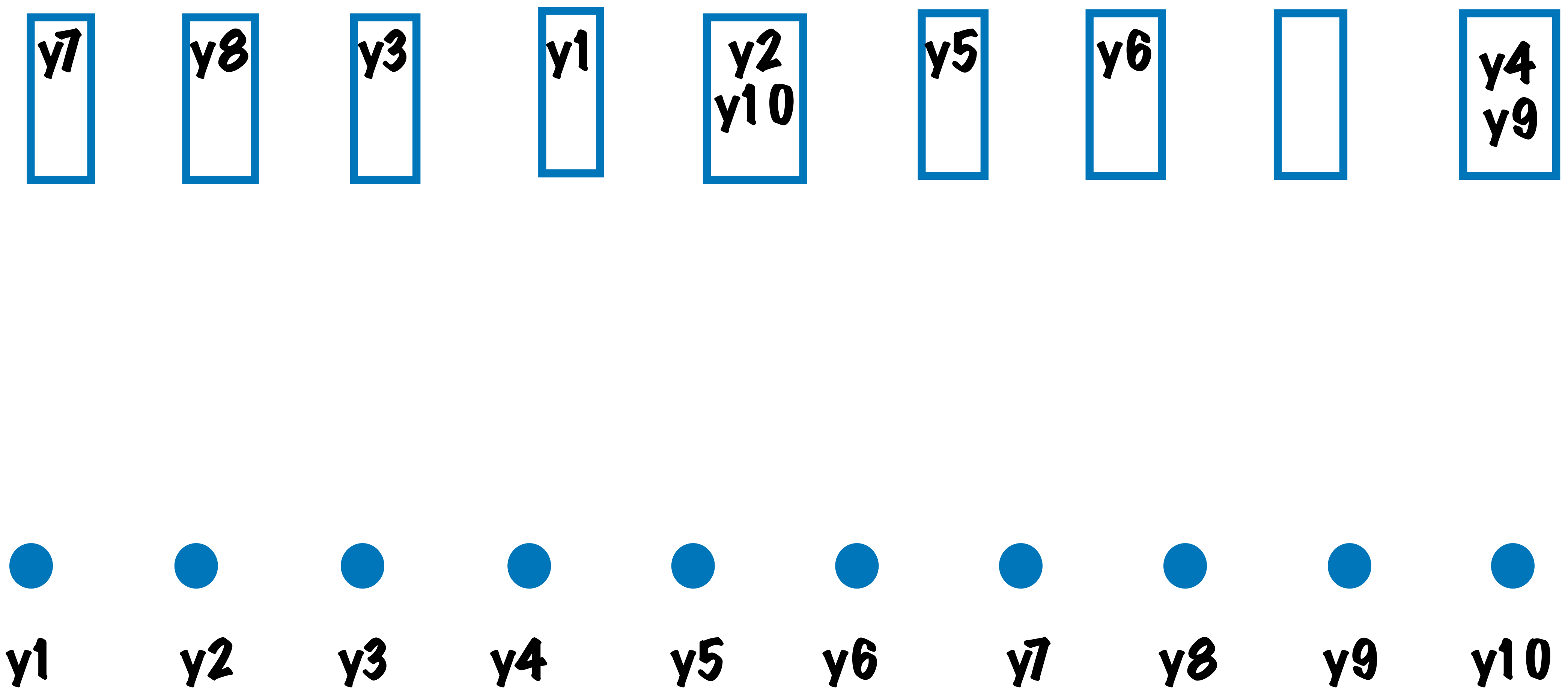


Feedback Vertex Set in Tournaments

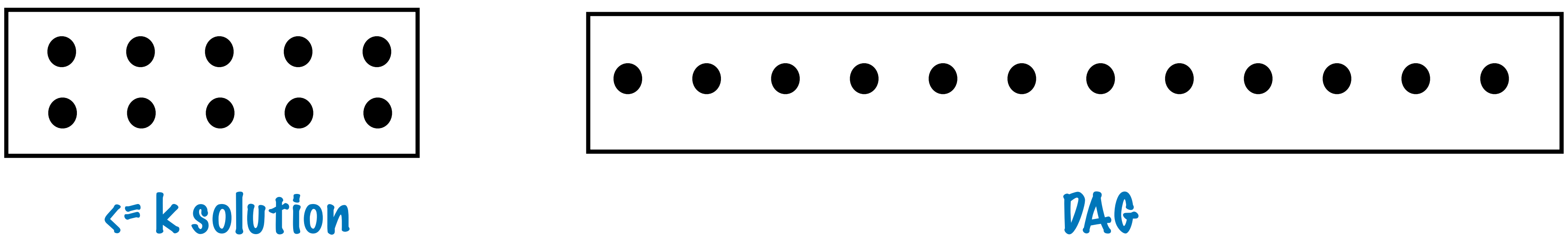
Disjoint Compression

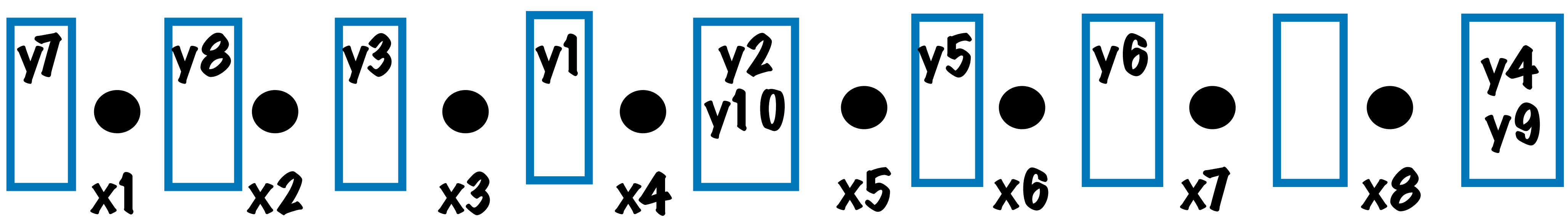


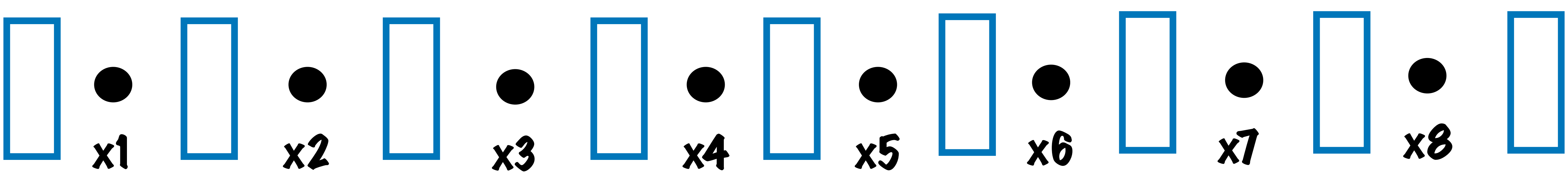
Feedback Vertex Set in Tournaments

Disjoint Compression

Consider solution DAG's topological order







Feedback Vertex Set in Tournaments

Disjoint Compression

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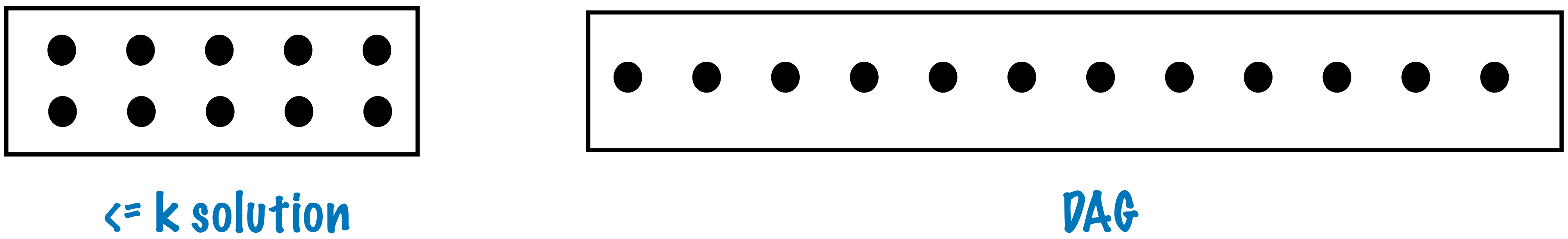

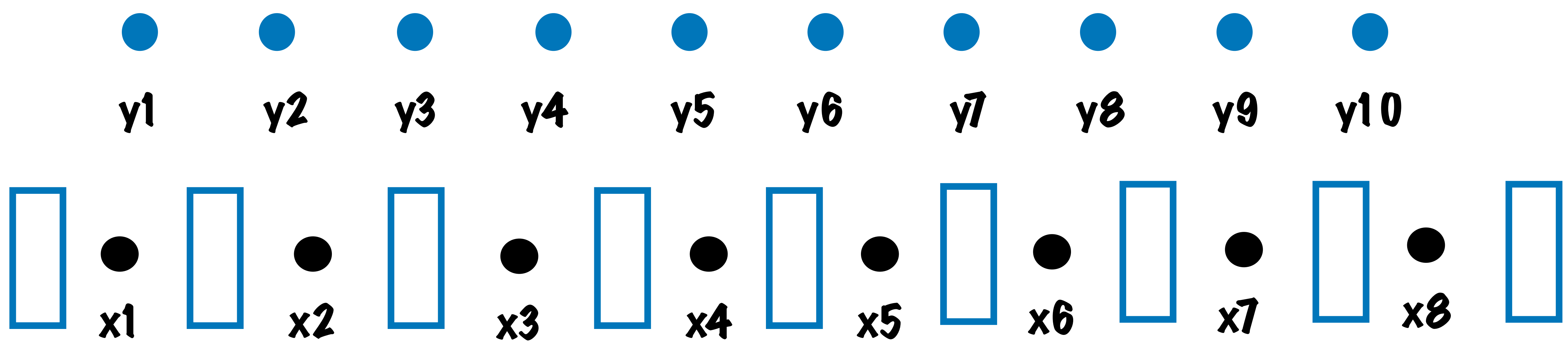

Feedback Vertex Set in Tournaments

Disjoint Compression

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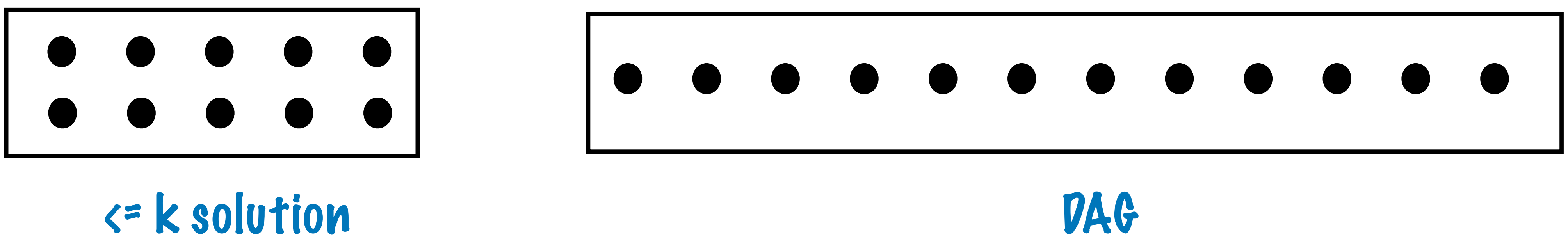
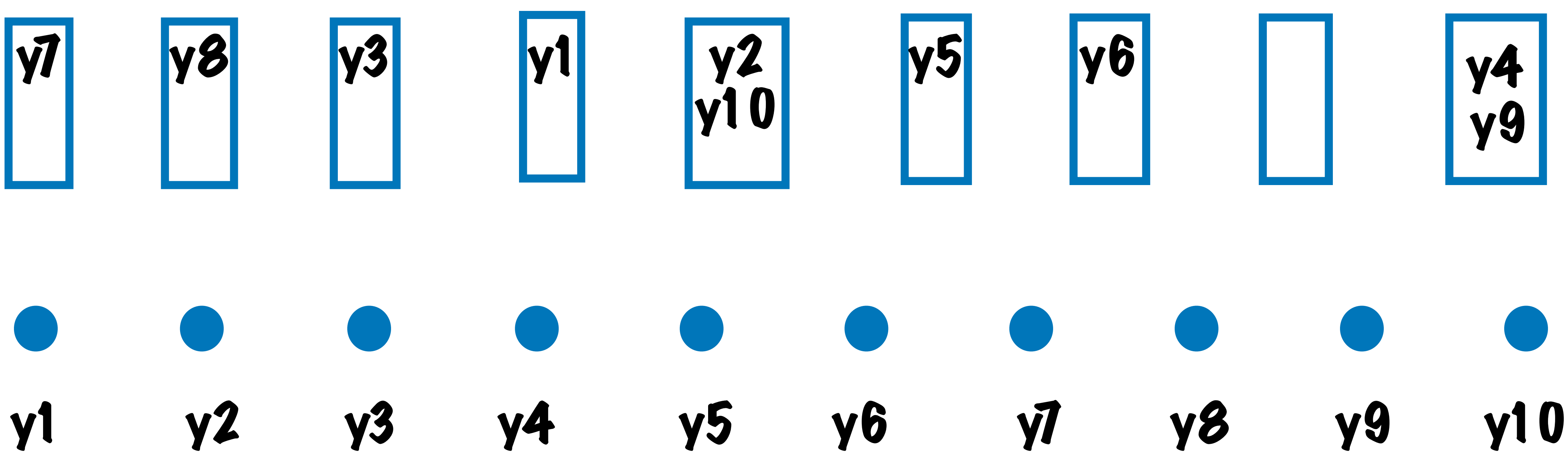
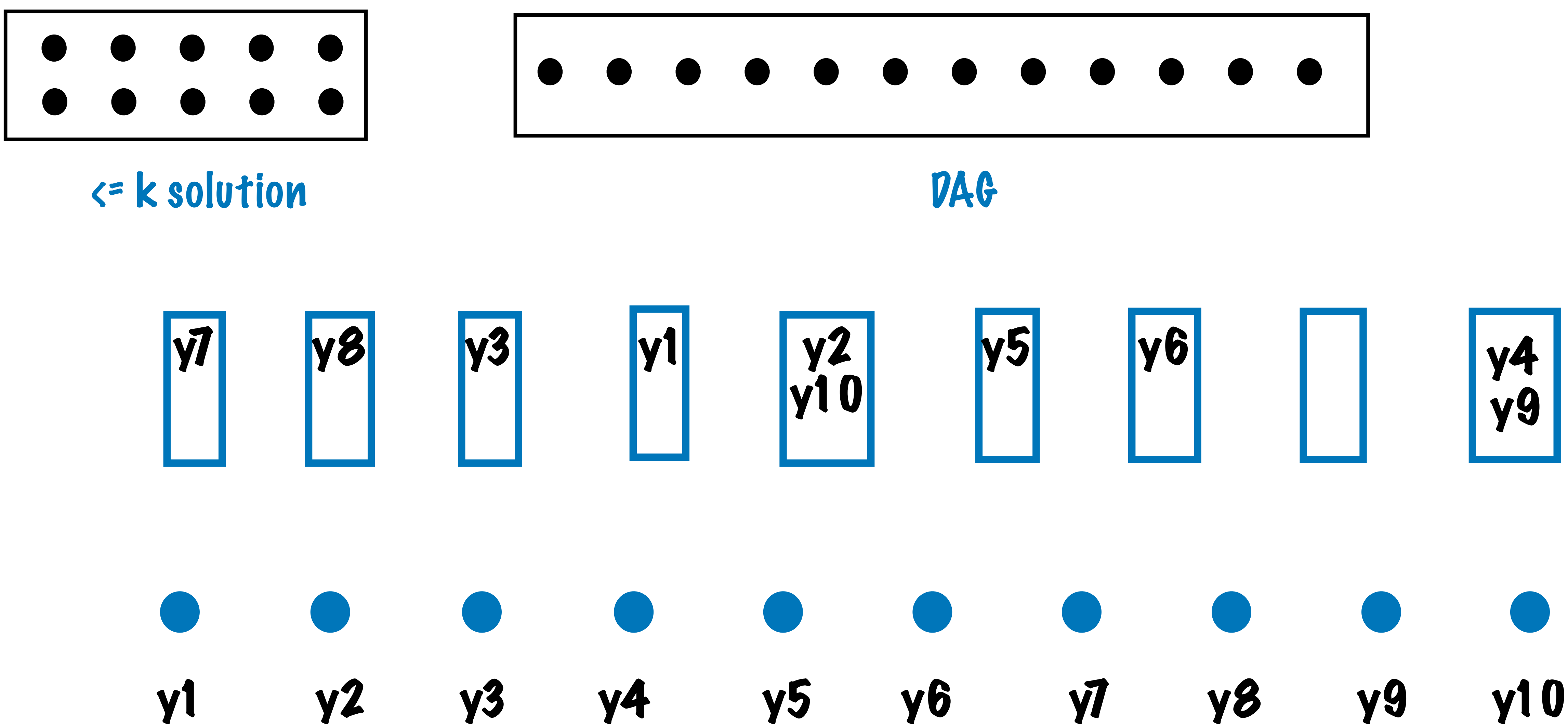




Feedback Vertex Set in Tournaments

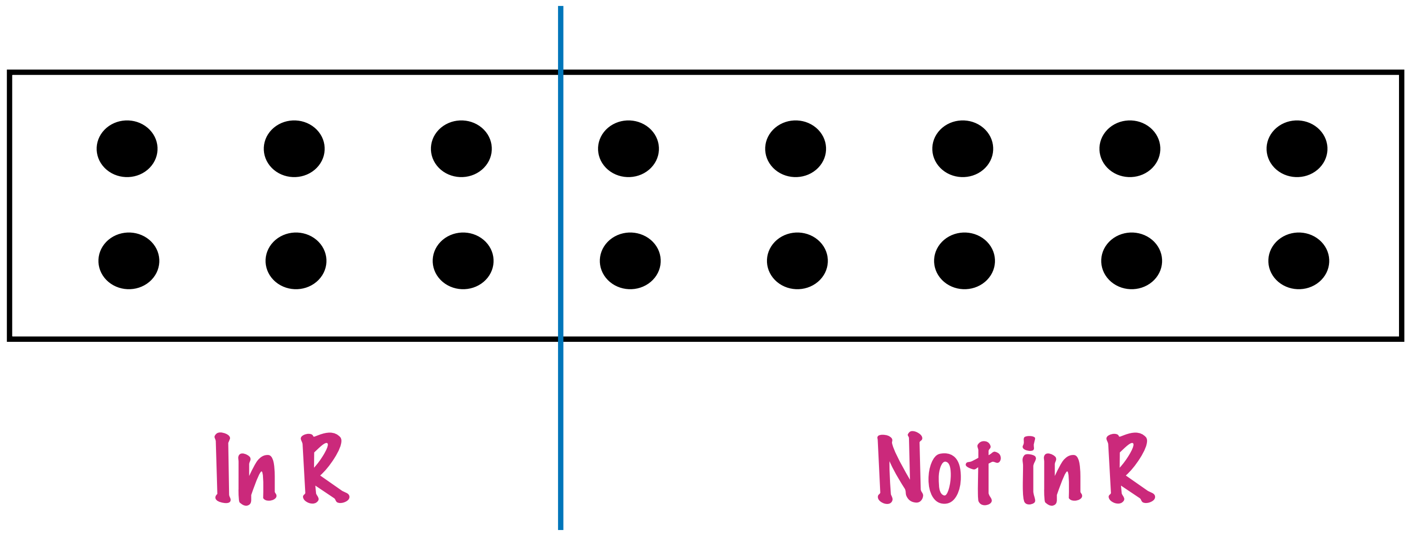
Disjoint Compression

Consider solution DAG's topological order



Find longest common subsequence

Feedback Vertex Set in Tournaments



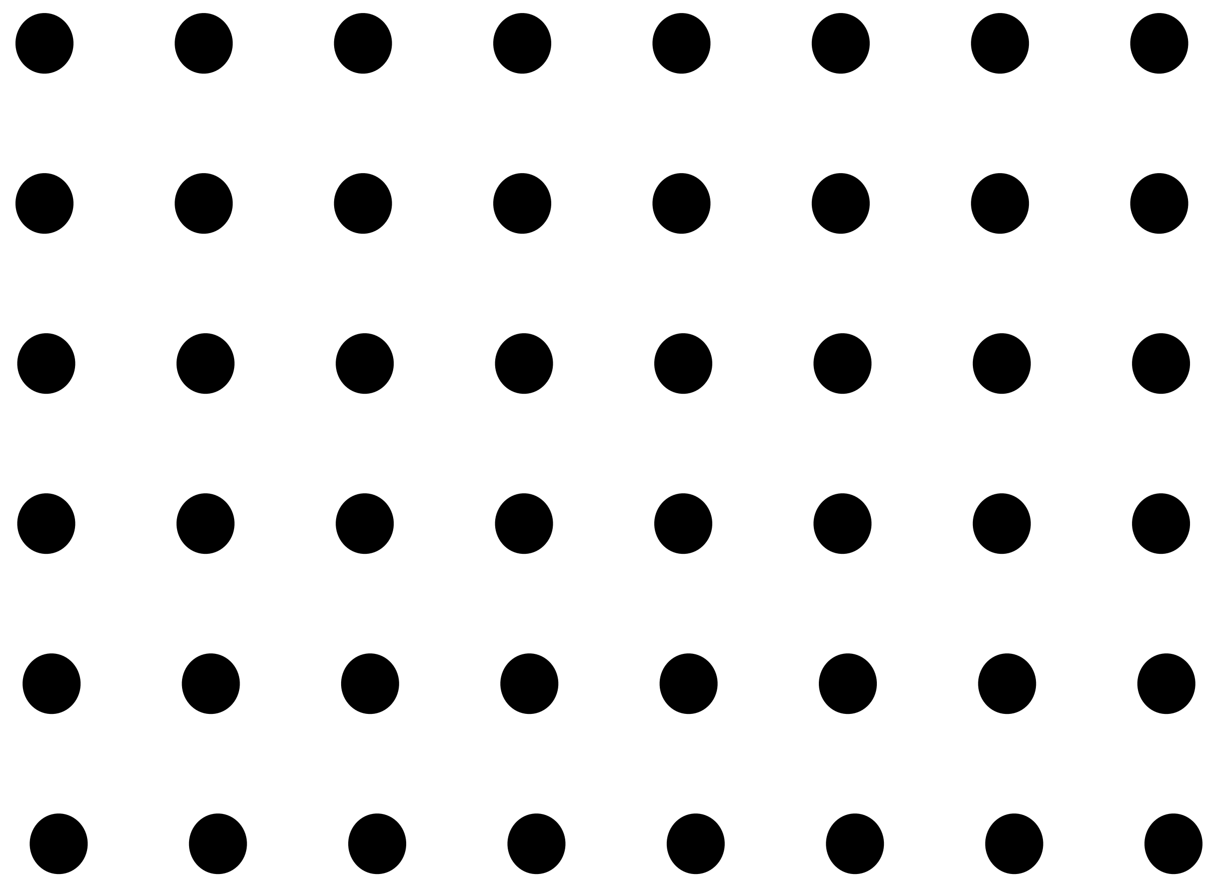
(k+1)-size solution S

* We want <= k size solution R
* Suppose we know $S \cap R$
  * We don't know $S \cap R$, guess! (2^{k+1} choices)
    * Solve Disjoint Compression
      * Longest Common Subsequence (in polynomial time)

$O^*(2^k)$ algorithm

Feedback Vertex Set in Tournaments

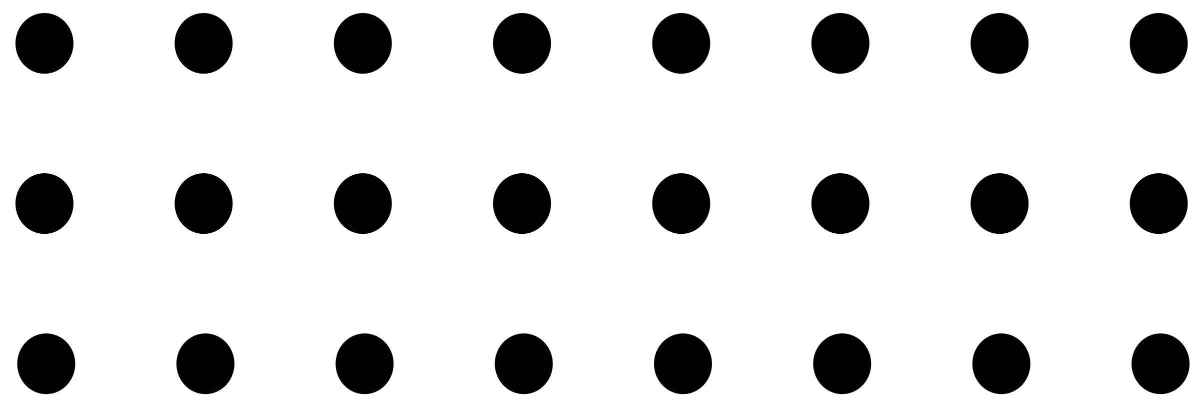
How to get a (k+1)-size solution S?

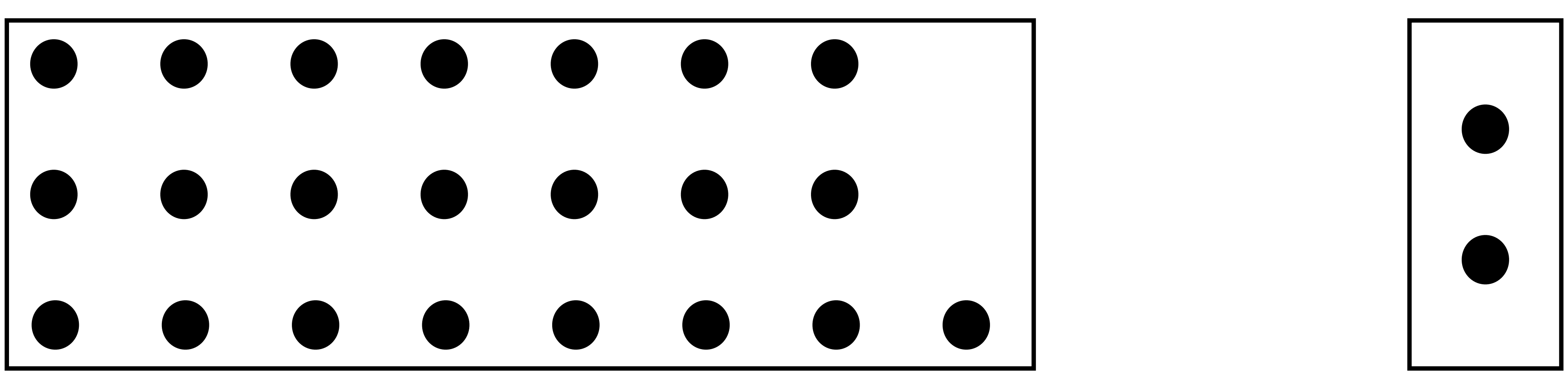


Tournament T

Feedback Vertex Set in Tournaments

Consider any k+3 vertices of T

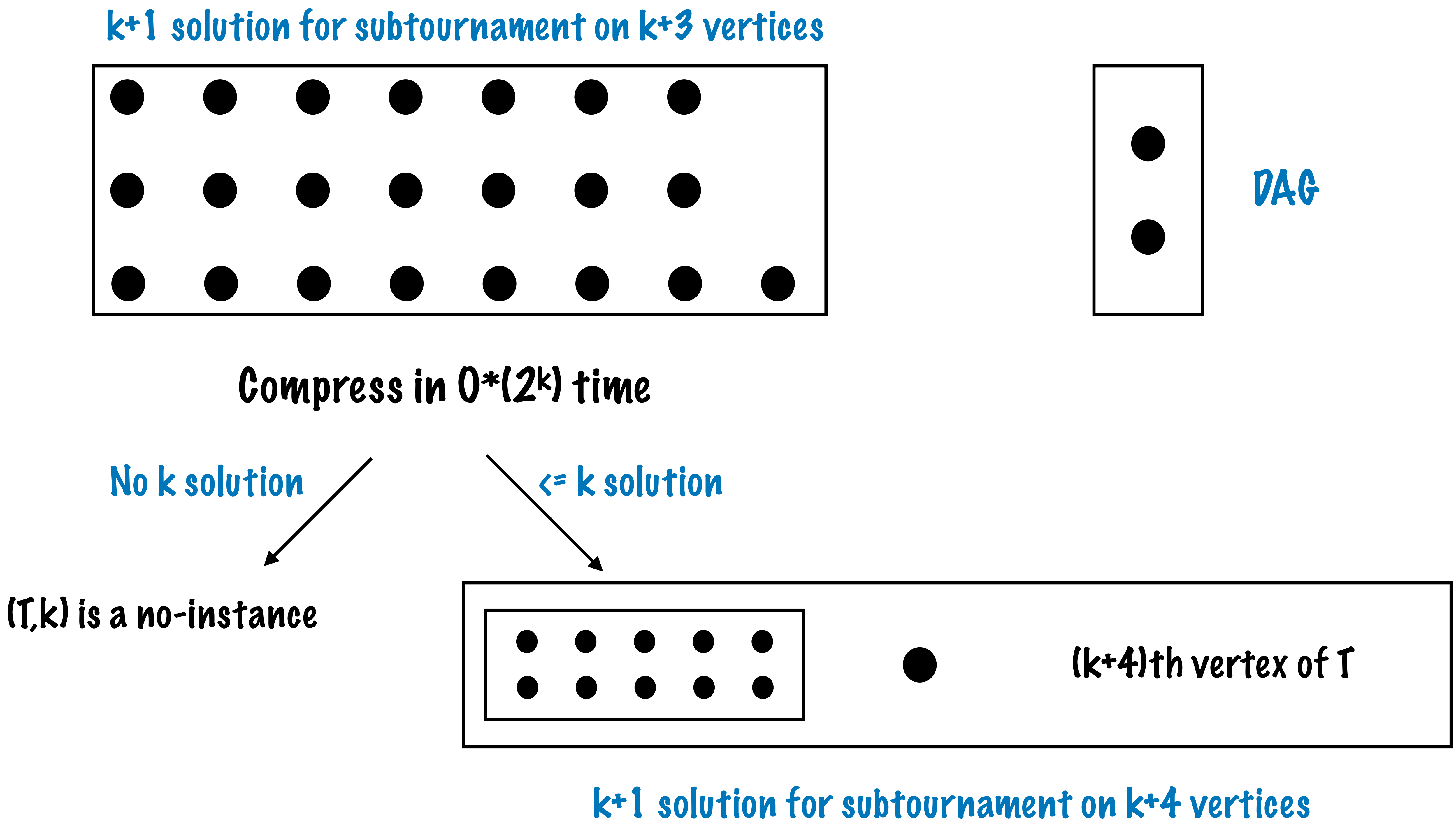




DAG

k+1 solution

Feedback Vertex Set in Tournaments



Iteratively Compress