

# CS3200: Computer Networks

## Lecture 9

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# Cyclic Redundancy Check (CRC)

- Sender and receiver must agree upon a **generator polynomial**,  $G(x)$ , in advance.
- Both the high- and low-order bits of the generator must be 1.
- To compute the CRC for some frame with  $m$  bits corresponding to the polynomial  $M(x)$ , the frame must be longer than the generator polynomial.
- The idea is to append a CRC to the end of the frame in such a way that the polynomial represented by the checksummed frame is divisible by  $G(x)$ .

# Cyclic Redundancy Check (CRC)

## Algorithm for computing CRC

- 1 Let  $r$  be the degree of  $G(x)$ . Append  $r$  zero bits to the low-order end of the frame so it now contains  $m + r$  bits and corresponds to the polynomial  $x^r M(x)$ .
- 2 Divide the bit string corresponding to  $G(x)$  into the bit string corresponding to  $x^r M(x)$ , using modulo 2 division.
- 3 Subtract the remainder (which is always  $r$  or fewer bits) from the bit string corresponding to  $x^r M(x)$  using modulo 2 subtraction. The result is the checksummed frame to be transmitted. Call its polynomial  $T(x)$ .

# Cyclic Redundancy Check (CRC)

- Why show the low-order bits of  $G(x)$  be 1?
- Why do we consider  $x^r M(x)$  instead of  $M(x)$ ?
- What kind of errors will be detected?

# CRC Error Detection

- Imagine that a transmission error occurs, so that instead of the bit string for  $T(x)$  arriving,  $T(x) + E(x)$  arrives.
- Each 1 bit in  $E(x)$  corresponds to a bit that has been inverted.
- Upon receiving the checksummed frame, the receiver divides it by  $G(x)$ ; that is, it computes  $[T(x) + E(x)]/G(x)$ .
- $T(x)/G(x)$  is 0, so the result of the computation is simply  $E(x)/G(x)$ .

# CRC Error Detection

- Suppose the  $i^{\text{th}}$  bit was received in error. Then,  $E(x) = x^i$ .
- When will this be detected?
- What about two isolated single-bit errors, i.e.,  $E(x) = x^i + x^j$ , where  $i > j$ ?
- For example,  $x^{15} + x^{14} + 1$  will not divide  $x^k + 1$  for any value of  $k$  below 32,768.

# CRC Error Detection

- What about odd number of errors? Then,  $E(X)$  contains an odd number of terms (e.g.,  $x^5 + x^2 + 1$ )?
- Interestingly, no polynomial with an odd number of terms has  $x + 1$  as a factor in the modulo 2 system.
- What about burst errors?
- A burst error of length  $k$  can be represented by  $x^i(x^{k-1} + \dots + 1)$ .
- Can detect burst errors of length  $\leq r$ , where  $r$  is the degree of  $G(x)$ .