

CS 5003: Parameterized Algorithms

Lecture 16

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Reference Books: Parameterized Algorithms by Cygan et al. and Kernelization by Fomin et al.

Vertex Cover Above LP

Vertex Cover Above LP

Instance: A graph G on n vertices m edges and integer k

Question: Does G have a vertex cover of size at most k ?

Parameter: $k - \text{lp}(G)$

$$\text{lp}(G) \geq |M|$$

Vertex Cover Above Matching

Instance: A graph G on n vertices m edges, integer k , a matching M

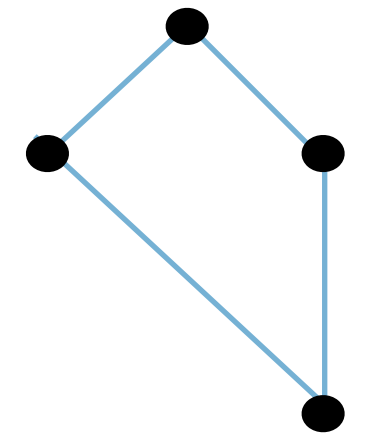
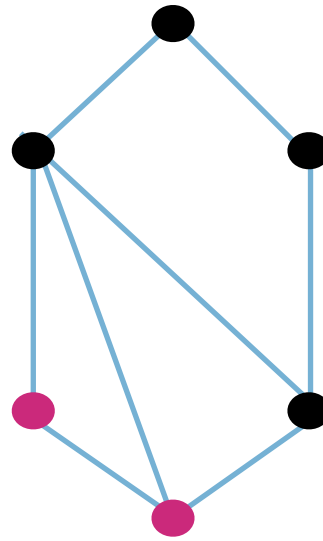
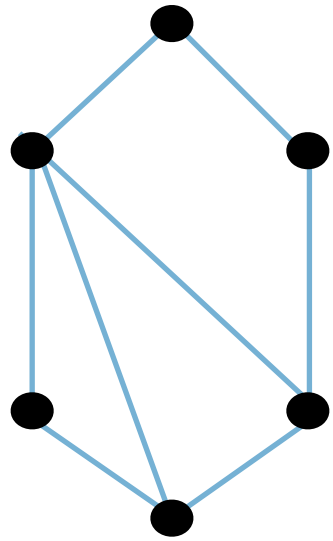
Question: Does G have a vertex cover of size at most k ?

Parameter: $k - |M|$

$4^{(k - \text{lp}(G))} n^{O(1)}$ time algorithm is a $4^{(k - |M|)} n^{O(1)}$ time algorithm

Odd Cycle Transversal

OCT - set of vertices that has at least one vertex of every odd length cycle



bipartite

A graph is bipartite iff it has no odd cycle

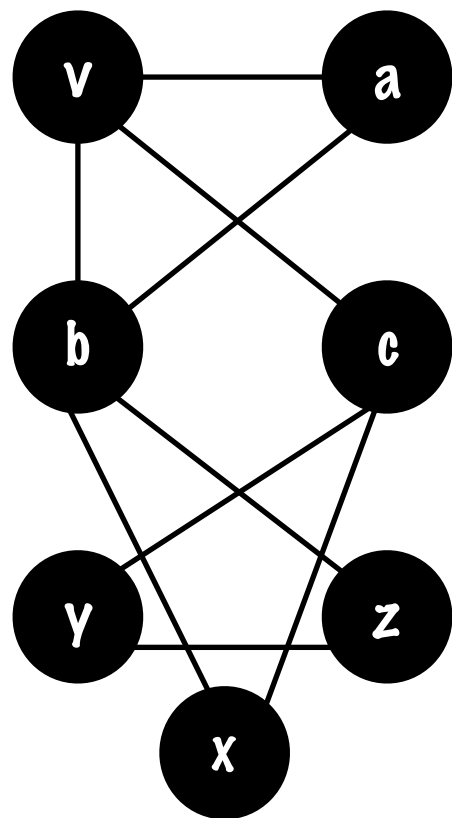
Odd Cycle Transversal

Instance: A graph G on n vertices m edges and integer k

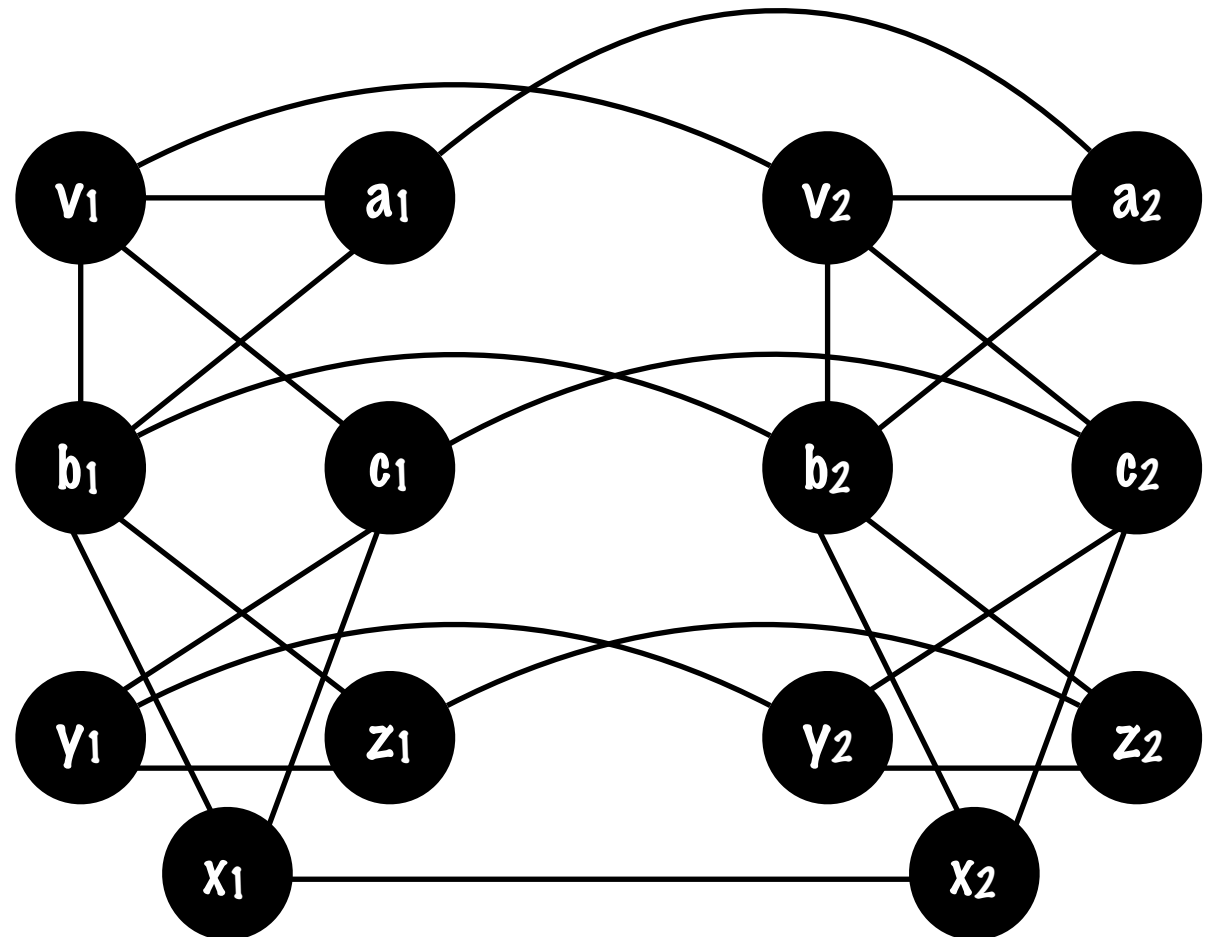
Question: Does G have an oct of size at most k ?

Parameter: k

OCT Reduces to VC Above LP



(G, k) OCT

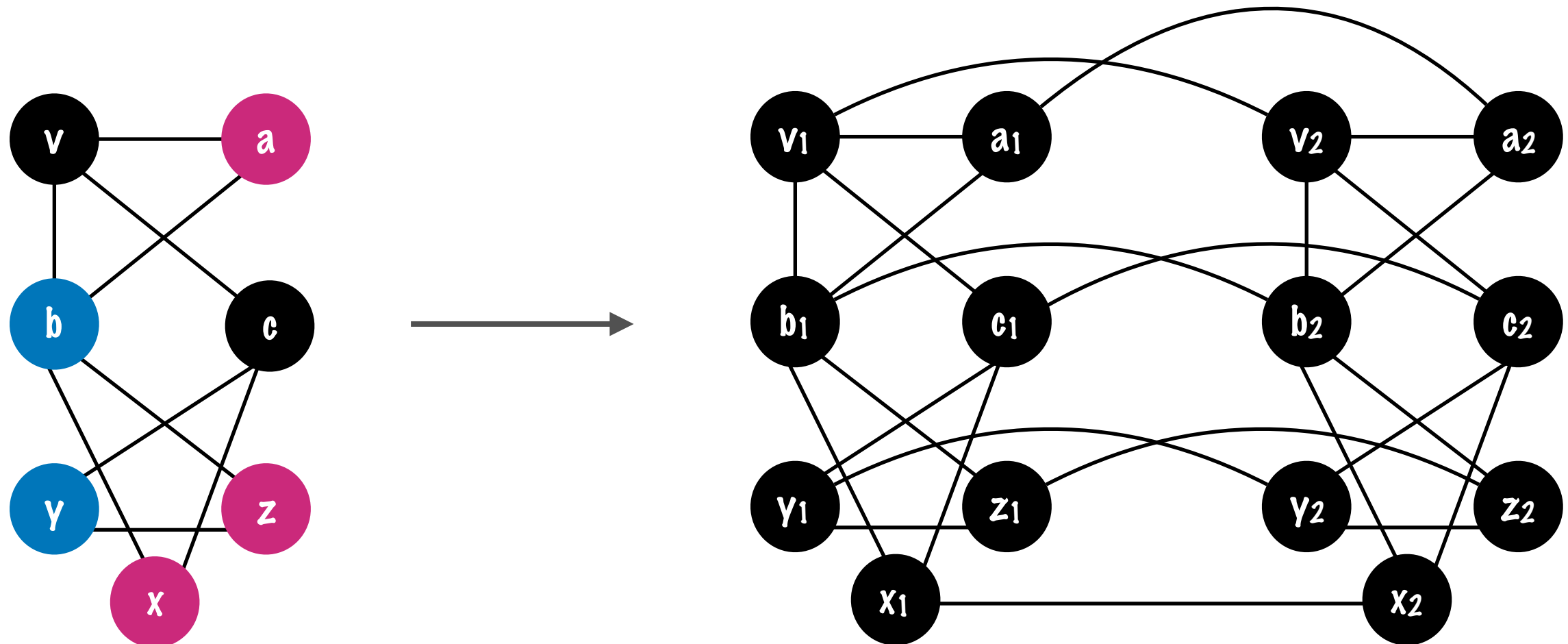


$(H, |V(G)|+k)$ Vertex Cover

G has an OCT of size k iff H has a VC of size $|V(G)|+k$

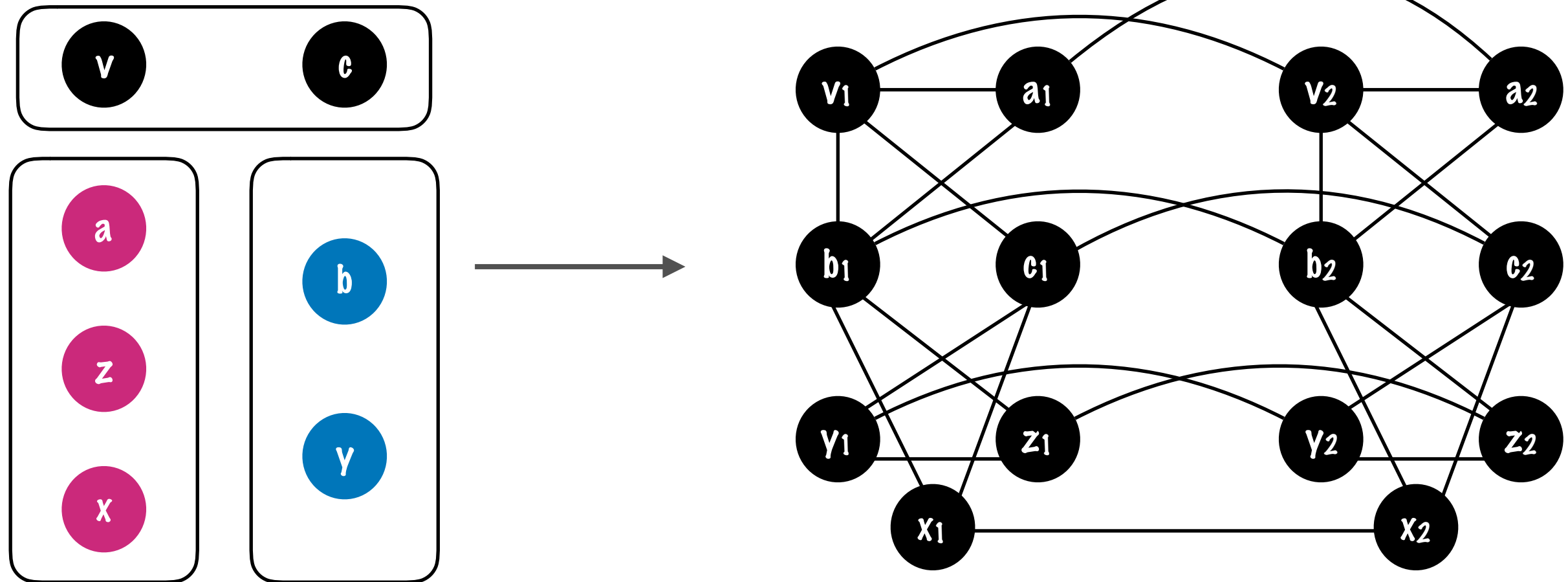
OCT Reduces to VC Above LP

Suppose G has an OCT of size k



OCT Reduces to VC Above LP

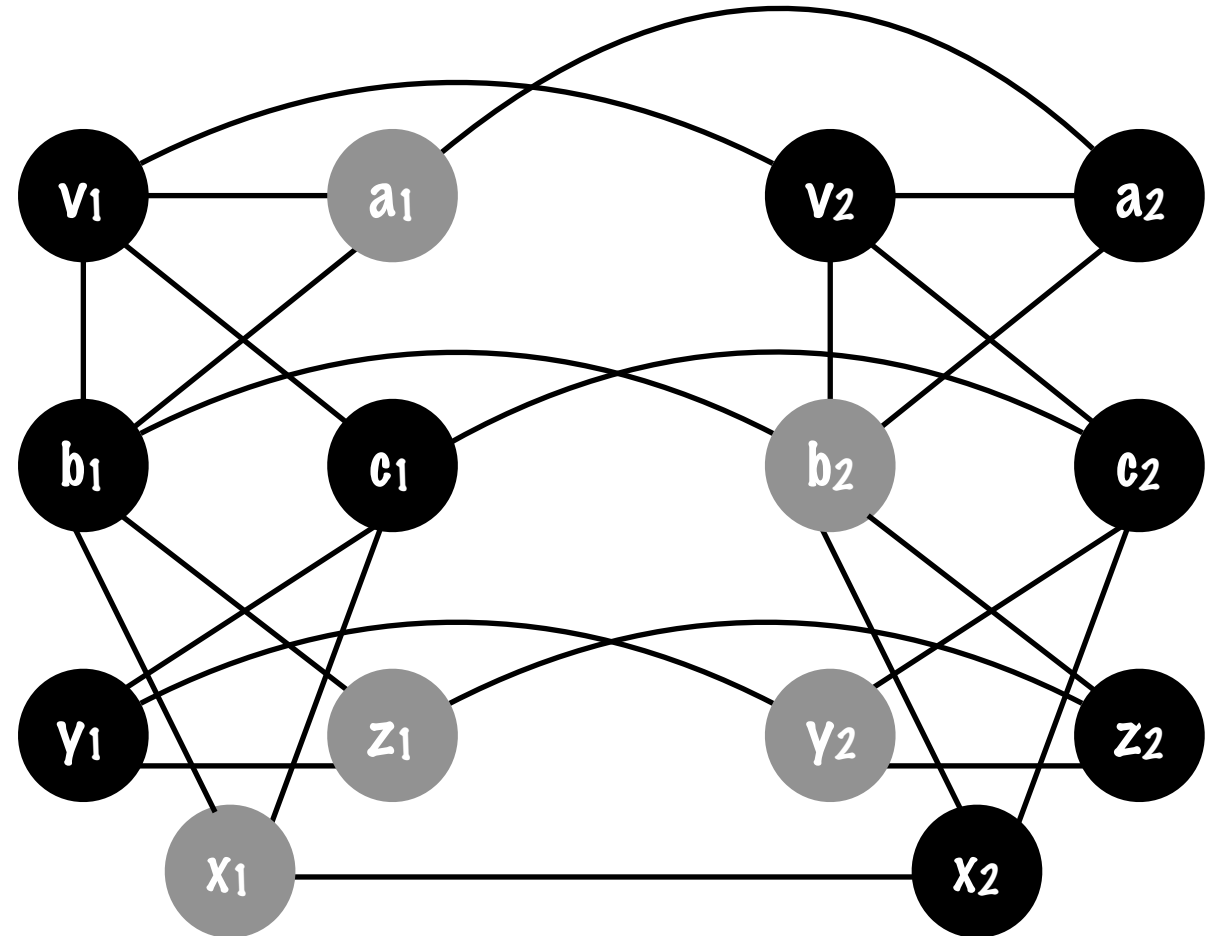
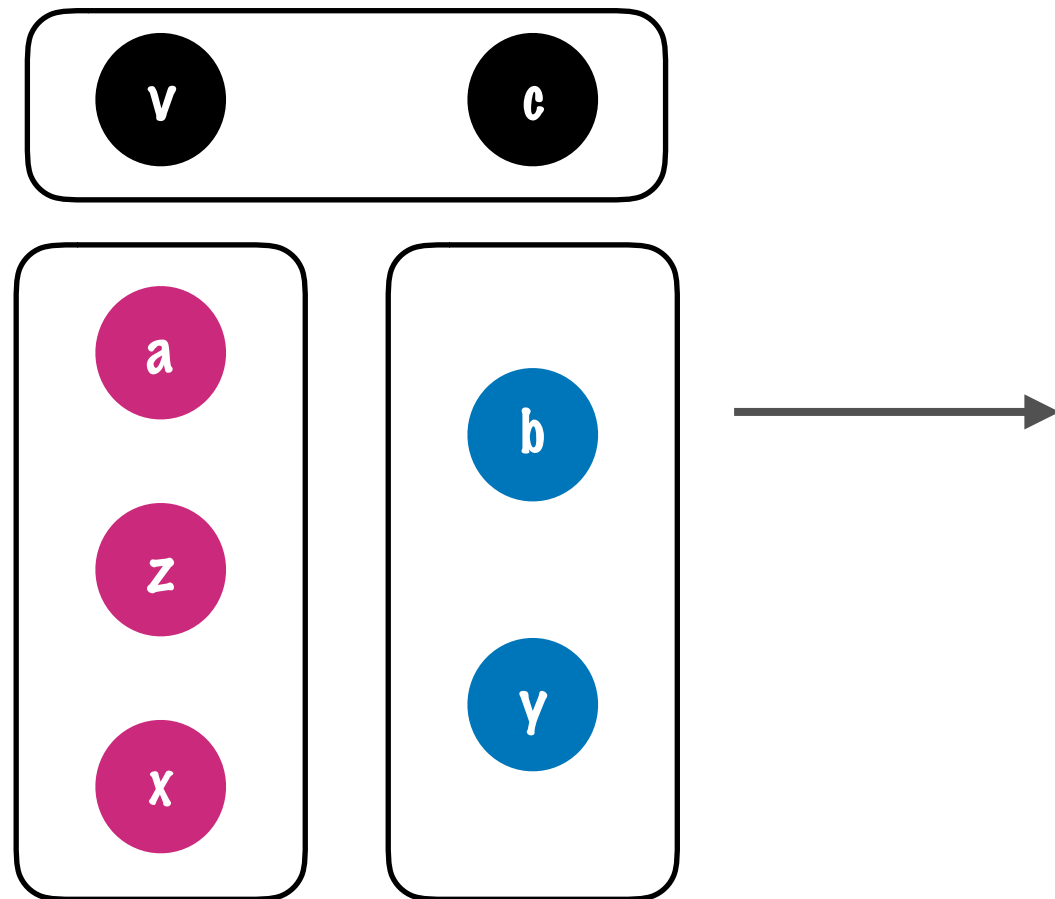
Suppose G has an OCT of size k



G has a bipartite graph of size $|V(G)| - k$

OCT Reduces to VC Above LP

Suppose G has an OCT of size k

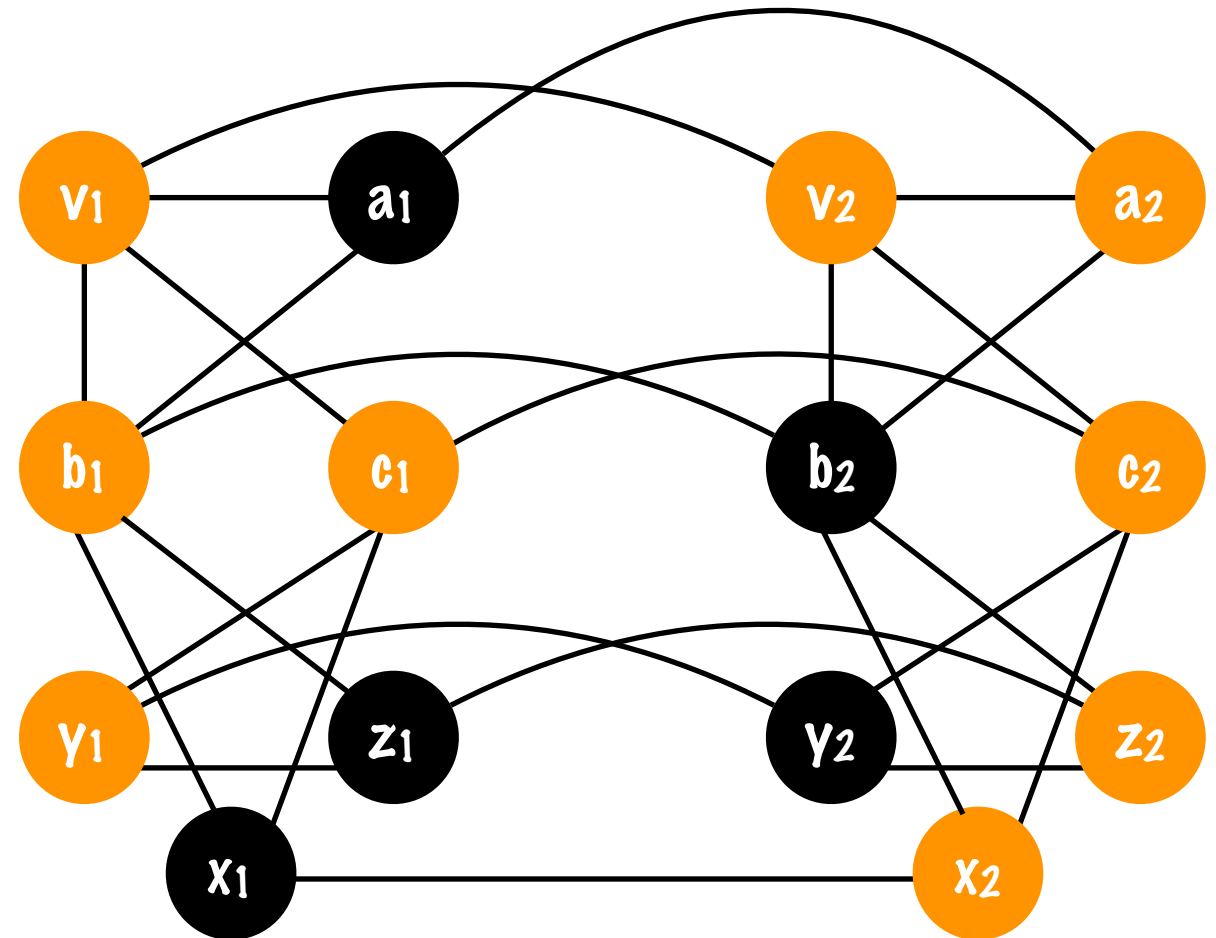
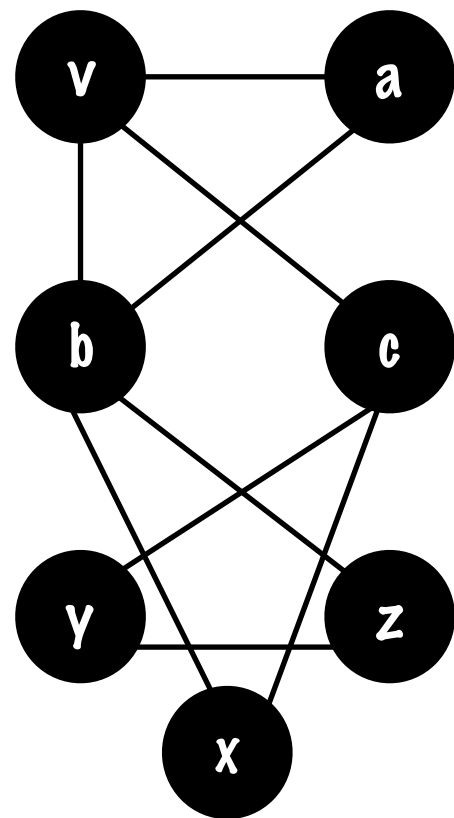


H has an IS of size $|V(G)| - k$

H has a VC of size $|V(G)| + k$

OCT Reduces to VC Above LP

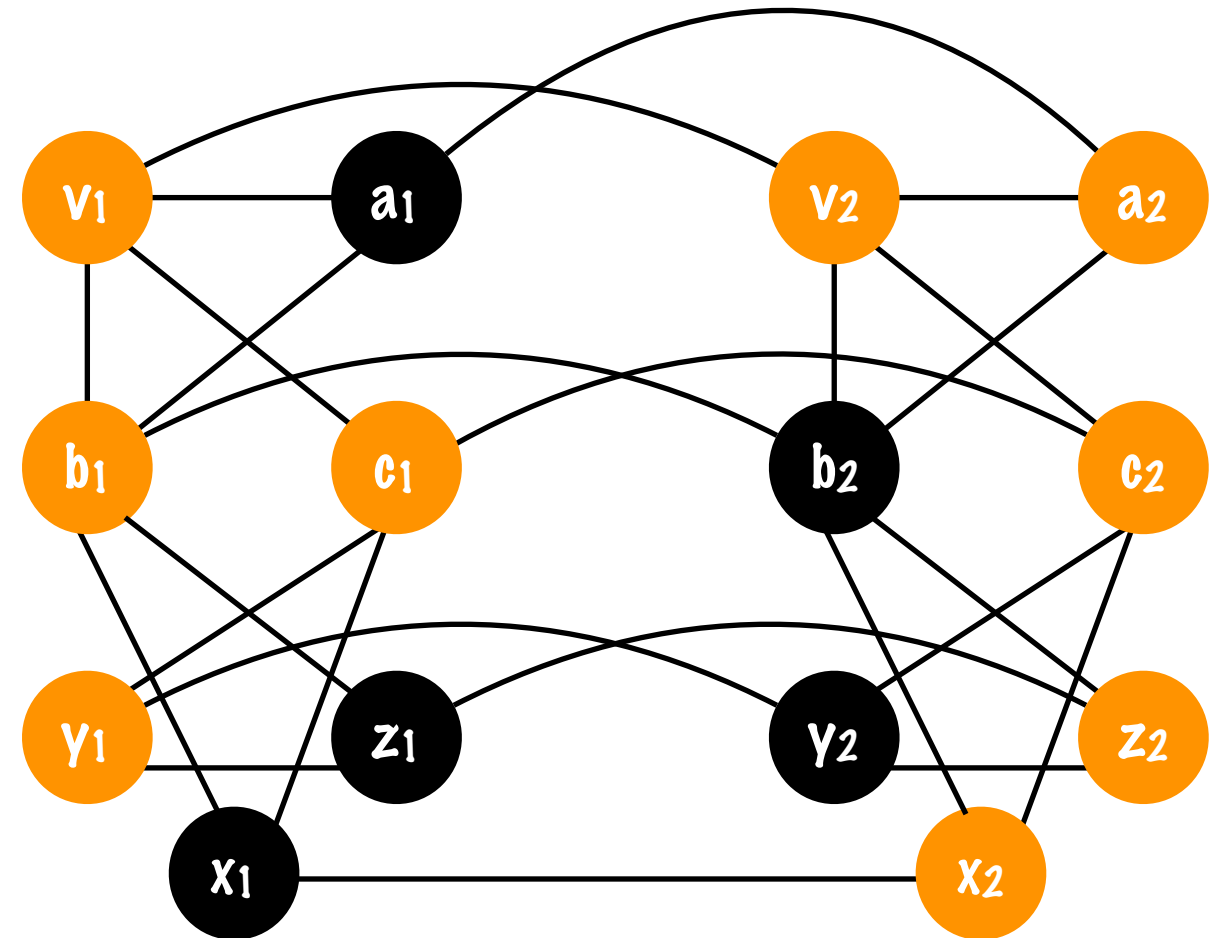
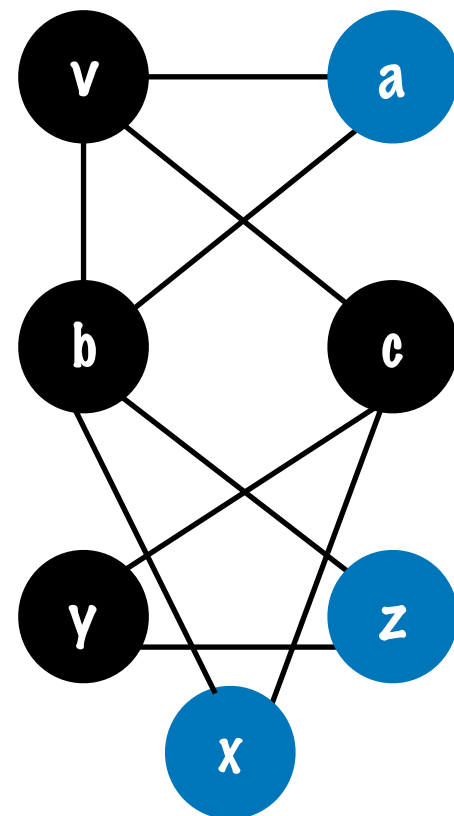
Suppose H has a VC of size $|V(G)|+k$



H has an IS of size $|V(G)|-k$

OCT Reduces to VC Above LP

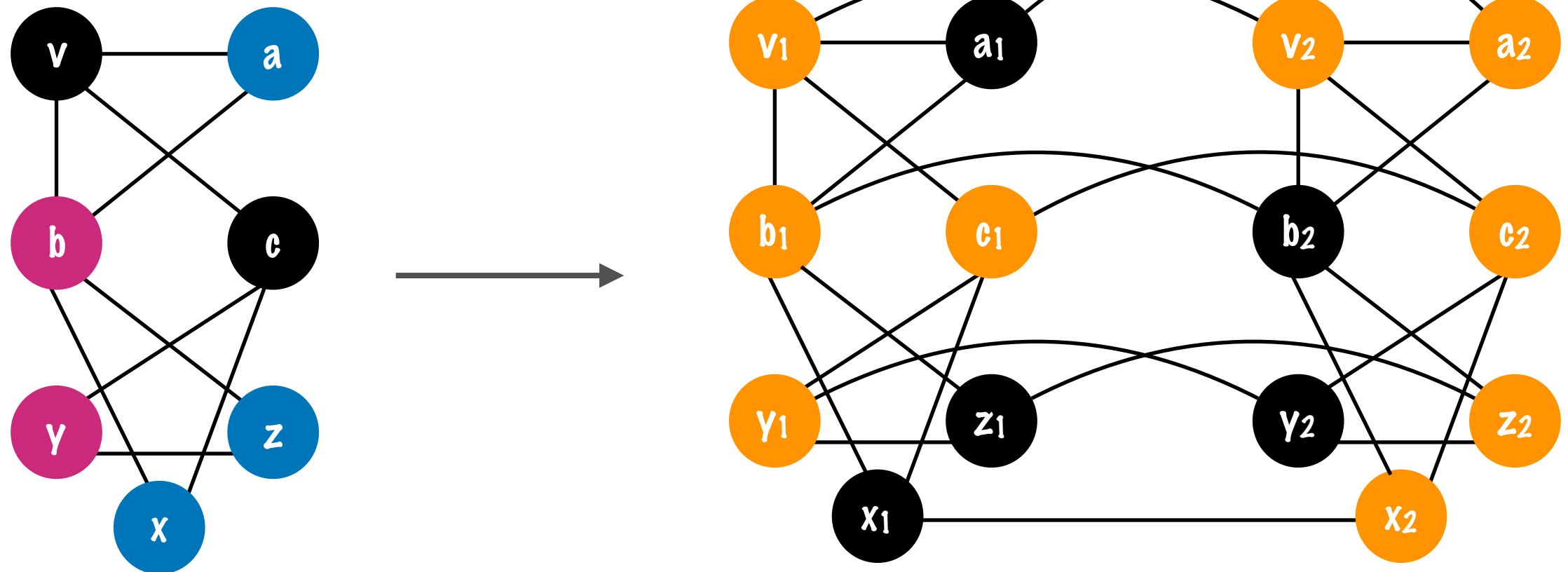
Suppose H has a VC of size $|V(G)|+k$



H has an IS of size $|V(G)|-k$

OCT Reduces to VC Above LP

Suppose H has a VC of size $|V(G)|+k$



G has a bipartite graph of size $|V(G)|-k$

G has an OCT of size k

OCT Reduces to VC Above LP



G has an OCT of size k iff H has a VC of size $|V(G)|+k$

- * To determine if G on n vertices has an OCT of size k ,
 - * Construct H from G
 - * Matching M of size n in H
 - * Determine if H has a VC of size $n+k$
 - * Use the Vertex Cover Above LP algorithm on H that has matching M
 - * $(4^{n+k-|popt(H)|}) n^{O(1)}$ time which is $4^{n+k-|M|=k} n^{O(1)}$ time)