CS 5003: Parameterized Algorithms

Lectures 5-7

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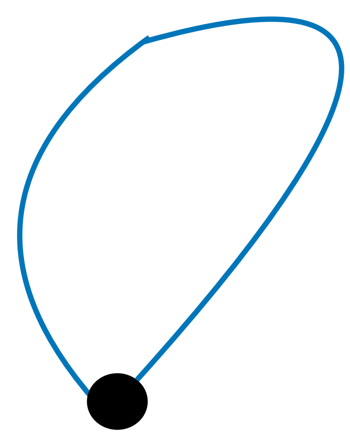
Reference Books: Parameterized Algorithms by Cygan et al. and Kernelization by Fomin et al.

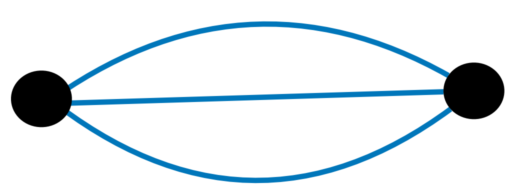
Feedback Vertex Set

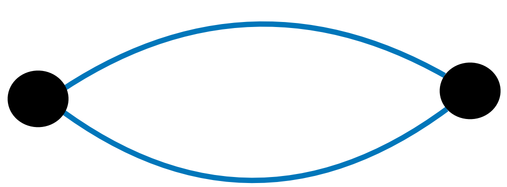
Assume graph is a multigraph

* **Reduction Rule 1:** Delete isolated vertices
* **Reduction Rule 2:** Delete degree-1 vertices
* **Reduction Rule 3:** If there is a loop at a vertex v, delete v from the graph and reduce the parameter by 1



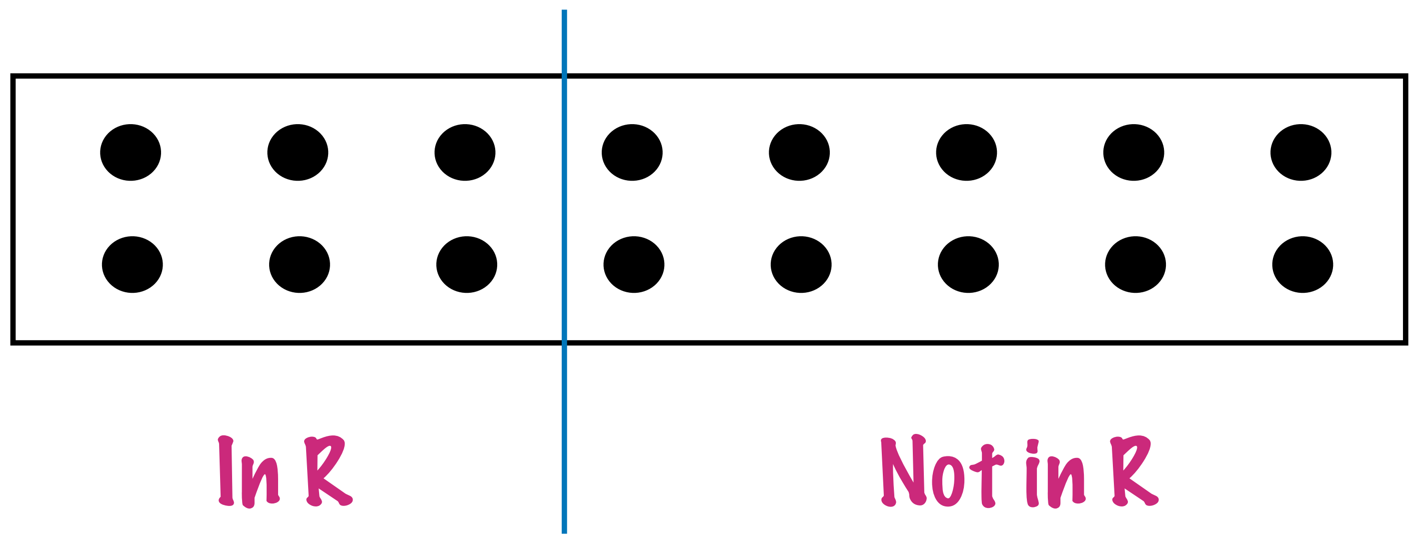
* **Reduction Rule 4:** If there is an edge with multiplicity > 2, reduce it to 2



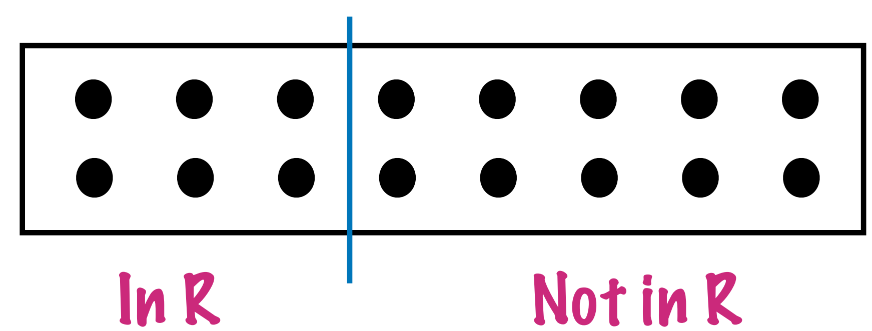


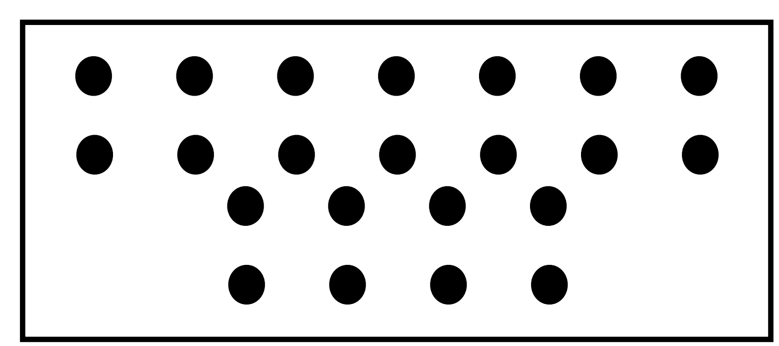
Feedback Vertex Set

Suppose we have a (k+1)-size solution S



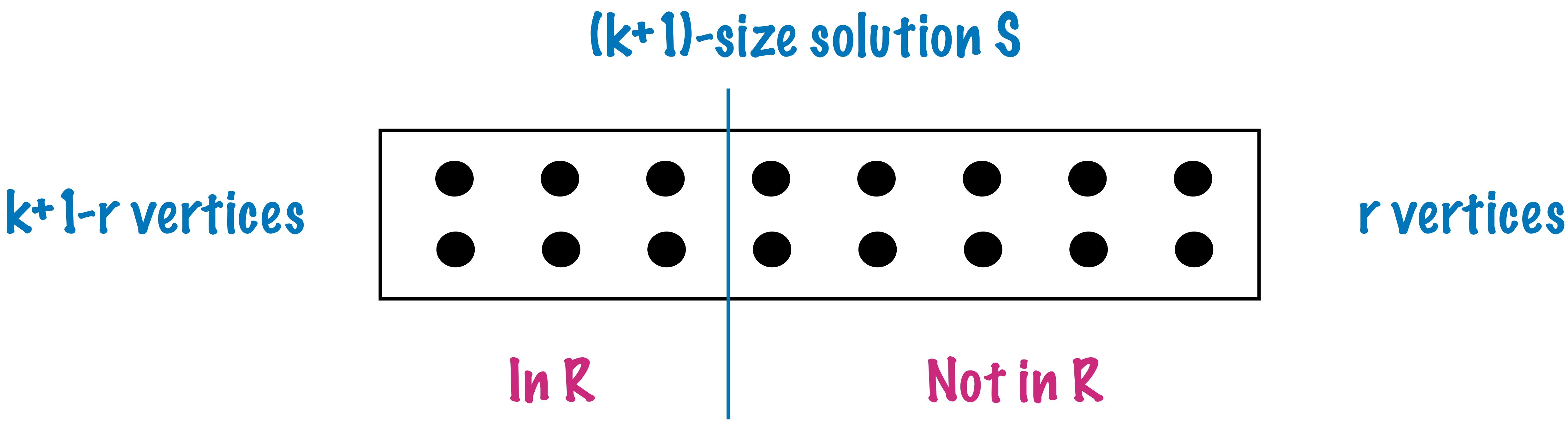
* We want <= k size solution R
* Suppose we know $S \cap R$
  * If we don't know $S \cap R$, guess!
    * 2^{k+1} choices

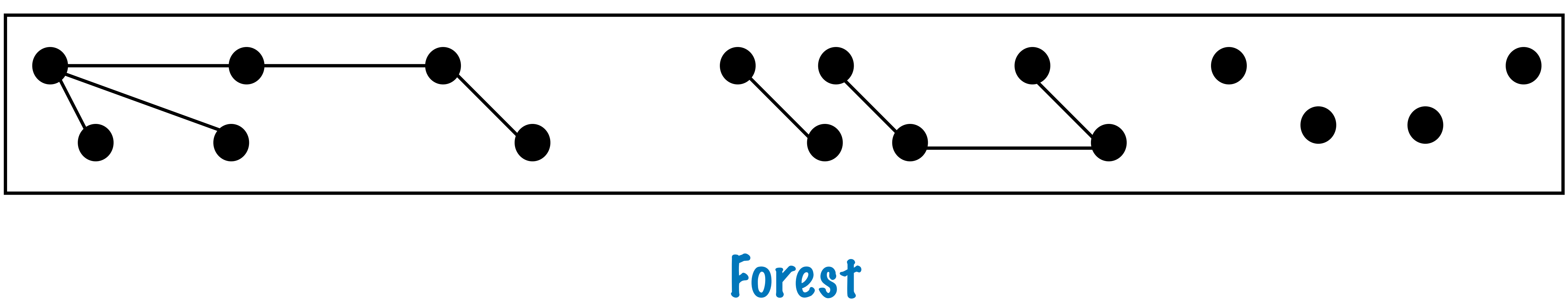




Forest

Feedback Vertex Set



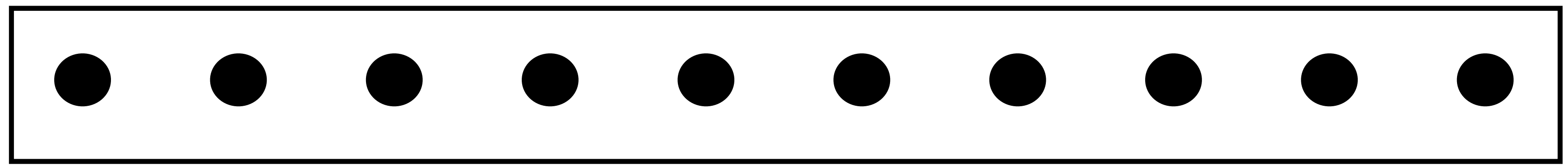


To find a set of <= r-1 vertices here

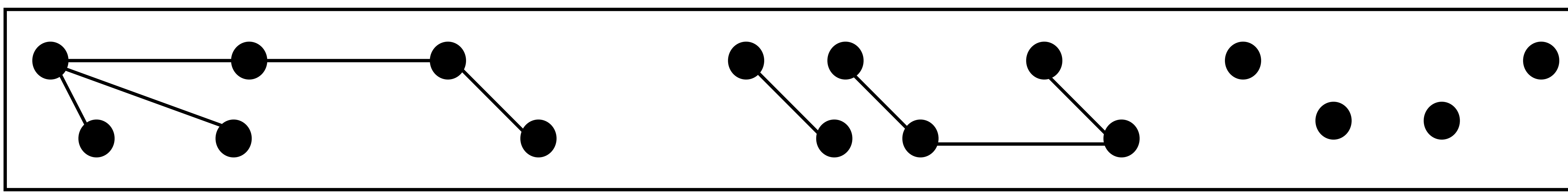
Feedback Vertex Set

If these r vertices cannot make a forest in themselves that means we nee

r-size solution



Forest



Forest

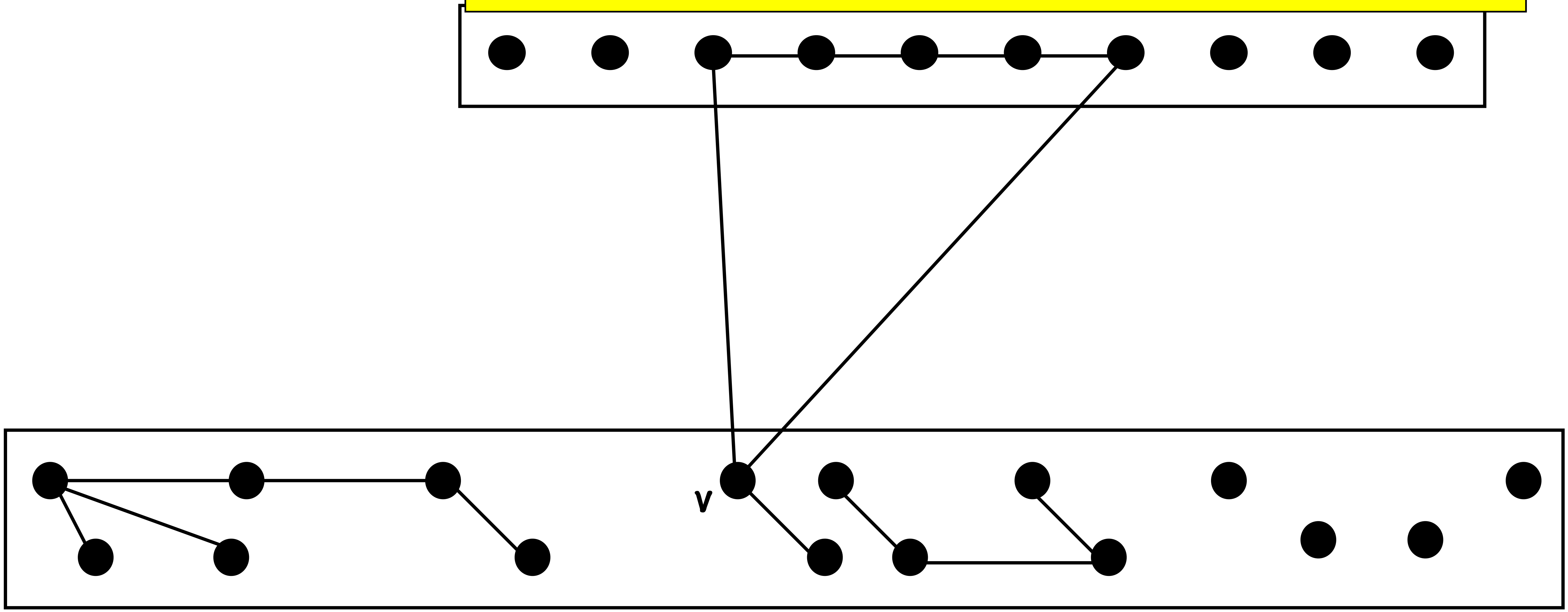
Find a disjoint (r-1) size solution

Disjoint Compression

Feedback Vertex Set

Disjoint Compression

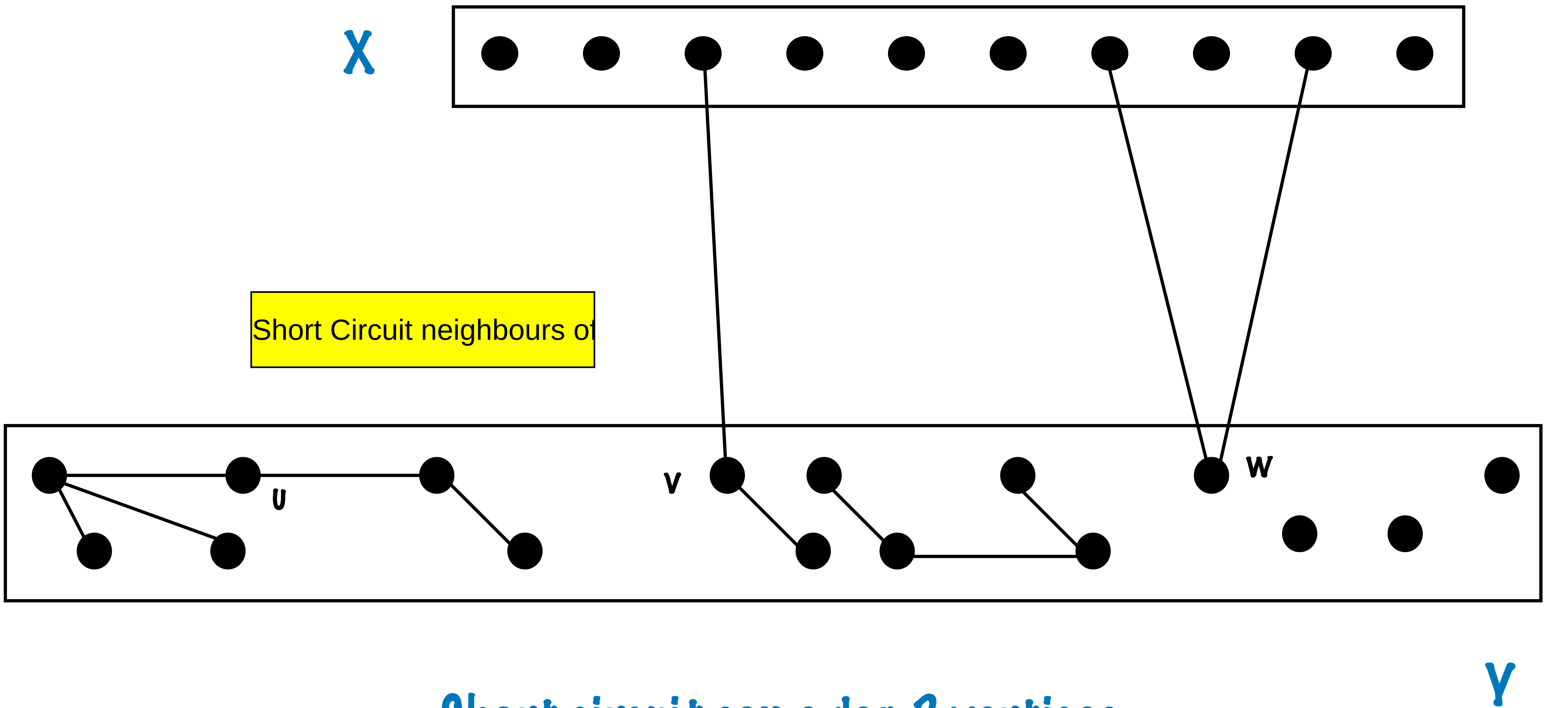
to break a cycle like this, we must remove v ...



Add v into solution and reduce parameter by 1

Feedback Vertex Set

Disjoint Compression

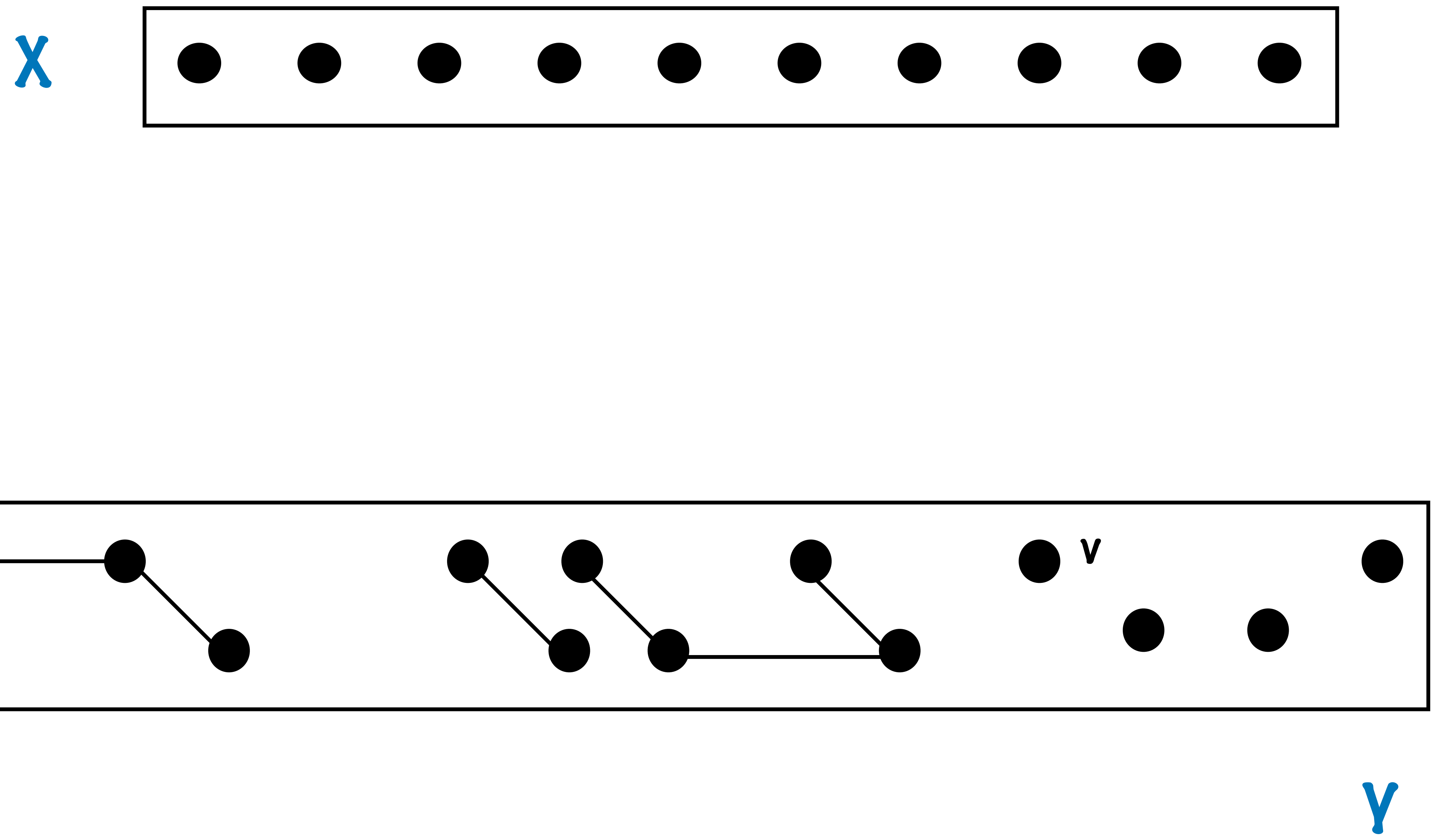


Short circuit some deg 2 vertices

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)



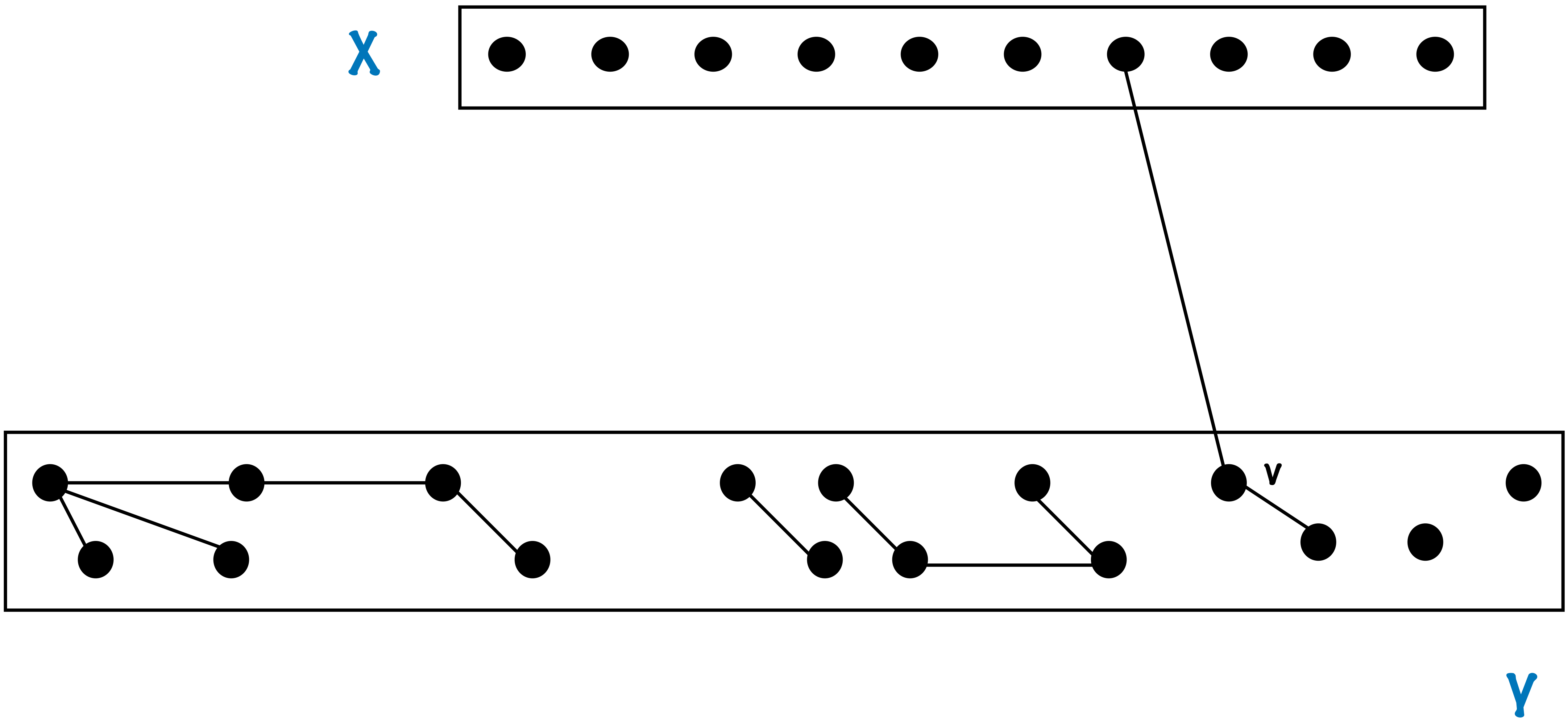
v : vertex of degree (in Y) <=1

There will exist such a vertex as forest always have leaves.

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)

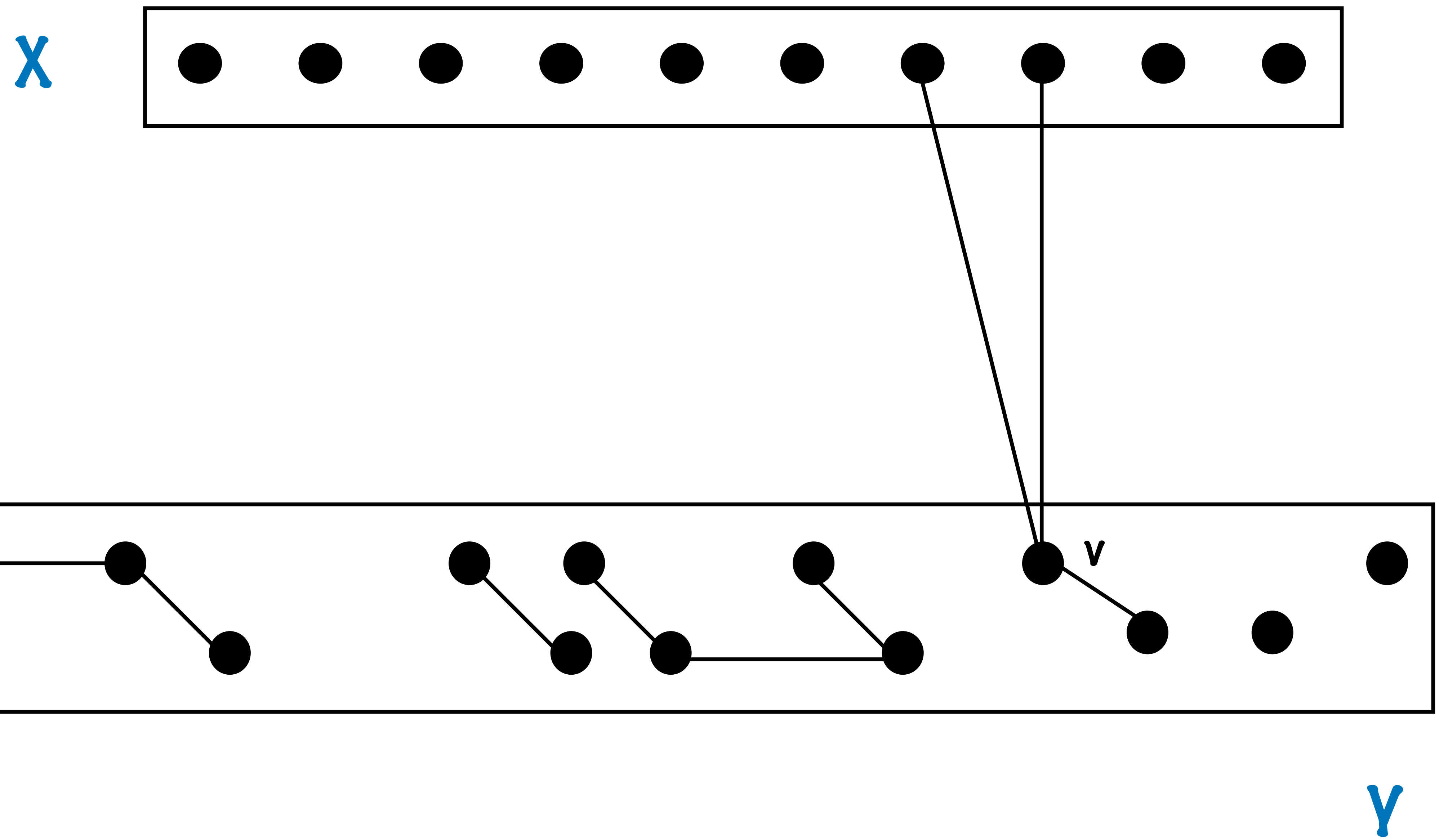


v has >=2 neighbours in X

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)

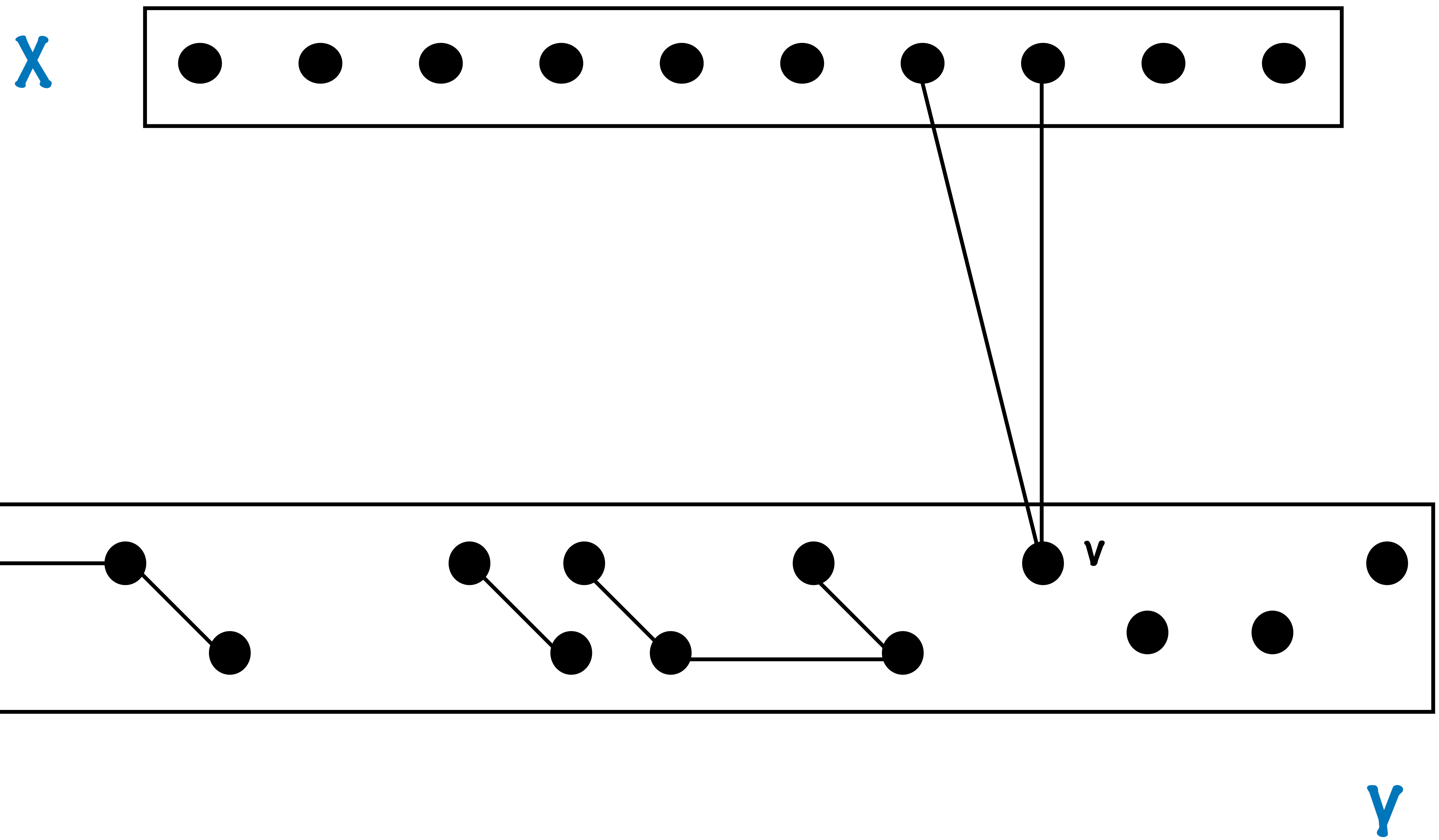


v has >=2 neighbours in X

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)

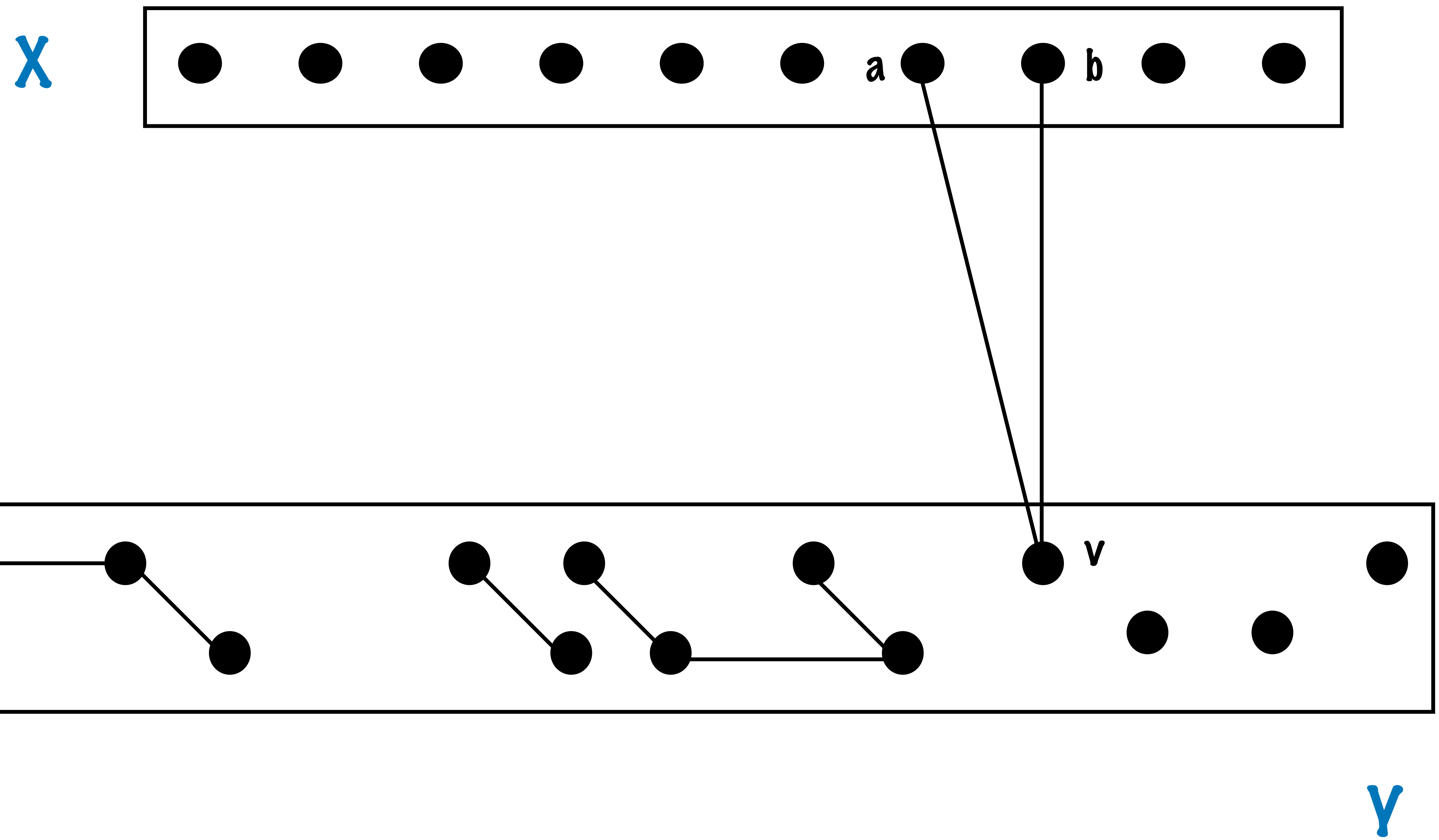


v has >=2 neighbours in X

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)

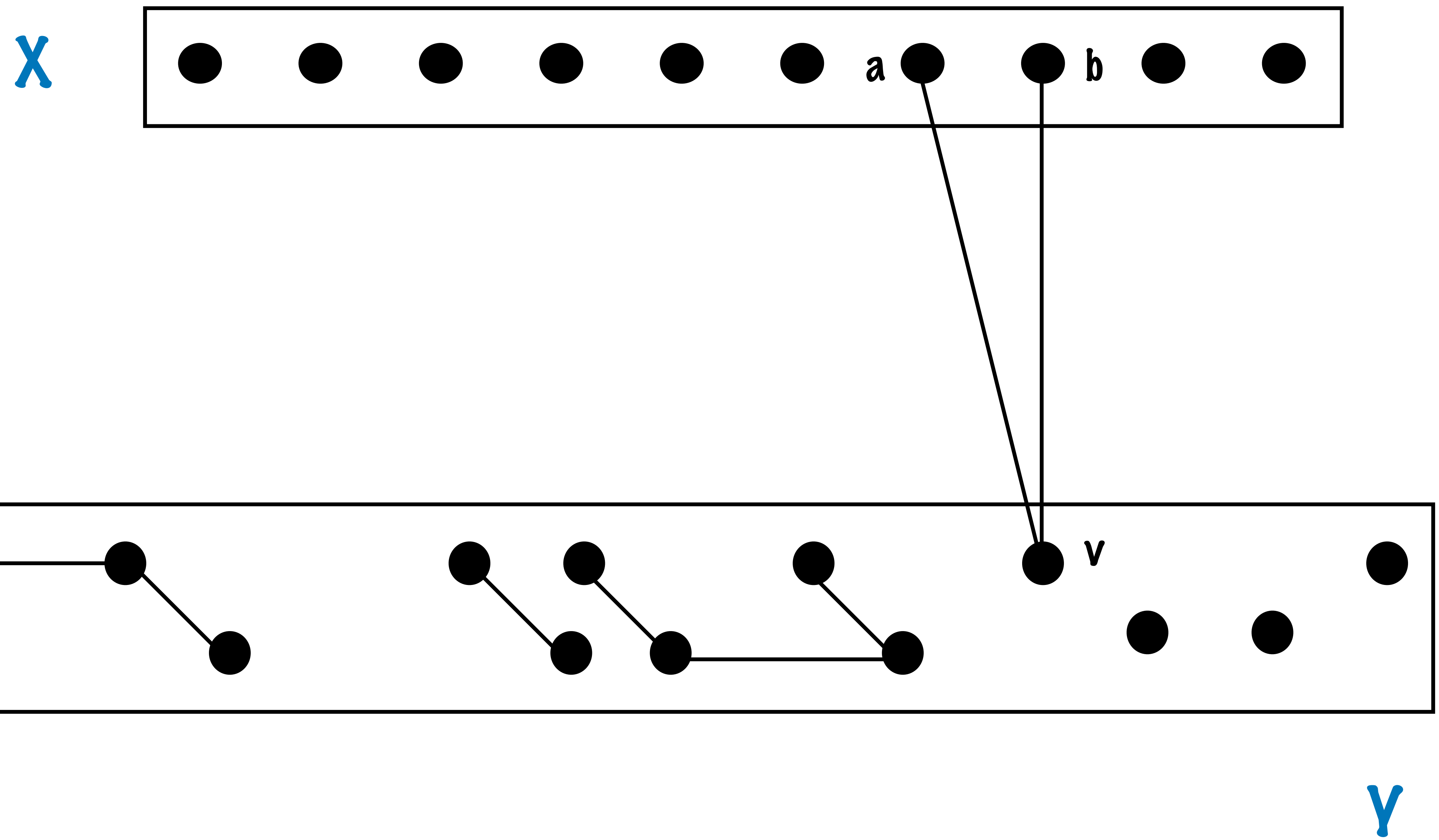


v has >=2 neighbours in X

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)

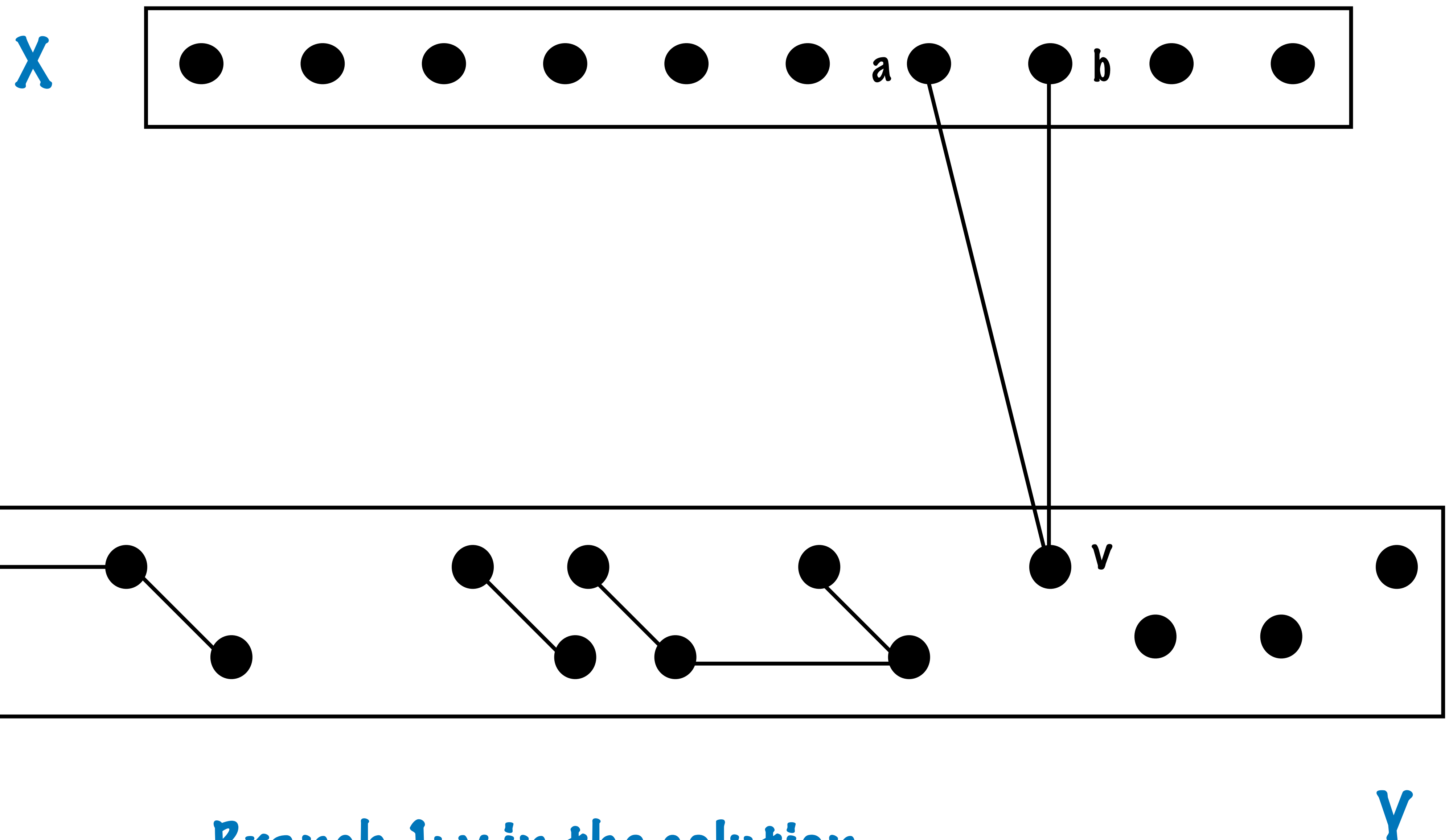


a and b are in different components

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)



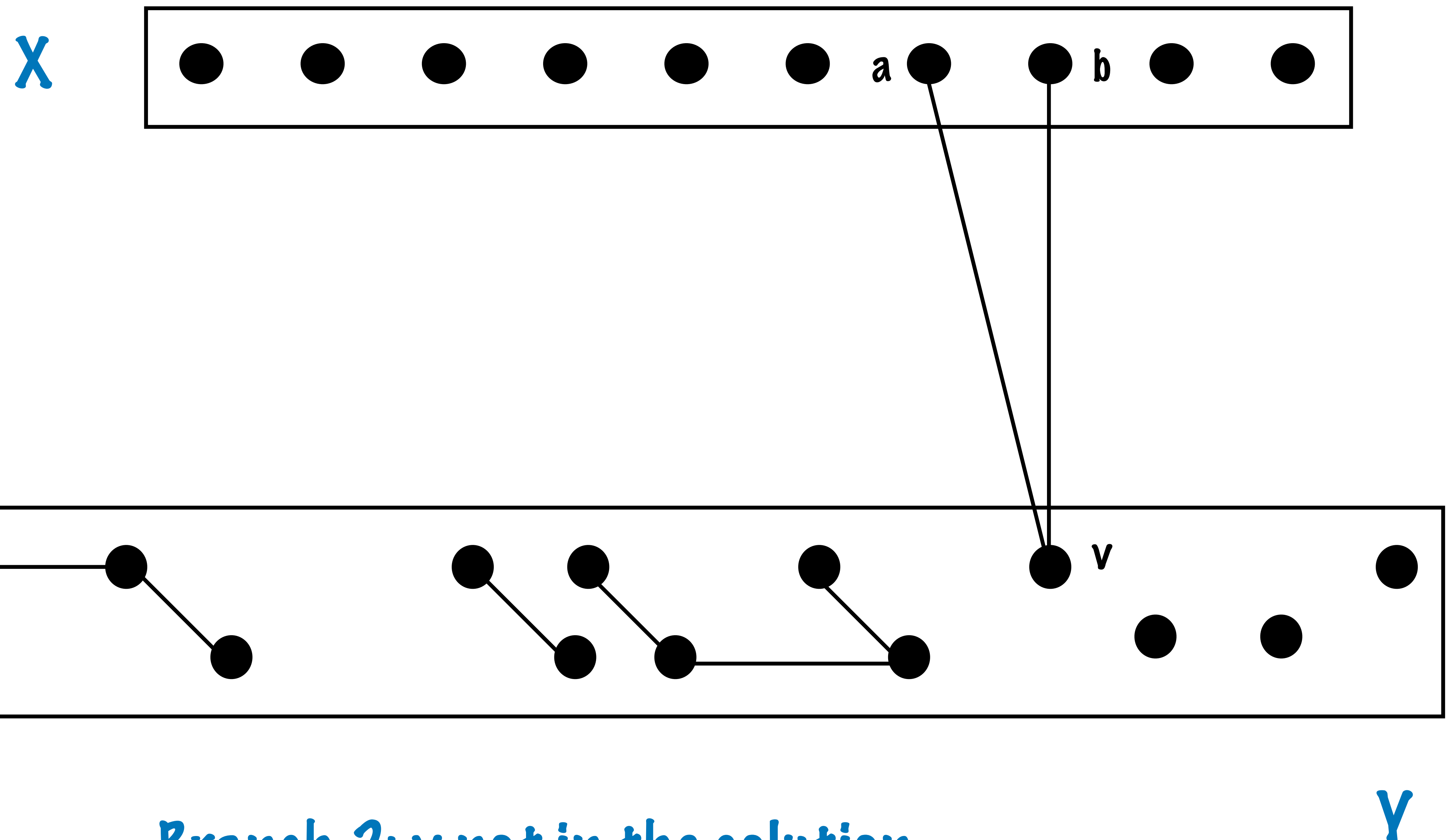
Branch 1: v in the solution

Instance: (X,Y-{v},r-2)

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)



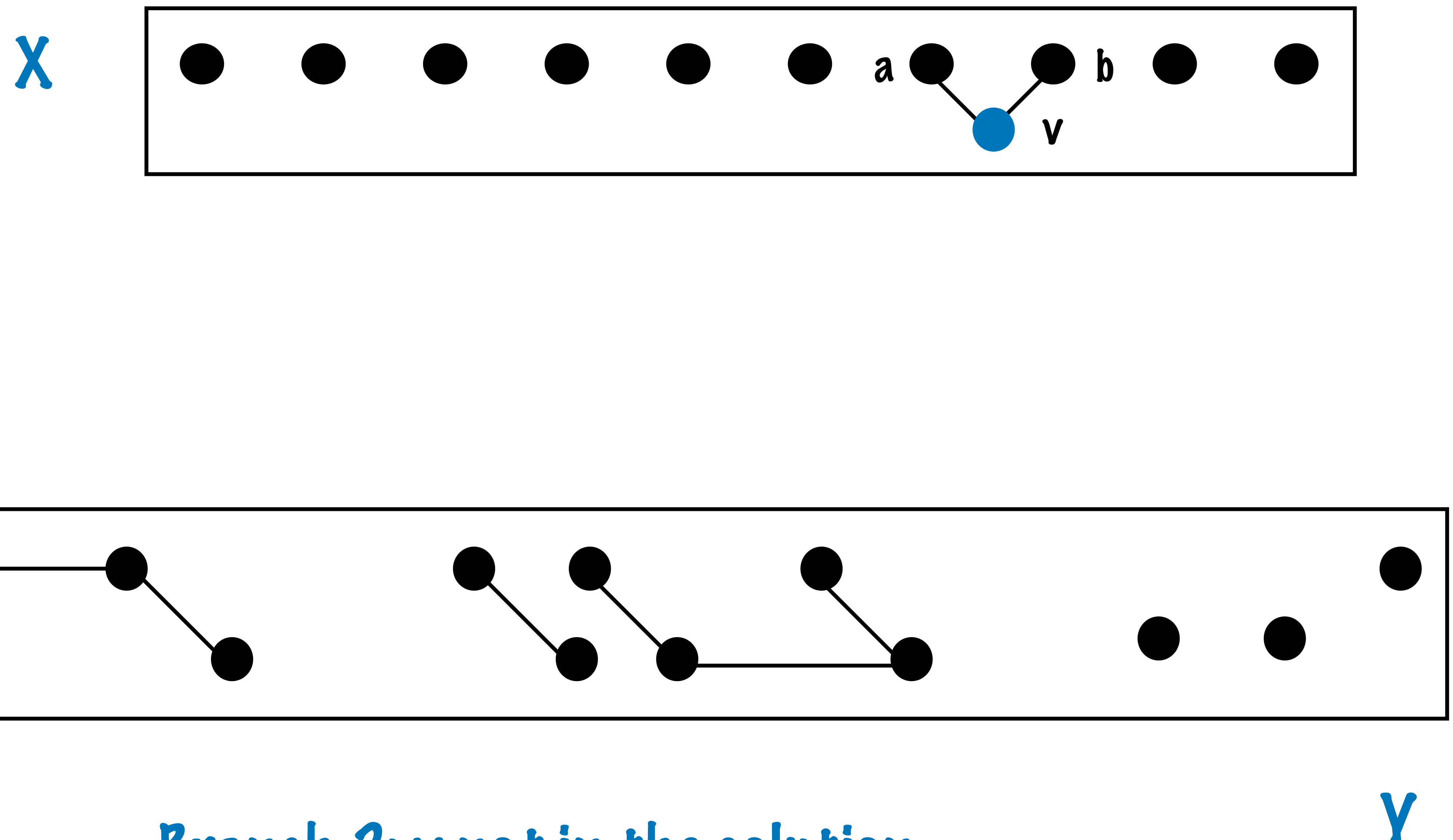
Branch 2: v not in the solution

Instance: $(X \cup \{v\}, Y - \{v\}, r-1)$

Feedback Vertex Set

Disjoint Compression

Instance: (X,Y,r-1)



Branch 2: v not in the solution

Instance: $(X \cup \{v\}, Y - \{v\}, r-1)$

Feedback Vertex Set

Disjoint Compression

$(X, Y, r-1)$

v

$(X, Y-\{v\}, r-2)$ $\quad$ $(X \cup \{v\}, Y, r-1)$

r drops by 1

#C(X) drops by 1

C(X) is number of compo

. . . .

$r + \#C(X) <= k+1 + k <= 2k+1$

Feedback Vertex Set

Disjoint Compression

$(X,Y,r-1,l)$

v

l drops by 1 $(X,Y-\{v\},r-2,l-1)$ $(X\cup\{v\}, Y, r-1,l-1)$ l drops by 1

. . . .

As C(X) <= r therefore l <= 2r.AND

$l = r + \#C(X) <= k+1+k <= 2k+1$

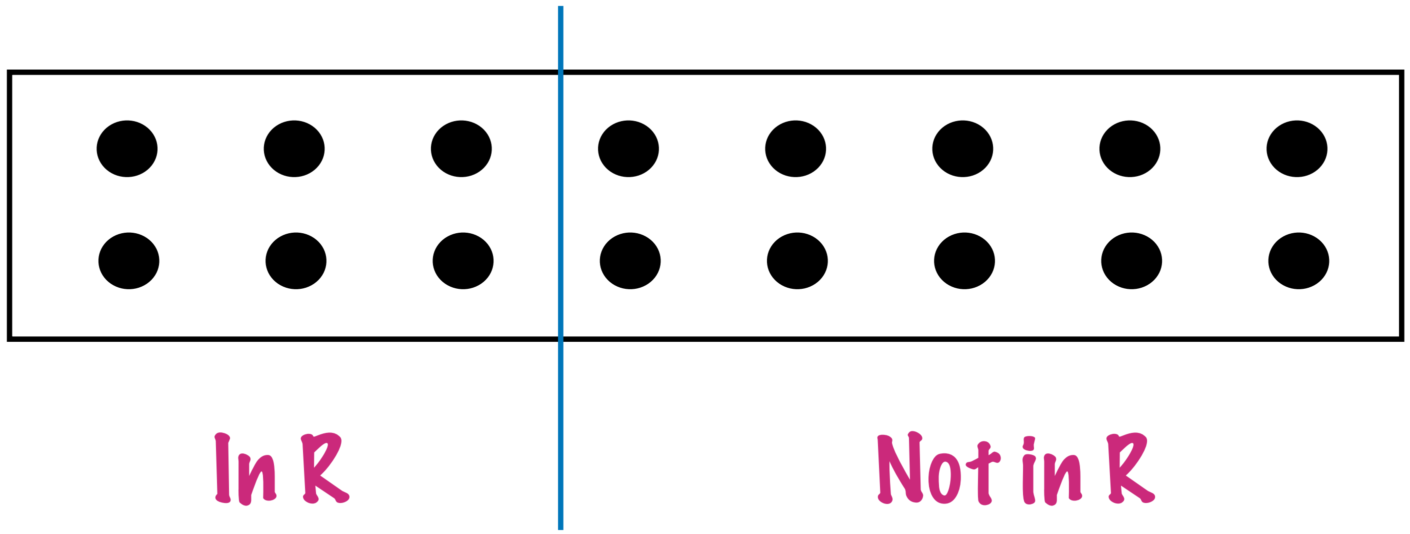
Let T(l) denote the no. of leaves in the tree rooted at instance with measure l

$$T(l) <= \begin{cases} 2T(l-1) & \text{if } l>=1 \\ 1 & \text{otherwise} \end{cases}$$

$T(l) <= 2^l$

$O^*(2^{2k})$ time for Disjoint Compression

Feedback Vertex Set



(k+1)-size solution S

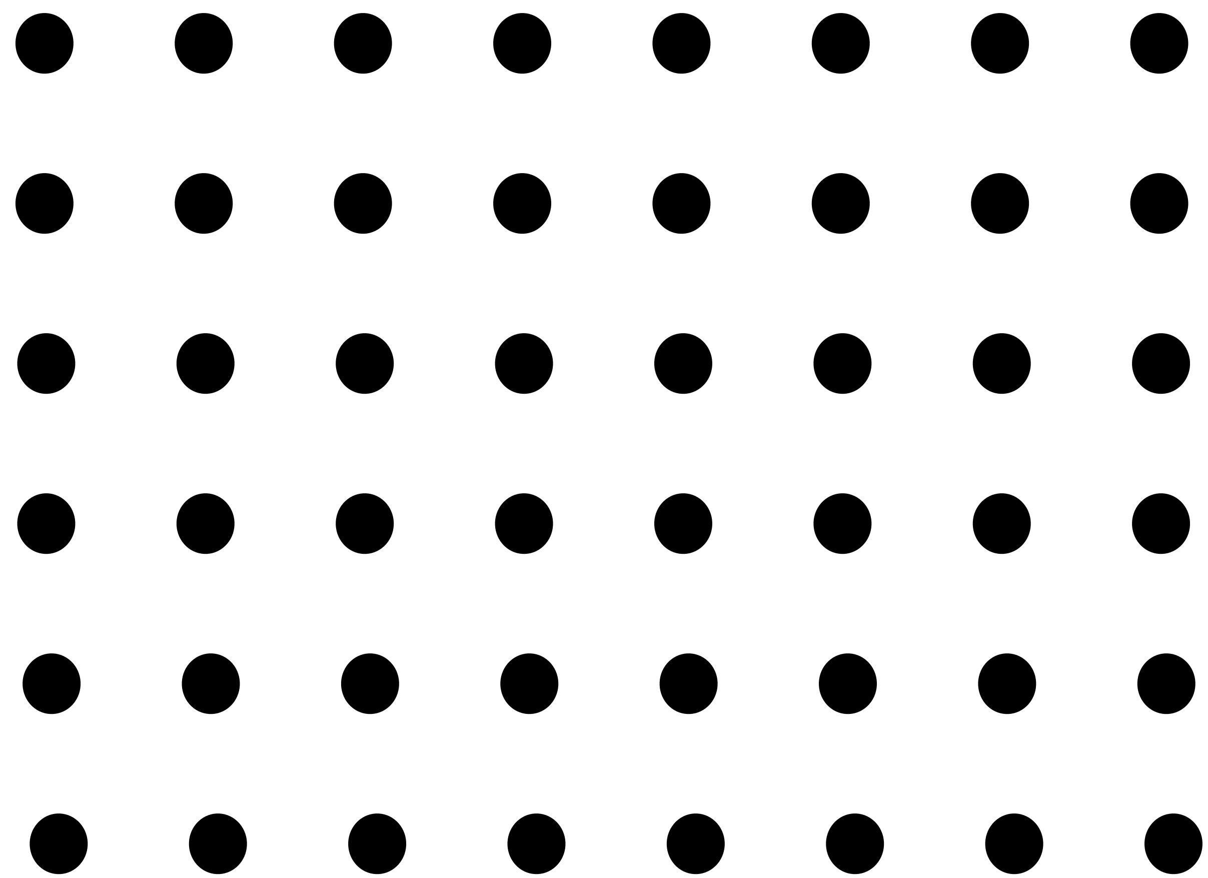
- We want <= k size solution R
- Guess S ∩ R (2^{k+1} choices)
  - Solve Disjoint Compression in $O^*(4^k)$ time

Summation i = 0 to k + 1, (k+1)C(i)4^{k + 1 - i} = 5^{k + 1}.

$O^*(5^k)$ algorithm

Feedback Vertex Set

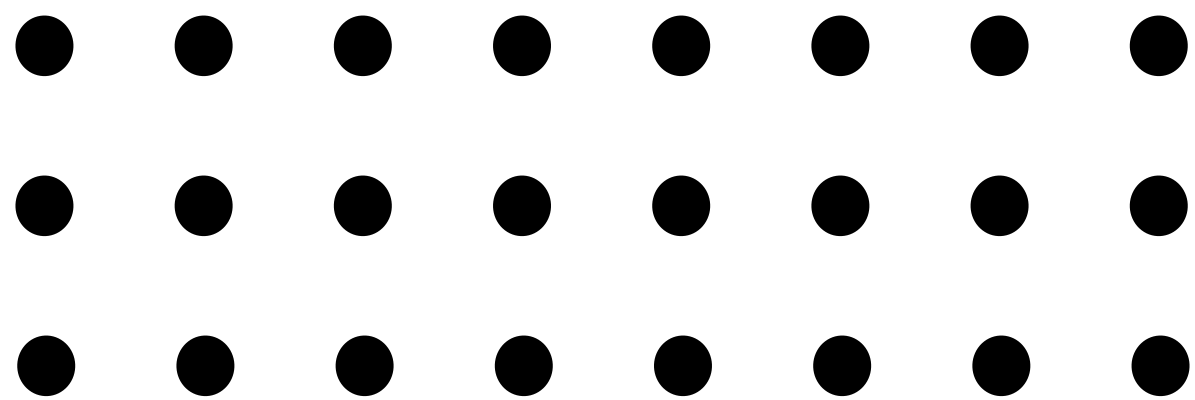
How to get a (k+1)-size solution S?

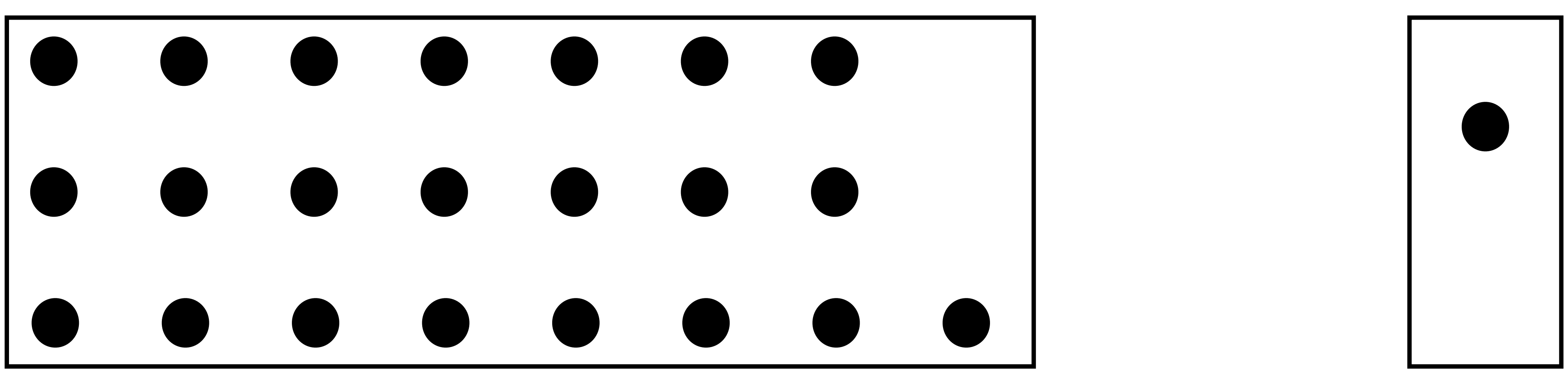


Graph G

Feedback Vertex Set

Consider any k+2 vertices of G



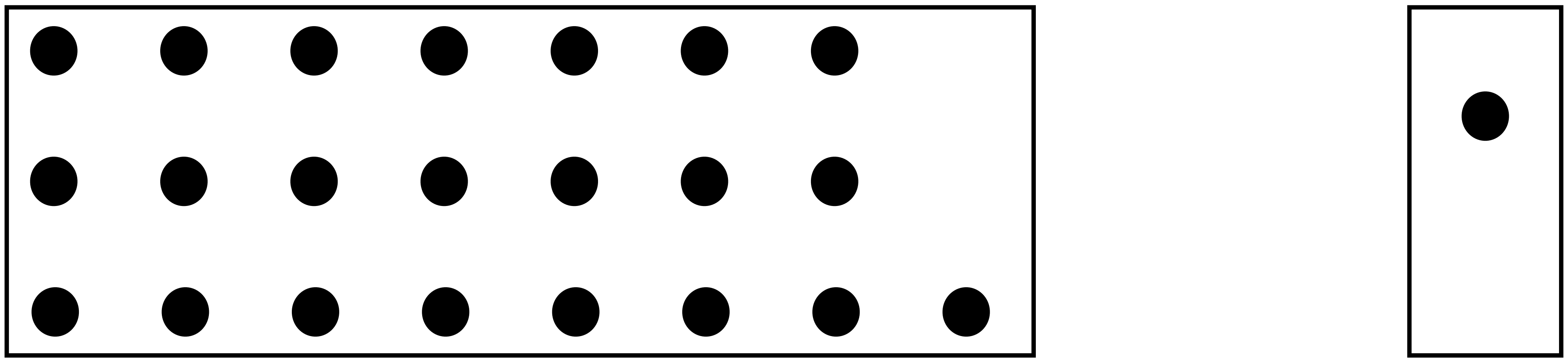


k+1 solution

Forest

Feedback Vertex Set

k+1 solution for subgraph on k+2 vertices



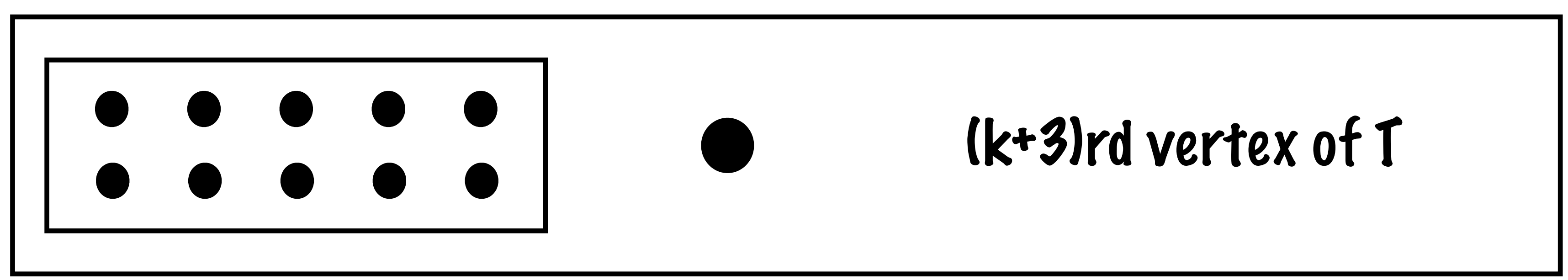
Compress in $O^*(5^k)$ time

No k solution

<= k solution

Add the (k + 3)rd vertex to this <= k soln found.

(G,k) is a no-instance



k+1 solution for subgraph on k+3 vertices

Iterative Compression