CS 5003: Parameterized Algorithms

Lecture 7

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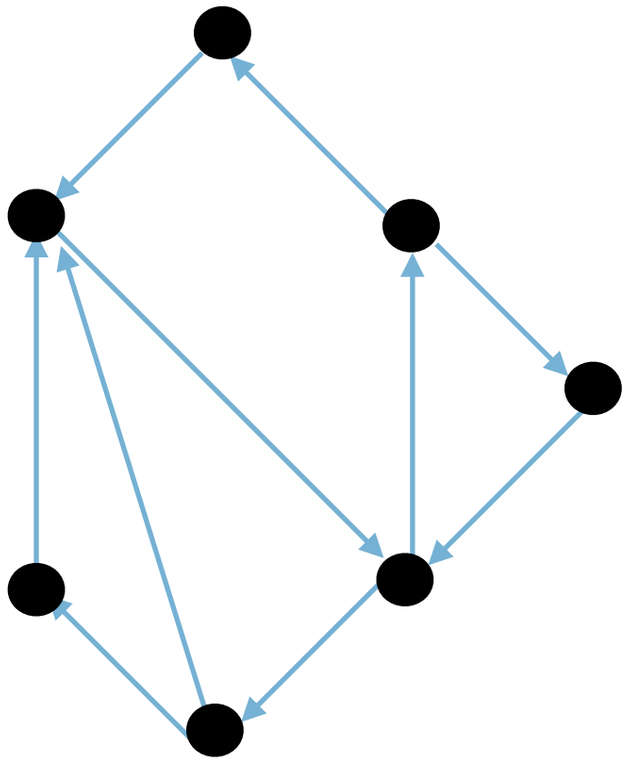
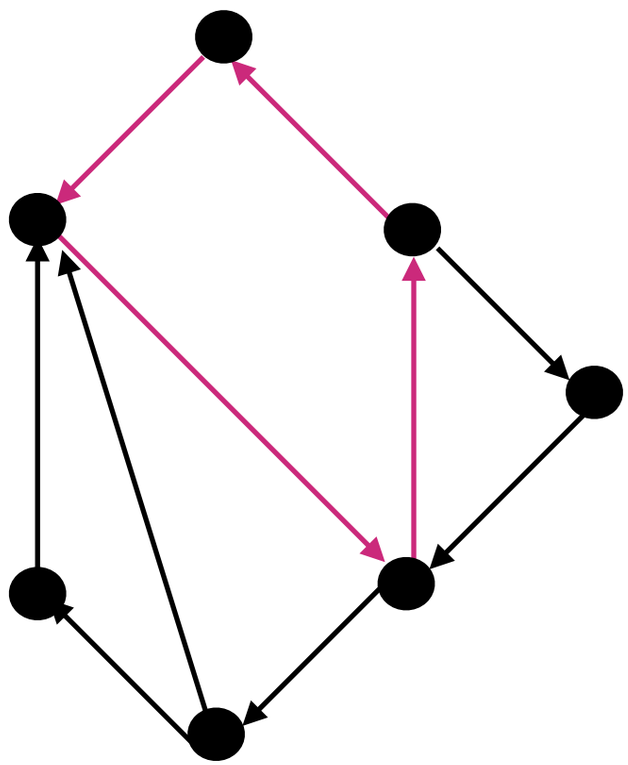
Reference Books: Parameterized Algorithms by Cygan et al. and Kernelization by Fomin et al.

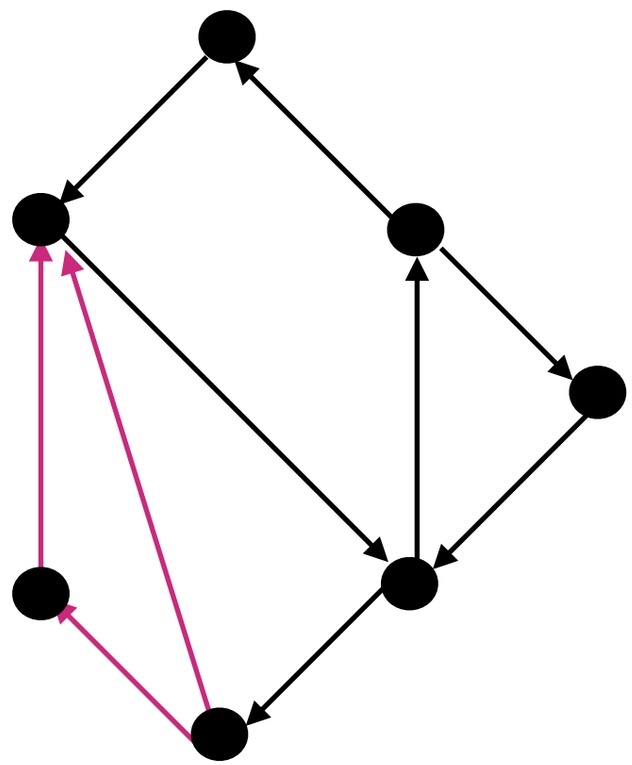
Directed Graphs

Every edge has direction: arc

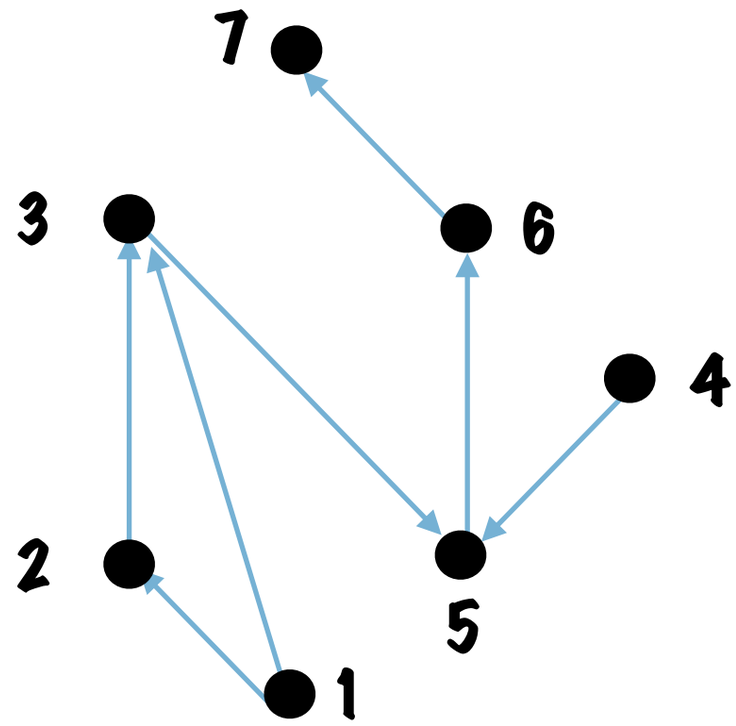


Directed cycle





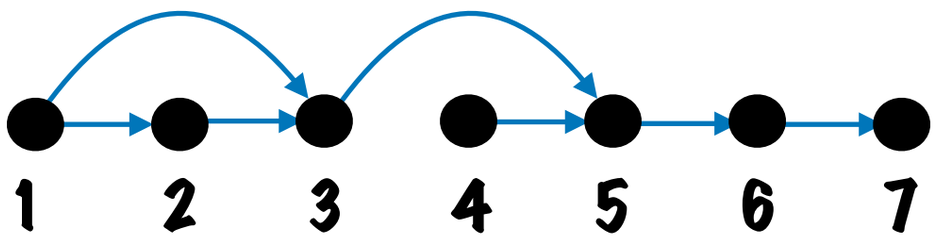
Directed Acyclic Graphs



Directed graphs with no directed cycle

<- as given TO, to have a cycle there must be a backward edge which is

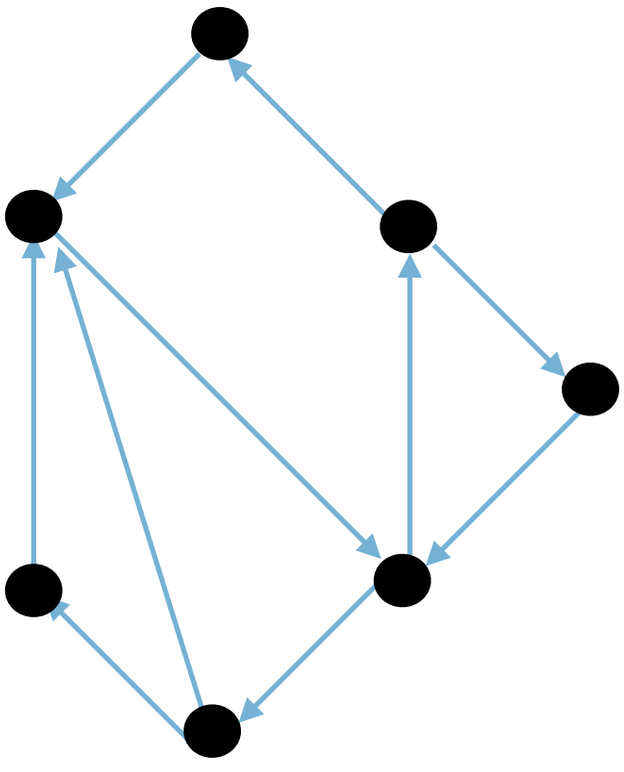
Lemma: A digraph is a DAG iff it has a topological ordering



Topological ordering

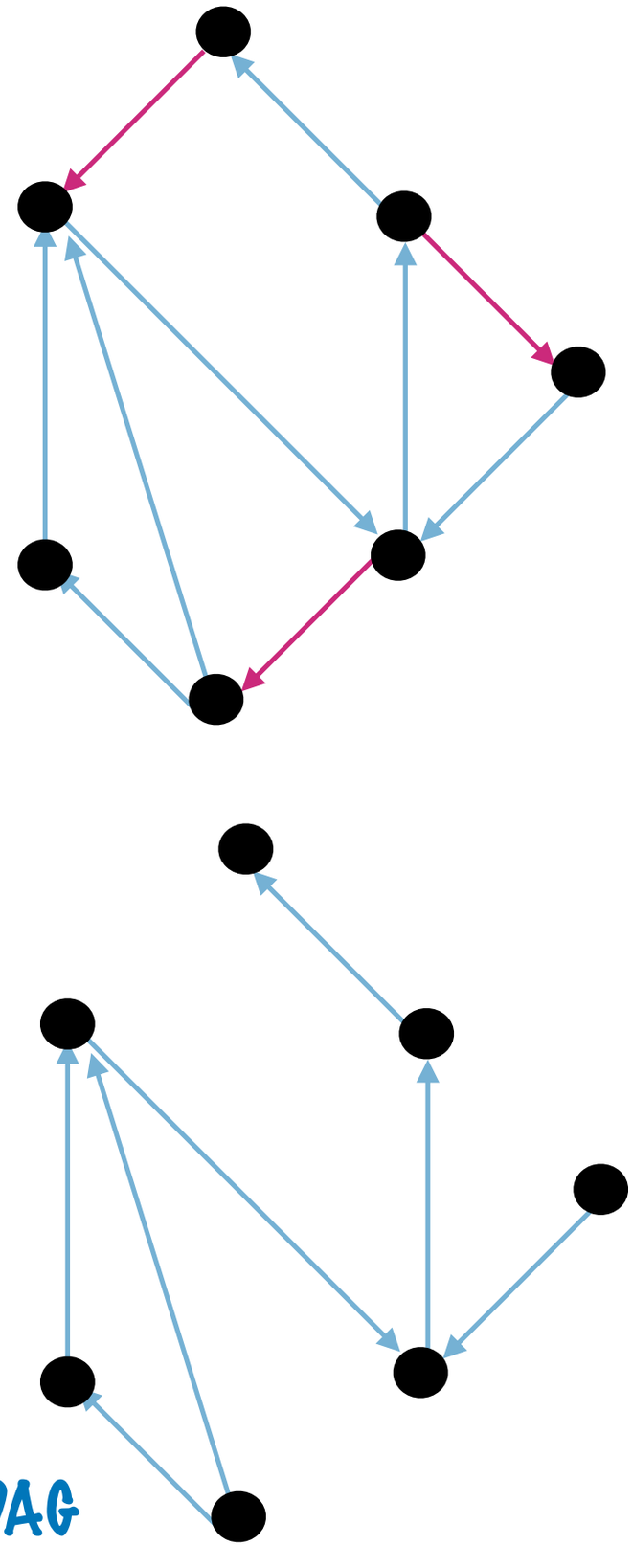
Feedback Arc Set

FAS - set of arcs that has at least one arc of every directed cycle



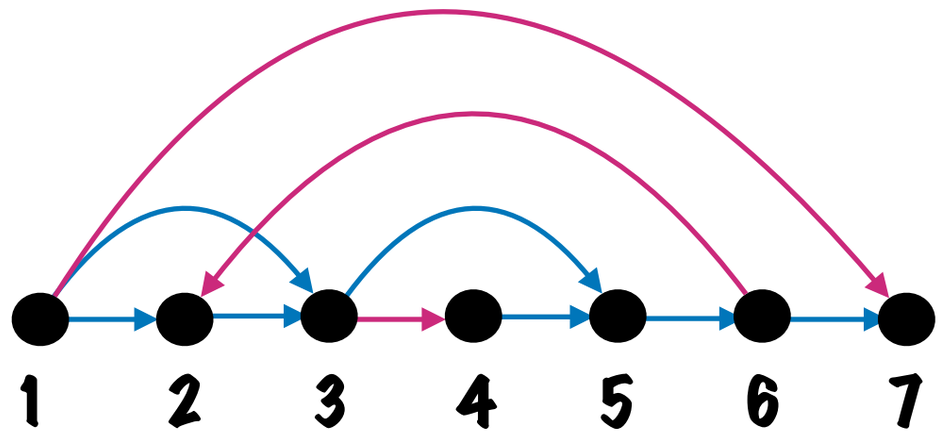
Digraph

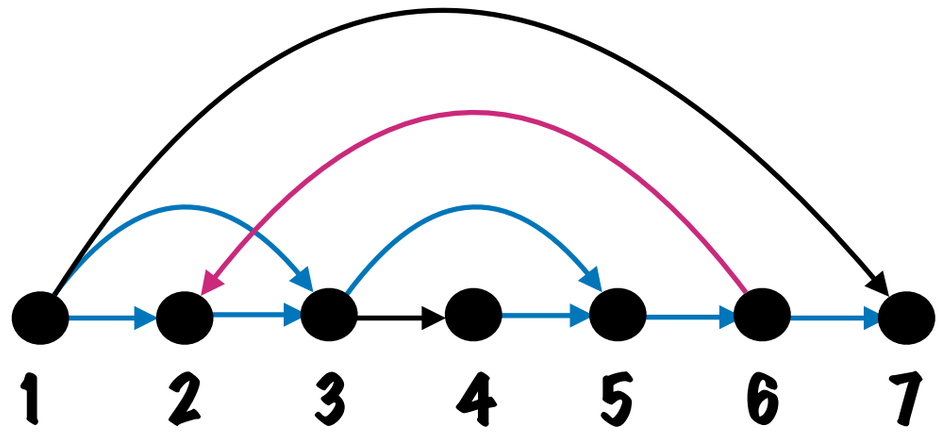
deletion



DAG

Minimal Feedback Arc Set

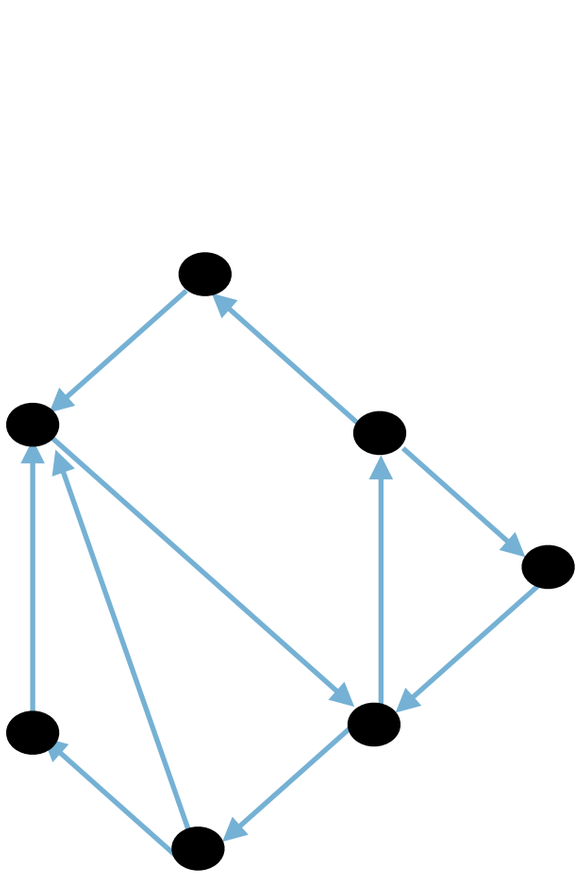


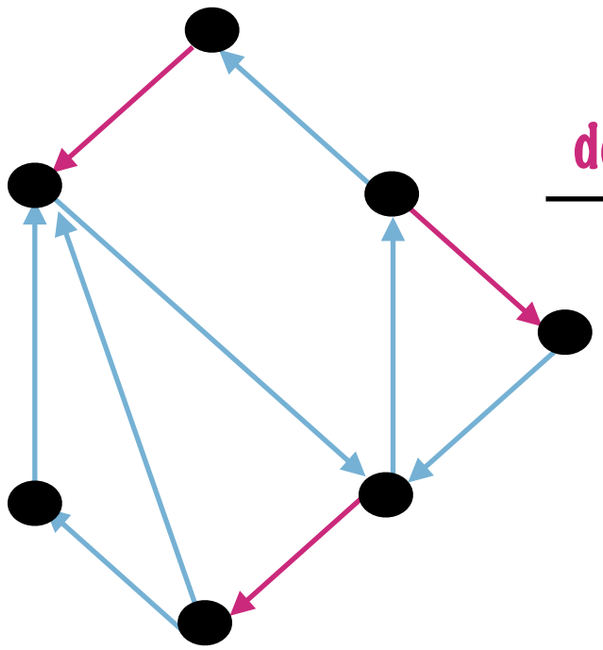


Lemma: A set of arcs F is a minimal FAS iff it is a minimal set of arcs whose reversal results in a DAG

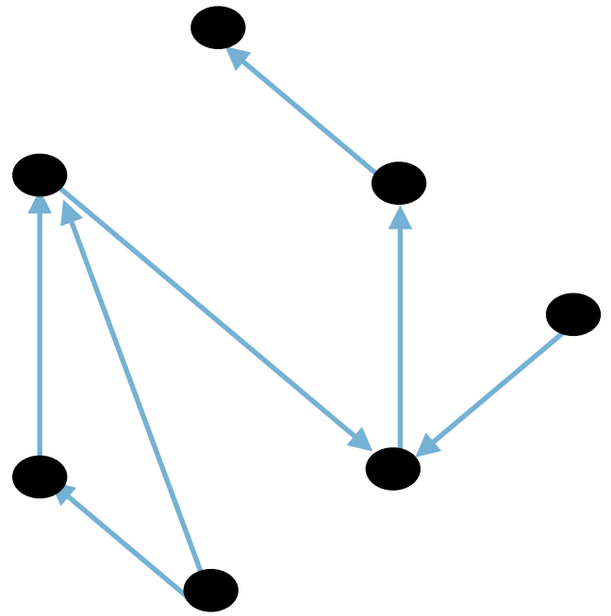
Feedback Arc Set: Reversal and Deletion

Lemma: A set of arcs F is a minimal FAS iff it is a minimal set of arcs whose reversal results in a DAG

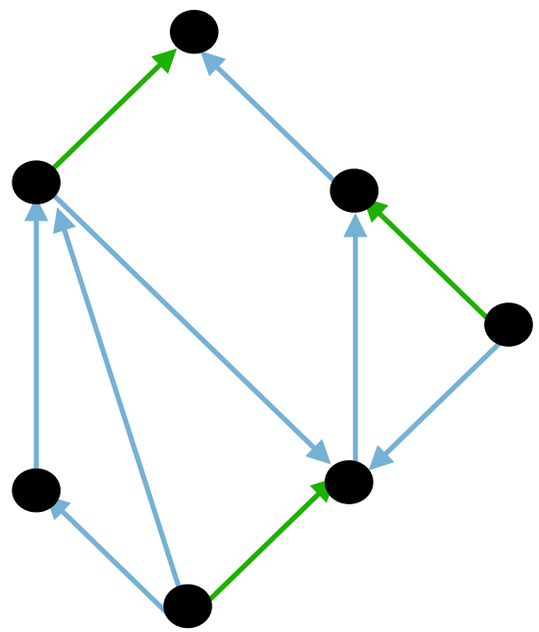




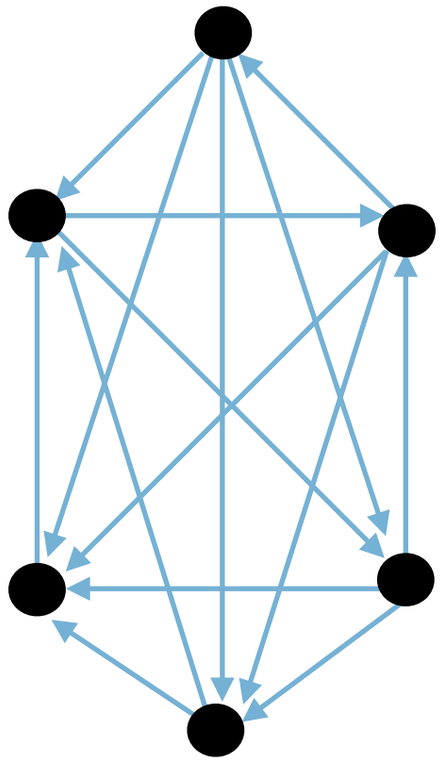
deletion



reversal



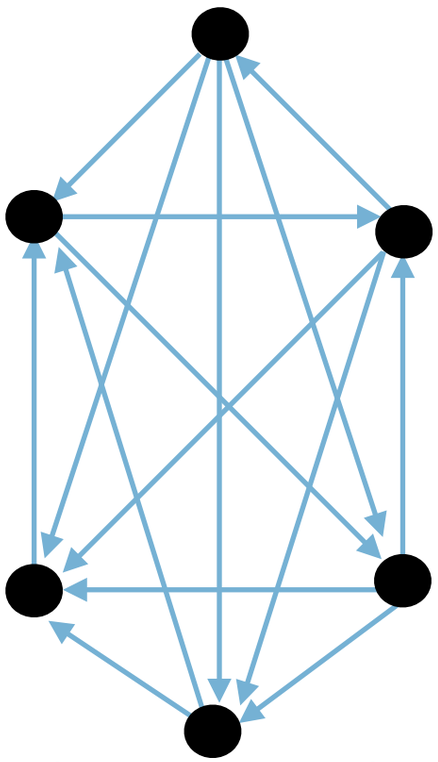
Tournaments



tournament

Lemma: Acyclic tournaments have unique topological ordering

Feedback Arc Set in Tournaments



tournament

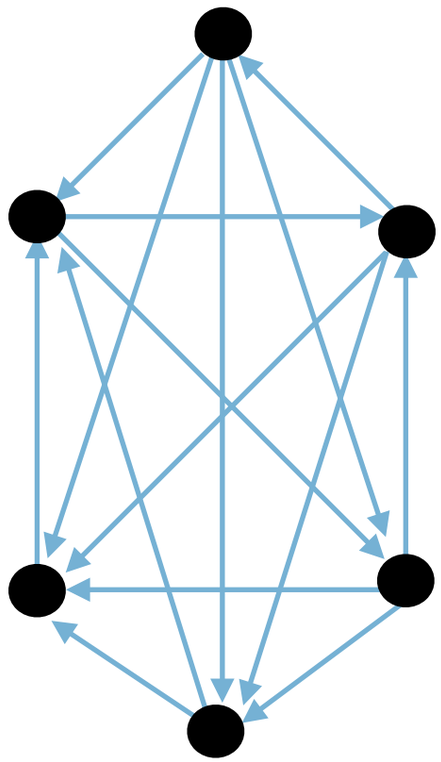
Feedback Arc Set in Tournaments

Instance: A tournament T and an integer k

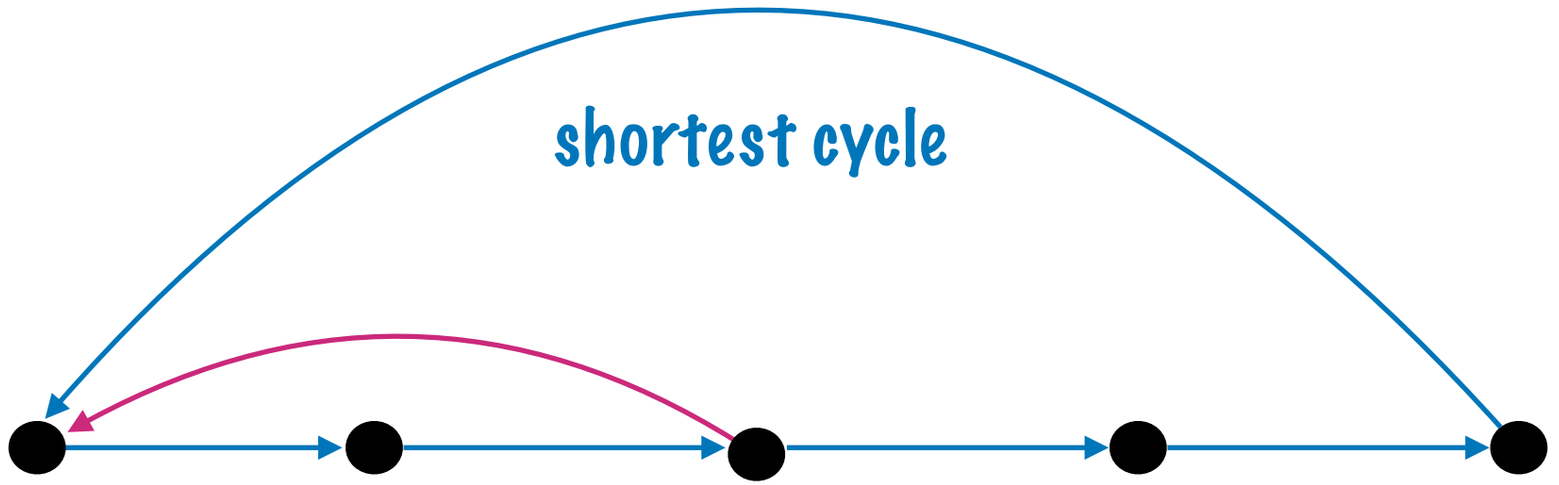
Question: Does there exist a feedback arc set of T of size at most k?

Parameter: k

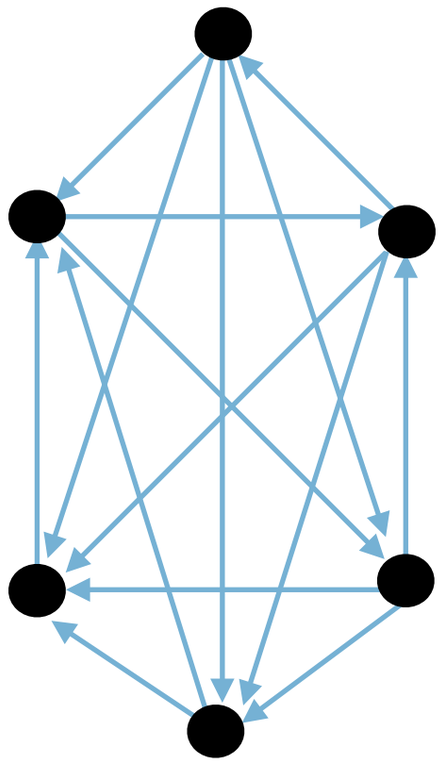
Feedback Arc Set in Tournaments



Lemma: A tournament is acyclic iff it has no triangle



Feedback Arc Set in Tournaments



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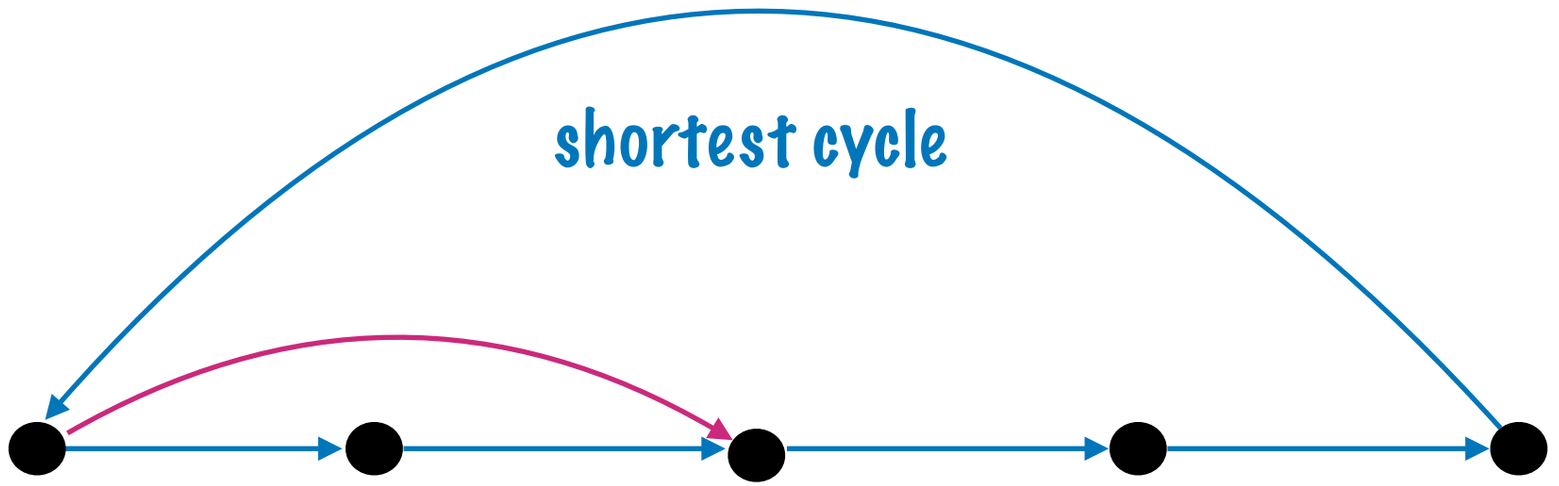
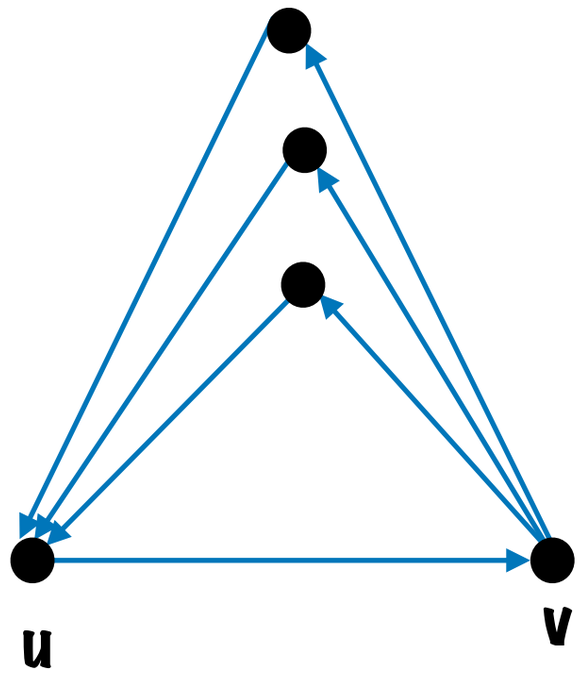


Feedback Arc Set in Tournaments: Quadratic Kernel

* **Reduction Rule 1:** If an arc e=(u,v) is in k+1 triangles, reverse it and reduce k by 1



Resulting digraph is still a tournament

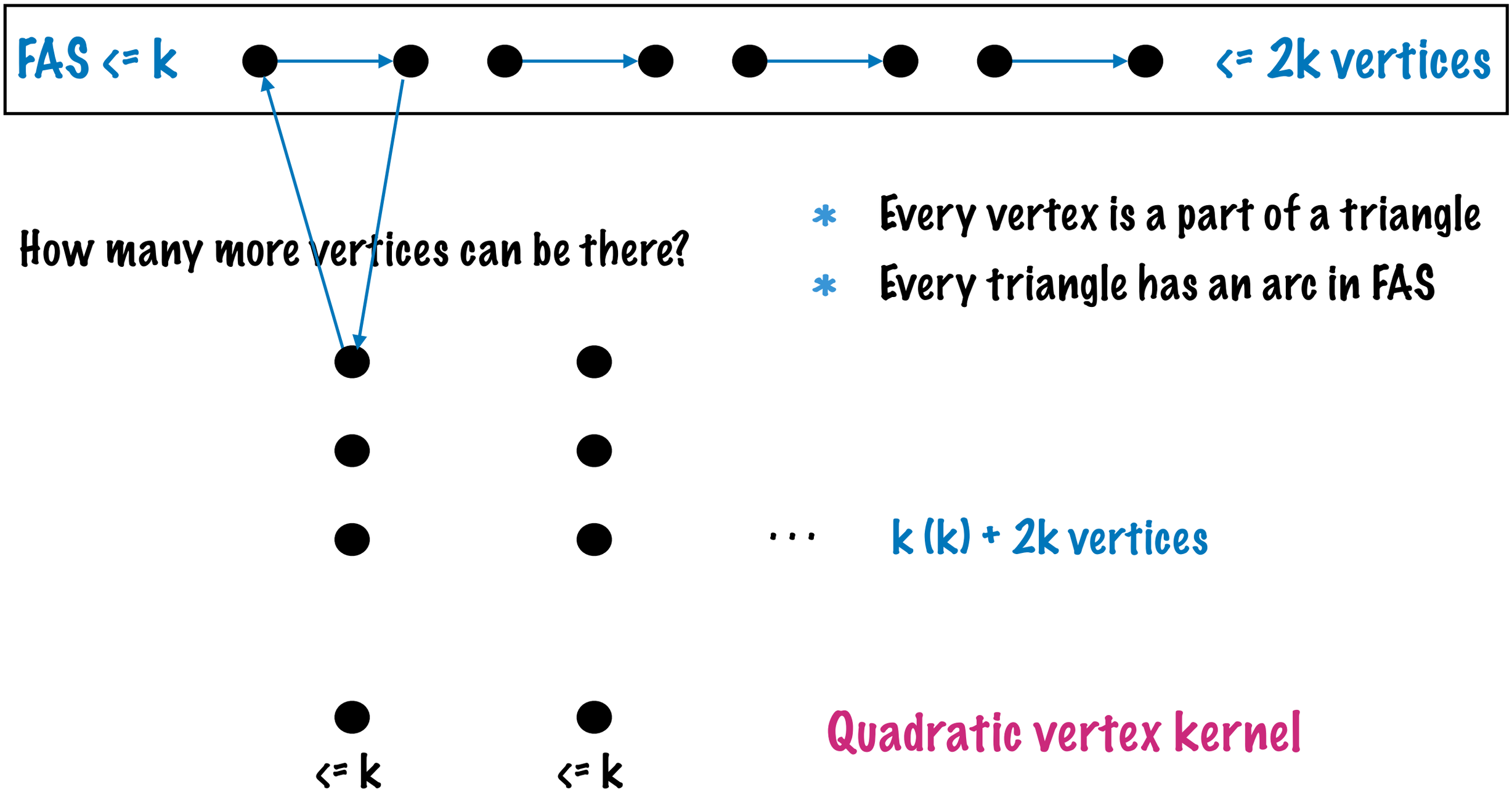
(T,k) is an yes-instance iff (T',k-1) is an yes-instance

* **Reduction Rule 2:** Delete vertices not in any triangle

(T,k) is an yes-instance iff (T-v,k) is an yes-instance

Feedback Arc Set in Tournaments: Quadratic Kernel

Suppose (T, k) is an yes-instance



* Every vertex is a part of a triangle
* Every triangle has an arc in FAS

Quadratic vertex kernel

Feedback Arc Set in Tournaments: $O^*(3^k)$ Algorithm

Lemma: A tournament is acyclic iff it has no triangle

Branching Algorithm?

What are the recursive subproblems?

$O^*(3^k)$ algorithm