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## (CS 5008) Reinforcement Learning : Assignment 3

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### Markov Chain

Q1) Consider a random walk on the set of integer  $\mathbb{Z}$  described as follow:  $Prob(s_{t+1} = s + 1 | s_t = s) = 0.5$  and  $Prob(s_{t+1} = s - 1 | s_t = s) = 0.5$ . Start from various initial distributions  $\mu_0$  (remember  $s_0 \sim \mu_0$ ), and find out  $\mu_4$ , the distribution after 4 time steps.

Q2) Consider a single queue with maximum length  $n = 9$ . What is the total number of states? Let the queue evolve in discrete time steps  $t = 0, 1, \dots$ , and let probability of arrival of a customer between times  $t$  and  $t + 1$  be  $p$ , and let the probability that a customer is serviced between  $t$  and  $t + 1$  be  $q$ . Also, let arrival and service be independent of each other. Describe the probability transition matrix for this system.

Q3) We know that  $\mu_{t+1}^\top = \mu_t^\top \mathcal{P}$ . Verify that  $\mu_{t+1}$  is a distribution if  $\mu_t$  is a distribution.

Q4) Consider the filtering problem discussed in the class? Verify that  $Prob(s_0 | o_0, s_0 \sim \mu_0)$  is nothing but the Bayes rule.

Q5) Consider the filtering problem discussed in the class? How will we find  $Prob(s_t | o_k, s_0 \sim \mu_0)$ , where  $k < t$ .

Q6) Please work out the “rain and umbrella” explained in class for i) Filtering ii) Prediction iii) Smoothing and iv) Maximum likelihood sequence (Viterbi Algorithm). Play around with different numbers.