Spectral Geometric Methods for Deformable 3D Shape Retrieval

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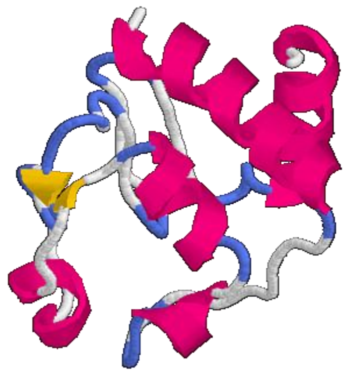
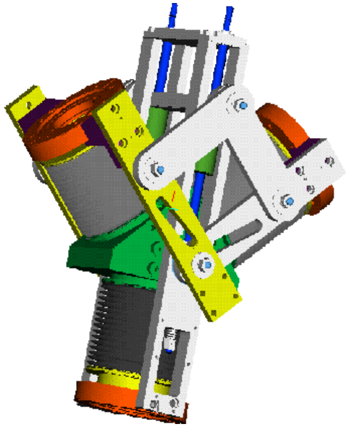
Outline

3D Shape Retrieval

Background



Molecular Biology
(Protein Databank)



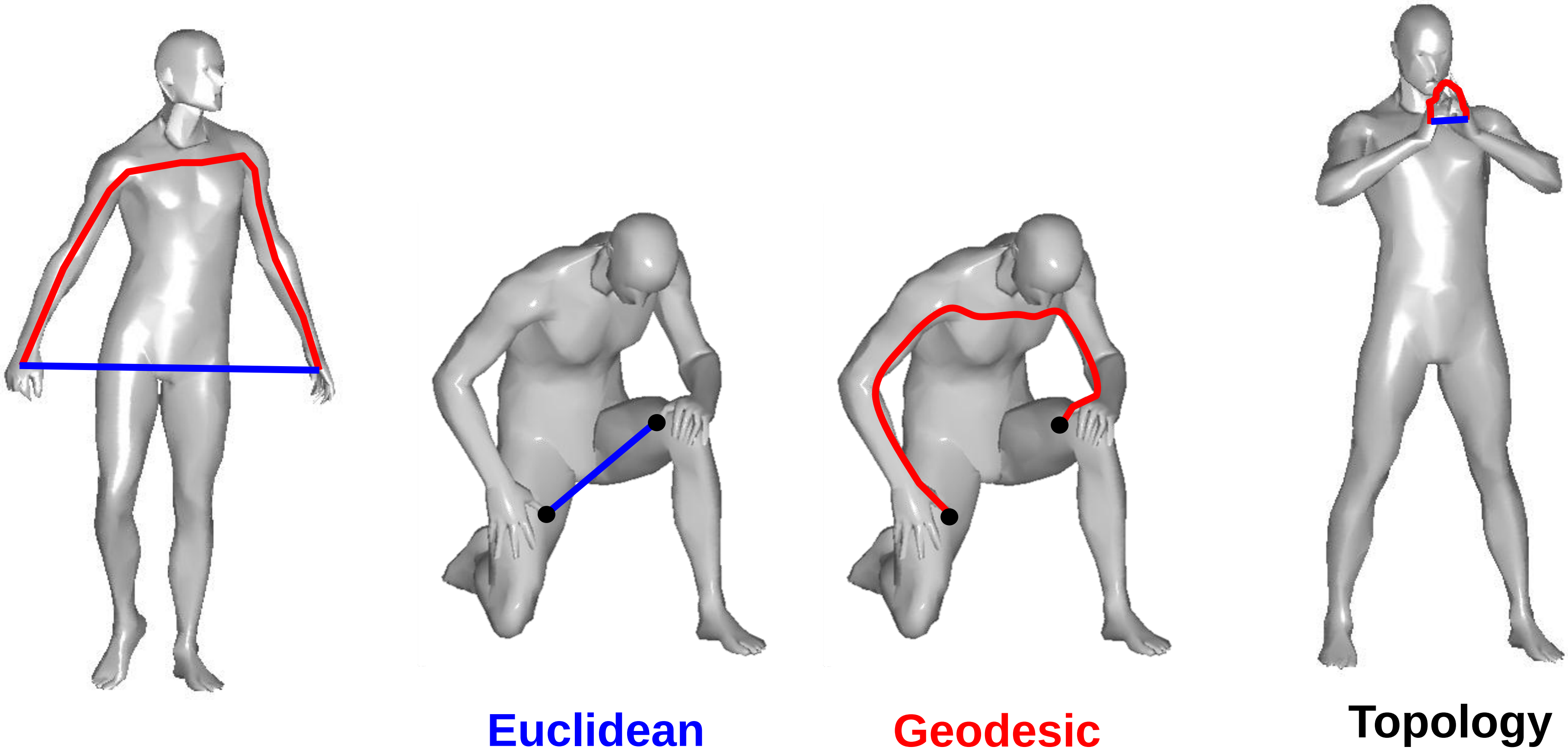
Mechanical CAD
(National Design Repository)

Vision & Graphics

Related Works

- ShapeDNA [*Reuter et al.'06*]
- Global Point Signature [*Rustamov '07*]
- Heat Kernel Signature [*Sun et al.'09*]
- Wave Kernel Signature [*Aubry et al.'11*]
- Shape Google [*Bronstein et al.'11*]
- Many others !

Deformable Shape



Human images adapted from Bronstein et al.

Spectral Geometry

- Laplace-Beltrami (LB) operator

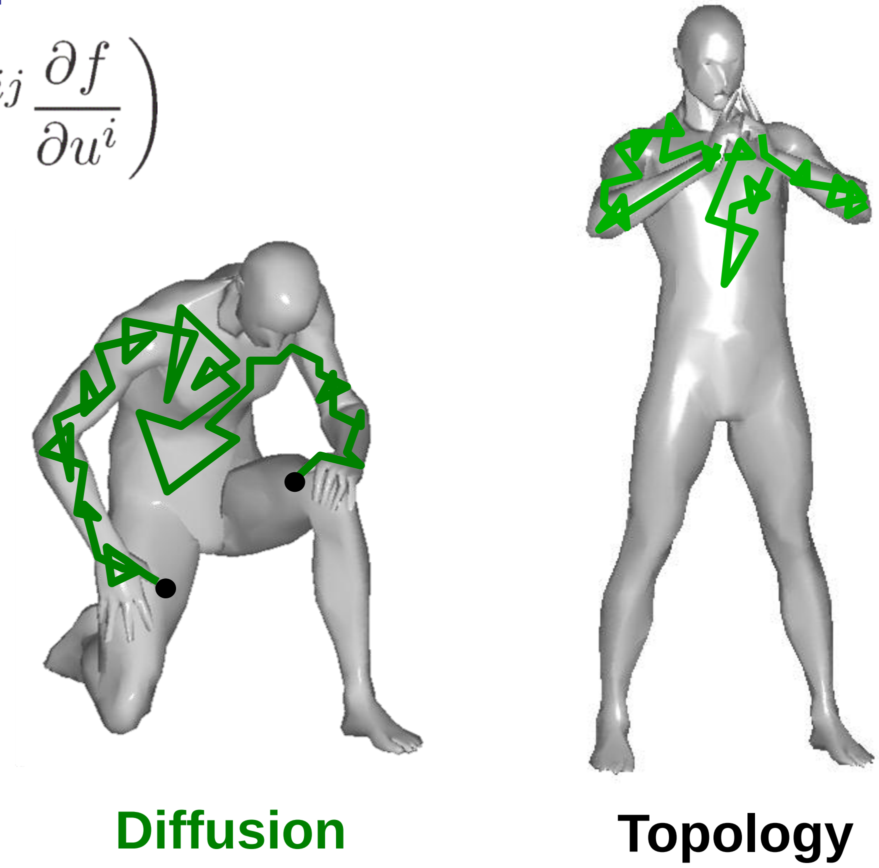
$$\Delta_{\mathbb{M}} f = -\frac{1}{\sqrt{|g|}} \sum_{i,j=1}^{2} \frac{\partial}{\partial u^j}\left(\sqrt{|g|}\, g^{ij} \frac{\partial f}{\partial u^i}\right)$$

- Eigen-decomposition

$$\Delta \varphi_i = \lambda_i \varphi_i$$

Properties

- *intrinsic: isometric invariant*
- *a complete orthonormal basis*



Human images adapted from Bronstein et al.

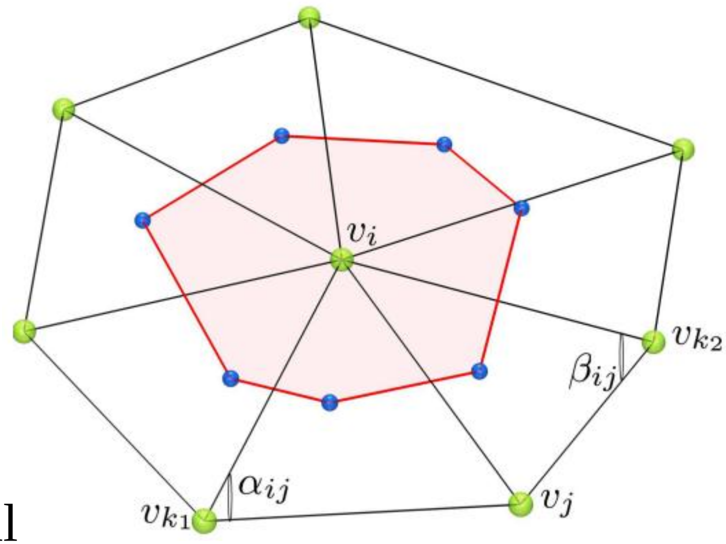
Discretization

- Generalized eigenvalue problem

$$C\varphi_i = \lambda_i A \varphi_i$$

$$0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_m$$

- Cotangent weight scheme



Area Matrix

$A = \mathrm{diag}(a_i)$, a_i is the area of the voronoi cell

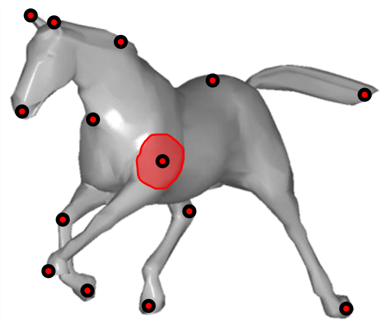
Weight Matrix

$$C = \begin{cases} \sum_{i=1}^{m} c_{ij} & \text{if } i = j \\ -c_{ij} & \text{if } i \sim j \\ 0 & \text{o.w.} \end{cases} \qquad c_{ij} = \begin{cases} \dfrac{\cot \alpha_{ij} + \cot \beta_{ij}}{2} & \text{if } i \sim j \\ 0 & \text{o.w.} \end{cases}$$

Pipeline

Shape Google

Analogy to Images/documents



Signatures
e.g. HKS

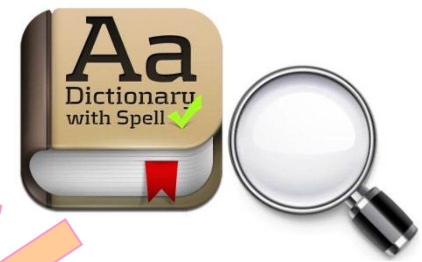
Geometric words

Horse Image adapted from Bronstein et al.

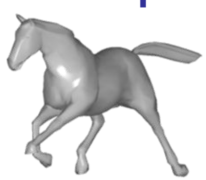
Contributions

- Dictionary Learning (offline)

Descriptors → K-means →



- Comparison (online)



Descriptors → Vector Quantization → Spatial Pooling → Histogram Distance

Descriptors Review

Ambiguity Modeling

Spectral Graph Wavelet Signature

Intrinsic Spatial Pyramid Matching

Intrinsic Spatial Partition Matching

Spectral Signatures (cont.)

- **ShapeDNA and Eigenvalue Descriptor** (Reuter CAD'06, Jain GMP'06)

  Normalized sequence of the first eigenvalues

- **Global Point Signature (GPS)** (Rustamov SGP'07)

$$\mathrm{GPS}(x) = \left(\frac{\varphi_2(x)}{\sqrt{\lambda_2}}, \frac{\varphi_3(x)}{\sqrt{\lambda_3}}, \ldots, \frac{\varphi_i(x)}{\sqrt{\lambda_i}}, \ldots \right)$$

- **Heat Kernel Signature (HKS)** (Sun SGP'09)

$$\mathfrak{p}_t(x, y) = \sum_{i=1}^{\infty} e^{-\lambda_i t} \varphi_i(x) \varphi_i(y)$$

Spectral Signatures

Scale Invariant Heat Kernel Signature (SIHKS) (Bronstein CVPR'10)

$$F\left[\tilde{\mathfrak{p}}'\right](\omega) = \tilde{H}'(\omega) = \tilde{H}(\omega)e^{-j\omega 2\log_\alpha a} \qquad |\tilde{H}'(\omega)| = |\tilde{H}(\omega)|$$

- *Remove the dependence of scale effect in HKS*

Wave Kernel Signature (WKS) (Aubry ICCVW'11)

$$\mathrm{P}(x,t) = \sum_{k=1}^{\infty} C_t \exp\left(\frac{-(\log t - \log \lambda_k)^2}{\sigma^2}\right)\varphi_k(x)^2$$

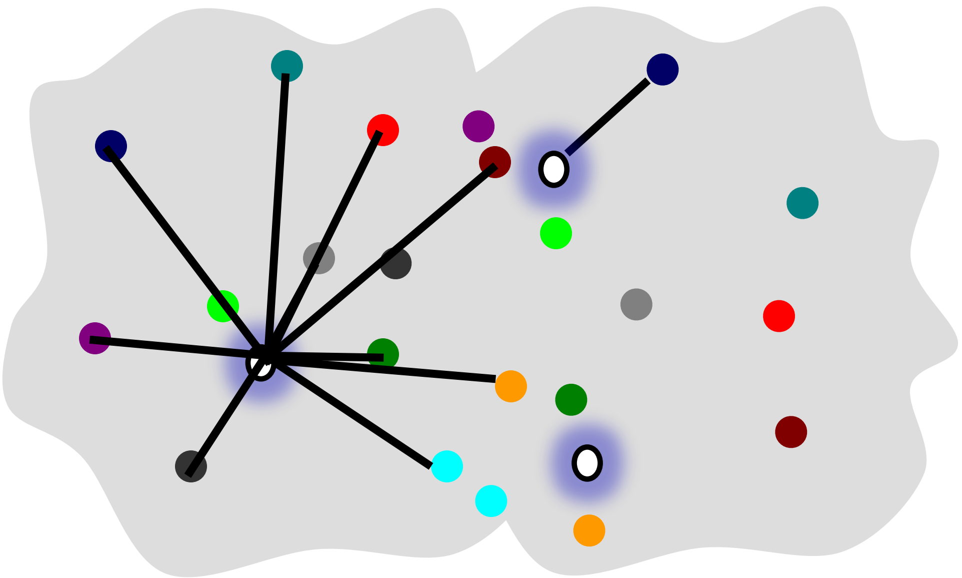
- *WKS is band-pass, while HKS is low-pass*

Heat Mean Signature (HMS) (Fang CVPR'11)

$$\mathrm{HMS}_t(x) = \frac{1}{m}\sum_{y\neq x} \mathfrak{p}_t(x,y)$$

- *HMS is the average of heat kernels used by HKS*

Codebook Models



Fitting Kernels

- *Laplace Kernel*
- *Gaussian Kernel*

Ambiguity Modeling

(Gemert PAMI'09)

	Best Candidate	Multiple Candidates
Constant Weight	*Traditional Codebook*	*Kernel Codebook*
Kernel Weighted	*Codeword Plausibility*	*Codeword Uncertainty*

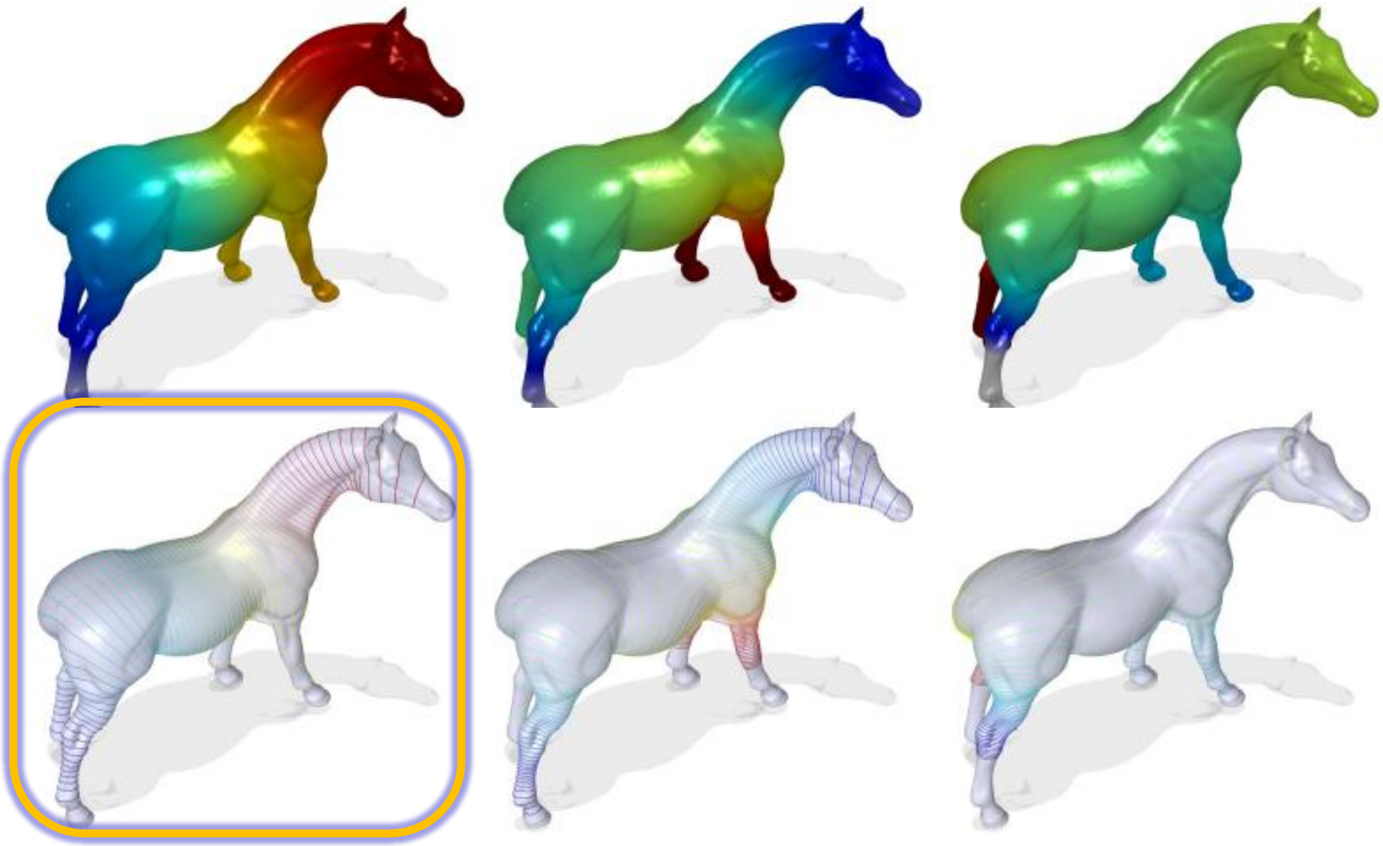
Motivation

- Spatial information

- Proper generalization

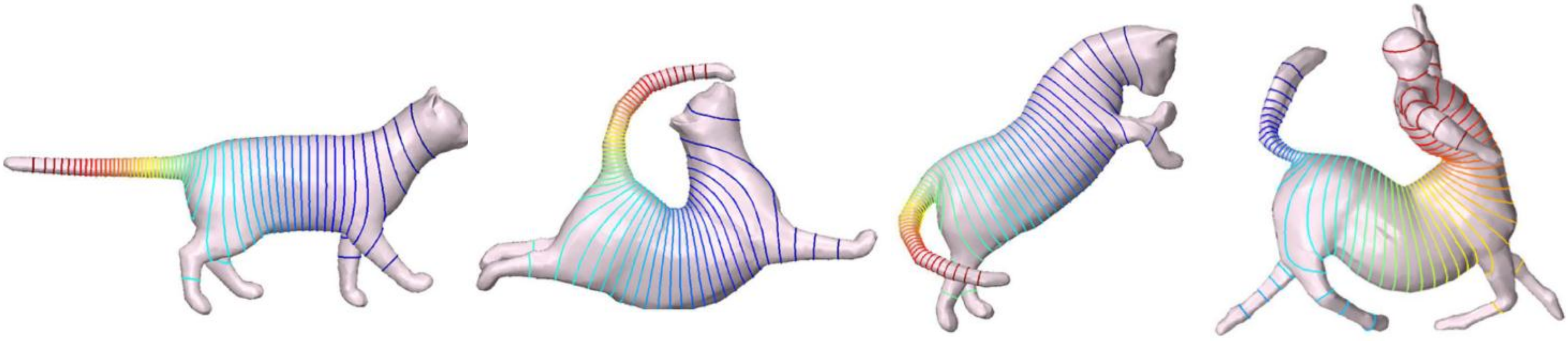
	Descriptor (HOG)	Bag of Feature	Shape Contexts	SPM
Image	*Dalal et al. CVPR 2005*	*Sivic et al. ICCV 2003*	*Belongie et al. PAMI 2002*	*Lazebnik et al. CVPR 2006*
Surface	*Zaharescu et al. CVPR 2009* ✓	*Bronstein et al. SIGGRAPH 11* ✓	*Kokkinos et al. CVPR 2012* ✓	?

Isocoutours



— The second LB eigenfunction, because
- *the smoothest map from the manifold to real line*

Intrinsic Spatial Partition

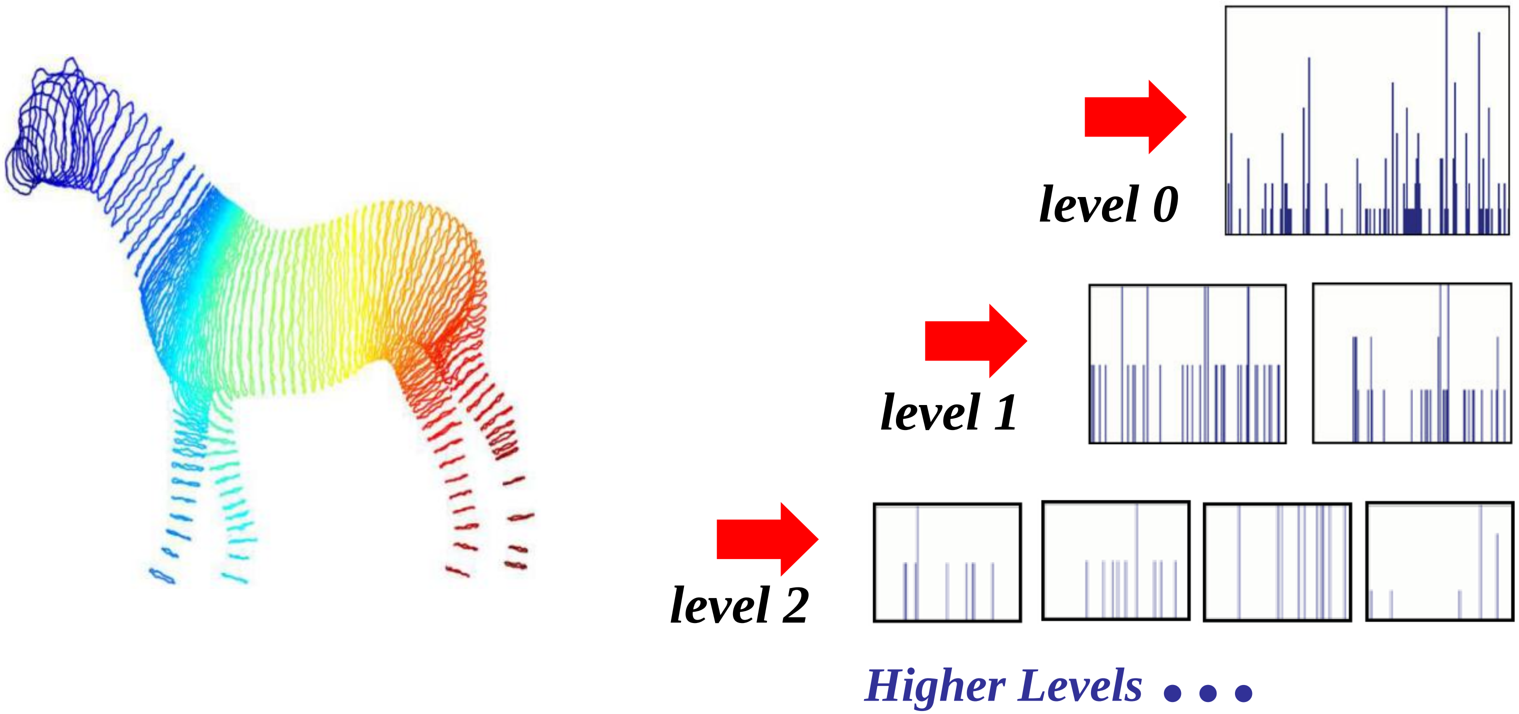


- **Intrinsic Cuts**
  - *consistent with isometric deformation*
  - *correspondence of isocontours*

- **Matching with R partitions**

$$\mathcal{B}^{\mathrm{R}}(P,Q) = \min(\mathcal{A}^{\mathrm{R}}(H_P, H_Q), \mathcal{A}^{\mathrm{R}}(H_P, T_Q))$$

Intrinsic Spatial Pyramid Matching



- Dissimilarity with L levels

$$\mathcal{D}^L(P,Q) = \mathcal{B}^L(P,Q) + \sum_{\ell=0}^{L-1} \frac{1}{2^{L-\ell}} \left(\mathcal{B}^\ell(P,Q) - \mathcal{B}^{\ell+1}(P,Q)\right)$$

SHREC 2010: Comparisons

- Evaluation Measure: Discounted Cumulative Gain (DCG)
- Descriptors

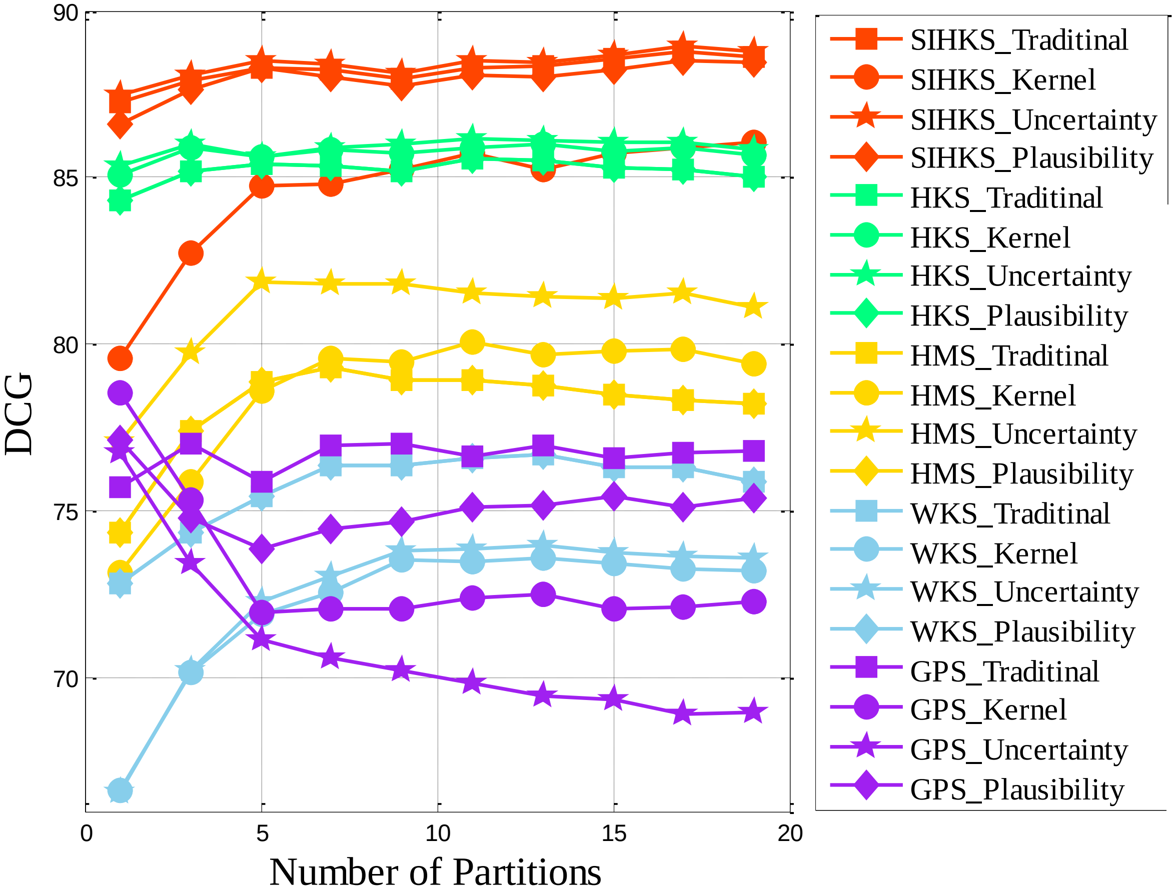
Spectral Descriptors	HKS	SIHKS	HMS	WKS	GPS	ShapeDNA	EVD
DCG	0.848	**0.877**	0.754	0.727	0.757	0.801	0.636

- Ambiguity Modeling

Ambiguity Type	Traditional	Kernel	Uncertainty	Plausibility
DCG	0.872	0.850	**0.874**	0.872

SHREC 2010: Improvement with ISPM

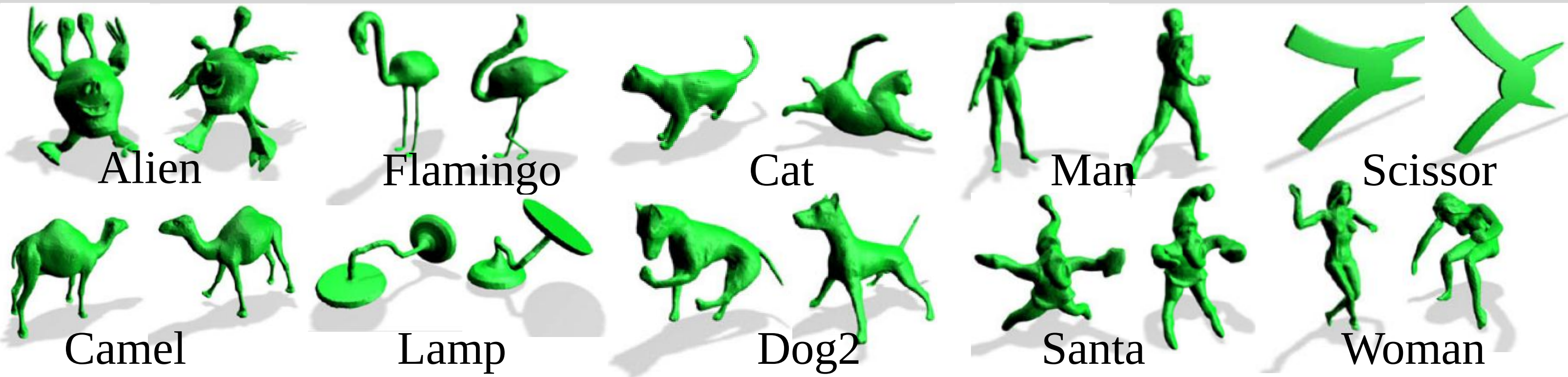
Single Level Partition

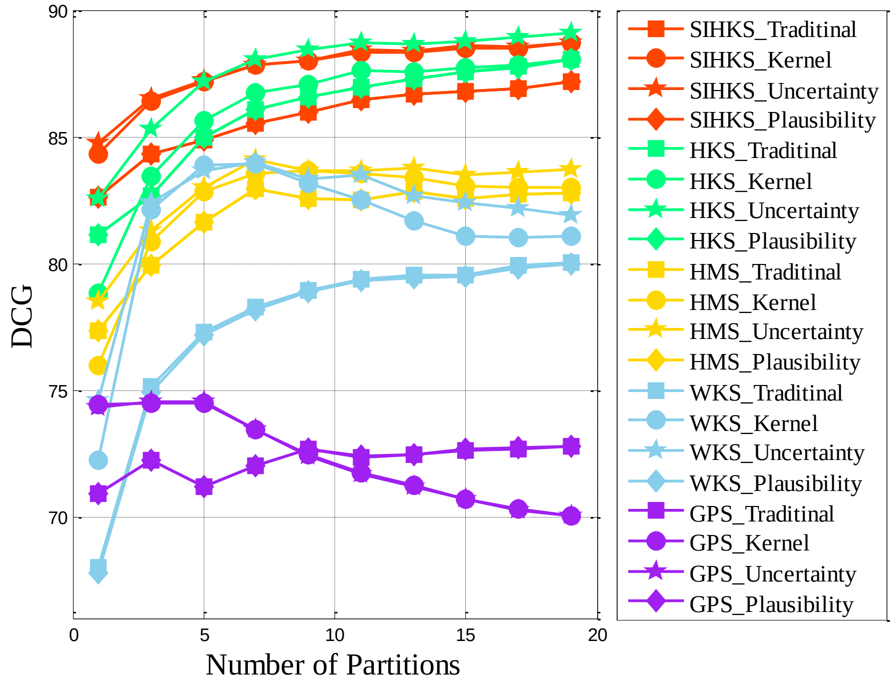


Single *v.s.* Pyramid

Spectral Descriptor	Level L (Partitions)	Codeword Uncertainty Single	Codeword Uncertainty Pyramid
HKS	1 (2)	0.8449	0.8496
	2 (4)	0.8593	0.8603
	3 (8)	0.8602	0.8628
	4 (16)	0.8597	**0.8644**
SIHKS	1 (2)	0.8747	0.8759
	2 (4)	0.8787	0.8823
	3 (8)	0.8797	0.8830
	4 (16)	0.8847	**0.8851**

SHREC 2011





Spectral Descriptor	Level L (Partitions)	Codeword Uncertainty Single	Codeword Uncertainty Pyramid
HKS	1 (2)	0.8497	0.8431
	4 (16)	0.8892	0.8870
	7 (128)	0.8892	**0.8903**
	9 (512)	0.8885	0.8891
SIHKS	1 (2)	0.8714	0.8661
	4 (16)	0.8862	0.8853
	7 (128)	0.8887	**0.8890**
	9 (512)	0.8888	0.8890

Spectral Graph Wavelet Signature

Graph Fourier Transform

- Forward and inverse graph Fourier

$$\hat{f}(\ell) = \langle \chi_\ell, f \rangle = \sum_{i=1}^{n} \chi_\ell^*(i) f(i), \quad \ell = 1, \ldots, n$$

$$f(j) = \sum_{\ell=1}^{n} \hat{f}(\ell) \chi_\ell(j), \quad j \in \mathcal{V}$$

– **Eigensystem**
  - *eigenvalues act as the frequencies, eigenfunctions as basis functions*

– **Spectral Signatures**
  - *HKS & WKS*

– **Limitation**
  - *the time information of a signal is lost*

Spectral Graph Wavelet Transform (SGWT)

- Expressing continuous wavelet transform in Fourier domain

$$(T^t f)(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \boxed{\hat{\psi}^*(t\omega)} \hat{f}(\omega) e^{\imath\omega x} d\omega$$

- SGWT

  - **Wavelet function: generating kernel** g
    - *analogous to the Fourier domain wavelet*
    - *band-pass filter to analog wavelet function*

  - **Scaling function: generating kernel** h
    - *low-pass filter to analog scaling function*

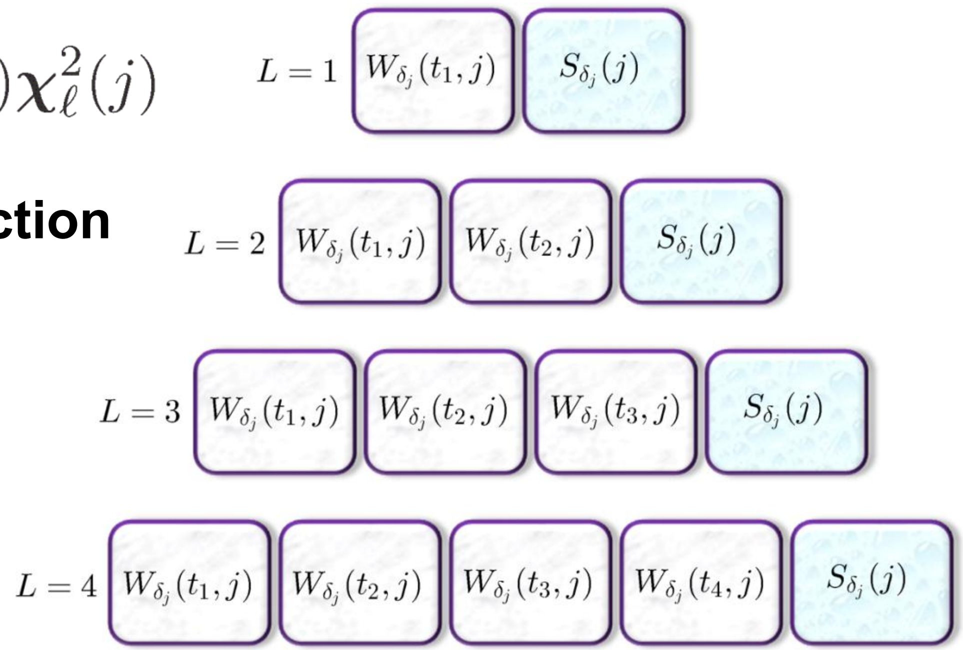
Proposed Multiresolution Shape Signature

- Assuming the signal is a unit impulse function
  - **Coefficients of wavelet function**

$$W_{\delta_j}(t,j) = \langle \psi_{t,j}, \delta_j \rangle = \sum_{\ell=1}^{n} g(t\lambda_\ell)\chi_\ell^2(j)$$

  - **Coefficients of the scaling function**

$$S_{\delta_j}(j) = \sum_{\ell=1}^{n} h(\lambda_\ell)\chi_\ell^2(j)$$



- The Signature

$$\mathcal{S}_R(j) = \{\boldsymbol{s}_L(j) \mid L = 1, \ldots, R\}$$

$$\boldsymbol{s}_L(j) = \{W_{\delta_j}(t_k, j) \mid k = 1, \ldots, L\} \cup \{S_{\delta_j}(j)\}$$

Design wavelet functions for retrieval

Desired descriptor properties

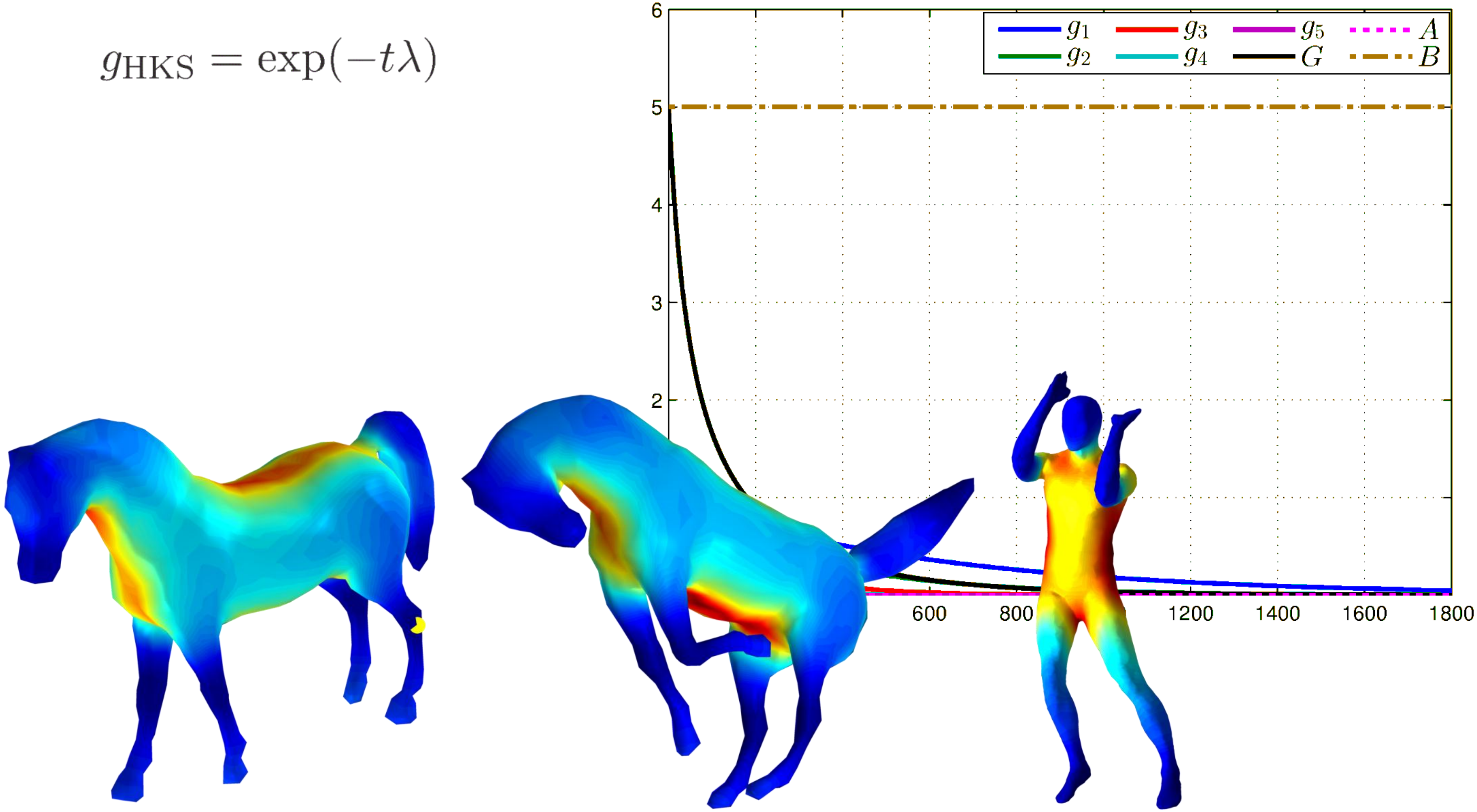
- **Invariance**
  - *Insensitive to isometric transformation*
- **Discriminative Power**
  - *Micro and macro structures*
- **Efficiency**
  - *Compact; Polynomial; No overlap*

Cubic spline wavelet and scaling function kernels

$$g(x) = \begin{cases} x^2 & \text{if } \; x < 1 \\ -5 + 11x - 6x^2 + x^3 & \text{if } \; 1 \le x \le 2 \\ 4x^{-2} & \text{if } \; x > 2 \end{cases} \qquad h(x) = \gamma \exp\left(- \left(\frac{x}{0.6\lambda_{\min}} \right)^4 \right)$$

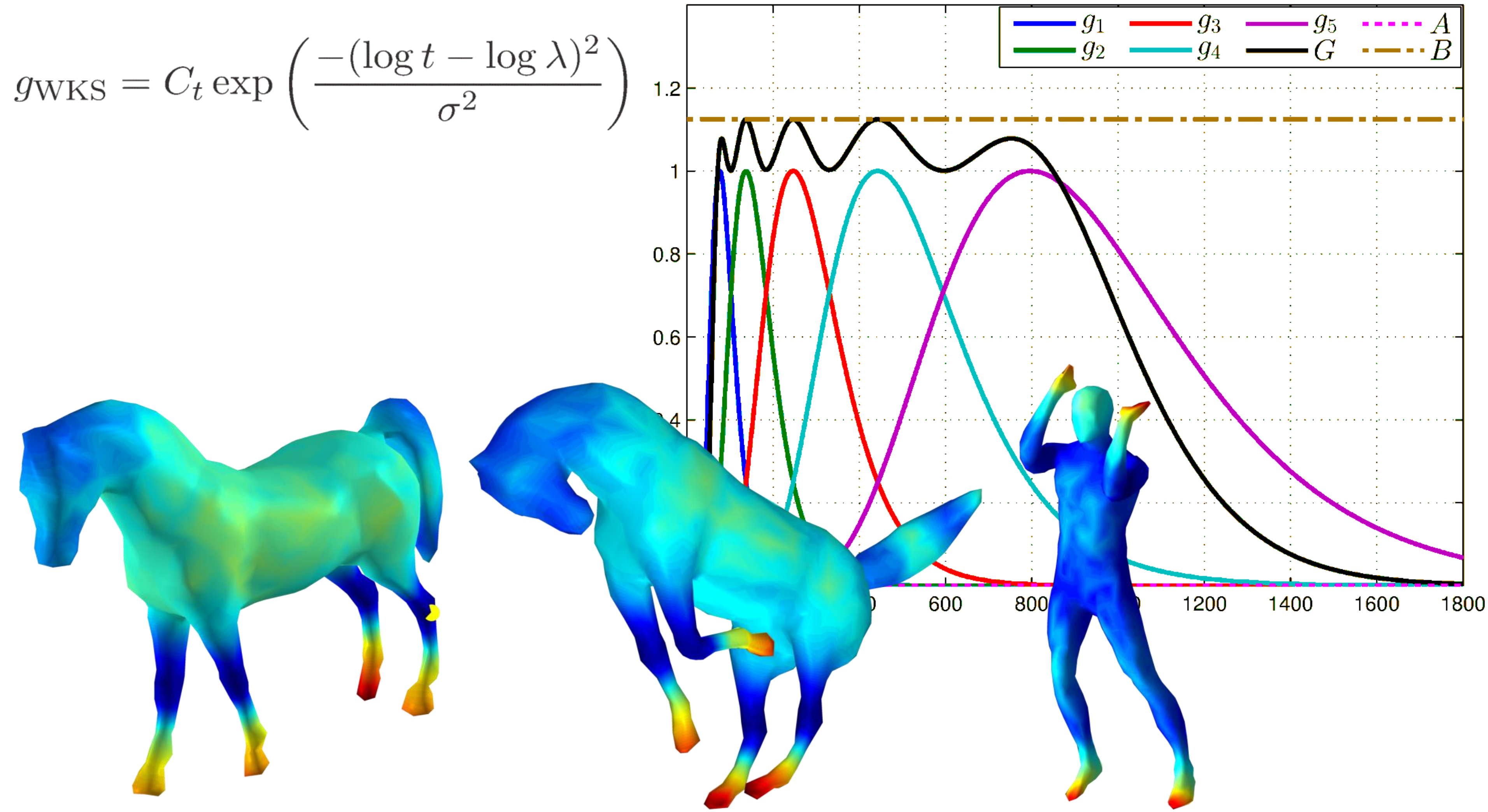
Relation to HKS

$$g_{\text{HKS}} = \exp(-t\lambda)$$



Relation to WKS

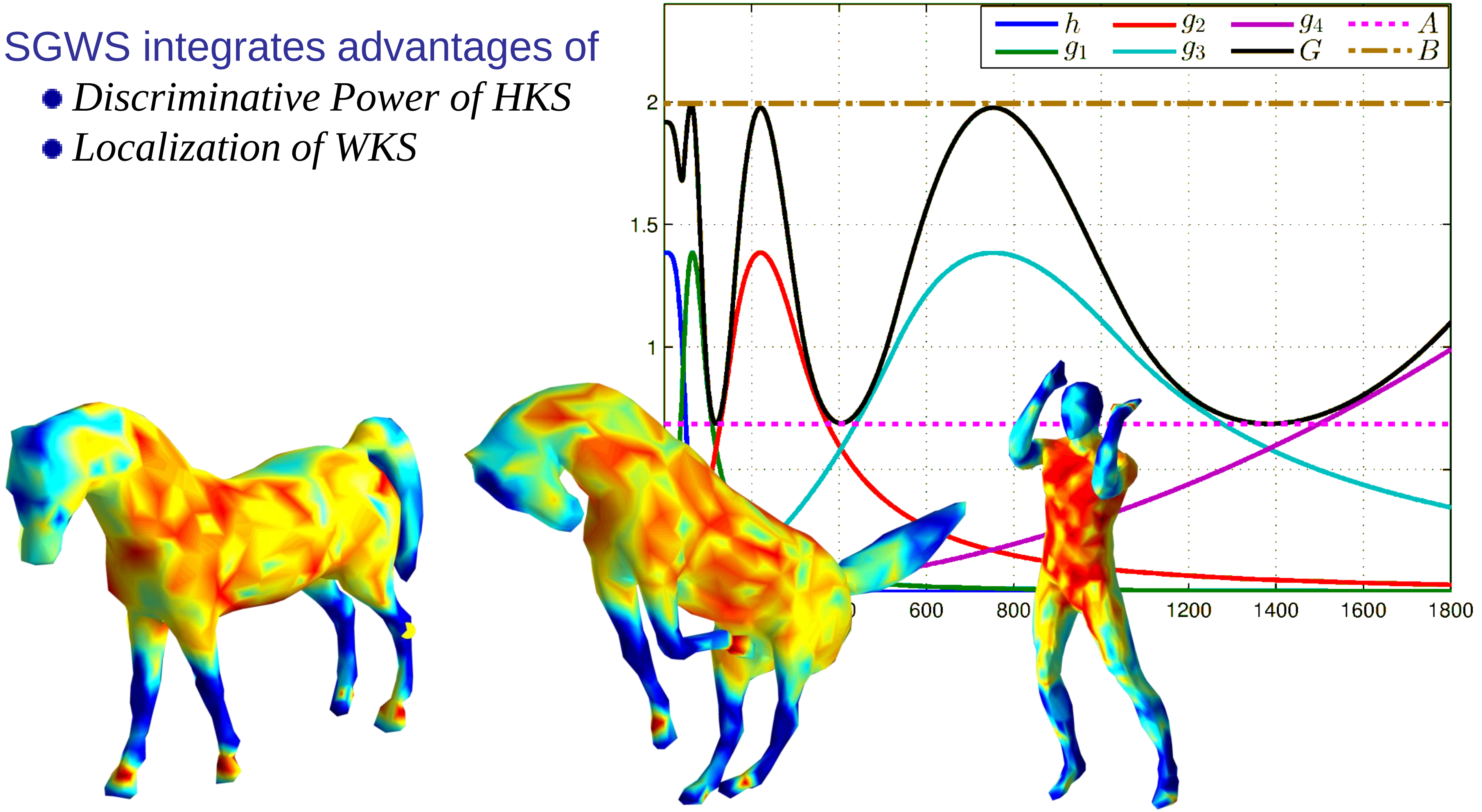
$$g_{\mathrm{WKS}} = C_t \exp\left(\frac{-(\log t - \log\lambda)^2}{\sigma^2}\right)$$



Proposed Signature Revisited

SGWS integrates advantages of

- *Discriminative Power of HKS*
- *Localization of WKS*



SHREC 2010

Performance with varying resolutions

- *It yields high accuracy with a very compact representation*
- *Stable results and simple parameters' selection*

Behavior with ISPM

- *Consistent high improvement*
- *Achieve best accuracy on a lower level of ISPM*

Level l (Partitions)	SIHKS	HKS	WKS	Spectral Graph Wavelet Signature (R)					
				1	2	3	4	5	6
0 (1)	0.8719	0.8448	0.7280	0.8124	**0.8635**	0.8541	0.8454	0.8547	0.8497
1 (2)	0.8734	0.8420	0.7290	0.8172	0.8593	0.8603	0.8498	0.8554	0.8527
2 (4)	0.8793	0.8509	0.7509	0.8789	0.8792	0.8732	0.8681	0.8665	0.8634
3 (8)	0.8817	0.8531	0.7612	0.8781	0.8737	0.8682	0.8651	0.8604	0.8615
4 (16)	0.8841	0.8520	0.7634	0.8687	0.8721	0.8636	0.8640	0.8563	0.8590
Improvement	0.0122	0.0083	0.0354	0.0665	0.0157	0.0191	0.0227	0.0118	0.0137

SHREC 2011

- Higher accuracy than SIHKS, HKS and WKS
- Best result compared with signatures in the framework of diffusion geometry

Level l (Partitions)	SIHKS	HKS	WKS	Spectral Graph Wavelet Signature (R)					
				1	2	3	4	5	6
0 (1)	0.8262	0.8114	0.6801	0.8043	**0.8948**	0.8536	0.8731	0.8613	0.8633
1 (2)	0.8436	0.8277	0.7097	0.8419	0.9208	0.8858	0.9002	0.8873	0.8911
4 (16)	0.8671	0.8721	0.7933	0.9203	0.9508	0.9443	**0.9526**	0.9471	0.9517
7 (128)	0.8771	0.8878	0.8042	0.9132	0.9427	0.9366	0.9383	0.9344	0.9359
9 (512)	0.8793	0.8902	0.8029	0.8982	0.9344	0.9241	0.9319	0.9277	0.9272
Improvement	0.0531	0.0788	0.1241	0.1160	0.0560	0.0907	0.0795	0.0858	0.0884

Conclusions

Contributions

- Intrinsic Spatial Partition Matching
- Spectral Graph Wavelet Signature

Future directions

- Unifying Topology and Geometry
- Applications of the Global Intrinsic Coordinate System
- Design of Wavelet Generating Kernels
- From Image Processing to Geometry Processing

The end

Thanks!

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